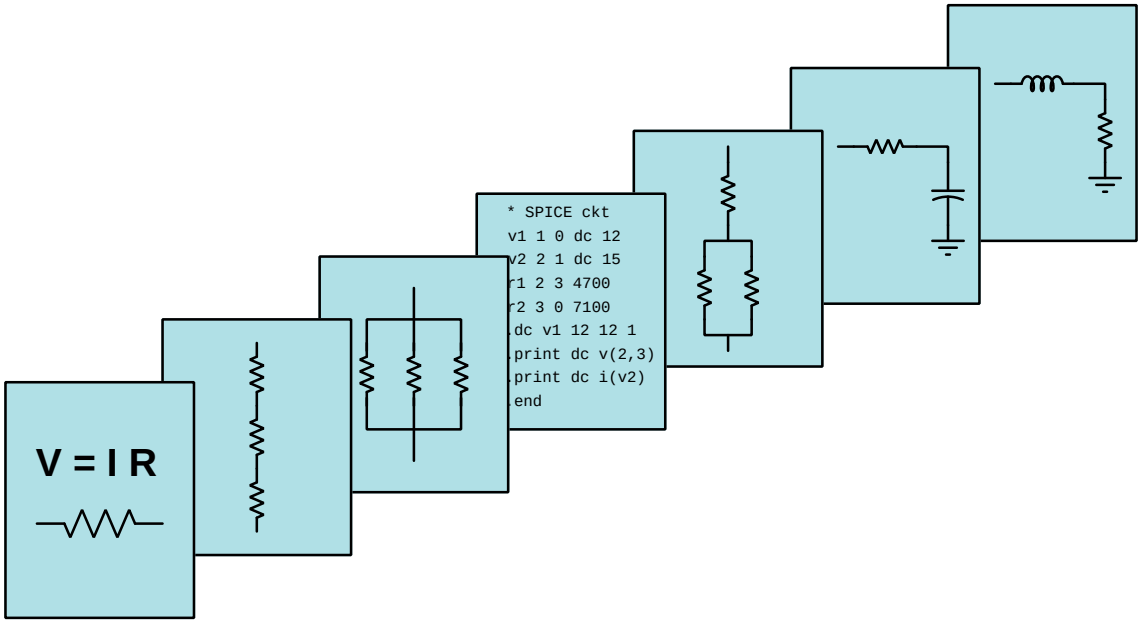
Modular Electronics Learning (ModEL) project



Matter and Energy

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Chapter 1

Introduction

This learning module introduces some basic concepts of matter and energy useful to students of electricity and electronics. The Simplified Tutorial is very brief compared to the Full Tutorial, which touches on basic chemistry and thermodynamics. Electricity is fundamentally regards the exchange of energy using mobile electrical charges, and so a firm conceptual foundation in the concept of energy, how it is conserved, as well as conservation laws related to electric charge and mass is very important.

When first learning anything new, it is extremely helpful to challenge yourself to express what you have learned into your own words, using your own examples, and to discover how these new ideas connect to ideas previously learned. You can do this by writing, by speaking, and also by direct application (by doing). Then, share your thoughts and actions with others to gain their perspective. If you are a student in a formal educational environment, bringing this level of engagement to the conversation will help you (as well as your fellow students) tremendously. You will find this not only helps you remember, but it also gives you greater depth of understanding.

Important principles discussed in the Tutorial chapters include **conservation laws** (energy, mass, electric charge), basic **atomic structure**, the relationship of energy to **force** and **motion**, different **forms of energy** and how they relate to motion, and the distinction between energy versus **power**. In particular, *energy* in its different forms is a unifying principle throughout the study of electricity and electronics.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the concept of energy conservation? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to explore the concept of mass conservation? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How do we define *energy*, and how may we calculate its quantity?
- What does it mean for a physical property to be *conserved*?

- What is *work*, and what does it mean in the active and passive form (i.e. what does it mean to *do work* on something, versus *have work done* on something)?
- How does *power* differ from *energy*, and why are these concepts easy to confuse with one another?
- How does an electrostatic force differ from a gravitational force?

An excellent "active reading" strategy to apply in this text as well as all technical tutorials is to actually perform the mathematical work shown by the author, for the purpose of reinforcing your mathematical abilities as well as illuminating the author's points. Look for opportunties in the text to do this, where you can take the information given in the set-up of a mathematical problem, then independently compute your own answer, checking against the text to see if your result matches the author's.

You will find many other "active reading" strategies in the "Reading outline and reflections" subsection of the Conceptual reasoning section of the *Questions* chapter. Additionally, the "Foundational concepts" subsection in that same chapter lists many of the important principles contained in this learning module. A good self-check of your reading comprehension is to see if you are able to define each of those listed concepts.

Common misconceptions include the following. For each of these, challenge yourself to prove why the misconception is incorrect:

- Equating *force* with *energy*
- Equating *power* with *energy*

In the *Derivations and Technical References* chapter you will find resources for exploring important concepts in greater depth. Examples include *Newton's Laws of Motion* where we find Isaac Newton's expressions of fundamental principles related to the motion of matter, and *Dimensional analysis* where we explore a powerful technique for validating the application of mathematics to physical phenomena.

Chapter 2

Simplified Tutorial

Matter is anything possessing *mass*, "mass" being defined as the opposition to acceleration (changes in velocity). Mass is related to *weight*, but not identical to it because "weight" is the downward force exerted on a mass by a gravitational field. Matter in outer space, for example, would be weightless since it experiences negligible gravity, but it would still have mass.

Isaac Newton, the famous scientist who lived from 1642-1727, formulated some *laws of motion*, the first and second of which describe these properties of mass:

1. An object at rest tends to stay at rest; an object in motion tends to stay in motion[1]
2. The acceleration of an object is directly proportional to the net force acting upon it and inversely proportional to the object's mass

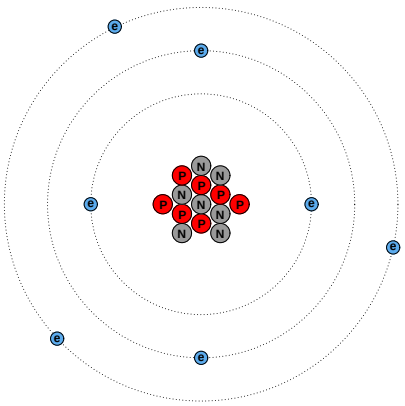
A very important property of mass is that it is *conserved*, which means it is eternal and cannot be created or destroyed. Mass may change form, but it cannot be eliminated nor can it appear from nothing. This is known as the *Law of Mass Conservation*, or simply the *Conservation of Mass*.

The standard metric unit of measurement for mass is the *kilogram*, one kilogram being equal to one thousand *grams* (a much smaller measure of mass). In the Imperial (British) measurement system mass is measured either in *slugs* or *pounds*.

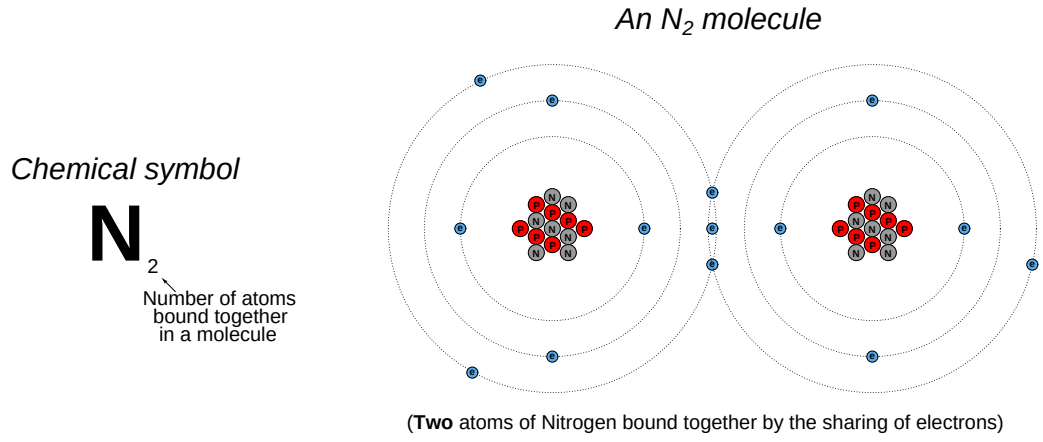
Through many centuries of experimentation, people discovered that matter may be subdivided into ever-smaller components. What may appear to be a completely solid and homogeneous object is actually comprised of *molecules*, which are in turn formed of even smaller *atoms*, which in turn are formed of even smaller *sub-atomic particles*. The separation of molecules into individual atoms, as well as the combining of atoms to form molecules, is the domain of *chemistry*. The separation of atoms into particles and combining particles to make atoms is the domain of *nuclear physics*.

[1] Adding Tony's corollary to this, *an object on vacation tends to stay on vacation.*

A tremendously simplified model of a single atom of the element *nitrogen* is shown here, each of its constituent particles represented by colored circles: *protons*, *neutrons* and *electrons*. Seven protons and seven neutrons comprise the nucleus of a nitrogen atoms, and seven electrons "orbit" around it:



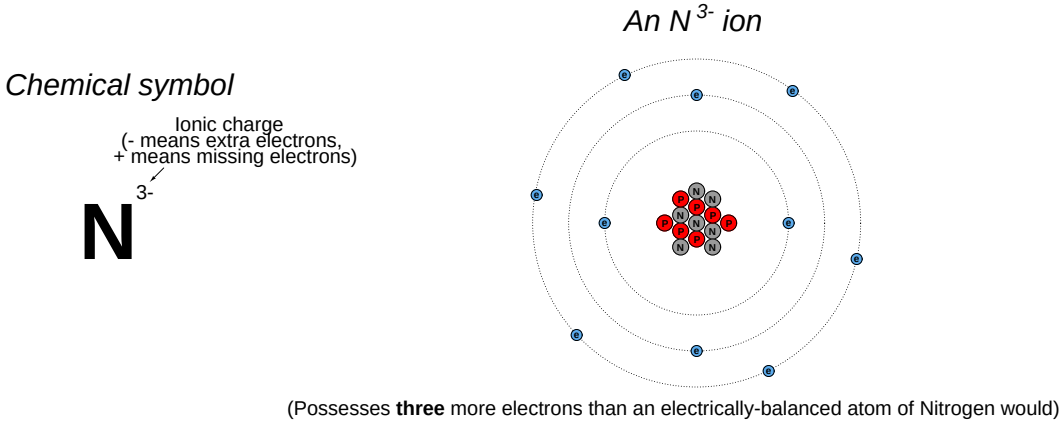
The chemical identity of an atom is defined entirely by the number of protons (called the *atomic number*) within an atom's nucleus. Those protons and neutrons comprising an atom's nucleus are very tightly bound together, so tight in fact as to be almost inseparable from each other. This is why atoms are almost impossible to modify[2]. The orbiting electrons are much easier to dislodge or to add, and in fact it is precisely this process that results in atoms merging together to form molecules, the following illustration showing a nitrogen molecule formed of two nitrogen atoms bonded together by the sharing of three electrons:



[2]In fact, the word "atom" comes from the Greek word *atomos*, which means "indivisible". Ancient philosophers of nature speculated that atoms themselves were conserved (i.e. incapable of creation or destruction) but modern scientific techniques have shown this to be untrue. Now it actually is possible to pluck particles out of or add particles to atomic nuclei, thereby altering those atoms' identities, but it is still very difficult process.

Electrons and protons are mutually attracted by a force known as an *electrostatic force*. This is similar to gravity, but very different in one regard: whereas gravity can only attract matter together, the electrostatic force can both *attract* and *repel* matter. The equivalent of *mass* for gravitational fields is *charge* for electrostatic fields, and particles having the same "polarity" of charge repel each other while particles having opposite-polarity charges attract. By convention, electrons are said to have a *negative charge* and protons a *positive charge*. Electrons are attracted to protons, but repelled by other electrons. Protons also tend to repel each other, but this repulsive electrostatic force is much weaker than the *strong nuclear force* binding protons and neutrons together in the nucleus. Neutrons have no charge.

In an electrically "balanced" atom, the number of electrons exactly equals the number of protons, leaving the atom which no net charge. If an atom is stripped of one or more of its electrons, it will have a net positive charge; if extra electrons join the atom, it will have a net negative charge. In both cases we refer to the "imbalanced" atom as an *ion*, one example being a negatively-charged nitrogen ion represented in the illustration below:



(Possesses **three** more electrons than an electrically-balanced atom of Nitrogen would)

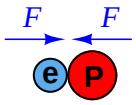
In some types of solid material, most notably *metals*, the atoms bind together in such a way that their outer-most electrons are free to drift apart from their parent atoms and actually move like a fluid within the solid. These mobile charges are the stuff of *electricity*. Ions existing within a liquid or a gas – where whole atoms are free to drift independently of each other – may also serve as mobile electric charges just like individual electrons moving about within a solid metal object.

Electric charge, like mass, is also a conserved quantity. That is to say, charges can neither be created nor destroyed but are eternal. This is known as the *Law of Electric Charge Conservation*, or simply the *Conservation of Electric Charge*.

To summarize, all matter has *mass* and experiences an attractive force (called the gravitational force) toward other masses. Some types of matter also have *charge* and experience either an attractive or a repulsive force with other charged matter (called the electrostatic force). Both mass and charge are *conserved* quantities in our universe. Charges that are free to move about is what *electricity* is made of.

A concept more abstract than either mass or charge is *energy*, and for the purposes of our discussion will be defined as *that which sets matter into motion*. Imagine an electron and a proton separated a short distance from each other in otherwise empty space, their electrostatic attraction causing them to move toward each other:

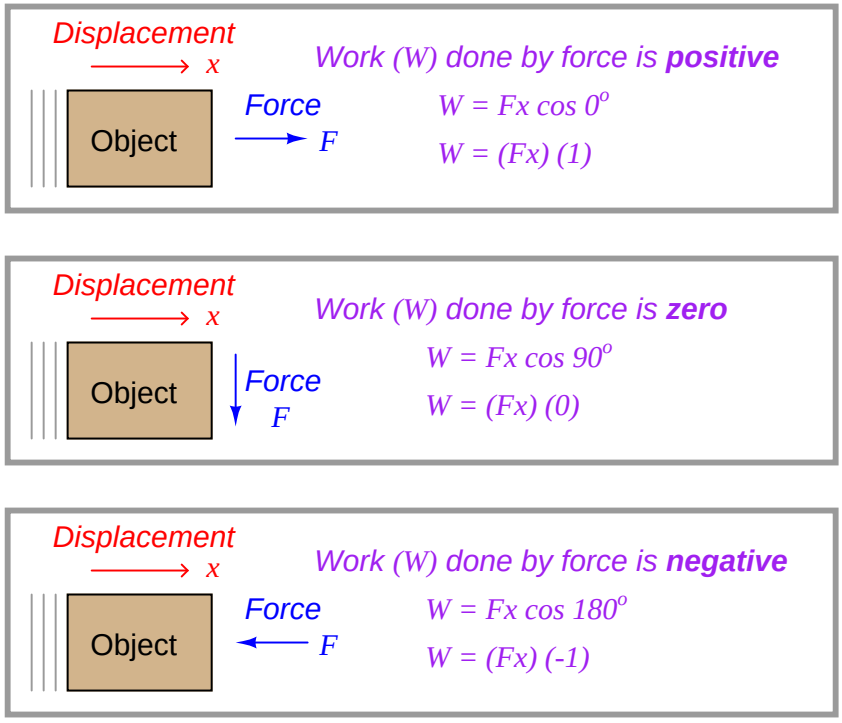
In the absence of any external restraint, these two charged particles *will* move toward each other, and thus we may conclude energy is at work here as per our definition of *energy being that which sets matter into motion*. At first one might be tempted to identify the source of this energy as the electrostatic force acting on these particles, but this is not entirely accurate. The next illustration should make this clear:



Here, we see the electron and proton have moved together until they touch, at which point they cannot travel any closer to each other. The attractive force of their opposing electrostatic charges is still there, but since there is no longer room for motion they cannot move closer. If we define energy as "that which sets matter into motion" we must conclude energy is no longer present in this system, even though force still is. Therefore, energy cannot be the same thing as force.

Force is clearly involved in energy – for without a force attracting the two particles together they would have no impetus to begin moving toward each other – but there must be more to the concept of energy than force alone. What we need for there to be energy is both *force* and *displacement* (travel, or at least the potential of travel).

In the study of physics, the combined action of force and displacement is called *work*, and it may be calculated as the product (multiplication) of force and displacement and the cosine of the angle between those two vectors[3].



When work occurs – i.e. a force acting parallel with a displacement – energy is exchanged. Returning to our electron-proton system, one could imagine some external force prying those two particles apart from each other against the electrostatic force of their mutual attraction. In this case that external force would be *doing work on the particles*, and in so doing that work those two particles would end up having more energy (i.e. more potential to travel toward each other) than before. If we could somehow harness both of those separated particles to make them motivate some other matter as they fell toward each other again, the electron and proton would be *doing work on that other matter*, and in so doing that work their energy would be transferred there.

Interestingly, energy – just like mass and just like charge – is a *conserved* quantity, incapable of being created or destroyed. Any time work occurs in the physics sense of that word, energy merely moves from one location or state to another, but the grand total of energy in the universe remains exactly the same. This is called the *Law of Energy Conservation*, or simply the *Conservation of Energy*.

