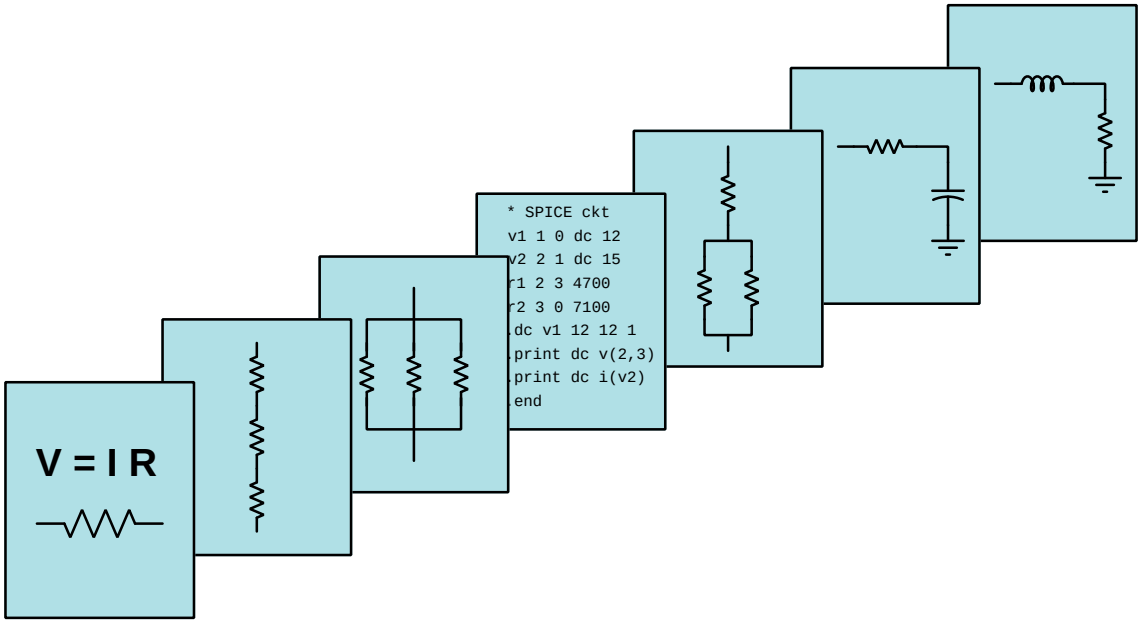
Modular Electronics Learning (ModEL) project



Linear Current Regulators

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Chapter 1

Introduction

True current *sources* are a rarity compared to voltage sources, and therefore we must generally devise some form of current-regulation circuit to use in conjunction with available voltage sources in order to mimic a true current source. This module explains how to do this using transistors.

Important concepts related to linear current regulator circuits include **current sources**, semiconductor **diode** behavior, **Ohm's Law**, **Joule's Law**, bipolar junction **transistor** behavior, **beta** ratio, **majority** and **minority** carriers, the Shockley **diode equation**, **temperature effects** on semiconductor junctions, **amplification**, **sourcing** versus **sinking**, field-effect **transistor** behavior, and **transconductance**.

Here are some good questions to ask of yourself while studying this subject:

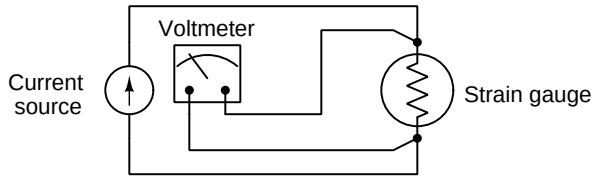
- What practical applications are there for current regulation?
- How do bipolar junction transistors function?
- Why does a bipolar junction transistor tend to regulate current through it?
- Why does a field-effect transistor tend to regulate current through it?
- What is the "pinch-off" voltage for a FET and why does it matter?
- How does temperature affect the behavior of a transistor?
- Why must the two transistors forming a current mirror circuit be thermally bonded together?
- How do we set the current-regulation point of a current mirror circuit?
- How do we set the current-regulation point of a JFET current regulator?

Chapter 2

Tutorial

Most types of electrical sources tend to output a fixed voltage rather than a fixed current. Chemical batteries, for example, output a voltage determined by the electrochemical potential of the elements and compounds intrinsic to the battery's design (e.g. approximately 2.2 Volts for a lead-acid cell of any physical size). Electromechanical generators output a voltage based on Faraday's Law of Electromagnetic Induction ($V = N\frac{d\Phi}{dt}$), where the induced voltage (V) is a function of the magnetic flux's rate of change ($\frac{d\Phi}{dt}$) as well as the number of turns of wire in the coil (N) experiencing that changing magnetic flux. Thus, the voltage output of a generator tends to remain fixed for a given magnetic field strength and a given shaft speed.

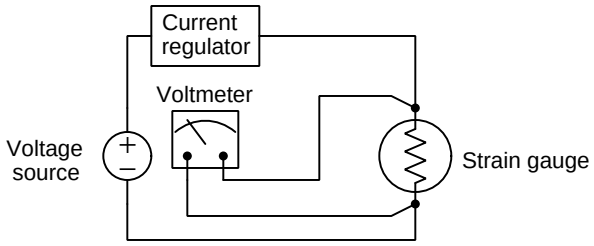
However, some electrical and electronic applications demand a source of constant (or nearly constant) current. A good example of this is in *rheometric* measurement circuits where the sensor translates its physical measurement into an electrical resistance, and we need to accurately measure that resistance to infer the physical measurement. Voltage is a fairly easy quantity to measure with high accuracy, and so to convert the sensor's resistance into a measurable voltage we need to pass a known (constant) current through that sensor. The following example circuit shows a particular type of resistance-based sensor called a *strain gauge* connected to such a measurement circuit:



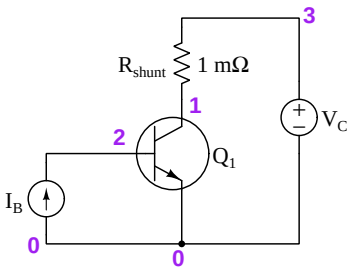
As physical stress is applied to the strain gauge, its electrical resistance changes. The constant current energizing this strain gauge produces a voltage drop across the gauge proportional to that resistance (Ohm's Law, $V = IR$), and therefore proportional to the amount of physical strain. The voltmeter's indication may then be read as a proportion of mechanical strain.