[3] A *vector* in physics is a quantity that has both a magnitude and a direction. This is true of both forces and displacements.

Energy is usually defined as being one of two types, *potential* or *kinetic*. "Potential" energy is capable of setting matter into motion, without any motion occurring at the moment. "Kinetic" energy, by contrast, is what moving matter possesses – i.e. able to set other matter into motion by its own motion (by collision, for example).

Both forms of energy are easy to calculate, potential energy being a function of force and potential displacement parallel to that force, and kinetic being a function of mass and velocity:

$$E_p = Fx \qquad\qquad E_k = \frac{1}{2}mv^2$$

Where,

E_p = Potential energy in Joules or Newton-meters (metric), or foot-pounds (British)
F = Force in Newtons (metric) or pounds (British)
x = Potential displacement in meters (metric) or feet (British)

E_k = Kinetic energy in Joules or Newton-meters (metric), or foot-pounds (British)
m = Mass of object in kilograms (metric) or slugs (British)
v = Velocity of mass in meters per second (metric) or feet per second (British)

Some quantitative examples illustrate both types of energy:

- A boulder with a weight of 500 Newtons (gravitational force) resting at the top of a 30 meter hill possesses 15000 Newton-meters (or 15000 Joules) of potential energy
- A bullet having a mass of 2.9 grams (0.0029 kilograms) traveling at a velocity of 1100 meters per second possesses 1754.5 Newton-meters (or 1754.5 Joules) of kinetic energy

Energy may freely exchange between these forms, and when such an exchange takes place the sum total of energy remains constant due to Conservation. For example, if that bullet were fired straight up[4] it would leave the gun barrel with 1754.5 Joules of kinetic energy, which would convert to potential energy as it climbed to maximum altitude and minimum velocity, then would convert back into kinetic energy as the bullet lost altitude and fell toward Earth. Air friction during its ascent would result in the bullet attaining less than 1754.5 Joules of potential energy at its apogee, and air friction while falling would result in even less kinetic energy when it reached ground level again. However, *total* energy would remain the same: 1754.5 Joules total in kinetic energy plus potential energy plus thermal and acoustic energy[5] in the air disturbed by the bullet's flight.

Power is often confused with *energy*, but it means something different. Whereas "energy" is that which sets matter into motion, "power" is defined as *the rate at which energy gets exchanged from one form to another*. In the metric system the Newton-meter or Joule is used to measure energy, but the *Watt* is the unit for power, with one Watt being equal to one Joule (or Newton-meter) per second. Whereas a measurement of energy tells you how much work that energy could do, a measurement of power tells you how *rapidly* work can get done.

[4] Please, do not attempt this experiment!

[5] Thermal energy is actually another example of potential energy, as *temperature* consists of random molecular motion.

Chapter 3

Full Tutorial

Physics is really the study of matter and energy, and of the laws describing the interactions of both. The movement of matter largely follows the laws described by Isaac Newton (so much so that this category of physics is often referred to as *Newtonian physics*). The flow and transformation of energy is a category called *thermodynamics*. In a sense, this Tutorial may be thought of as an extremely condensed lesson in physics.

It may also be thought of as an extremely condensed lesson in *chemistry*, the study of how different types of atoms interact with each other. Matter consists of atoms, but atoms rarely exist in isolation. Usually atoms are found *bonded* to each other to form larger collections of matter called *molecules*.

3.1 Matter and atomic structure

Matter is anything possessing *mass*, and "mass" is an opposition to acceleration. Masses have *weight* when exposed to a gravitational field such as on Earth, but even when far removed from sources of gravity (e.g. deep in space) matter still possesses mass[1].

In everyday scenarios we find matter existing in the form of very large numbers of sub-microscopic *atoms* and *molecules*. Common *phases* of matter include solid, liquid, and gas. Solids are materials where the molecules are more or less locked together into a rigid form. In liquids, the molecules are not locked together per se, but exist adjacent to one another while retaining individual mobility. Gases consist mostly of empty space interspersed with molecules that are entirely mobile.

[1] A good demonstration of this fact would be to attempt to *shake* a massive object such as an anvil if brought about an orbiting space station or other spacecraft. Though weightless, that object would still resist being shaken, and the more mass it has the more resistance it would offer to a shaking force.

3.1.1 Atoms and particles

Scientific experimentation has led humanity to the discovery that matter is comprised of *atoms*, and that every atom consists of just three types of "elementary" building-blocks called *particles*: *electrons*, *protons*, and *neutrons*. Combinations of these three particle types in various whole-number quantities constitute every type of atom. These fundamental particles are absolutely miniscule in comparison to the macroscopic existence of human beings. Just to illustrate, the mass of a single proton is approximately 1.67×10^{-27} kilograms: written without scientific notation, it would be 0.00000000000000000000000000167 kg. An electron is even smaller: weighing in at 9.11×10^{-31} kg (about *1800 times* less mass than a proton!). Being far smaller in size than a wavelength of visible light[2], we cannot see these particles even with the most powerful optical microscope.

Protons and neutrons are very tightly bound together in the nucleus (center) of an atom. This bond is so secure that only extraordinary forces are able to pry an atom's nucleus apart. Suffice it to say, one cannot disturb the stability of an atomic nucleus by rubbing, cutting, grinding, heating, smashing, or any other macroscopic physical process. The force binding protons and neutrons together in the nucleus is known as the *strong nuclear force*.

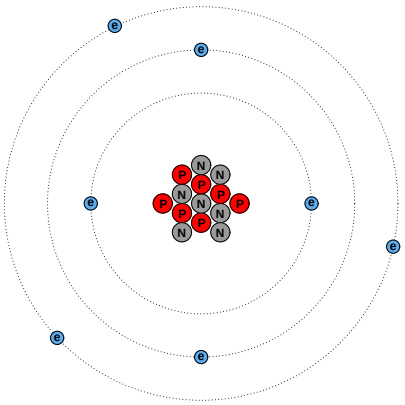
Electrons "orbit" the nucleus of atoms, and are held in proximity to those nuclei by electrostatic attraction (the so-called *electromagnetic force*), which is many orders of magnitude weaker than the strong nuclear force. Thus, electrons *can* be dislodged from or added to atoms through the action of macroscopic forces such as rubbing, cutting, grinding, heating, etc. It is the changeable configurations of electrons that accounts for different atoms joining together to form *molecules*.

The chemical identity of any atom is a simple and direct function of how many protons that atom has in its nucleus. Each nitrogen atom, for example, has seven (7) protons in its nucleus. This quantity is called the *atomic number* of an atom. In order for an atom to have a net neutral electric charge, there must be as many electrons orbiting the nucleus as there are protons in the nucleus, since protons are positively charged and electrons are negatively charged (equal and opposite electric charges, each). Therefore, a neutral atom of nitrogen will have seven electrons orbiting around the nucleus, electrically balancing the seven protons within the nucleus.

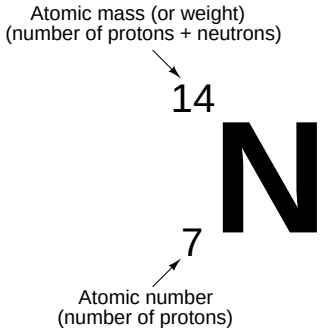
The number of neutrons within the nucleus of an atom does not affect the atom's chemical identity, but it may affect its nuclear properties (e.g. whether or not it is radioactive; to what degree it captures certain forms of radiation, etc.). For example, most nitrogen atoms have seven neutrons along with seven protons in their nuclei, giving a total nuclear particle count of fourteen – the atomic *mass* of the atom, sometimes called the atomic *weight*. However, it is possible for a nitrogen atom to have eight neutrons (an atomic mass of fifteen) and still be "nitrogen," with all the same chemical properties.

[2] In order for a wave of light to be influenced at all by an object, that object must be at least the size of the wave's length. To use an analogy with water waves, it would be comparing the interaction of a water wave on a beach against a large rock (a disturbance in the wave pattern) versus the non-disturbance of that same wave as it encounters a small buoy.

A tremendously simplified model of a common nitrogen atom is shown here, with 7 protons and 7 neutrons in the nucleus, and 7 electrons in "orbit" around the nucleus:



The atomic number of this atom (the number of protons in the nucleus) is seven, which is what defines it as nitrogen. The *atomic mass* of this atom (the sum of protons and neutrons in the nucleus) is fourteen. The chemical symbol for this atom is shown here:

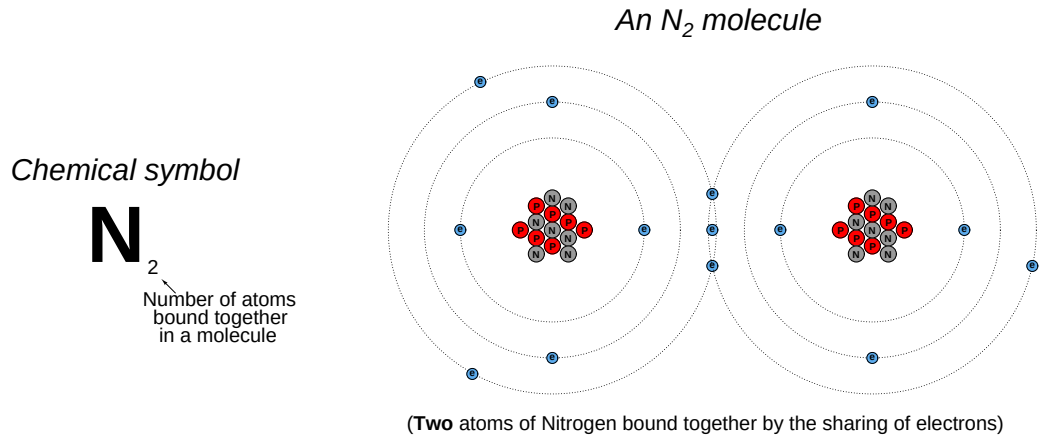


The atomic number is redundant to the letter "N" for nitrogen, since only the element nitrogen can have an atomic number of seven. The atomic mass is only relevant when we need to distinguish one *isotope* of nitrogen from another (variations of elements having the same number of protons but different numbers of neutrons), and this is seldom because the chemical properties of isotopes are identical – only their masses differ. For these reasons, you will usually find no left-hand subscripts or superscripts placed near chemical symbols of elements in chemical expressions.

3.1.2 Molecules and ions

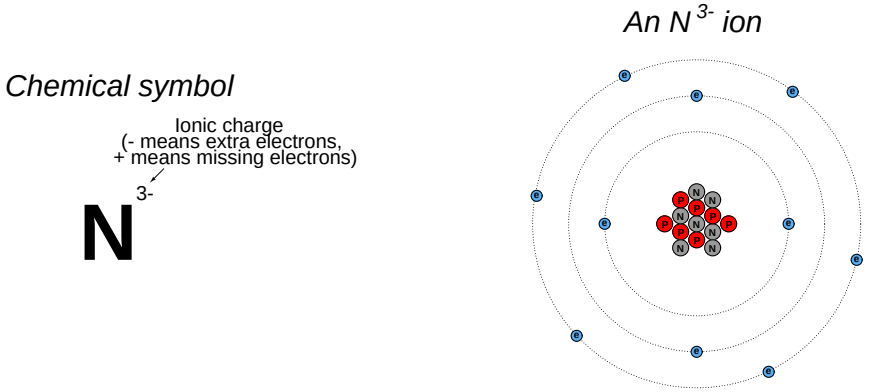
By contrast, subscripts and superscripts placed to the right of a chemical symbol have very important meanings in chemistry. A right-hand subscript refers to the number of atoms bound together to form a molecule. A right-hand superscript refers to the electrical charge possessed by an atom (or by a molecule) by virtue of the number of electrons not matching the number of protons:

An N_2 molecule may be represented simplistically as follows, the two nitrogen atoms joined by a mutual sharing of three of its highest-energy (valence) electrons, shown in this illustration as those electrons residing in the largest-diameter "orbits". Incidentally, this "triple bond" characterizing the nitrogen molecule is very strong (the more electrons participating in the joining of two atoms, the stronger the bond, all other factors being equal):



(**Two** atoms of Nitrogen bound together by the sharing of electrons)

An N^{3-} ion is an atom of nitrogen having three more electrons than it normally would when electrically balanced:



(Possesses **three** more electrons than an electrically-balanced atom of Nitrogen would)

3.1.3 Chemical formulae

A chemical *formula* is a written description of a molecule's constituent atoms. Ethanol (ethyl alcohol), for example, is a conglomerate of two carbon atoms, six hydrogen atoms, and one oxygen atom. One way to express this structure is to write the following formula for ethanol, the right-hand subscripts showing the relative quantities of atoms in each ethanol molecule:

$$C_2H_6O$$

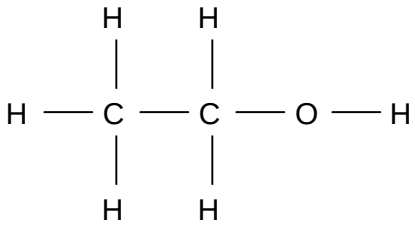
This is called a *molecular formula*, because it shows the proportions of atom types comprising each molecule.

A more common way to write the formula for ethanol, though, is this:

$$C_2H_5OH$$

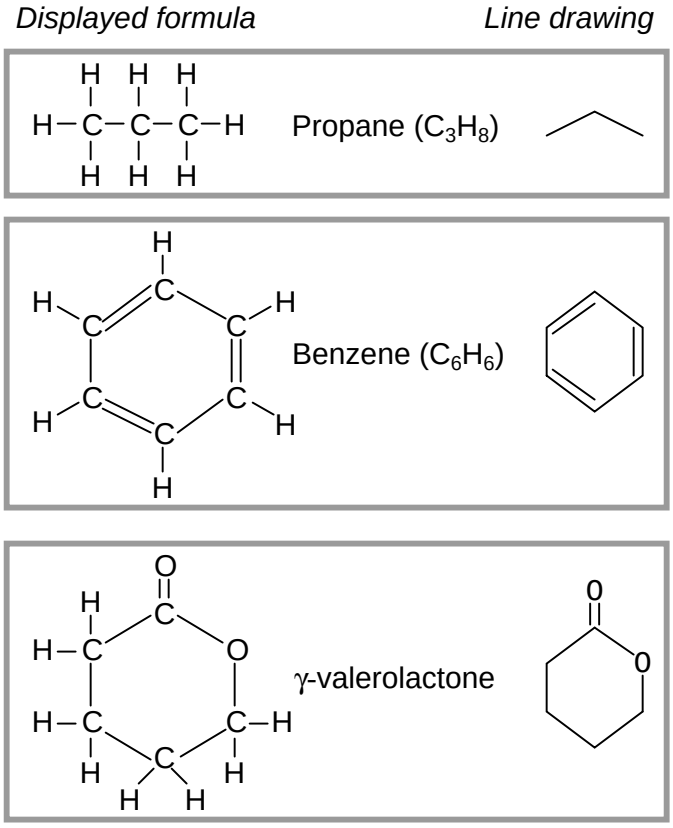
Here, an attempt is made to show the physical structure of the ethanol molecule, where one of the hydrogen atoms is located further away from the others. This is called a *structural formula*. If more detail of the bonds between atoms in a molecule is needed, a semi-graphic representation called a *displayed formula* (also known as an *expanded structural formula*) may be used in lieu of a structural formula:

Displayed formula for ethanol
(C_2H_5OH)



Each letter in a displayed formula represents a single atom within that molecule, and each line segment in a displayed formula represents a bond between two.

In organic chemistry – the study of molecules principally centered around *carbon* atoms – a special type of notation is used to show the structural detail of the molecule with fewer lines and letters than a displayed formula. This notation is called a *line drawing*, where each line segment represents a single electron bond[3] to a carbon atom, each vertex and line-end represents the location of a carbon atom, and any hydrogen atoms directly bound to a carbon atom are simply omitted for simplicity. Compare and contrast these displayed formulae and line drawings for a few different organic compounds:



An important principle in organic chemistry is that carbon atoms prefer to form exactly *four* bonds with surrounding atoms[4]. This fact is exploited in line-drawing notation where any bonds not explicitly shown at a vertex are assumed to be single-bonds with hydrogen atoms, enough of them to bring the total number of bonds with that carbon atom to four. Since a great many organic compounds are principally comprised of carbon and hydrogen, the line-drawing symbols for these molecules tend to be more lines than letters.

[3] One line represents a single bond, which is one electron shared per bound atom. Two parallel lines represent a *double bond*, where each carbon atom shares two of its valence electrons with the neighboring atom. Three parallel lines represent a triple bond, where each atom shares three of its outer electrons with the neighboring atom.

[4] Incidentally, nitrogen atoms preferentially form exactly *three* bonds, and oxygen atoms exactly *two* bonds. The reason for this pattern is the particular patterns of electrons orbiting each of these atoms, and their respective energy levels. For more information on this, see section 3.2 beginning on page 19.

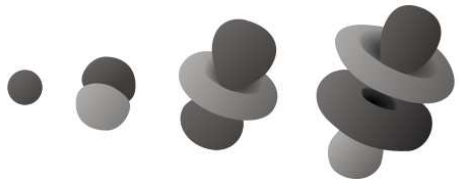
Chemical engineers often perform mass and energy balance calculations for processes where *mixtures* of similar compounds exist. Wastewater treatment is one example, where an array of organic compounds must all be treated through oxidation (chemical reaction with oxygen). In such cases, it is common for chemical engineers to write formulae expressing the *average* ratios of elements, so that they may calculate the quantity of reactant(s) needed to complete the desired chemical reaction with compounds in the mixture. Primary sludge clarified from municipal wastewater, for example, may be represented by the *compositional formula* $C_{22}H_{39}O_{10}N$. This does not suggest the existence of some monstrous molecule consisting of twenty-two carbon atoms, thirty-nine hydrogen atoms, ten oxygen atoms, and a lone nitrogen atom somewhere in a sample of sludge, but rather that the combined *average* carbon, hydrogen, oxygen, and nitrogen quantities in that sludge exist in a variety of molecular forms in these approximate proportions. This aggregate formula expression helps the engineer quantify the gross chemical characteristics of the sludge, and from that determine how much oxygen will be necessary to completely oxidize it.

Sometimes, compositional formulae are written with non-integer subscripts. An example of this would be the compositional formula $C_{4.8}H_{8.4}O_{2.2}$, which also happens to be an average composition for municipal wastewater sludge (ignoring nitrogen content). The same formula could just as well have been written $C_{48}H_{84}O_{22}$, or even $C_{24}H_{42}O_{11}$, because these subscript values all express the exact same proportions.

3.2 Electronic structure

Earlier in this chapter you were shown a model of a nitrogen atom with a dense nucleus (comprised of protons and neutrons) surrounded by electrons whirling around like satellites around a planet. While there are some useful features of this model, it is largely in error. A more realistic view of atomic structure begins with the realization that electrons do not exist as discrete particles, but rather as wave packets. In a sense, they orbit the nucleus within certain *areas of probability*, as described by the principles of quantum mechanics. One way to envision this is to think of an electron's placement around the nucleus in the same way you might picture a city shrouded by a layer of fog. The electron does not have a discrete location (even though there *is* a discrete number of them found in every atom), but rather may be found anywhere within a certain region to varying degrees of probability.

Things get even stranger the more electrons there are in an atom. No two electrons may share the same quantum states in the same atom – a principle called the *Pauli Exclusion Principle*. This means the electrons surrounding a nucleus must exist in distinct patterns. Just a few of these patterns are shown here as *orbitals* (regions of high probability where up to two electrons may be found surrounding a nucleus):[5]



Electrons situate themselves around the nucleus of any atom according to one basic rule: the minimization of potential energy. That is, the electrons seek the lowest-energy positions available around the nucleus. Given the electrostatic attraction between negative electrons and the positive nucleus of an atom, there is potential energy stored in the "elevation" between an orbiting electron and the nucleus, just as there is gravitational potential energy in any object orbiting a planet. Electrons lose energy as they situate themselves closer to the nucleus, and it requires an external input of energy to move an electron farther away from its parent nucleus.

In a sense, most of chemistry may be explained by this principle of minimized potential energy. Electrons "want" to "fall" as close as they can to the positively-charged nucleus. However, there is limited "seating" around the nucleus. As described by Pauli's Exclusion Principle, electrons cannot simply pile on top of each other in their quest for minimum energy, but rather must occupy certain regions of space allowed by their unique quantum states.

An analogy[6] for visualizing this is to picture an atom as if it were an amphitheater, with the stage being the nucleus and the concentric array of seats being places where electrons may reside. All spectators (electrons) desire to be as close to the stage (nucleus) in an amphitheater (atom) as possible, but since everyone cannot occupy the best seat, people are forced to choose seats at

[5] These orbitals just happen to be the 1s, 2p, 3d, and 4f orbitals, as viewed from left to right. In each case, the nucleus lies at the geometric center of each shape. In a real atom, all orbitals share the same center, which means any atom having more than two electrons (that's all elements except for hydrogen and helium!) will have multiple orbitals around one nucleus. This four-set of orbital visualizations shows what some orbitals would look like if viewed in isolation.

[6] Please understand that like all analogies, this one merely illustrates a complex concept in terms that are easier to recognize. Analogies do not *explain* why things work, but merely liken an abstract phenomenon to something more accessible to common experience.

different positions around the stage. As a result, the inner seats fill first, with most empty seats being furthest away from the stage. The concept of energy fits neatly into this analogy as well: just as electrons give up *energy* to "fall into" lower-energy regions around the nucleus, people must give up *money* to purchase seats closest to the action on stage.

The energy levels available for orbiting electrons are divided into categories of *shells* and *subshells*. A "shell" (or, *principal quantum number*, n) describes the main energy level of an electron. In our amphitheater analogy, this is equivalent to a *tier* or *seating level*. A "subshell" (or, *subsidiary quantum number*, l) further divides the energy levels within each electron shell, and assigns different shapes to the electrons' probability "clouds." In the amphitheater analogy, a subshell would be a row of seats within a particular tier. To make the analogy accurate, we would have to imagine each row of seats in a tier having a different shape (not all arcs or straight lines), with varying degrees of viewing comfort afforded by each shape. The first row in each tier faces uniformly toward the stage, allowing easy viewing. Successive rows (subshells) in each tier (shell) contain more seats, but are bent in such a way that the stage is not as easy to view, making these rows less desirable to occupy. Electron subshells always have an even-numbered electron capacity, analogous to theater rows containing even-numbered quantities of seats, because atomic electrons tend to gather in pairs called *orbitals*.

Chemists identify electron shells both by number (the value of the quantum number n) and/or by capital letters: the first shell by the letter K, the second by L, the third by M, and the fourth by N. Higher-order shells exist for atoms requiring[7] many electrons (high atomic number), and the lettering pattern is alphabetic (fifth shell is O, sixth is P, etc.). Each successive shell has a greater number of subshells available, like successive amphitheater tiers having more rows: the low-level tiers closest to the stage having the fewest rows, and the high-level tiers furthest from the stage having the most rows.

A numbering and lettering system is also used by chemists to identify subshells within each shell (the l quantum number value starting with zero, and lower-case letters beginning with "s"): the first subshell ($l = 0$) in any shell represented by the letter s, the second ($l = 1$) by p, the third ($l = 2$) by d, the fourth ($l = 3$) by f, and all others by successive lower-case letters of the alphabet[8]. Each subshell of each shell has an even-numbered capacity for electrons, since the electrons in each subshell are organized in "orbital" regions, each orbital handling a maximum of two[9] electrons. The number of orbitals per shell is equal to twice the l value plus one. An "s" subshell has one orbital holding up to two electrons. A "p" subshell has three orbitals holding up to six electrons total. A "d" subshell has five orbitals holding up to ten electrons total. An "f" subshell has seven orbitals holding up to 14 electrons total. A "g" subshell has nine orbitals holding up to 18 electrons total.

The number of subshells in any shell is the same as that shell's n value. Thus, the first (K) shell has only one subshell, "s". The second (L) shell has two subshells, an "s" and a "p". The third (M) shell has three subshells available, an "s", a "p", and a "d"; and so on.

[7] Truth be told, higher-order shells exist even in simple atoms like hydrogen, but are simply not occupied by that atom's electron(s) unless they are "excited" into a higher energy state by an external input of energy.

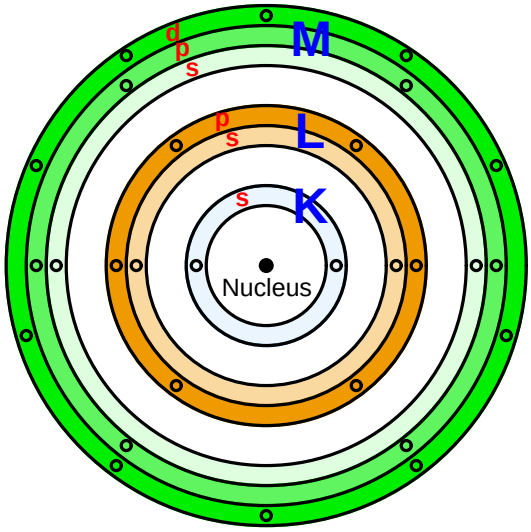
[8] The letters *s*, *p*, *d*, and *f* refer to the words *sharp*, *principal*, *diffuse*, and *fundamental*, used to describe the appearance of spectral lines in the early days of atomic spectroscopy research. Higher-order subshells are labeled alphabetically after *f*: *g*, *h*, and *i*.

[9] The two electrons of any orbital have opposite *spin* values.

This table shows the first few shells, their subshells, and electron capacities of each:

Shell (n value)	**Subshell** (l value)	**Subshell electron capacity** $= 2(2l + 1)$
$n = 1$; K	$l = 0$; s	$2 \times (2 \times 0 + 1) = 2$
$n = 2$; L	$l = 0$; s	$2 \times (2 \times 0 + 1) = 2$
	$l = 1$; p	$2 \times (2 \times 1 + 1) = 6$
$n = 3$; M	$l = 0$; s	$2 \times (2 \times 0 + 1) = 2$
	$l = 1$; p	$2 \times (2 \times 1 + 1) = 6$
	$l = 2$; d	$2 \times (2 \times 2 + 1) = 10$
$n = 4$; N	$l = 0$; s	$2 \times (2 \times 0 + 1) = 2$
	$l = 1$; p	$2 \times (2 \times 1 + 1) = 6$
	$l = 2$; d	$2 \times (2 \times 2 + 1) = 10$
	$l = 3$; f	$2 \times (2 \times 3 + 1) = 14$

Reviewing our amphitheater analogy, atomic shells are like seating tiers (levels), subshells are like rows of seats within each tier, and subshell electron capacity is like the number of seats in each row. This simple illustration shows an atom with three shells (K, L, and M) with the respective subshells (s, p, and d) represented by differently-shaded rings within each shell, having different numbers of places for electrons within each one:



This illustration is vastly over-simplified, failing to show the diverse shapes of each subshell, serving only to show how each successive shell grows in subshells and electron capacities.

The complete electron configuration for an atom may be expressed using *spectroscopic notation*, showing the shell numbers, subshell letters, and number of electrons residing within each subshell as a superscript. For example, the element helium (with an atomic number of 2) would be expressed as $1s^2$, with just two electrons in the "s" subshell of the first shell. The following table shows the electron structures of the first nineteen elements in the periodic table, from the element hydrogen (atomic number[10] = 1) to potassium (atomic number = 19):

Element	**Atomic number**	**Electron configuration**
Hydrogen	1	$1s^1$
Helium	2	$1s^2$
Lithium	3	$1s^2 2s^1$
Beryllium	4	$1s^2 2s^2$
Boron	5	$1s^2 2s^2 2p^1$
Carbon	6	$1s^2 2s^2 2p^2$
Nitrogen	7	$1s^2 2s^2 2p^3$
Oxygen	8	$1s^2 2s^2 2p^4$
Fluorine	9	$1s^2 2s^2 2p^5$
Neon	10	$1s^2 2s^2 2p^6$
Sodium	11	$1s^2 2s^2 2p^6 3s^1$
Magnesium	12	$1s^2 2s^2 2p^6 3s^2$
Aluminum	13	$1s^2 2s^2 2p^6 3s^2 3p^1$
Silicon	14	$1s^2 2s^2 2p^6 3s^2 3p^2$
Phosphorus	15	$1s^2 2s^2 2p^6 3s^2 3p^3$
Sulfur	16	$1s^2 2s^2 2p^6 3s^2 3p^4$
Chlorine	17	$1s^2 2s^2 2p^6 3s^2 3p^5$
Argon	18	$1s^2 2s^2 2p^6 3s^2 3p^6$
Potassium	19	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

In order to avoid having to write unwieldy spectroscopic descriptions of each element's electron structure, it is customary to write the notation only for subshells that are unfilled. For example, instead of writing the electron structure of the element aluminum[11] as $1s^2 2s^2 2p^6 3s^2 3p^1$, we might just as well write a condensed version showing only the last subshell ($3p^1$), since all the previous subshells are completely full.

Another way to abbreviate the spectroscopic notation for elements is to condense all the shells below the newest (unfilled) shell as the corresponding noble element, in brackets. To use the example of aluminum again, we could write its spectroscopic notation as $[Ne]3s^2 3p^1$ since its shell 1 and shell 2 configurations are completely described by the electron configuration of neon.

[10] The atomic number is the quantity of protons found in an atom's nucleus, and may only be a whole number. Since any electrically balanced atom will have the same number of electrons as protons, we may look at the atomic number of an element as being the number of electrons in each atom of that element.

[11] Building on the amphitheater analogy for one atom of the element aluminum, we could say that there are two electrons occupying the "s" seating row on the first level, plus two electrons occupying the "s" seating row on the second level, plus six electrons occupying the "p" seating row on the second level, plus two electrons occupying the "s" seating row on the third level, plus one electron occupying the "p" seating row on the third level.

Re-writing our electron shell table for the first nineteen elements using this condensed notation:

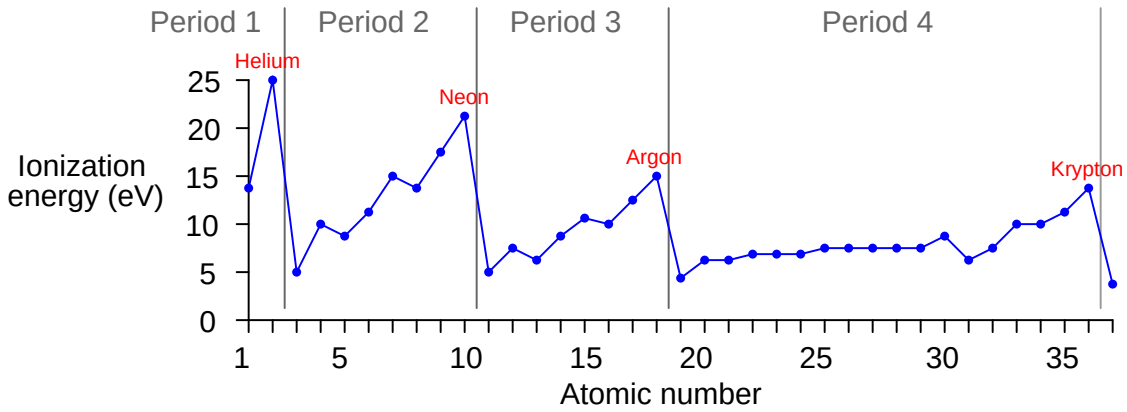
Element	Atomic number	Electron configuration
Hydrogen	1	$1s^1$
Helium	2	$1s^2$
Lithium	3	[He]$2s^1$
Beryllium	4	[He]$2s^2$
Boron	5	[He]$2s^2 2p^1$
Carbon	6	[He]$2s^2 2p^2$
Nitrogen	7	[He]$2s^2 2p^3$
Oxygen	8	[He]$2s^2 2p^4$
Fluorine	9	[He]$2s^2 2p^5$
Neon	10	[He]$2s^2 2p^6$
Sodium	11	[Ne]$3s^1$
Magnesium	12	[Ne]$3s^2$
Aluminum	13	[Ne]$3s^2 3p^1$
Silicon	14	[Ne]$3s^2 3p^2$
Phosphorus	15	[Ne]$3s^2 3p^3$
Sulfur	16	[Ne]$3s^2 3p^4$
Chlorine	17	[Ne]$3s^2 3p^5$
Argon	18	[Ne]$3s^2 3p^6$
Potassium	19	[Ar]$4s^1$

If we progress from element to element in increasing atomic number, we see that no new shell begins to form until after we reach the noble element for that period[12] at the far right-hand column. With the beginning of each new period at the far-left end of the Table, we see the beginning of the next higher-order electron shell. The shell(s) below are represented by whichever noble element shares that same configuration[13], indicating a "noble core" of electrons residing in extremely stable (low-energy) regions around the nucleus.

[12] Recall the definition of a "period" in the Periodic Table being a horizontal row, with each vertical column being called a "group".

[13] Building on the amphitheater analogy once again for one atom of the element aluminum, we could say that all seats within levels 1 and 2 are occupied (just like an atom of neon), plus two electrons occupying the "s" seating row on the third level, plus one electron occupying the "p" seating row on the third level.

The beginning of the next higher-order shell is what accounts for the periodic cycle of ionization energies we see in elements of progressing atomic number. The first electron to take residence in a new shell is very easy to remove, unlike the electrons residing in the "noble" configuration shell(s) below:



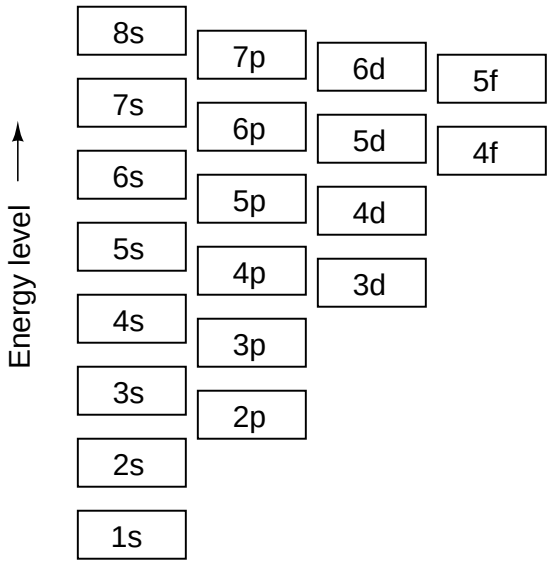
Noble elements found at the end of each period have the most stable (i.e. difficult-to-ionize) electron configurations.