Clearly, we need a current source to measure strain in this manner, yet most electrical sources output constant voltage rather than constant current. A solution to this problem is a *current regulator* circuit designed to receive voltage from an electrical source and act to restrict current to some pre-determined level regardless of variations in that voltage source or in the load being

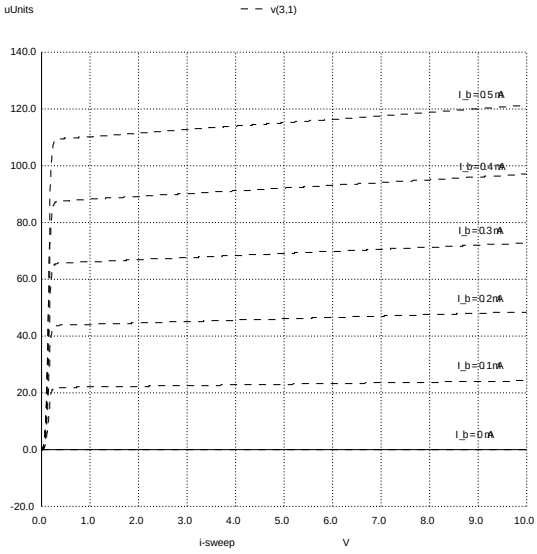
energized by the source. A current regulator, shown as a "box" symbol in the following diagram, allows us to perform this strain gauge measurement using a normal voltage source:



Fortunately for us, some electronic components naturally exhibit current-regulating properties. Perhaps the most evident of these is the bipolar junction transistor (BJT) operating in its "active" region. We see this current-regulating behavior in the characteristic curves of a BJT, plotted as collector current (I_C) over collector-emitter voltage (V_{CE}). The following SPICE simulation shows an NPN bipolar transistor being tested for collector current as the supply voltage (V_C) sweeps from 0 to 10 Volts. The simulation tests the transistor for six different values of base current (I_B), stepping from 0 mA to 0.5 mA in 0.1 mA increments:



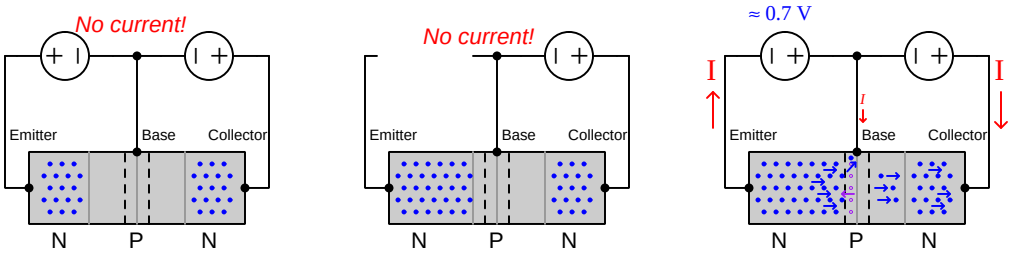

```
* SPICE netlist
vc 3 0 dc
ib 0 2 dc
q1 1 2 0 realistic
rshunt 3 1 1e-3
.model realistic npn
+ is=15f vaf=90 bf=220
+ cjc=2p mjc=0.3 vjc=0.72
+ cje=5p mje=0.3 vje=0.72
+ nc=2 rc=0
.dc vc 0 10 0.01 ib 0 0.5m 0.1m
.plot dc v(3,1)
.end
```



Note how each of the curves remain relatively "flat" from a low V_C value all the way up to 10 Volts. This is the transistor behaving as a current *regulator*, holding the value of current relatively constant despite large changes in supply voltage. The current's regulated value is strongly determined by base current (I_B) and only very weakly affected by collector-emitter voltage (V_{CE}).

It is well worth reviewing why bipolar junction transistors regulate current, to better understand how and why current-regulating circuits function. Recall that a bipolar junction transistor consists of three layers of semiconductor material of different doping, in this case *NPN*. The middle layer, called

the *base* of the transistor, naturally has two depletion regions, one at each PN junction, and for this reason a BJT's normal state is non-conducting from collector to emitter. If, however, an external source forward-biases the base-emitter junction and collapses that depletion region, mobile charge carriers from the emitter layer will become injected into the base. Some of these charge carriers exit the transistor through the base terminal, but most of them drift into the base-collector depletion region where they are pulled into the collector layer by the electric field of the collector-emitter voltage, as shown in the illustration below:

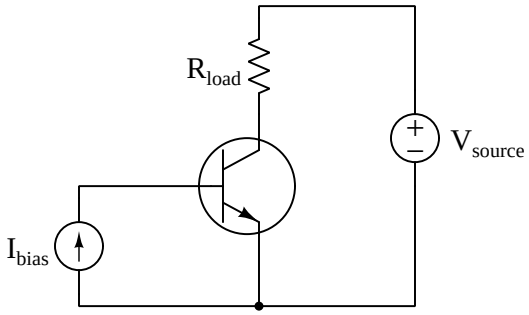


In order to understand why it *regulates* current, we must take a closer look at the charge-injection action happening within the transistor. As previously mentioned, the conductivity of a transistor between emitter and collector terminals depends on charge carriers being injected into the base region. These are called *minority carriers* because they are foreign to the base's doping. For example, an NPN transistor has a P-doped base which means holes are the majority carrier there and free electrons are in the minority. Yet, electrons are the majority charge carrier type in the N-type emitter and collector regions. Making the transistor conductive from end-to-end (i.e. between emitter and collector) requires that those charge carriers prevalent in the emitter and collector regions traverse the base as "aliens" within that region. We can get free electrons injected into the base region by forward-biasing the emitter-base PN junction. From there, the base-collector voltage drop "sweeps" those electrons into the collector region where they complete their journey from emitter to collector, and the transistor conducts load current. However, the important point to realize here is that these injected "minority" charge carriers within the base region are sparse, not plentiful as they would be in a regular conductor such as copper wire. Increasing the voltage drop between emitter and collector may cause those injected carriers to be "swept" through the base and into the collector at higher velocities, but it has little effect on the total number of charge carriers per second (i.e. current). The degree to which the emitter-base junction is forward-biased controls that. Therefore, a transistor's maximum end-to-end current is strongly determined by V_{BE} (which controls the rate of minority charge injection) and only weakly[1] determined by V_{CE}. This is how the BJT acts as a constant-current regulator, maintaining a nearly-constant collector current despite wide variations in V_{CE}.

Now that we see how and why a bipolar junction transistor regulates current, we must figure out how to build practical BJT circuits to perform this task.

[1] In an ideal BJT there would be no increase at all, and each characteristic curve would be perfectly flat over most of the domain. The reason a slight positive slope exists is due to the *Early effect* named after the discoverer (Dr. Early of Bell Laboratories). Increasing V_{CE} causes the base-collector depletion region to widen, and thus expands the "net" with which minority carriers are "captured" from the base to the collector.

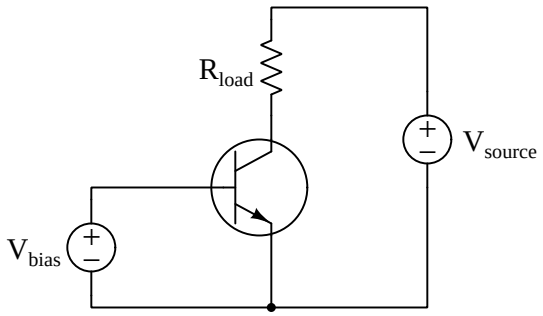
Starting with the demonstration circuit used in the previous SPICE analysis, we may place our constant-current load (e.g. strain gauge) in series with the transistor's collector terminal, and forward-bias the transistor's base-emitter junction with a constant current. The goal is to have the transistor maintain a constant current through R_{load} despite variations in the resistance of R_{load} as well as variations in the supply voltage (V_{source}):



Unfortunately, this simple circuit is impractical for two important reasons. First, it requires a constant-current source to activate the base-emitter junction (I_{bias}), and the whole reason we are trying to develop a current regulator circuit is because true current sources are rare. Second, even if we were somehow able to send a constant current through this transistor's base terminal, the collector (load) current would still end up varying with *temperature*. Recall that the ratio between a BJT's collector current and its base current is called *beta* (β) or h_{FE}, and that this ratio while large is unstable. Beta varies with many factors, one of the most significant being the temperature of the PN junctions within the transistor. If the transistor's temperature becomes warmer or cooler, its beta will change accordingly, and along with that the load current will drift away from the value we desire.