Not only is the "noble core" notation convenient for tersely describing the electron structure of an element, but it also reveals an important concept in chemistry: the idea of *valence*. Electrons residing in lower-order shells are, by definition, at lower energy states than electrons residing in higher-order shells and are therefore much more difficult to dislodge. Therefore, the electrons in unfilled shells, being easier to dislodge, play a far greater role in chemical bonds than electrons residing in filled shells below. These "outer" electrons are called *valence electrons*, and their number determines how readily an atom will chemically interact with another atom. This is why elements found in the same group (having similar outer-electron configurations) bear similar chemical characteristics: the electrons lying below in the "noble core" configurations have little effect on how the atom will bond with other atoms. A lithium atom, with its outer-most electron configuration being $2s^1$, reacts in much the same way as an atom of sodium having an outer-most configuration of $3s^1$, and much the same as a potassium atom having an outer-most configuration of $4s^1$, all because those outer-shell electrons are the most available for interaction with electrons of other atoms.

If we examine the electron structures of atoms with successively greater atomic numbers (more protons in the nucleus, therefore more electrons in orbit to balance the electrical charge), we notice that the shells and subshells fill up in an interesting pattern. One might think that all the lower-order shells get completely filled before any electrons go into a higher-order shell – just as we might expect people to fill every seat in all the lower tiers of an amphitheater before filling seats in any of the higher tiers – but this is not always the case. Instead, the energy levels of subshells within shells is split, such that certain subshells within a higher shell will have a lower energy value than certain subshells within a lower shell. Referring back to our amphitheater analogy, where seating tiers represented shells and seat rows of various shape represented subshells, it is as though people choose to fill the more comfortable rows in higher-level tiers before sitting in the less-comfortable rows in lowest available tiers, the desire for comfort trumping the desire for proximity to the stage.

A rule commonly taught in introductory chemistry courses called the *Madelung rule* (also referred to as *Aufbau order*, after the German verb *aufbauen* meaning "to build up") is that subshells fill with increasing atomic number in such an order that the subshell with the lowest $n + l$ value, in the lowest shell, gets filled before any others.

The following graphic illustrates this ordering:



Madelung filling order: 1s → 2s → 2p → 3s → 3p → 4s → 3d → 4p → 5s → 4d → 5p → 6s → 4f → 5d → 6p → 7s → 5f → 6d → 7p → 8s → *(etc.)*

It should be noted that exceptions exist for this rule. We see one of those exceptions with the element chromium ($_{24}$Cr). Strictly following the Madelung rule in progressing from vanadium (atomic number = 23, valence electron structure $3d^3 4s^2$) to chromium (atomic number = 24), we would expect the next electron to take residence in the "3d" subshell making chromium's valence structure be $3d^4 4s^2$, but instead we find *two more* electrons residing in chromium's 3d subshell with one less in the 4s subshell ($3d^5 4s^1$). The sequence resumes its expected progression with the next element, manganese (atomic number = 25, valence electron structure $3d^5 4s^2$). The general principle

of energy minimization still holds true . . . it's just that the relative energies of succeeding subshells do not follow a simple rule structure. In other words, the Aufbau order is an over-simplified view of reality. To use the amphitheater analogy again, it's as if someone gave up one of the nice chairs in tier 4 to sit next to a friend who just occupied one of the less comfortable chairs in tier 3.

The practical importance of electron configurations in chemistry is the potential energy possessed by electrons as they reside in different shells and subshells. This is extremely important in the formation and breaking of chemical bonds, which occur due to the interaction of electrons between two or more atoms. A chemical bond occurs between atoms when the outer-most (valence) electrons of those atoms mutually arrange themselves in energy states that are collectively lower than they would be individually. The ability for different atoms to join in chemical bonds completely depends upon the default energy states of electrons in each atom, as well as the next available energy states in the other atoms. Atoms will form stable bonds only if the union allows electrons to assume stable (low-energy) levels. This is why different elements are very selective regarding which elements they will chemically bond with to form compounds: not all combinations of atoms result in favorable potential energy levels.

The amount of energy required to break a chemical bond (i.e. separate the atoms from each other) is the same amount of energy required to restore the atoms' electrons to their previous (default) states before they joined. This is the same amount of energy released by the atoms as they come together to form the bond. Thus, we see the foundation of the general principle in chemistry that forming chemical bonds releases energy, while breaking chemical bonds requires an input of energy from an external source. We also see in this fact an expression of the Conservation of Energy: the amount of energy "invested" in breaking bonds is precisely the same as the amount of energy "returned" when those same bonds re-form.

In summary, the whole of chemistry is a consequence of electrons not being able to assume arbitrary positions around the nucleus of an atom. Instead, they seek the lowest possible energy levels within a framework allowing them to retain unique quantum states. Atoms with mutually agreeable electron structures readily bond together to form molecules, and they release energy in the process of joining. Molecules may be broken up into their constituent atoms, if sufficient energy is applied to overcome the bond. Atoms with incompatible electron structures do not form stable bonds with each other.

3.3 Work, energy, and power

Two very fundamental and closely-related concepts in physics are *work* and *energy*. *Work* is simply what happens when any force acts through a parallel motion, such as when a weight is lifted against gravity or when a spring is compressed. *Energy* is a more abstract concept and therefore more difficult to define. One definition[14] of energy is "that which permits or results in motion," where the word "motion" is used in a very broad sense including even the motion of individual atoms within a substance. Energy exists in many different forms, and may be transferred between objects and/or converted from one form to another, but cannot be created from nothing or be destroyed and turned into nothing (this is the *Law of Energy Conservation*). *Power* is the rate at which work is done, or alternatively the rate at which energy transfer occurs.

First, just a little bit of math. Work (W) is mathematically defined as the dot-product of force (F) and displacement (x) vectors[15], written as follows:

$$W = \vec{F} \cdot \vec{x}$$

Where,

W = Work, in newton-meters (metric) or foot-pounds (British)

$\vec{F}$ = Force vector (force and direction exerted) doing the work, in Newtons (metric) or pounds (British)

$\vec{x}$ = Displacement vector (distance and direction traveled) over which the work was done, in meters (metric) or feet (British)

[14] A common definition of energy is the "ability to do work" which is not always true. There are some forms of energy which may *not* be harnessed to do work, such as the thermal motion of molecules in an environment where all objects are at the same temperature. Energy that has the ability to do work is more specifically referred to as *exergy*. While energy is always conserved (i.e. never lost, never gained), exergy is a quantity that can never be gained but can be lost. The inevitable loss of exergy is closely related to the concept of *entropy*, where energy naturally diffuses into less useful (more homogeneous) forms over time. This important concept explains why no machine can never be perfectly ($100.\overline{0}\%$) efficient, among other things.

[15] A *vector* is a mathematical quantity possessing both a magnitude and a direction. Force (F), displacement (x), and velocity (v) are all vector quantities. Some physical quantities such as temperature (T), work (W), and energy (E) possess magnitude but no direction. We call these directionless quantities "scalar." It would make no sense at all to speak of a temperature being "79 degrees Celsius due North" whereas it *would* make sense to speak of a force being "79 Newtons due North". Physicists commonly use a little arrow symbol over the variable letter to represent that variable as a vector, when both magnitude and direction matter. Thus $\vec{F}$ represents a force vector with both magnitude and direction specified, while plain F merely represents the magnitude of that force without a specified direction. A "dot-product" is one way in which vectors may be multiplied, and the result of a dot-product is always a scalar quantity.

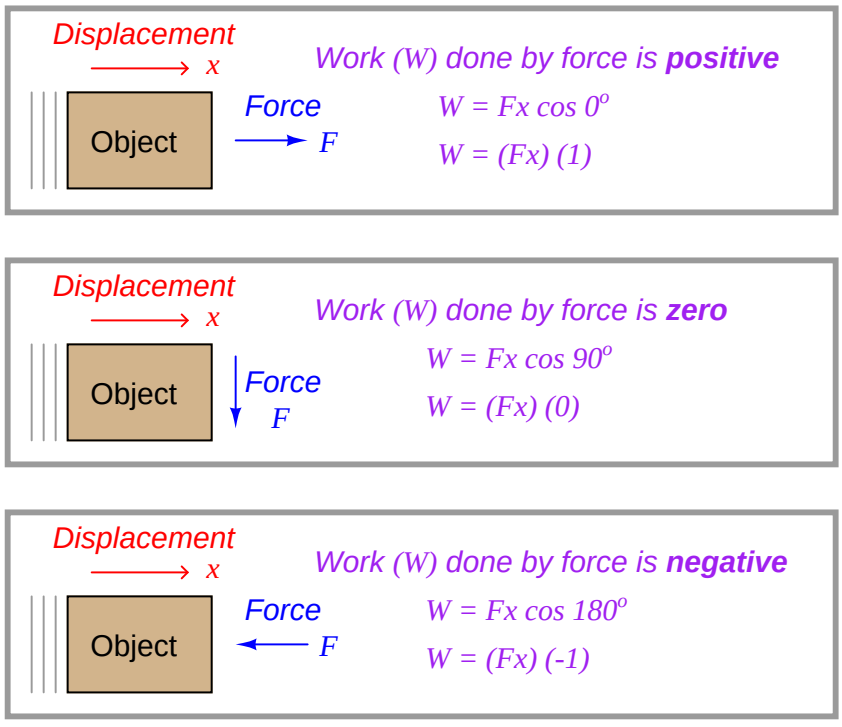
3.3.1 Force and displacement vectors

The fact that both force and displacement appear as *vectors* tells us their relative directions are significant to the calculation of work. If the force and displacement vectors point in exactly the same direction, work is simply the product of F and x magnitudes ($W = Fx$). If the force and displacement vectors point in opposite directions, work is the *negative* product ($W = -Fx$). Another way to express the calculation of work is as the product of the force and displacement magnitudes (F and x) and the cosine of the angle separating the two force vectors ($\cos\theta$):

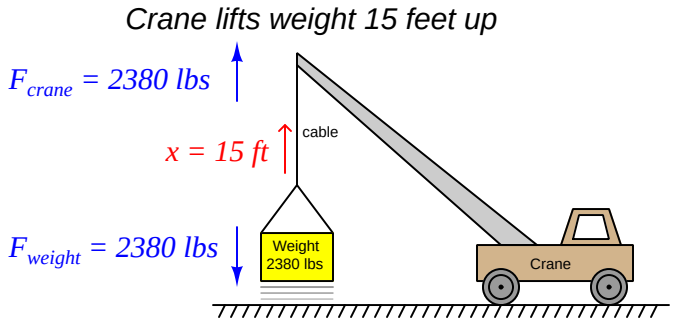
$$W = Fx\cos\theta$$

When the two vectors $\vec{F}$ and $\vec{x}$ point the same direction, the angle θ between them is zero and therefore $W = Fx$ because $\cos 0^o = 1$. When the two vectors point in opposite directions, the angle between them is 180^o and therefore $W = -Fx$ because $\cos 180^o = -1$.

This means if a force acts in the same direction as a motion (i.e. force and displacement vectors pointing in the same direction), the work being done by that force will be a *positive* quantity. If a force acts in the opposite direction of a motion (i.e. force and displacement vectors pointing in opposite directions), the work done by that force will be a *negative* quantity. If a force acts perpendicularly to the direction of a motion (i.e. force and displacement vectors at right angles to each other), that force will do *zero* work.



Illustrations are helpful in explaining these concepts. Consider a crane hoisting a 2380 pound weight 15 feet up into the air by means of an attached cable:



Crane does work on weight W_{crane} *= (2380 lb)(15 ft)(cos* 0^o*) = +35700 ft-lbs*
Work is done on the weight W_{weight} *= (2380 lb)(15 ft)(cos* 180^o*) = -35700 ft-lbs*

The amount of work done from the crane's perspective is $W_{crane} = +35700$ ft-lbs, since the crane's force ($F_{crane} = 2380$ lbs, up) points in the same upward direction as the cable's motion ($x =$ 15 ft, up) and therefore there is no angular difference between the crane force and motion vectors (i.e. $\theta = 0$). The amount of work done from weight's perspective, however, is $W_{weight} = -35700$ ft-lbs because the weight's force vector ($F_{weight} = 2380$ lbs, down) points in the *opposite* direction as the cable's motion vector ($x = 15$ feet, up), yielding an angular difference of $\theta = 180^o$. Another way of expressing these two work values is to state the crane's work in the active voice and the weight's work in the passive voice: *the crane did 35700 ft-lbs of work, while 35700 ft-lbs of work was done on the weight.* This language is truly appropriate, as the crane is indeed the *active* agent in this scenario, while the weight *passively opposes* the crane's efforts. In other words, the crane is the *motive source* of the work, while the weight is a *load*.

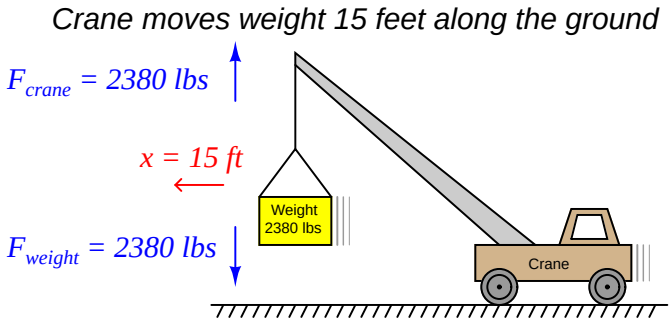
3.3.2 Energy and power

Now, how does *energy* fit into this illustration? Certainly we see that motion occurred, and therefore energy must have been involved. The energy lifting the 2380 pound weight 15 feet upward didn't come from nowhere – it must have been present somewhere in the universe prior to the weight's ascension if the Law of Energy Conservation is indeed true. If the crane is powered by an internal combustion engine, the energy came from the fuel stored in the crane's fuel tank. If the crane is electric, the energy to hoist the weight came from a battery or from an electrical generator somewhere sending power to the crane through an electric cable. The act of lifting the weight was actually an act of *energy transfer* from the crane's energy source, through the crane motor and cable, and finally to the weight. Along the way, some of the initial energy was also converted into heat through inefficiencies in the crane's motor and mechanism. However, if we were to calculate the sum of all the energy transferred to the lifted weight plus all energy "dissipated" in the form of heat, that total must be precisely equal to the initial energy of the fuel (or electricity) used by the crane in doing this lift. In other words, the sum total of all energy in the universe is the same after the lift as it was before the lift.

Power applies in this scenario to how *quickly* the weight rises. So far, all we know about the weight's lifting is that it took 35700 ft-lbs of energy to do that work. If we knew how long it took the crane to do that work, we could calculate the crane's power output. For example, a crane with a power rating of 35700 ft-lbs per second could complete this 15-foot lift in only one second of time. Likewise, a crane with a power rating of only 3570 ft-lbs per second would require 10 seconds of time to execute the same lift.

3.3.3 Perpendicular force and displacement

An interesting thing happens if the crane moves sideways along the ground after lifting the weight 15 feet up into the air. No work is being done by the crane on the weight, or by the weight on the crane, because the displacement vector is now perpendicular ($\theta = 90^o$) to both force vectors:



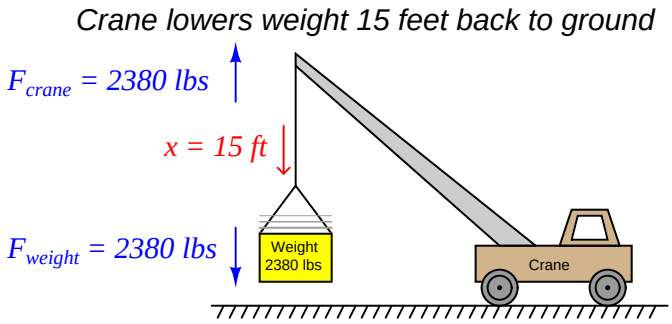
No work done on or by the crane $W_{crane} = (2380\ lb)(15\ ft)(\cos 90^o) = 0\ ft\text{-}lbs$
No work done on or by the weight $W_{weight} = (2380\ lb)(15\ ft)(\cos -90^o) = 0\ ft\text{-}lbs$

It should be noted that the crane's engine *will* do work as it overcomes rolling friction in the wheels to move the crane along, *but this is not work done on or by the hoisted weight.* When we calculate work – as with all other calculations in physics – we must be very careful to keep in mind where the calculation(s) apply in the scenario. Here, the forces and displacement with regard to the hoisted weight are perpendicular to each other, and therefore no work is being done there. The only work done anywhere in this system as the crane rolls 15 feet horizontally involves the horizontal force required to roll the crane, which is unspecified in this illustration.

Similarly, there is no transfer of energy to or from the hoisted weight while the crane rolls along. Whatever energy comes through the crane's engine only goes into overcoming rolling friction at the wheels and ground, not to do any work with the weight itself. Generally this will be a very small amount of energy compared to the energy required to hoist a heavy load.

3.3.4 More energy transfer

A good question to ask after hoisting the weight is "Where did that 35700 ft-lbs of energy go after the lift was complete?" The Law of Energy Conservation tells us that energy cannot be destroyed, and so the 35700 ft-lbs of work must be accounted for somehow. In this case, the energy is now *stored* in the weight where it may be released at some later time. We may demonstrate this fact by slowly lowering the weight back down to the ground and watching the energy transfer:



Work is done on the crane W_{crane} = (2380 lb)(15 ft)(cos 180^o) = -35700 ft-lbs
Weight does work on crane W_{weight} = (2380 lb)(15 ft)(cos 0^o) = +35700 ft-lbs

Notice how the weight is now the actively-working object in the system, doing work on the crane. The crane is now the passive element, opposing the work being done by the weight. Both the crane and the weight are still pulling the same directions as before (crane pulling up, weight pulling down), but now the direction of displacement is going down which means the weight is "winning" and therefore doing the work, while the crane is "losing" and opposing the work.

If we examine what is happening inside the crane as the weight descends, we see that energy is being transferred from the descending weight to the crane. In most cranes, the descent of a load is controlled by a *brake* mechanism, regulating the speed of descent by applying friction to the cable's motion. This brake friction generates a great deal of heat, which is a form of energy transfer: energy stored in the elevated weight is now being converted into heat which exits the crane in the form of hot air (air whose molecules are now vibrating at a faster speed than they were at their previous temperature). If the crane is electric, we have the option of *regenerative braking* where we recapture this energy instead of dissipating it in the form of heat. This works by switching the crane's electric motor into an electric generator on demand, so the weight's descent turns the motor/generator shaft to generate electricity to re-charge the crane's battery or be injected back into the electric power grid to do useful work elsewhere.

Referring back to the illustration of the crane hoisting the weight, it is clear that the weight stored energy while it was being lifted up by the crane, and released this energy back to the crane while it was being lowered down to the ground. The energy held by the elevated weight may therefore be characterized as *potential energy*, since it had the *potential* to do work even if no work was being done by that energy at that moment.

A special version of the general work formula $W = \vec{F} \cdot \vec{x}$ exists for calculating this *gravitational potential energy*. Rather than express force and displacement as vectors with arbitrary directions, we express the weight of the object as the product of its mass and the acceleration of gravity ($F = mg$) and the vertical displacement of the object simply as its height above the ground ($x = h$). The amount of potential energy stored in the lift is simply equal to the work done during the lift ($W = E_P$):

$$E_p = mgh$$

Where,

E_p = Gravitational potential energy in newton-meters (metric) or foot-pounds (British)

m = Mass of object in kilograms (metric) or slugs (British)

g = Acceleration of gravity in meters per second squared (metric) or feet per second squared (British)

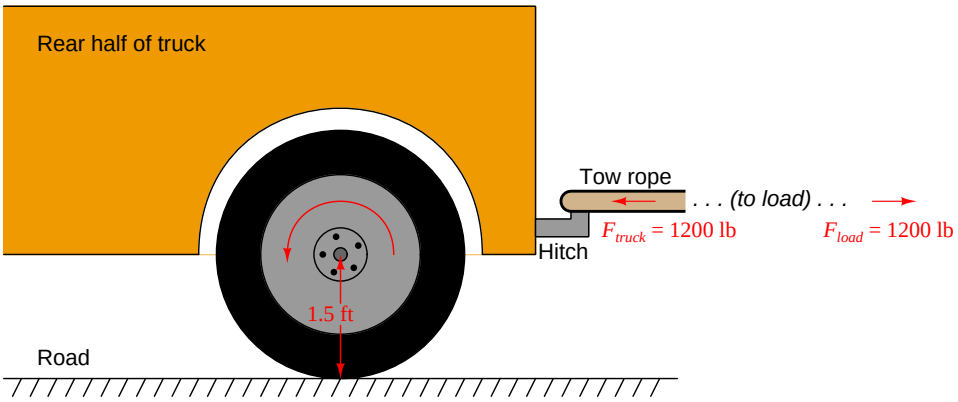
h = Height of lift in meters (metric) or feet (British)

There is no need for vectors or cosine functions in the $E_p = mgh$ formula, because gravity and height are always guaranteed to act along the same axis. A positive height (i.e. *above* ground level) is assumed to yield a positive potential energy value for the elevated mass.

Many different forms of potential energy exist, although the standard "textbook" example is of a mass lifted to some height against the force of gravity. Compressed (or stretched) springs have potential energy, as do pressurized fluids, chemical bonds (e.g. fuel molecules prior to combustion), hot masses, electrically-charged capacitors, and magnetized inductors. Any form of energy with the potential to be released into a different form at some later time is, by definition, *potential* energy.

3.3.5 Rotational work

An important application of work, energy, and power is found in *rotational* motion. Consider the application of a truck towing some load by a rope. The truck exerts a linear (i.e. straight-line) pulling force on the load being towed, but it must do so by applying a *rotary* (i.e. turning) force through the axle shaft powering the drive wheels:



Clearly, the truck *does work* on the load by exerting a pulling force in the direction of motion. If we wish to quantify this work, we may consider the work done by the truck as it tows the load a distance of 40 feet:

$$W_{truck} = Fx \cos \theta$$

$$W_{truck} = (1200 \text{ lb})(40 \text{ ft}) \cos 0^o$$

$$W_{truck} = 48000 \text{ lb-ft}$$

It should also be clear that *work is being done on* the load, since the load's force on the tow rope points in the opposite direction of its motion:

$$W_{load} = Fx \cos \theta$$

$$W_{load} = (1200 \text{ lb})(40 \text{ ft}) \cos 180^o$$

$$W_{load} = -48000 \text{ lb-ft}$$

For the sake of this example, it matters not what happens to the energy delivered to the load. Perhaps it gets converted to heat through the mechanism of friction at the load (e.g. friction from road contact, friction from wind resistance), perhaps it gets converted into potential energy in the case of the road inclining to a greater altitude, or perhaps (most likely) it is some combination of all these.

In order to quantify this amount of work from the perspective of the truck's rotating wheel, we must cast the variables of pulling force and pulling distance into rotary terms. We will begin by first examining the distance traveled by the wheel. Any circular wheel has a radius, and the wheel must turn a certain number of revolutions in order to pull the load any distance. The obvious function of a wheel is to convert between linear and rotary motion, and the common measure between these two motions is the *circumference* of the wheel: each revolution of the wheel equates directly to one circumference's worth of linear travel. In our truck example, the wheel has a radius of 1.5 feet, which means it must have a circumference of 9.425 feet (i.e. $2 \times \pi \times 1.5$ feet, since $C = \pi D = 2\pi r$). Therefore, in order to travel 40 linear feet this wheel must rotate 4.244 times (i.e. 40 feet $\div$ 9.425 feet/revolution = 4.244 revolutions).

Next, we must relate pulling force to rotational force. The 1200 pounds of pulling force exerted by the wheel[16] originates from the twisting force exerted by the axle at the wheel's center. The technical term for this twisting force is *torque* (symbolized by the Greek letter "tau", τ), and it is a function of both the linear pulling force and the wheel's radius:

$$\tau = rF$$

Solving for torque (τ) in this application, we calculate 1800 pound-feet given the wheel's linear pulling force of 1200 pounds and a radius of 1.5 feet. Please note that the unit of "pound-feet" for torque is *not* the same as the unit of "foot-pounds" for work. Work is the product of force and displacement (distance of motion), while torque is the product of force and radius length. Work requires motion, while torque does not[17].

The torque value of 1800 lb-ft and the turning of 4.244 revolutions should be sufficient to calculate work done by the wheel, since torque equates directly to pulling force and the number of revolutions equates directly to pulling distance. It should be obvious that the product of 1800 lb-ft and 4.244 revolutions does not, however, equal 48000 ft-lbs of work, and so our torque-revolutions-work formula must contain an additional multiplication factor k:

$$W = k\tau x$$

Substituting 48000 ft-lbs for work (W), 1800 lb-ft for torque (τ), and 4.244 for the number of revolutions (x), we find that k must be equal to 6.283, or 2π. Knowing the value for k, we may re-write our rotational work formula more precisely:

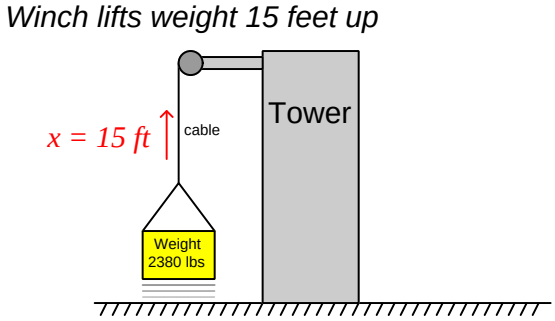
$$W = 2\pi\tau x$$

This formula is correct no matter the wheel's size. A larger-radius wheel will certainly travel farther for each revolution, but that larger radius will proportionately reduce the pulling force for any given amount of torque, resulting in an unchanged work value.

[16] Note that this calculation will assume all the work of towing this load is being performed by a *single* wheel on the truck. Most likely this will not be the case, as most towing vehicles have multiple driving wheels (at least two). However, we will perform calculations for a single wheel in order to simplify the problem.

[17] Consider the example of applying torque to a stubbornly seized bolt using a wrench: the force applied to the wrench multiplied by the radius length from the bolt's center to the perpendicular line of force yields torque, but absolutely no work is done on the bolt until the bolt begins to move (turn).

Applying this work formula to another application, let us consider an electric *winch* where a load is lifted against the force of gravity. A "winch" is a mechanism comprised of a tubular drum and cable, the cable wrapping or unwrapping around the drum as the drum is turned by an electric motor. For comparison we will use the same weight and lifting distance of the crane example, merely replacing the crane with a winch located atop a tower. In this case we will make the winch drum 3 inches in diameter, and calculate both the torque required to rotate the drum as well as the number of rotations necessary to lift the weight 15 feet:



From the crane example we already know the amount of work which must be done on the weight to hoist it 15 feet vertically: *35700 ft-lbs*. Our rotational work formula ($W = 2\pi\tau x$) contains *two* unknowns at this point in time, since we know the value of W but not τ or x. This means we cannot yet solve for either τ or x using this formula. If we also knew the value of τ we could solve for x and vice-versa, which means we must find some other way to solve for one of those unknowns.

A drum radius of 3 inches is equivalent to 0.25 feet, and since we know the relationship between radius, force, and torque we may solve for torque in that manner:

$$\tau = rF$$

$$\tau = (0.25 \text{ ft})(2380 \text{ lb})$$

$$\tau = 595 \text{ lb-ft}$$

Now that we have a value for τ we may substitute it into the rotational work formula to solve for the number of necessary drum rotations:

$$W = 2\pi\tau x$$

$$x = \frac{W}{2\pi\tau}$$

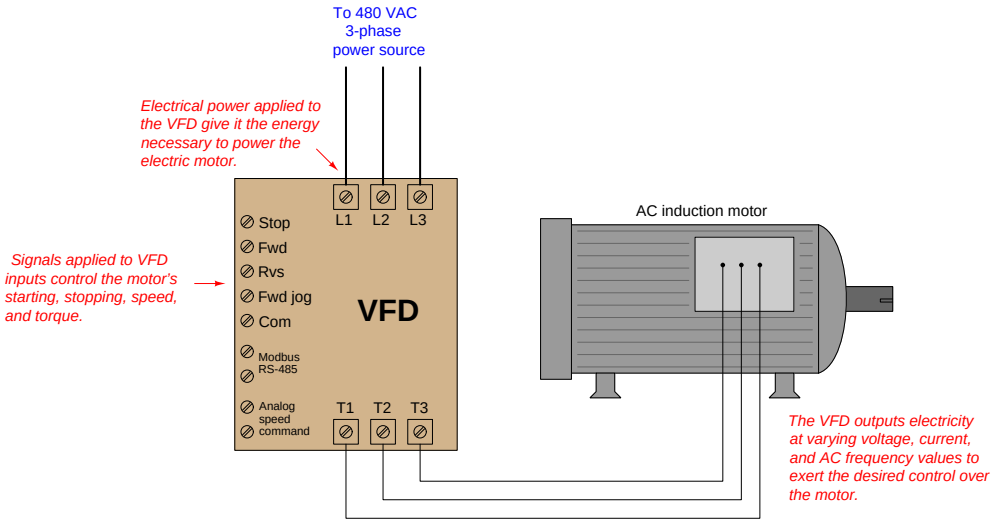
$$x = \frac{35700 \text{ ft-lb}}{(2\pi)(595 \text{ lb-ft})}$$

$$x = 9.549 \text{ revolutions}$$

As always, energy is conserved in this electric winch system just as it is in any other system. The 35700 foot-pounds of energy invested in the gravitational potential energy of the weight had to originate from somewhere, which in the case of an electric winch is the electrical power source feeding the winch motor. If that winch motor were allowed to turn in reverse and act as a generator, it would convert the descending weight's loss in potential energy into electrical power to be returned to the source, whether that source by a rechargeable battery or an electrical power "grid" with other electrical loads that may use that energy.

The same, of course, is true for the tow truck example. The energy expended in towing the load must come from somewhere, and in the case of a combustion-type truck engine that source is the fuel powering the engine. For electric vehicles, the energy source is a rechargeable battery, and the reversible nature of electric motors means the vehicle may use its drive motor as a generator to "brake" (slow down), returning the vehicle's kinetic energy into electrical energy to recharge the battery for later use. This is why all-electric and hybrid-electric vehicles are remarkably efficient in stop-and-go traffic conditions: they utilize their drive motors as *regenerative brakes* to recover the vehicle's kinetic energy rather than dissipating that same energy in the form of heat using friction-based brake mechanisms.

Many applications exist within the industrial world for rotational work and electric motors. Conveyor belts, pumps, fans, compressors, and a host of other mechanisms may be powered via electric motors, the speed and torque of those electric motors controlled using electronic circuits called *motor drives*. Alternating-current (AC) electric motors are controlled by electronic devices called *Variable Frequency Drives*, or *VFDs*, which achieve precise speed control by varying the frequency of the AC power feeding the motor, and achieve precise torque control by varying the voltage and current feeding the motor. VFDs are very important "final control" devices used in a wide variety of industrial control systems.

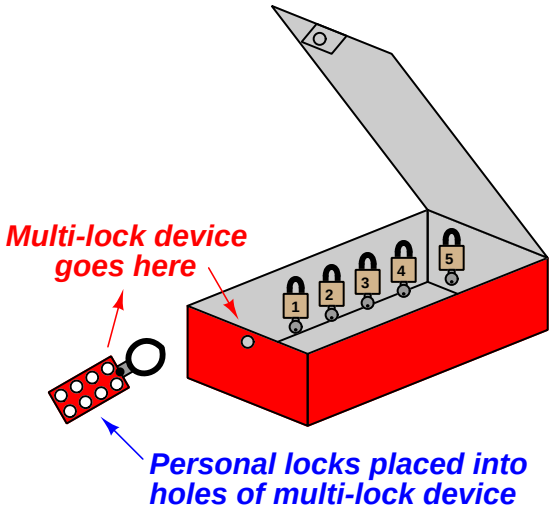


Sophisticated VFDs control regeneration as well (i.e. using the motor as a generator to "brake" a rotating machine), allowing energy to be extracted from the mechanism.

3.3.6 Energy and safety

Potential energy is an important principle not just in the study of physics, but also for workplace safety. When large amounts of potential energy are released, the effects may be hazardous to personnel and/or destructive to equipment. An industrial maintenance procedure known as *lock-out/tag-out* (LOTO) requires that all potential energy sources on a system must either be exhausted or otherwise secured to that there will be negligible risk to maintenance personnel as they perform work on a system. The most common way to ensure this is to place a padlock on each energy-disconnect device (e.g. switch, valve, etc.) to secure its position so that potential energy cannot be released in a hazardous or destructive way. Each maintenance worker places a padlock on that disconnect device to prevent its actuation, and also places a tag on the device explaining when and why it was secured. Each and every person performing work on the system must use their own personal padlock, so that the system cannot be re-activated without the active consent of all persons working on that system.