Temperature changes that concern us here are not just ambient temperature, but also the result of heat dissipation within the transistor itself. This is a serious concern if the transistor is to function as a regulator for some substantial amount of current. Power dissipation for any device is the product of voltage drop and current (Joule's Law, $P = IV$), and this means the transistor will be dissipating energy in the form of heat as it functions to regulate current through the load. If changes in transistor temperature will cause load current to deviate from the intended value, then our current-regulator circuit will not be very effective.

Modifying this circuit, we may replace I_{bias} with a low-value *voltage* source which we will call V_{bias}:



The amount of current passed by a PN semiconductor junction is a function of voltage as described by the Shockley diode equation:

$$I = I_S \left(e^{\frac{qV}{nKT}} - 1 \right)$$

Where,

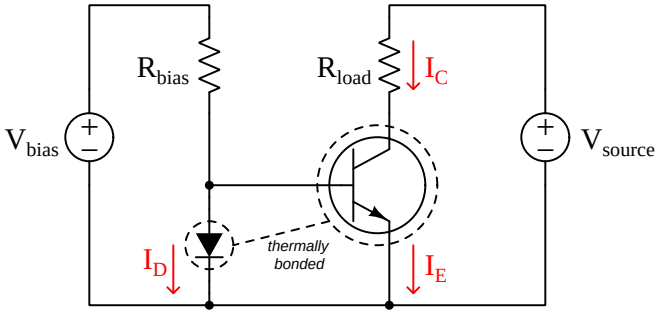
I = Forward-bias current through the diode, Amperes
I_S = Reverse-bias saturation current[2] through the diode, Amperes
e = Euler's constant (≈ 2.71828)
V = Voltage applied to the PN junction externally, Volts
q = Elementary charge of an electron (1.602×10^{-19} Coulombs)
n = Ideality factor (1 for a perfect junction)
k = Boltzmann's constant ($1.3806504 \times 10^{-23}$ J / K)
T = Absolute temperature (Kelvin), 273.15 more than degrees Celsius

Therefore, we ought to be able to calculate the necessary base-emitter voltage[3] (V) to create the emitter current value we desire, which will be very nearly equal to the collector current (since I_B is typically so small). However, even if we manage to achieve this feat of applying just the right amount of constant voltage across the transistor's base-emitter junction, we are still faced with the problem of temperature changes. Whether taking the form of ambient (environmental) temperature changes, temperature changes resulting from internal power dissipation, or some combination of these, this current-regulator circuit will perform very poorly as a result.

[2] A very small amount of current will still flow in the reverse-biased condition, due to so-called *minority carriers* in the P and N halves of the diode. This tiny current, usually in the range of *nano-Amperes* is referred to as the *reverse saturation current* because its value does not increase appreciably with greater reverse-bias voltage but rather "saturates" or "plateaus" at a constant value. This saturation current, while fairly independent of applied voltage, varies greatly with changes in device *temperature*.

[3] It is very important to note here that the forward voltage drop of a silicon PN junction is *not* a constant 0.7 Volts! The assumption of a 0.7 Volt drop is exactly that: a *simplifying assumption* intended to make certain calculations easier, but it is not strictly the truth of the matter. BJT current regulators depend entirely on the exponential relationship between voltage and current for a PN junction, and so we must abandon the fixed 0.7 Volt assumption if we are to ever understand how these circuits function.

A very clever solution to the problem of temperature is to use a second PN junction operating at the same temperature as the transistor to generate the necessary V_{bias}, rather than relying on an external voltage source for that bias voltage. Consider the following circuit, where a semiconductor diode and an NPN transistor exist in thermal equilibrium:



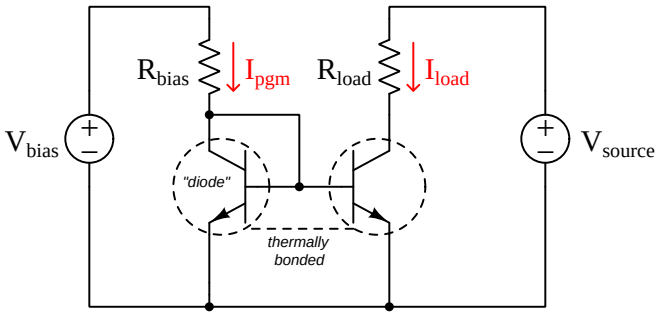
The thermal "bond" between the diode and transistor junctions consists of a material bridge allowing heat to easily flow between the two devices. Such a bond ensures the two devices remain at the same temperature[4] even if they dissipate different amounts of power, and therefore they should exhibit the same voltage/current characteristics (i.e. the Shockley diode equation where T is equal). Moreover, the diode junction has been connected in parallel with the transistor's base and emitter terminals, and this parallel connection guarantees they must experience the same voltage.

Suppose we select V_{bias} and R_{bias} to generate a certain amount of constant current through the diode, approximately equal to $\frac{V_{bias}-0.7}{R_{bias}}$. The vast majority of this current will pass through the diode's PN junction, with only a very small amount diverting away from the diode to flow into the transistor's base terminal, and with that bulk of current through the diode we expect to see a voltage drop across the diode junction in accordance with the Shockley diode equation. Meanwhile, the transistor's base-emitter junction experiences the exact same voltage drop as the diode by virtue of their parallel connection. If the transistor's base-emitter junction is physically identical to the diode's PN junction (e.g. same material, same doping concentrations, same temperature) and it experiences the same voltage drop as the diode, the transistor's emitter current must be equal (or nearly equal) to that of the diode. In summary, $I_E \approx I_D$, and most importantly *this will remain true even as temperature varies*. By extension, for any transistor having a sufficiently large beta ratio, $I_C \approx I_D$. Thus, the circuit functions as a current regulator for R_{load}, the target value for that regulated current set by V_{bias} and R_{bias}.

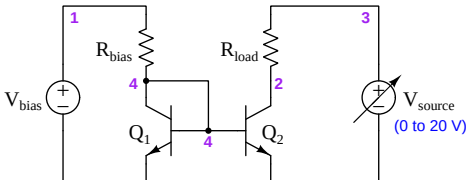
[4] If both devices are implanted adjacent to each other on the same wafer of silicon as an *integrated circuit*, the thermal bonding will be natural.