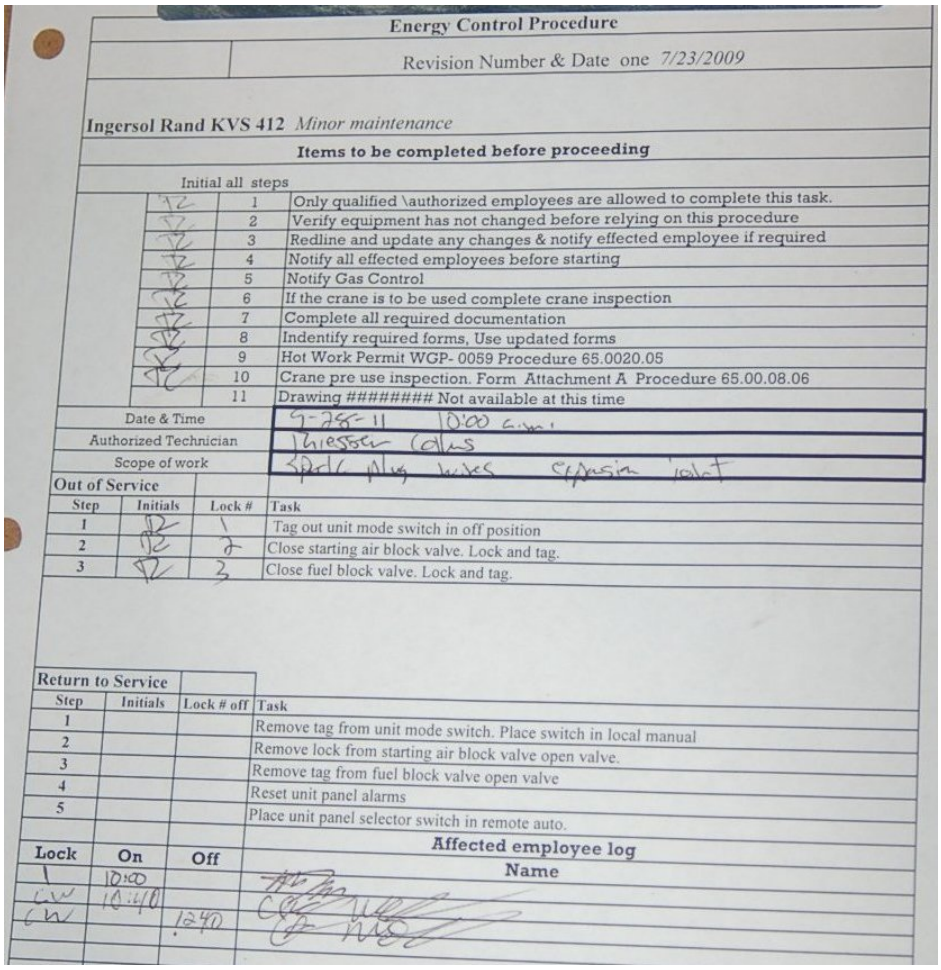
An efficient strategy for safely locking out a large number of safety-disconnect devices on a system with multiple personal locks is to use a sheet-metal box containing a numbered padlock (with matching key) for each energy-flow device to be secured on the equipment, as well as a list identifying which lock goes on which energy-flow device. Each energy-disconnect device is placed in the safe position and then locked in that position with a dedicated padlock. After that, all the padlock keys are placed back inside the sheet-metal box. The lid of this box is then shut and secured with a multi-lock device, permitting multiple peoples' personal locks to be applied so the lid cannot be opened unless *all* personal locks are removed from it:



None of the energy-securing devices may be altered unless *all* personal locks have been removed from the lock box, thereby ensuring the safety of all persons working on the system.

Procedures created and maintained at the worksite will identify the energy-flow devices in need of securing prior to commencement of work on a piece of equipment. These procedures are literally life-saving documents, as they ensure no energy-securing device is overlooked by personnel doing work on the equipment or system.

A photograph of such a document – appropriately titled an "Energy Control Procedure" – shows the steps mandated to secure all potential energy sources prior to commencing work on a large industrial engine. This procedure also serves to document which locks were used to secure which energy flow devices during the procedure, as well as who performed the procedure:



Energy Control Procedure

Revision Number & Date one *7/23/2009*

Ingersol Rand KVS 412 *Minor maintenance*

Items to be completed before proceeding

Initial all steps		
	1	Only qualified \authorized employees are allowed to complete this task.
	2	Verify equipment has not changed before relying on this procedure
	3	Redline and update any changes & notify effected employee if required
	4	Notify all effected employees before starting
	5	Notify Gas Control
	6	If the crane is to be used complete crane inspection
	7	Complete all required documentation
	8	Indentify required forms, Use updated forms
	9	Hot Work Permit WGP- 0059 Procedure 65.0020.05
	10	Crane pre use inspection. Form Attachment A Procedure 65.00.08.06
	11	Drawing ######## Not available at this time
Date & Time		9-25-11 10:00 a.m.
Authorized Technician		Thiessen Collins
Scope of work		Spark plug wires cylinder valve

Out of Service

Step	Initials	Lock #	Task
1		1	Tag out unit mode switch in off position
2		2	Close starting air block valve. Lock and tag.
3		3	Close fuel block valve. Lock and tag.

Return to Service

Step	Initials	Lock # off	Task
1			Remove tag from unit mode switch. Place switch in local manual
2			Remove lock from starting air block valve open valve.
3			Remove tag from fuel block valve open valve
4			Reset unit panel alarms
5			Place unit panel selector switch in remote auto.

Affected employee log

Lock	On	Off	Name
1	10:00		
LW	10:40		
LW		12:40	

Note the particular lock-out steps required in this procedure: switching the control mode to the "off" position and tagging it, closing the fuel gas valve supplying fuel to the engine and locking/tagging it, and finally closing the valve supplying high-pressure air for engine starting and locking/tagging it. The closure of the starting air valve prevents the engine from being pneumatically turned while personnel are performing work on it. The closure of the fuel gas valve eliminates hazards resulting from the pressure of the fuel gas as well as its chemical energy (i.e. fire hazard) and/or biological threats (poisoning or asphyxiation). Note also how this procedure lists steps of notification to be taken prior to locking or tagging anything on the engine, as well as any other procedures possibly necessary (e.g. inspecting the maintenance crane if that will be needed for work on the engine).

The following is a set of incomplete lists of various energy-securing devices and energy sources which may be "locked out" for maintenance on a larger system:

Electrical energy

- Circuit breaker (locked in "off" position, also "racked out" if possible)
- Grounding switch (locked in "on" position to positively ground power conductors)
- Power cord (plastic cap locked onto plug, covering prongs)

Mechanical energy

- Block valve (locked in "shut" position) to prevent pressurized fluid motion
- Flange blind (installed in line) to prevent pressurized fluid motion
- Vent valve (locked in "open" position) to prevent fluid pressure buildup
- Mechanical clutch (disengaged, and locked in that position) to prevent prime mover from moving something
- Mechanical coupling (disassembled, and locked in that state) to prevent prime mover from moving something
- Mechanical brake (engaged, and locked in that position) to prevent motion
- Mechanical locking pin (inserted, and held in position by a padlock) to prevent motion
- Raised masses lowered to ground level, with lifting machines locked out

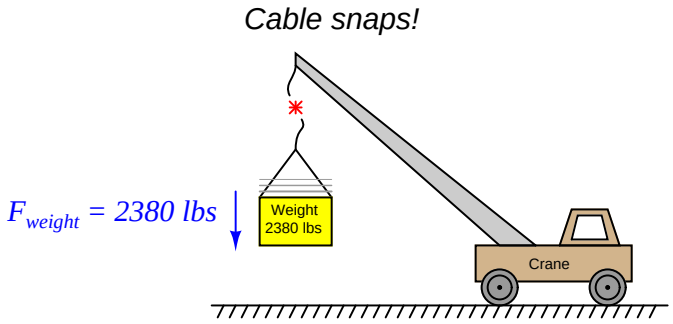
Chemical energy

- Block valve (locked in "shut" position) to prevent chemical fluid entry
- Vent valve (locked in "open" position) to prevent chemical fluid pressure buildup
- Ventilation fan (locked in "run" state) to prevent chemical vapor buildup

With all these preventive measures, the hope is that no form of potential energy great enough to pose danger may be accidently released.

3.3.7 Kinetic energy

Let us return to the crane illustration to explore another concept called *kinetic energy*. As you might guess by the word "kinetic," this form of energy exists when an object is in the process of *moving*. Let's imagine the crane lifting the 2380 pound weight 15 feet up into the air, and then the cable snapping apart so that the weight free-falls back to the ground:



The Conservation of Energy still (and always!) holds true: all the potential energy stored in the elevated weight must be accounted for, even when it free-falls. What happens, of course, is that the weight accelerates toward the ground at the rate determined by Earth's gravity: 32.2 feet per second per second (32.2 ft/s^2). As the weight loses height, its potential energy decreases according to the gravitational potential energy formula ($E_p = mgh$), but since we know energy cannot simply disappear we must conclude it is taking some other form. This "other form" is based on the weight's *velocity* (v), and is calculated by the kinetic energy formula:

$$E_k = \frac{1}{2}mv^2$$

Where,

E_k = Kinetic energy in Joules or newton-meters (metric), or foot-pounds (British)
m = Mass of object in kilograms (metric) or slugs (British)
v = Velocity of mass in meters per second (metric) or feet per second (British)

Thus, the Conservation of Energy explains why a falling object must fall faster as it loses height: kinetic energy must increase by the same amount that potential energy decreases, if energy is to be conserved. A very small amount of this falling weight's potential energy will be dissipated in the form of heat (as air molecules are disturbed) rather than get converted into kinetic energy. However, the vast majority of the initial 35700 ft-lbs of potential energy gets converted into kinetic energy, until the weight's energy is all kinetic and no potential the moment it first touches the ground.

When the weight finally slams into the ground, all that (nearly 35700 ft-lbs) of kinetic energy once again gets converted into other forms. Compression of the soil upon impact converts much of the energy into heat (molecular vibrations). Chunks of soil ejected from the impact zone possess their own kinetic energy, carrying that energy to other locations where they slam into other stationary objects. Sound waves rippling through the air also convey energy away from the point of impact. All in all, the 35700 ft-lbs of potential energy which turned into (nearly) 35700 ft-lbs of kinetic energy at ground level becomes dispersed.

The Law of Energy Conservation is extremely useful in projectile mechanics problems, where we typically assume a projectile loses no energy and gains no energy in its flight. The velocity of a projectile, therefore, depends on its height above the ground, because the sum of potential and kinetic energies must remain constant:

$$E_p + E_k = \text{constant}$$

As a projectile rises in altitude (i.e. its gravitational potential energy increases), its velocity must slow down (i.e. its kinetic energy decreases) in order that its total energy remain the same in accordance with the Law of Energy Conservation.

In free-fall problems, where the only source of energy for a projectile is its initial height, the initial potential energy must be equal to the final kinetic energy:

$$E_p \text{ (initial)} = E_k \text{ (final)}$$

$$mgh_i = \frac{1}{2}mv_f^2$$

We can see from this equation that mass cancels out of both sides, leaving us with this simpler form:

$$gh_i = \frac{1}{2}v_f^2$$

It also leads to the paradoxical conclusion that the mass of a free-falling object is irrelevant to its velocity. That is, both a heavy object and a light object in free fall hit the ground with the same velocity, and fall for the same amount of time, if released from the same height under the influence of the same gravity[18].

[18] In practice, we usually see heavy objects fall faster than light objects due to the resistance of air. Energy losses due to air friction nullify our assumption of constant total energy during free-fall. Energy lost due to air friction never translates to velocity, and so the heavier object ends up hitting the ground faster (and sooner) because it had much more energy than the light object did to start.

3.3.8 Dimensional analysis of energy

Dimensional analysis confirms the common nature of energy whether in the form of potential, kinetic, or even mass (as described by Einstein's equation). First, we will set these three energy equations next to each other for comparison of their variables:

$$E_p = mgh \qquad \text{Potential energy due to elevation}$$

$$E_k = \frac{1}{2}mv^2 \qquad \text{Kinetic energy due to velocity}$$

$$E = mc^2 \qquad \text{Mass-to-energy conversion}$$

Next, we will dimensionally analyze them using standard SI metric units (kilogram, meter, second). Following the SI convention, mass (m) is always expressed in kilograms [kg], distance (h) in meters [m], and time (t) in seconds [s]. This means velocity (v, or c for the speed of light) in the SI system will be expressed in meters per second [m/s] and acceleration (a, or g for gravitational acceleration) in meters per second squared [m/s^2]:

$$\frac{[\text{kg}][\text{m}^2]}{[\text{s}^2]} = [\text{kg}]\left[\frac{\text{m}}{\text{s}^2}\right][\text{m}] \qquad \text{Potential energy due to elevation}$$

$$\frac{[\text{kg}][\text{m}^2]}{[\text{s}^2]} = [\text{kg}]\left[\frac{\text{m}}{\text{s}}\right]^2 \qquad \text{Kinetic energy due to velocity}$$

$$\frac{[\text{kg}][\text{m}^2]}{[\text{s}^2]} = [\text{kg}]\left[\frac{\text{m}}{\text{s}}\right]^2 \qquad \text{Mass-to-energy conversion}$$

In all three cases, the unit for energy is the same: kilogram-meter squared per second squared. This is the fundamental definition of a "joule" of energy (also equal to a "newton-meter" of energy), and it is the same result given by all three formulae.

Power is defined as the rate at which work is being done, or the rate at which energy is transferred. Mathematically expressed, power is the first time-derivative of work (W):

$$P = \frac{dW}{dt}$$

The metric unit of measurement for power is the *Watt*, defined as one joule of work performed per second of time. The British unit of measurement for power is the *Horsepower*, defined as 550 foot-pounds of work performed per second of time.

Although the term "power" is often colloquially used as a synonym for force or strength, it is in fact a very different concept. A "powerful" machine is not necessarily a machine capable of doing a great amount of work, but rather (more precisely) a great amount of work *in a short amount of time*. Even a "weak" machine is capable of doing a great amount of work given sufficient time to complete the task. The "power" of any machine is the measure of *how rapidly* it may perform work.

3.4 Elementary thermodynamics

Thermodynamics is the study of heat, temperature, and their related effects in physical systems. As a subject, thermodynamics is quite complex and expansive, usually taught as a course in itself at universities. The coverage in this Tutorial is limited to some of the more elementary and immediately practical facets of thermodynamics rather than a comprehensive overview.

3.4.1 Heat versus Temperature

Most people use the words *heat* and *temperature* interchangeably. This is unfortunate for every student of thermodynamics, because it means they must first deconstruct this false conception and replace it with one more scientifically accurate before any progress may be made. While "heat" and "temperature" are related concepts, they are not identical.

When people say something is "hot," what they really mean is that the object has a high temperature. Temperature is a direct function of random molecular motion within an object or a fluid sample. This is usually easiest to visualize for a gas, where unattached molecules have great freedom to vibrate, collide, and otherwise move about. The molecules of a substance at high temperature are moving more vigorously (higher velocity) than the molecules of the same substance at low temperature.

Heat, by contrast, is an expression of thermal energy transfer. By placing a pot of water over a fire, we are *adding heat* to that pot (transferring thermal energy to the water), the effect of which is to *raise its temperature* (making the water molecules' motions more vigorous). If that same pot is taken away from the fire and allowed to cool, its *loss of heat* (transferring energy out of the water to the surrounding air) will result in its *temperature lowering* (the individual water molecules slowing down).

Heat gain or loss often results in temperature change, but not always. In some cases heat may be gained or lost with negligible temperature change – here, the gain or loss of heat manifests as physical changes to the substance other than temperature. One example of this is the boiling of water at constant pressure: no matter how much heat is transferred to the water, its temperature will remain constant at the boiling point (100 degrees Celsius at sea level) until all the water has boiled to vapor. The addition of thermal energy to the boiling water does not raise its temperature (i.e. make the molecules move faster), but rather goes into the work of disrupting inter-molecular bonds so that the liquid turns into vapor. Another example is the heating of chemical reactants in an endothermic (heat-absorbing) reaction: much of the thermal energy added to the chemical solution goes into the work of separating chemical bonds, resulting in molecular changes but not (necessarily) increased temperature.

Heat transfer can *only* happen, though, where there is a difference of temperature between two objects. Thermal energy (heat) naturally flows from the "hotter" (higher-temperature) substance to the "colder" (lower-temperature) substance. To use the boiling water example, the only way to get heat transfer into the water is to subject the water to a hotter substance (e.g., a flame, or a hot electric heating element). If you understand temperature as being molecular motion within a substance, with a hotter object's molecules vibrating more vigorously than a colder object's molecules, this natural transfer of heat from hot to cold makes perfect sense: the molecular vibrations of the higher-temperature object literally transfer to the molecules of the lower-temperature object. As those respective molecules touch each other, with fast-vibrating molecules colliding against slow-vibrating molecules, the inter-molecular collisions transfer energy away from the fast-vibrating molecules (so they aren't vibrating as fast anymore) and toward the slow-moving molecules (so they begin to vibrate faster than before). It's like a vibrating tuning fork touched to a non-vibrating tuning fork: the vibrating fork loses some of its vibration by transferring energy to the (formerly) quiet tuning fork.

Much more attention will be directed to the concepts of heat and temperature in subsequent subsections.

3.4.2 Temperature

In an ideal, monatomic[19] gas (one atom per molecule), the mathematical relationship between average molecular velocity and temperature is as follows:

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Where,
m = Mass of each molecule
v = Velocity of a molecule in the sample
$\overline{v}$ = Average ("mean") velocity of all molecules in the sample
$\overline{v^2}$ = Mean-squared molecular velocities in the sample
k = Boltzmann's constant ($1.3806504 \times 10^{-23}$ J / K)
T = Absolute temperature (Kelvin), 273.15 more than degrees Celsius

Non-ideal gases, liquids, and solids are more complex than this. Not only can the atoms of complex molecules move to and fro, but they may also twist and oscillate with respect to each other. No matter how complex the particular substance may be, however, the basic principle remains unchanged: temperature is an expression of how rapidly molecules move within a substance.

There is a temperature at which all molecular motion ceases. At that temperature, the substance cannot possibly become "colder," because there is no more motion to halt. This temperature is called *absolute zero*, equal to −273.15 degrees Celsius, or −459.67 degrees Fahrenheit. Two temperature scales based on this absolute zero point, *Kelvin* and *Rankine*, express temperature relative to absolute zero. That is, zero Kelvin and zero degrees Rankine is equal to absolute zero temperature. Any temperature greater than absolute zero will be a positive value in either the Kelvin or the Rankine scales. A sample of freezing water at sea level, for example, is 0 degrees Celsius (32 degrees Fahrenheit) but could also be expressed as 273.15 Kelvin[20] (0 plus 273.15) or 491.67 degrees Rankine (32 plus 459.67).

[19] Helium at room temperature is a close approximation of an ideal, monatomic gas, and is often used as an example for illustrating the relationship between temperature and molecular velocity.

[20] Kelvin is typically expressed without the customary "degree" label, unlike the three other temperature units: (degrees) Celsius, (degrees) Fahrenheit, and (degrees) Rankine.

A table of melting and boiling points (at sea-level atmospheric pressure) for various substances appears in this table, labeled in these four different units of temperature measurement:

Melting or boiling substance	^{o}C	^{o}F	K	^{o}R
Melting point of water (H_2O)	0	32	273.15	491.67
Boiling point of water (H_2O)	100	212	373.15	671.67
Melting point of ammonia (NH_3)	−77.7	−107.9	195.45	351.77
Boiling point of ammonia (NH_3)	−33.6	−28.5	239.55	431.17
Melting point of gold (Au)	1063	1945	1336	2405
Melting point of magnesium (Mg)	651	1203.8	924.2	1663.5
Boiling point of acetone (C_3H_6O)	56.5	133.7	329.65	593.37
Boiling point of propane (C_3H_8)	−42.1	−43.8	231.05	415.87
Boiling point of ethanol (C_2H_6O)	78.4	173.1	351.55	632.77

Note how degrees Celsius and Kelvin for each point on the table differ by a constant (offset) of 273.15, while each corresponding degree Fahrenheit and degree Rankine value differs by 459.67 (note that many of the figures in this table are slightly rounded, so the offset might not be *exactly* that much). You might think of Kelvin as nothing more than the Celsius scale zero-shifted by 273.15 degrees, and likewise degrees Rankine as nothing more than the Fahrenheit scale zero-shifted by 459.67 degrees.

Note also how *increments* in temperature measured in degrees Fahrenheit are the same as *increments* of temperature measured in degrees Rankine. The same is true for degrees Celsius and Kelvin. The difference between the melting point of ammonia (−77.7 degrees C) and the melting point of water (0 degrees C) is the same difference as that between the melting points of ammonia and water expressed in Kelvin: 195.45 and 273.15, respectively. Either way, the difference in temperature between these two substances' melting points is 77.7 degrees (C or K). This is useful to know when dealing with temperature changes over time, or temperature differences between points in a system – if an equation asks for a temperature difference (ΔT) in Kelvin, it is the same value as the temperature difference expressed in Celsius. Likewise, a ΔT expressed in degrees Rankine is identical to a ΔT expressed in degrees Fahrenheit. This is analogous to differences between two fluid pressures expressed in PSIG versus PSIA: the differential pressure value (PSID) will be the same.

Most people are familiar with the Fahrenheit and Celsius temperature scales used to express temperature in common applications, but the absolute scales of Rankine and Kelvin have special significance and purpose in scientific endeavors. The fact that Rankine and Kelvin are *absolute* scales in the same manner that *atmospheres* and *torr* are units of absolute pressure measurement makes them uniquely suited for expressing temperature (molecular motion) in relation to the absence of thermal energy. Certain scientific laws such as the *Ideal Gas Law* and the *Stefan-Boltzmann Law* relate other physical quantities to absolute temperature, and so require the use of these absolute units of measurement.

3.4.3 Heat

Heat, being the transfer of energy in thermal (molecular motion) form, may be measured in the same units as any other form of energy is measured: *Joules* (metric) and *foot-pounds* (British). However, special units of measurement are often used for heat instead:

- calorie
- kilocalorie (or "dietary Calorie")
- British Thermal Unit (BTU)

A *calorie* of heat is defined as the amount of thermal energy transfer required to change the temperature of one gram of water by one degree Celsius ($\Delta T = 1\ ^{o}\text{C} = 1$ K). One calorie is equivalent to 4.186 Joules.

A *British Thermal Unit*, or *BTU* is defined as the amount of thermal energy transfer required to change the temperature of one pound of water by one degree Fahrenheit ($\Delta T = 1\ ^{o}\text{F} = 1\ ^{o}\text{R}$). One BTU is equivalent to 778.2 foot-pounds.

The unit of "dietary" calories is used to express the amount of thermal energy available in a sample of food by combustion[21]. Since the official unit of the "calorie" is so small compared to the typical amounts of energy contained in a meal, nutritionists use the unit of the kilocalorie (1000 calories, or 4186 Joules) and call it "Calorie" (with a capital letter "C").

Just as "Calories" are used to rate the energy content of food, the heat units of "calories" and "BTU" are very useful in describing the potency of various industrial fuels. The following table shows the *heat of combustion* for a few common fuels, in units of kilocalories per gram and BTU per pound:

Fuel	**Combustion heat** (kcal/g)	**Combustion heat** (BTU/lb)
Methane (CH_4)	13.3	23940
Methanol (CH_4O)	5.43	9767
Ethanol (C_2H_6O)	7.10	12783
Propane (C_3H_8)	12.1	21700
Carbon monoxide (CO)	2.415	4347

For example, if exactly one gram of methane gas were completely burnt, the resulting heat liberated in the fire would be sufficient to warm 13.3 kilograms of water from 20 degrees Celsius to 21 degrees Celsius (a temperature rise, or ΔT, of one degree Celsius).

If a meal rated at 900 Calories (900 "dietary calories," or 900 kilocalories) of energy were completely metabolized, the resulting heat would be sufficient to warm a pool of water 900 kilograms in mass (900 liters, or about 237 gallons) by one degree Celsius. This same amount of heat could raise half the amount of water twice the temperature rise: 450 liters of water warmed two degrees Celsius.

[21] Animals process food by performing a very slow version of combustion, whereby the carbon and hydrogen atoms in the food join with oxygen atoms inhaled to produce water and carbon dioxide gas (plus energy). Although it may seem strange to rate the energy content of food by measuring how much heat it gives off when *burnt*, burning is just a faster method of energy extraction than the relatively slow processes of biological metabolism.

3.4.4 Heat transfer

Heat spontaneously[22] flows from higher-temperature substances to lower-temperature substances. This is the phenomenon you experience standing next to a fire on a cold day. Your body is cold (low temperature), but the fire is much hotter (high temperature), and your proximity to the fire aids in heat transfer from the fire to you.

Three principal methods exist for heat to transfer from one substance to another:

- Radiation[23] (by light waves)
- Conduction (by direct contact)
- Convection (by intermediate contact with a moving fluid)

Practical examples of heat transfer often involve multiple modes rather than just one. For example, the transfer of heat to a person's body by sunlight obviously involves *radiation* from the Sun, but it also involves *conduction* through layers of clothing and *convection* by air passing from sun-warmed objects to the person.

Temperature sensors likewise rely on multiple heat-transfer modes to sample thermal energy from matter. Infrared thermometers detect temperature by sensing the intensity of infrared light *radiated* by hot objects. A contact-style sensor directly touching a hot object relies on *conduction* to sense the temperature of that object. A similar sensor inserted into a pipe carrying a hot fluid relies on *convection* to measure the average temperature of that fluid.

[22] Heat may be forced to flow from cold to hot by the use of a machine called a *heat pump*, but this direction of heat flow does not happen naturally, which is what the word "spontaneous" implies. In truth, the rule of heat flowing from high-temperature to cold-temperature applies to heat pumps as well, just in a way that is not obvious from first inspection. Mechanical heat pumps cause heat to be drawn from a cool object by placing an even cooler object (the *evaporator*) in direct contact with it. That heat is then transferred to a hot object by placing an even hotter object (the *condenser*) in direct contact with it. Heat is moved against the natural (spontaneous) direction of flow from the evaporator to the condenser by means of mechanical compression and expansion of a refrigerant fluid.

[23] In this context, we are using the word "radiation" in a very general sense, to mean thermal energy radiated away from the hot source via photons. This is quite different from nuclear radiation, which is what some may assume this term means upon first glance.

Radiation

If you have ever experienced the immediate sensation of heat from a large fire or explosion some distance away, you know how *radiation* works to transfer thermal energy. Radiation is also the method of heat transfer experienced in the Earth's receiving of heat from the Sun (and also the mechanism of heat loss from Earth to outer space). Radiation is the least efficient of the three heat transfer mechanisms. It may be quantified by the Stefan-Boltzmann Law, which states the rate of heat lost by an object ($\frac{dQ}{dt}$) is proportional to the *fourth power* of its absolute temperature, and directly proportional to its radiating area:

$$\frac{dQ}{dt} = e\sigma AT^4$$

Where,
$\frac{dQ}{dt}$ = Radiant heat loss rate (watts)
e = Emissivity factor (unitless)
σ = Stefan-Boltzmann constant (5.670400×10^{-8} W / m^2 $\cdot$ K^4)
A = Surface area (square meters)
T = Absolute temperature (Kelvin)

Here is one of the scientific applications where temperature expressed in *absolute* units is truly necessary. Radiant energy is a direct function of molecular motion, and so we would logically expect objects to radiate energy at any temperature above absolute zero. The temperature value used in this formula *must* be in units of Kelvin[24] in order for the resulting $\frac{dQ}{dt}$ value to be correct. If degrees Celsius were used for T instead of Kelvin, the formula would predict zero thermal radiation at the freezing point of water (0 ^{o}C) and *negative* radiation at any temperature below freezing, which is not true. Remember that the "zero" points of the Celsius and Fahrenheit scales were arbitrarily set by the inventors of those scales, but that the "zero" points of the Kelvin and Rankine scales reflect a fundamental limit of nature.

The emissivity factor varies with surface finish and color, ranging from one (ideal) to zero (no radiation possible). Dark-colored, rough surfaces offer the greatest emissivity factors, which is why heater elements and radiators are usually painted black. Shiny (reflective), smooth surfaces offer the least emissivity, which is why thermally insulating surfaces are often painted white or silver.

Like all heat-transfer modes, radiation is two-way. Objects may emit energy in the form of radiation, and they may also receive energy in the form of radiation. Everyone knows white-colored shirts are cooler than black-colored shirts worn on a hot, sunny day – this is an example of how emissivity affects heat absorption by radiant transfer. A black-colored shirt (high emissivity value) enhances the receiving of radiant energy by your body from the sun. What is not as obvious, though, is that a white-colored shirt will keep you warmer than a black-colored shirt on a cold, dark day because that same decreased emissivity inhibits body heat *loss* by radiation. Thus, high-emissivity objects both heat *and* cool more readily by radiation than low-emissivity objects.

[24] Or in degrees Rankine, provided a suitably units-corrected value for the Stefan-Boltzmann constant were used.

Conduction

If you have ever accidently touched a hot iron or stove heating element, you possess a very vivid recollection of heat transfer through *conduction*. In conduction, fast-moving molecules in the hot substance transfer some of their kinetic energy to slower-moving molecules in the cold substance. Since this transfer of energy requires collisions between molecules, it only applies when the hot and cold substances directly contact each other.

Perhaps the most common application of heat conduction in industrial processes is through the walls of a furnace or some other enclosure containing an extreme temperature. In such applications, the desire is usually to *minimize* heat loss through the walls, so those walls will be "insulated" with a substance having poor thermal conductivity.

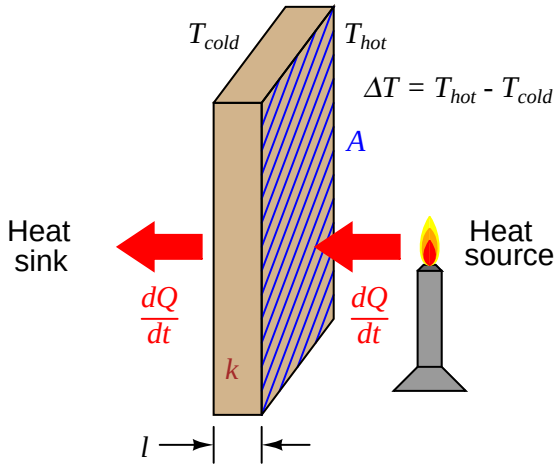
Conductive heat transfer rate is proportional to the difference in temperature between the hot and cold points, the area of contact, the distance of heat travel from hot to cold, and the thermal conductivity of the substance:

$$\frac{dQ}{dt} = \frac{kA\Delta T}{l}$$

Where,
$\frac{dQ}{dt}$ = Conductive heat transfer rate
k = Thermal conductivity
A = Surface area
ΔT = Difference of temperature between "hot" and "cold" sides
l = Length of heat flow path from "hot" to "cold" side

Note the meaning of "ΔT" in this context: it refers to the *difference* in temperature between two different locations in a system. Sometimes the exact same symbology ("ΔT") refers to a *change in temperature over time* in the study of thermodynamics. Unfortunately, the only way to distinguish one meaning of ΔT from the other is by context.

An illustration showing heat conduction through a wall gives context to the variables in the previous equation. As we see here, A refers to the surface area of the wall, ΔT refers to the difference of temperature between either surface of the wall, and l refers to the thickness of the wall:



In the United States, a common measure of insulating ability used for the calculation of conductive heat loss in shelters is the *R-value*. The greater the R-value of a thermally insulating material, the less conductive it is to heat (lower k value). "R-value" mathematically relates to k and l by the following equation:

$$R = \frac{l}{k}$$

Rearranging this equation, we see that $l = kR$, and this allows us to substitute kR for l in the conduction heat equation, then cancel the k terms:

$$\frac{dQ}{dt} = \frac{kA\Delta T}{kR}$$

$$\frac{dQ}{dt} = \frac{A\Delta T}{R}$$

R is always expressed in the compound unit of square feet · hours · degrees Fahrenheit per BTU. This way, with a value for area expressed in square feet and a temperature difference expressed in degrees Fahrenheit, the resulting heat transfer rate ($\frac{dQ}{dt}$) will naturally be in units of BTU per hour, which is the standard unit in the United States for expressing heat output for combustion-type heaters. Dimensional analysis shows how the units cancel to yield a heat transfer rate in BTUs per hour:

$$\frac{[\text{BTU}]}{[\text{h}]} = \frac{[\text{ft}^2][^\circ\text{F}]}{\frac{[\text{ft}^2][\text{h}][^\circ\text{F}]}{[\text{BTU}]}}$$

The utility of R-value ratings may be shown by a short example. Consider a contractor trailer, raised up off the ground on a mobile platform, with a total skin surface area of 2400 square feet (walls, floor, and roof) and a uniform R-value of 4 for all surfaces. If the trailer's internal temperature must be maintained at 70 degrees Fahrenheit while the outside temperature averages 40 degrees Fahrenheit, the required output of the trailer's heater will be:

$$\frac{dQ}{dt} = \frac{(2400 \text{ ft}^2)(30^o \text{ F})}{4 \text{ ft}^2 \cdot \text{h} \cdot^o \text{F/BTU}} = 18000 \text{ BTU per hour}$$

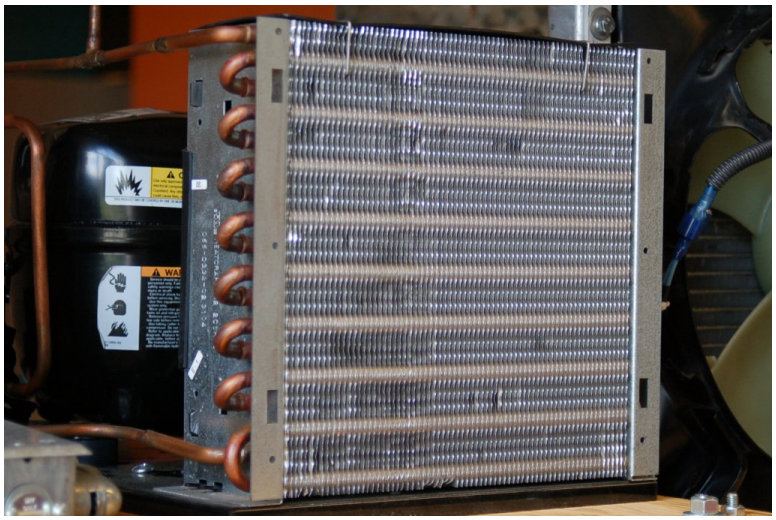
If the trailer's heater is powered by propane and rated at 80% efficiency (requiring 22500 BTU per hour of fuel heating value to produce 18000 BTU per hour of heat transfer into the trailer), the propane usage will be just over one pound per hour, since propane fuel has a heating value of 21700 BTU per pound.

Convection

Most industrial heat-transfer processes occur through *convection*, where a moving fluid acts as an intermediary substance to transfer heat from a hot substance (heat *source*) to a cold substance (heat *sink*). Convection may be thought of as two-stage heat conduction on a molecular scale: fluid molecules come into direct contact with a hot object and absorb heat from that object through conduction, then those molecules later release that heat energy through conduction by direct contact with a cooler object. If the fluid is recycled in a piping loop, the two-stage conduction process repeats indefinitely, individual molecules heating up as they absorb heat from the heat source and then cooling down as they release heat to the heat sink.

Special process devices called *heat exchangers* perform this heat transfer function between two different fluids, the two fluids circulating past each other on different sides of tube walls. A simple example of a heat exchanger is the radiator connected to the engine of an automobile, being a water-to-air exchanger, the engine's hot water transferring heat to cooling air entering the grille of the car as it moves.