A practical modification to this circuit replaces the diode with a transistor identical to the first, embedded on the same piece of silicon to ensure thermal bonding between the two[5]. The new transistor's base and collector terminals are "short-circuited" by a jumper wire so that only its base-emitter junction is active, ensuring it will function purely as a diode with properties identical to the other transistor's base-emitter junction. The resulting circuit is known as a *current mirror* because the active transistor works to duplicate (i.e. "mirror") the *programming current* (I_{pgm}) value at the load despite changes in load resistance and/or V_{source}:

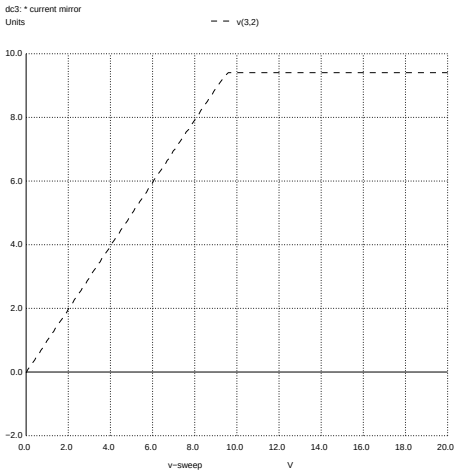
A current mirror circuit



A SPICE analysis shows load voltage as a function of V_{supply} over a range of 0 to 20 Volts:




```
* Current mirror
vsource 3 0 dc
vbias 1 0 dc 5
rbias 1 4 1000
q1 4 4 0 bjt
q2 2 4 0 bjt
rload 3 2 2200
.model bjt npn
+ is=15f bf=220
.dc vsource 0 20 0.1
.print dc v(3,2)
.end
```



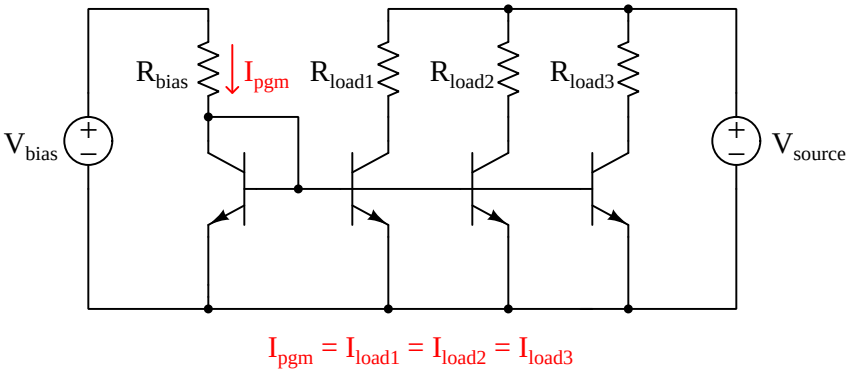
Note how the plot climbs linearly as V_{supply} increases from 0 Volts to approximately 9.5 Volts, but then suddenly levels off. This level portion of the curve shows load current being regulated at a constant value. The fact that load resistor voltage (and therefore load resistor current) is constant

[5] A similar circuit may be fashioned by physically attaching two discrete transistors back-to-back together, but the thermal bonding will not be as "close" as it would be if existing as two adjacent transistors within an integrated circuit. Matched pairs of transistors are manufactured as integrated circuits (ICs) for this very purpose.

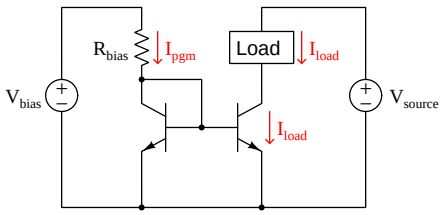
only after the supply voltage reaches a certain minimum value is worthy of discussion. A current mirror circuit is not a true *source* in that the transistors cannot provide energy to motivate current through the load. All Q_2 is able to do is *restrict* current to the "programmed" value, with V_{source} being the actual energy source for this current. With a V_{bias} of 5 Volts and R_{bias} of 1 kΩ, the "programming" current will be approximately 4.3 mA. Mirrored by transistor Q_2, 4.3 mA becomes the load current value as well, and with a 2.2 kΩ load resistance value this requires an absolute minimum of 9.46 Volts supplied by V_{supply} ($V = IR = (4.3\text{ mA})(2200\ \Omega) = 9.46$ V). Any amount of source voltage in excess of 9.46 Volts will be dropped across the collector-emitter terminals of Q_2 as it regulates load current at 4.3 mA and the load drops a constant 9.46 Volts.

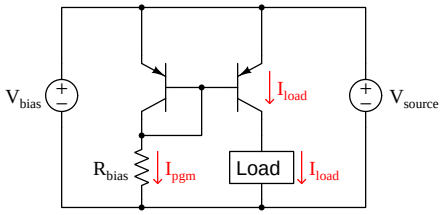
If a set of BJTs are constructed on an integrated circuit with paralleled base and emitter terminals, one programming current is able to set the regulation value for an arbitrary number of loads. With each regulating transistor receiving the exact same base-emitter voltage as dropped by the "diode" transistor, each load's current will be regulated to that same "programming" current value:

A multi-load current mirror

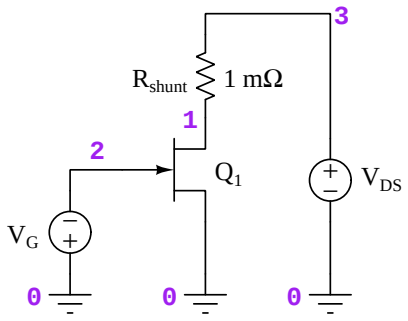


BJT current mirrors may be made in both *sourcing* and *sinking* versions, to suit different loads:

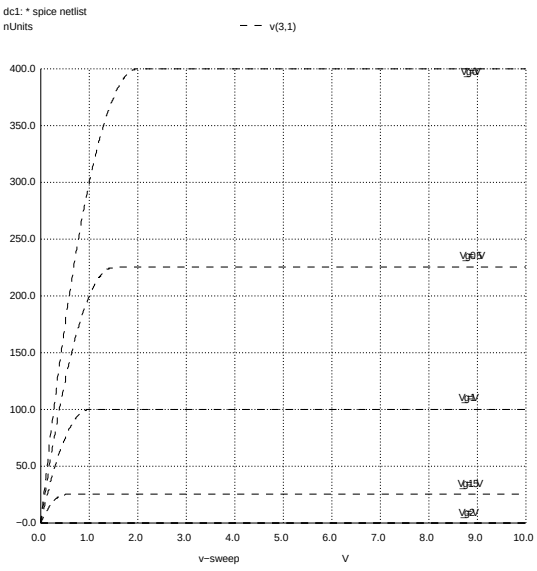




Bipolar junction transistors, of course, are not the only electronic component with a natural current-regulating characteristic. *Field-effect transistors* (FETs) do much the same, as shown in the following simulation showing a JFET biased by a DC voltage source (V_G) and powered by another DC source (V_{DS}) swept from 0 to 10 Volts:

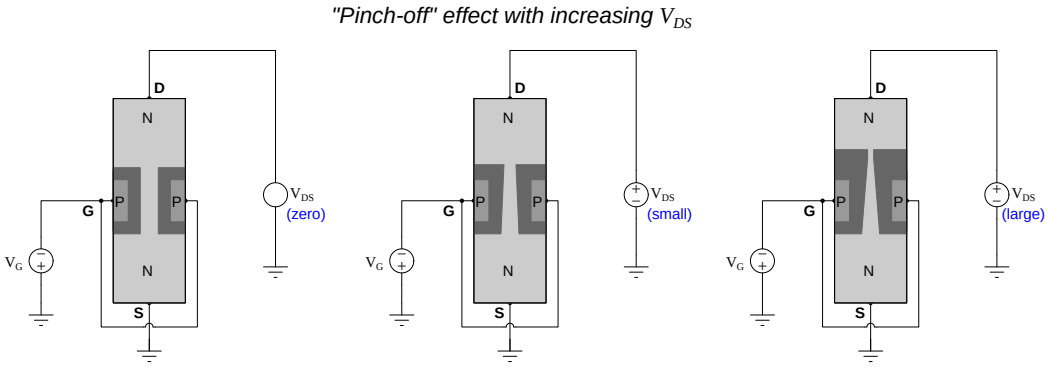



```
* SPICE netlist
vds 3 0 dc
vg 0 2 dc
rshunt 3 1 1e-3
j1 1 2 0 jfet
.model jfet njf vto=-2
.dc vds 0 10 0.01 vg 0 3 0.5
.plot dc v(3,1)
.end
```



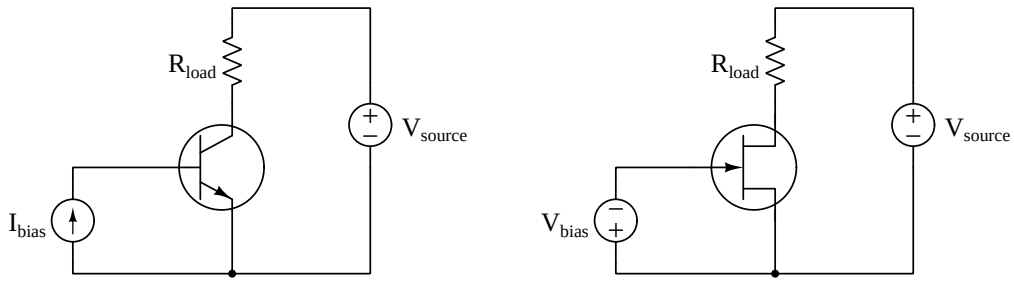
Any value of bias voltage -2 Volts or greater with respect to ground completely "pinches off" the JFET and halts load current. At bias voltages closer to zero, though, the JFET regulates current over a wide span of source voltage. Perhaps the most noticeable difference between the JFET versus the BJT is that the BJT reached its regulated current value at much lower source voltages, the BJT curves rising much more rapidly to their level regions as source voltage increased. The JFET, by contrast, exhibits a "gentler" approach to the current-regulation region. This fact makes the JFET inherently less precise as a current-regulating element than the BJT, although we shall shortly see a distinct advantage offered by the JFET.

Although similar in behavior, these two different types of transistors regulate current in entirely different manners. BJT current regulation is based on minority carrier injection, while FET current regulation is based on majority charge carriers passing through a channel narrowed by the presence of a depletion region between the gate and channel layers. The shape of these depletion regions alter with increased V_{DS}, becoming more constricting as V_{DS} grows stronger. The following illustration shows an N-channel junction field-effect transistor's depletion region with constant gate bias voltage and a changing drain-source voltage:



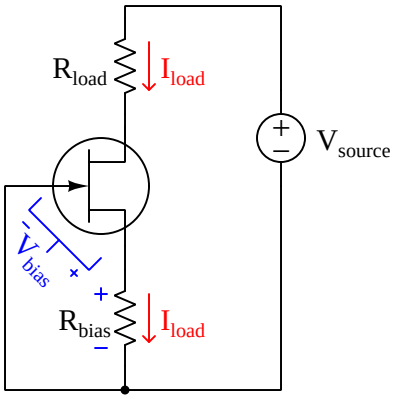
The potential difference between gate and source (V_{GS}) remains fixed by the bias voltage, while the potential difference between gate and drain increases as V_{DS} grows stronger. This enlarges the depletion region toward the more positive drain end of the channel, while the depletion region's width remains unchanged in shape or size toward the more negative end of the channel. Therefore, as V_{DS} increases, the depletion region grows to occupy more of the channel's width and constricts the flow to a greater degree, resulting in a nearly constant current over a range of V_{DS} values.

At first we may be tempted to design a JFET current regulator with a constant V_G bias voltage, similar to our first (impractical) BJT designs:



However, the same temperature dependence problem encountered with the BJT plagues the JFET circuit as well. Just as the beta ratio (β) of a BJT varies with temperature, so does the *transconductance* (g) of a FET. If we try to use either circuit for current regulation, we will be sadly disappointed with the performance.

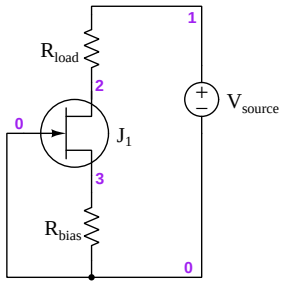
Fortunately, a very simple and clever solution exists for the JFET, consisting of placing a resistor in series with its source terminal and letting that resistor's voltage drop become the gate-biasing voltage:

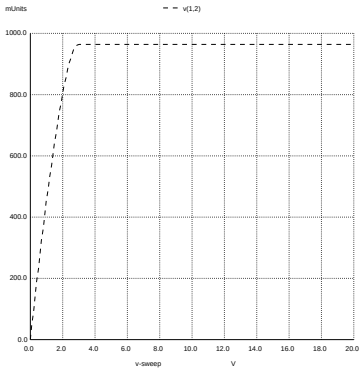


The principle of operation is quite simple: if load current happens to increase, so will V_{bias} dropped across the bias resistor, and the polarity of that voltage drop will be such as to expand the transistor's internal depletion regions and further restrict current[6]. The automatic response of this circuit to any attempted change in load current occurs regardless of the source of that disturbance. Increased load current due to a temperature change will be counteracted just the same as increased load current due to increased source voltage or decreased load resistance. An obvious advantage of this JFET current regulator circuit is that it only requires a single transistor, whereas all the practical BJT circuits require two transistors (one to "mirror" the other).

A SPICE simulation shows such a circuit regulating current through a load to a value of just under 0.3 mA as the supply voltage sweeps from 0 to 20 Volts:

```
* JFET current regulator
vsupply 1 0 dc
rload 1 2 3300
rbias 3 0 1000
j1 2 0 3 jfet
.model jfet njf vto=-2
.dc vsupply 0 20 0.1
.plot dc v(1,2)
.end
```





[6] This is another example of *negative feedback* at work, where the automatic response of a system is to counter-act a disturbance.

Chapter 3

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

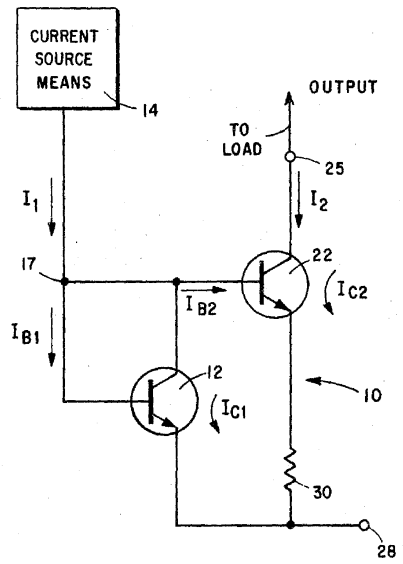
Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

3.1 Widlar current source patent

Bob Widlar, the famous analog IC designer, filed for a US Patent in May of 1965 describing a new form of current mirror circuit suitable for regulating very low values of current (in the microAmpere range) within integrated circuits. The patent (entitled *Low-Value Current Source for Integrated Circuits*, number 3,320,439) was granted in May of 1967 and began with this single illustration of the circuit:



The "current source means" is just an expression for the source of "programming current". The load is not shown, but would connect to the output terminal (25). This is clearly a current *sinking* regulator as we can see from the I_2 and I_{C2} arrows.

The initial paragraph of the patent is an *abstract*, tersely summarizing its contents:

> ABSTRACT OF THE DISCLOSURE
>
> A current source including a resistor and a pair of transistors having their emitters coupled to opposite ends of the resistor, one of the transistors having its collector and base shorted together; the device has a current supply coupled to the shorted collector and base of one of the transistors and to the base of the other transistor, allowing a small predictable difference in voltage to exist between the base-emitter voltages of the transistors which is coupled across the resistor to produce a small output current from the second transistor.

Widlar begins his discussion of the invention by describing a practical problem in integrated circuit design, namely how to design a low-current regulator using components practical to embed on a chip of silicon.

> This invention relates to a current source and in particular to an integrated circuit current source for providing current of a few hundred microamperes or less.
>
> In integrated circuits there is frequently a need for microampere current sources. To apply prior art techniques to obtain such a source would require the use of a relatively large resistor having a value of tens of thousands of Ohms or more. The use of such resistors in integrated circuits requires excessive surface area and makes the circuit difficult to manufacture regardless of whether thin film or bulk integrated circuit techniques are employed.
>
> To provide a microampere current source consistent with integrated circuit demands and to overcome the shortcomings of prior art circuits, a novel current source has been invented. This current source is based upon the principle that there is a predictable difference in the base emitter voltages of two transistors operating at different collector currents. Using this principle, a current source may be constructed to supply a microampere current requiring a resistor of only several kilo-Ohms or less. Such a resistor in an integrated circuit requires relatively small surface area and is easy to manufacture. Briefly, the structure of this invention comprises two transistors with their emitters coupled across a resistor and their bases and one of their collectors connected to a common point. By such an arrangement the difference in the base-emitter voltage of the transistors is coupled across the resistor. The resulting very small base-emitter difference voltage enables a microampere current to be supplied to the output terminal. This description of the invented structure will be better understood by reference to the detailed description which follows and to the sole drawing which is a schematic diagram of the invented circuit. `[page 2]`

Later in the patent text Widlar describes the magnitude of the voltage drop across the resistor (labeled component 30 in the diagram) as ΔV_{BE}, since by Kirchhoff's Voltage Law the resistor must drop the difference in voltage between the two transistors' base-emitter voltage drops (V_{BE1} and V_{BE2}). Widlar goes on to explain how this difference in base-emitter voltages is a function of the current ratio for the mirror circuit, beginning on page 3 of the patent.

> . . . Hence, the emitter-base voltage differential between adjacent identical integrated circuit transistors operating at different collector currents is given by:
>
> $$\Delta V_{BE} = \frac{kT}{q} \log_e \left(\frac{I_{C1}}{I_{C2}} \right) \qquad \text{(Equation 7)}$$
>
> It has been found that this expression is accurate within a few milliVolts. The collector currents I_{C1} and I_{C2} are different values as determined by current means 4 and the current-determining resistor 30 in accordance with the particular design requirements. Thus, a predictable value of ΔV_{BE} is generated as described by Equation 7. It should be noted that I_1 and I_2 may be considered the same as I_{C1} and I_{C2}, respectively, if

the current gain of the transistors is greater than about 10. It can now be seen that the current through resistor 30 approaches the current I_2. With the current I_2 passing through resistor 30:

$$\Delta V_{BE} = R_{30} \times I_2 \qquad \text{(Equation 8)}$$

$$R_{30} = \frac{\Delta V_{BE}}{I_2} \qquad \text{(Equation 9)}$$

In a typical design procedure employing the invention, the currents I_1 and I_2 would first be chosen and ΔV_{BE} computed by employing Equation 7. Then the value of resistor determined form ΔV_{BE} and I_2. With present technology, the value of resistor 30 is typically in the range of 200 Ohms to 20K Ohms and preferably less than 10K Ohms with output currents in the neighborhood of 5 to 200 microamperes and usually less than 200 microamperes. In one commercial embodiment, 20 μa. was obtained with a 5K ohm resistor.

From the above detailed description it can be seen that a low value current source has been invented which is completely suitable for integrated circuits. The circuit couples the base-emitter voltages of two similar transistors across a resistor. At predetermined operating collector currents the difference in these base-emitter voltages is predictable. This circuit utilizes this small predictable difference to lower the resistance value required for the current source by at least an order of magnitude. Such resistance lowering in turn brings the surface area of the integrated circuit within practical limits of the technology. In addition, the circuit with lower resistance requirements is easier to manufacture and is lower in cost. `[page 3]`

Widlar's patent is noteworthy for multiple reasons. First, it stands as a vivid example of how necessity is often the mother of invention: the Widlar current source was invented to solve the very practical problem of regulating small amounts of current using components reasonably fitting an integrated circuit wafer. Second, it demonstrates more usage of transistor approximations (e.g. resistor current being practically the same as I_2 for sufficiently large transistor beta values). Third, it illustrates some of the nonintuitive aspects of exponential functions: note how the difference in base-emitter voltages between the two transistors is a function of the two currents' *ratio* and not on the specific values of current. As an example of this last point, a Widlar current mirror with programming and load currents of 10 mA and 1 mA respectively would drop the same amount of voltage across the resistor as currents of 0.5 mA and 0.05 mA.

Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

4.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Faraday's Law of Electromagnetic Induction

Rheometric sensing

N-type semiconductor

P-type semiconductor

BJT principles

Majority versus Minority carriers

Shockley diode equation

Temperature effect on BJT action

Current mirror

Sourcing versus Sinking

FET principles

Threshold voltage

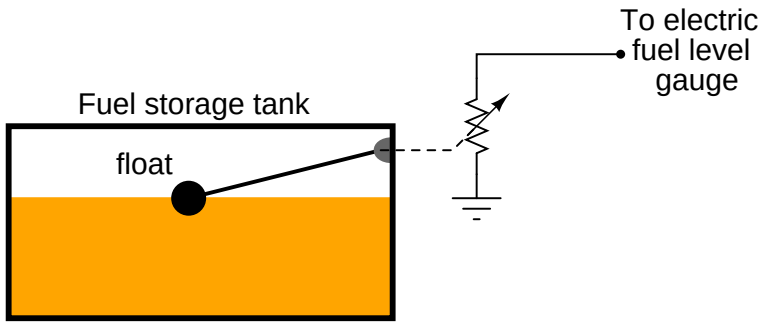
Transconductance

Beta

Negative feedback

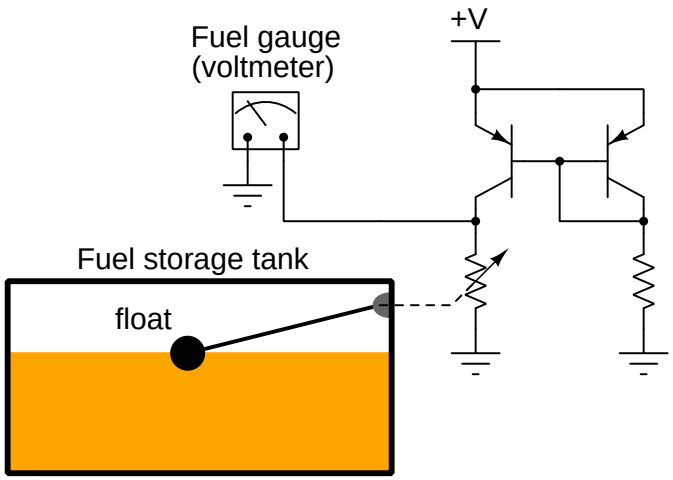
4.1.3 Fuel tank level sensing

Many different types of sensors work on the principle of a variable resistance representing a different physical quantity. One such sensor is the common fuel level sensor used in automotive, marine, and industrial fuel storage applications:



As the fuel level in the tank changes, the float position will change, changing the sensor's resistance. This resistance change is detected by an electric gauge (a special type of meter), which then provides visual indication of fuel level in the tank.

We must have some accurate way of measuring electrical resistance in order for this scheme to work. One common technique for doing this is to pass a constant current through the sensor resistance and then measure the voltage dropped across it. Since current mirror circuits function as current regulators, and therefore may be used as current sources if supplied with an external voltage, we could use a current mirror to force constant current through the fuel level sensor:



One problem with the circuit shown is that the sensor current will change as the supply voltage (+V) changes. This may be important to us, because the DC system voltage on an automobile may not be very stable, and this could lead to inaccuracies in fuel level measurement.

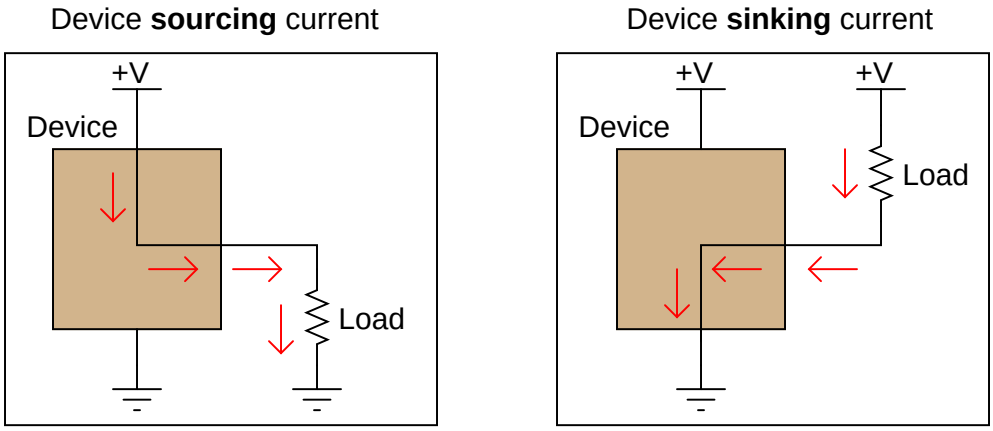
Figure out a way we could use a zener diode to stabilize the voltage in this current mirror circuit so that supply voltage changes would have minimal effect on the amount of current through the variable-resistance sensor.

Challenges

- Redesign this circuit to use a JFET instead of BJTs.

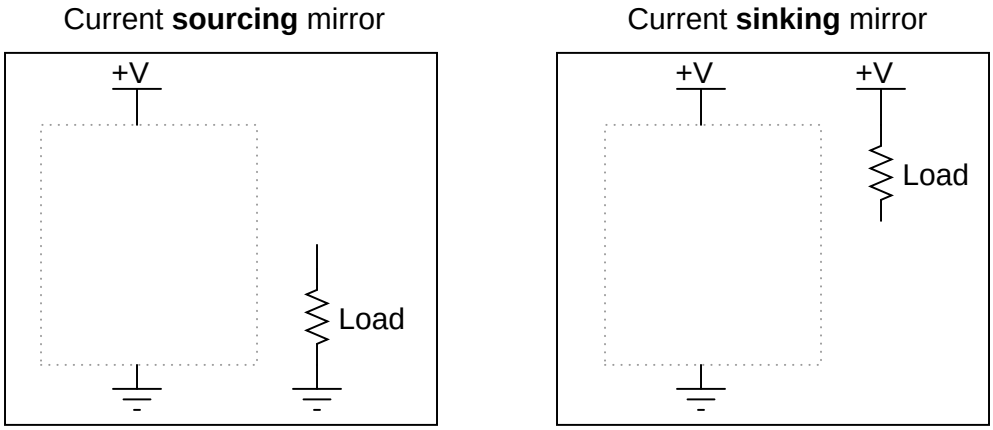
4.1.4 Sourcing and sinking current regulators

Two terms used commonly in electronics are *sourcing* and *sinking*, in reference to the direction of electric current between an active circuit and a load:

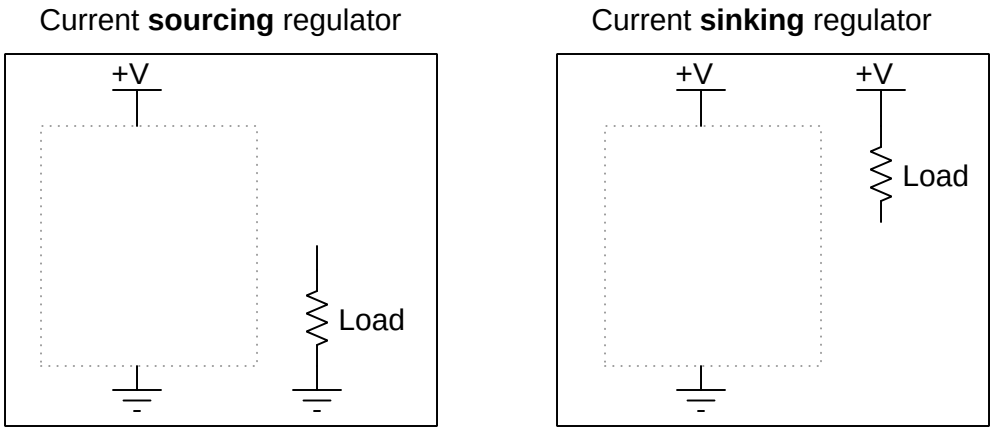


(All current directions shown in "conventional flow" notation)

Similarly, a current mirror circuit may be built to either source current or sink current, but not do both. Draw current mirror circuits within the dotted-line boxes suitable for sourcing and sinking current to a load resistor:



Now, draw JFET-based current regulator circuits within the dotted-line boxes:



Challenges

- Identify advantages and disadvantages of each current regulator type.

4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

4.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

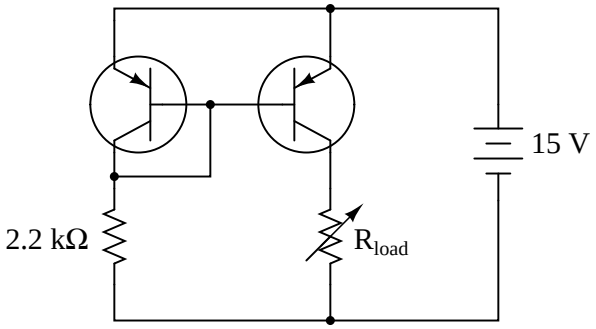
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

4.2.3 Current mirror load calculation

Calculate the approximate amount of current this current mirror circuit will try to maintain through R_{load}, assuming silicon transistors (0.7 Volts forward base-emitter junction drop):

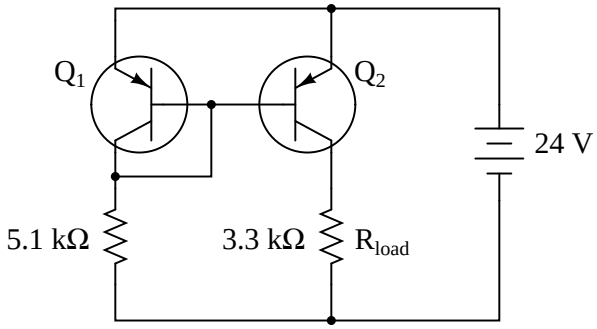


Challenges

- What would have to be changed in this circuit to increase the amount of current through the load resistor without changing the power supply voltage?

4.2.4 Load current and power dissipation

Calculate the approximate amount of current this current mirror circuit will try to maintain through R_{load}, assuming silicon transistors (0.7 Volts forward base-emitter junction drop):



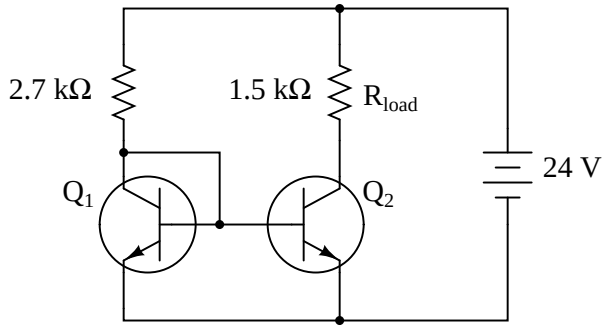
Also, calculate the approximate power dissipation of transistor Q_2.

Challenges

- Will P_{Q2} increase, decrease, or remain constant as R_{load} increases?
- Will P_{Q2} increase, decrease, or remain constant if the 5.1 kΩ resistor happens to decrease in value?

4.2.5 Current mirror transistor power dissipations

Calculate the power dissipated by each transistor in this current mirror circuit, assuming silicon transistors (0.7 Volts forward base-emitter junction drop):



Also, calculate the amount of current through each resistor.

Challenges

- Suppose the DC power source voltage decreased. What impact would this have on all currents and transistor power dissipations?
- What do these two power dissipation figures tell us about the relative power of the two transistors in a current mirror circuit? Explain why this is important in current mirror design, and why it is customary to thermally bond Q_1 and Q_2 together in discrete-component current mirrors.

4.2.6 Simple current mirror design

Choose a power supply voltage and programming resistance value for a simple current mirror circuit that will maintain approximately 15 mA of current through a load resistance of approximately 1 kΩ. Assume the use of a silicon transistor. Design your circuit such that at least 20 Volts will be dropped across the load's transistor (V_{CE}).

Challenges

- More than one correct answer exists for this problem. Propose at least one more set of values to satisfy all criteria.

4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

4.3.1 Broken C-B jumper

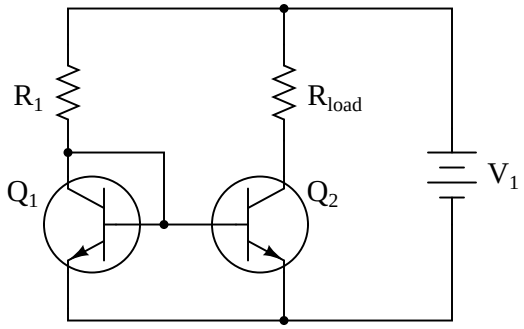
Current mirror circuits often employ a transistor with "jumpered" collector and base terminals to function as a plain PN junction with characteristics identical to another transistor. How would the operation of a current mirror be affected if this jumper was severed?

Challenges

- Would a current-sourcing mirror fail any differently than a current-sinking mirror given this particular fault?
- Suppose a regular rectifying diode were used in place of a C-B shorted transistor to make a current mirror. In what way(s), specifically, would the function of this mirror be worse than the function of a current mirror built using two identical transistors?

4.3.2 Faulted current mirror

Predict how all component voltages and currents in this circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):



- Resistor R_1 fails open:
- Transistor Q_1 fails open, collector to emitter:
- Transistor Q_1 fails shorted, collector to emitter:
- Transistor Q_2 fails open, collector to emitter:
- Transistor Q_2 fails shorted, collector to emitter:
- Load fails shorted:

Challenges

- How will load current be affected (if at all), supposing V_1 increases?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Widlar, Robert J., *US Patent 3,320,439*, "Low-Value Current Source for Integrated Circuits", application filed 26 May 1965, patent granted 16 May 1967.

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

9 May 2021 – commented out or deleted empty chapters.

18 April 2021 – changed all lower-case Greek letter "phi" symbols (ϕ) to upper-case (Φ).

22 March 2021 – added more digits to Boltzmann's constant, and clarified that Kelvin is 273.15 more than Celsius.

18 March 2021 – corrected multiple instances of "volts" that should have been capitalized "Volts".

11 February 2021 – corrected instances where "Ohms" was uncapitalized.

30 September 2020 – clarified why BJTs regulate current.

22 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

27 February 2020 – corrected yet another error, this time describing which terminals of a BJT need to be shorted together to make the diode junction for a current mirror.

26 February 2020 – corrected diagram errors where I showed a P-channel JFET instead of an N-channel JFET. Also corrected error in interpreting the last SPICE analysis. Instead of just under 1 mA load current, I should have said just under 0.3 mA load current, being slightly less than 1 Volt dropped across 3300 Ohms. Also fixed schematic error diagram in "Fuel tank level sensing" problem. 26 February 2020 was a banner day for errata.

27 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

23 December 2019 – minor edits to a question.

18 December 2019 – minor edits to diagnostic questions, replacing "no multiple faults" with "no coincidental faults".

13 December 2019 – finished writing Tutorial.

12 December 2019 – began writing Tutorial.

11 December 2019 – document first created.

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