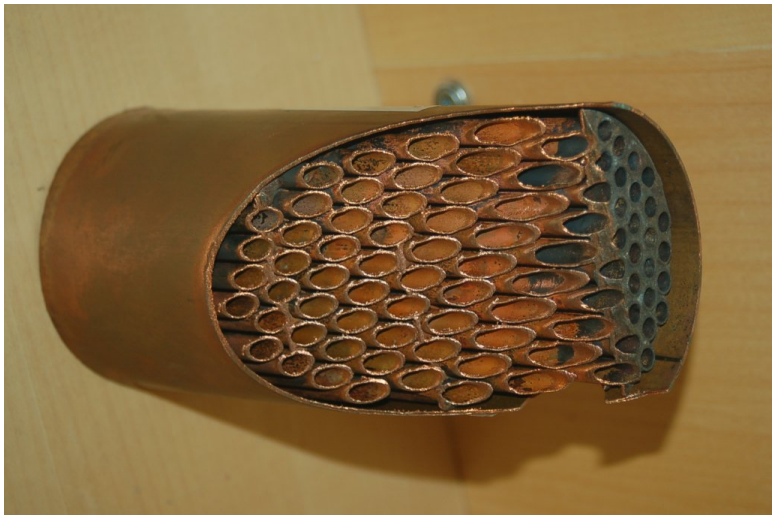
Another example of a liquid-to-air heat exchanger is the *condenser* on a heat pump, refrigerator, or air conditioner, a photograph appearing here:



Another common style of heat exchanger works to transfer heat between two liquids. A small example of this design used to transfer heat from a boat engine is shown here:

The purpose for this heat exchanger is to exchange heat between the liquid coolant of the boat engine and sea water, the latter being quite corrosive to most metals. An engine would soon be damaged if sea water were used directly as the coolant fluid, and so heat exchangers such as this provide a means to release excess heat to the sea without subjecting the engine block to undue corrosion. The heat exchanger, of course, *does* suffer from the corrosive effects of sea water, but at least it is less expensive and more convenient to replace than an entire engine when it reaches the end of its service life.

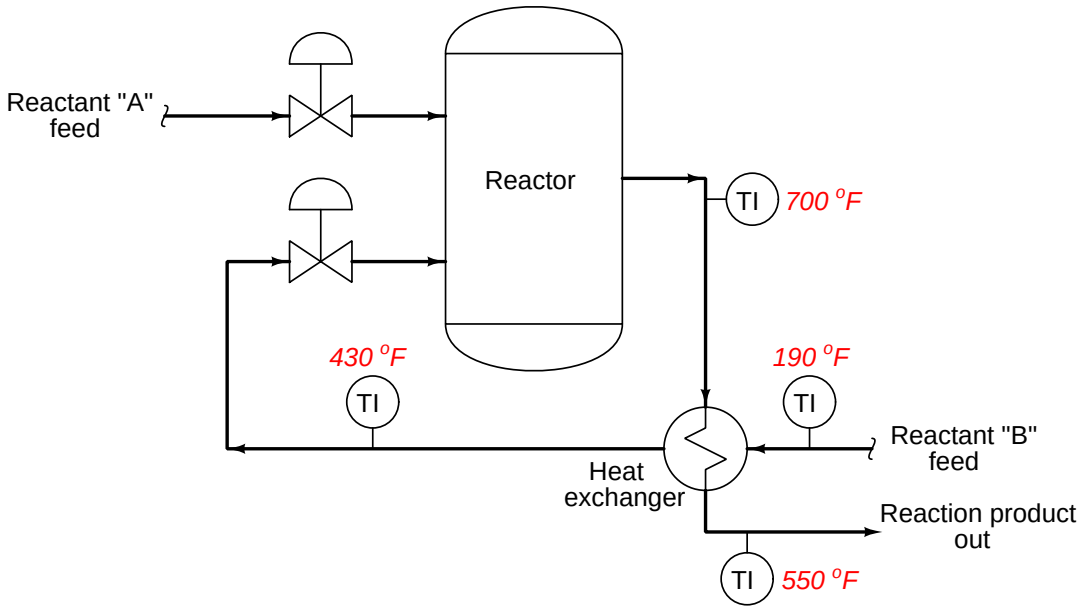
This marine engine heat exchanger is an example of a *shell-and-tube* design, where one fluid passes inside small tubes and a second fluid passes outside those same tubes, the tube bundle being contained in a shell. The interior of such an exchanger looks like this when cut away:



The tubes of this particular heat exchanger are made of copper, a metal with extremely high thermal conductivity (k), to facilitate conductive heat transfer.

Liquid-to-liquid heat exchangers are quite common in industry, where a set of tubes carry one process liquid while a second process liquid circulates on the outside of those same tubes. The metal walls of the tubes act as heat transfer areas for conduction to occur. Metals such as copper with very high k values (very low R values) encourage heat transfer, while long lengths of tube ensure ample surface area for heat exchange.

A common application of liquid-to-liquid heat exchangers is in exothermic (heat-releasing) chemical reaction processes where the reactants must be pre-heated before entering a reaction vessel ("reactor"). Since the chemical reaction is exothermic, the reaction itself may be used as the heat source for pre-heating the incoming feed. A simple process flow diagram[25] (PFD) shows how a heat exchanger accomplishes this transfer of heat:

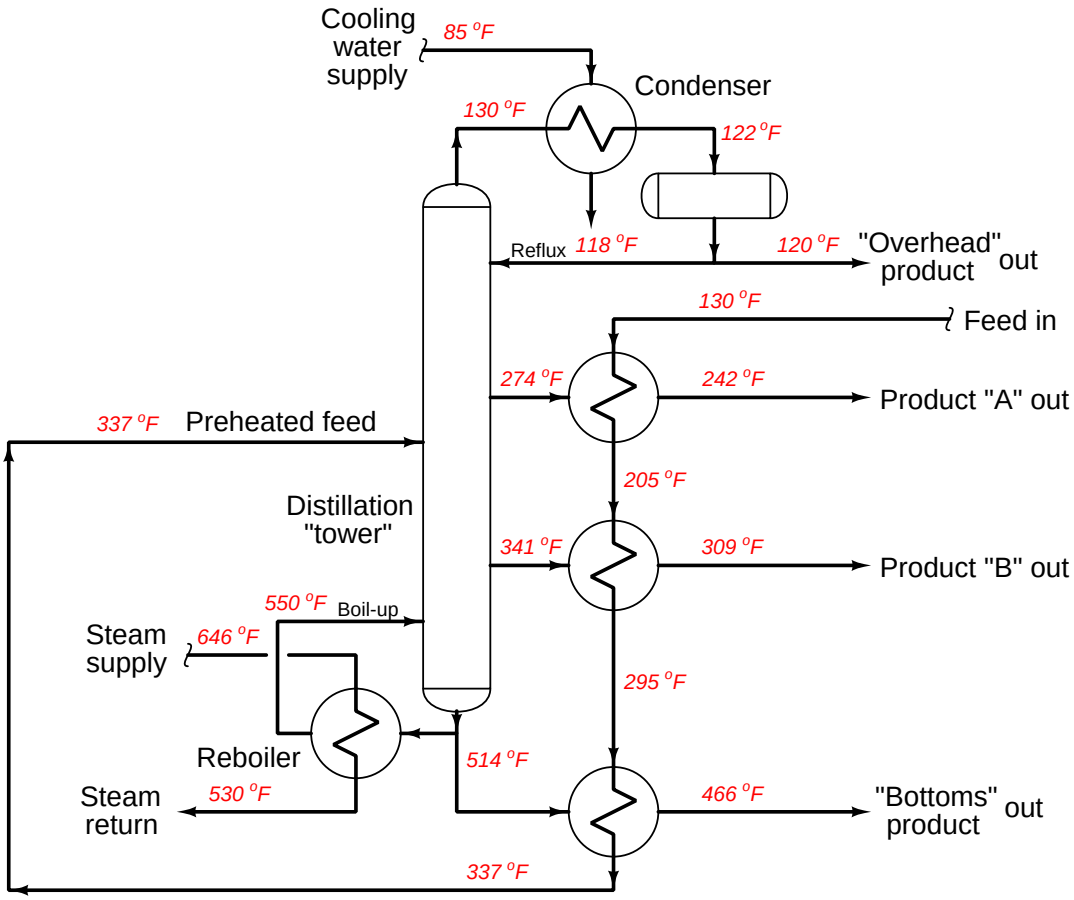


Another industrial application of heat exchangers is in *distillation* processes, where mixed components are separated from each other by a continuous process of boiling and condensation. Alcohol purification is one example of distillation, where a mixture of alcohol and water are separated to yield a purer (higher-percentage) concentration of alcohol. Distillation (also called *fractionation*) is a very energy-intensive[26] process, requiring great inputs of heat to perform the task of separation. Any method of energy conservation typically yields significant cost savings in a distillation process, and so we often find heat exchangers used to transfer heat from outgoing (distilled, or fractionated) products to the incoming feed mixture, pre-heating the feed so that less heat need be added to the distillation process from an external source.

[25] In this diagram, each line tipped by an arrow is a tube or pipe conveying some fluid. Circle-symbols with "TI" written inside represent temperature indicators (sensors). Each "bow-tie" symbol is a *valve* used to exert control over the flow rate of a fluid. The reactor itself is simply a hollow, sealed vessel inside of which the chemical reaction ensues.

[26] Jim Cahill of Emerson wrote in April 2010 ("Reducing Distillation Column Energy Usage" Emerson Process Expert weblog) about a report estimating distillation column energy usage to account for approximately 6% of the total energy used in the United States. This same report tallied the number of columns in US industry to be approximately 40000 total, accounting for about 19% of all energy used in manufacturing processes!

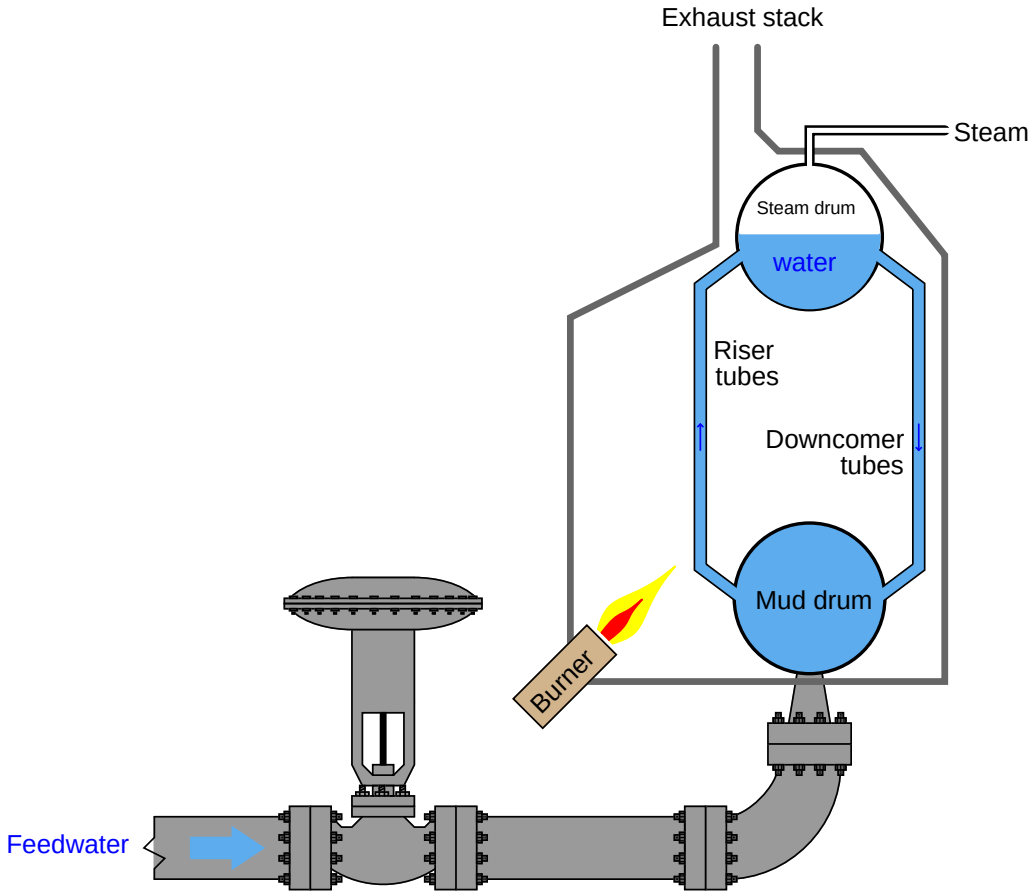
The following PFD shows a simple distillation process complete with heat exchangers for reboiling (adding heat to the bottom of the distillation column), condensing (extracting heat from the "overhead" product at the top of the column), and energy conservation (transferring heat from the hot products to the incoming feed):



Distillation "columns" (also called *fractionating towers* in the industry) are tall vessels containing sets of "trays" where rising vapors from the boiling process contact falling liquid from the condensing process. Temperatures increase toward the bottom of the column, while temperatures decrease toward the top. In this case, steam through a "reboiler" drives the boiling process at the bottom of the column (heat input), and cold water through a "condenser" drives the condensing process at the top of the column (heat extraction). Products coming off the column at intermediate points are hot enough to serve as pre-heating flows for the incoming feed. Note how the "economizing" heat exchangers expose the cold feed flow to the cooler Product A before exposing it to the warmer Product B, and then finally the warmest "Bottoms" product. This sequence of cooler-to-warmer maximizes the efficiency of the heat exchange process, with the incoming feed flowing past products of increasing temperature as it warms up to the necessary temperature for distillation entering the

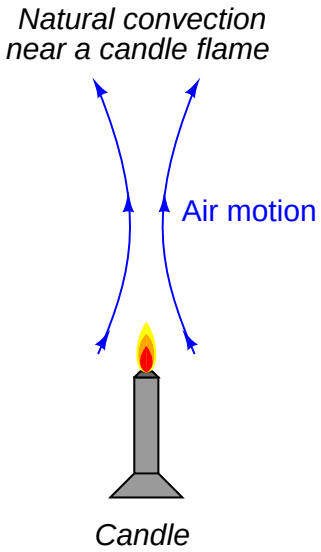
column.

Some heat exchangers transfer heat from hot gases to cool(er) liquids An example of this type of heat exchanger is the construction of a steam boiler, where hot combustion gases transfer heat to water flowing inside metal tubes:



Here, hot gases from the combustion burners travel past the metal "riser" tubes, transferring heat to the water within those tubes. This also serves to illustrate an important convection phenomenon: a *thermal siphon* (often written as *thermosiphon*). As water heats in the "riser" tubes, it becomes less dense, producing less hydrostatic pressure at the bottom of those tubes than the colder water in the "downcomer" tubes. This difference of pressure causes the colder water in the downcomer tubes to flow down to the mud drum, and hot water in the riser tubes to flow up to the steam drum. This natural *convection current* will continue as long as heat is applied to the riser tubes by the burners, and an unobstructed path exists for water to flow in a loop.

Natural convection also occurs in heated air, such as in the vicinity of a lit candle:



This thermally forced circulation of air helps *convect* heat from the candle to all other points within the room it is located, by carrying heated air molecules to colder objects.

3.4.5 Specific heat and enthalpy

Earlier, we saw how units of heat measurement were defined in terms of the amount of energy gain or loss required to alter the temperature of a water sample by one degree. In the case of the *calorie*, it was the amount of heat gain/loss required to heat/cool one gram of water one degree Celsius. In the case of the BTU, it was the amount of heat gain/loss required to heat/cool one pound of water one degree Fahrenheit.

As one might expect, one heat unit might be similarly defined as the amount of heat gain or loss to alter the temperature one-half of a degree for twice as much water, or two degrees for half as much water. We could express this as a proportionality:

$$Q \propto m\Delta T$$

Where,
Q = Heat gain or loss
m = Mass of sample
ΔT = Temperature change (rise or fall) over time

The next logical question to ask is, "How does the relationship between heat and temperature change work for substances other than water?" Does it take the same amount of heat to change the temperature of one gram of *iron* by one degree Celsius as it does to change the temperature of one gram of *water* by one degree Celsius? The answer to this question is a resounding *no!* Different substances require vastly different amounts of heat gain/loss to alter their temperature by the same degree, even when the masses of those substances happen to be identical.

We have a term for this ability to absorb or release heat, called *heat capacity* or *specific heat*, symbolized by the variable c. Thus, our heat/mass/temperature change relationship may be described as a true formula instead of a mere proportionality:

$$Q = mc\Delta T$$

Where,
Q = Heat gain or loss (metric calories or British BTU)
m = Mass of sample (metric grams or British pounds)
c = Specific heat of substance
ΔT = Temperature change (metric degrees Celsius or British degrees Fahrenheit)

Pure water, being the standard by which all other substances are measured, has a specific heat value of 1. The smaller the value for c, the less heat gain or loss is required to alter the substance's temperature by a set amount. That substance (with a low value of c) has a low "heat capacity" because each degree of temperature rise or fall represents a relatively small amount of energy gained or lost. Substances with low c values are easy to heat and cool, while substances having high c values require much heat in order to alter their temperatures, assuming equal masses.

A table of specific heat values (at room temperature, 25 degrees Celsius[27]) for common substances appears here:

Substance	**Specific heat value** (c) cal/g·^{o}C or BTU/lb·^{o}F
Aluminum (solid)	0.215
Iron (solid)	0.108
Copper (solid)	0.092
Lead (solid)	0.031
Ice (solid)	0.50
Water (liquid)	1.00
Methanol (liquid)	0.609
Ethanol (liquid)	0.587
Acetone (liquid)	0.521
Hydrogen (gas)	3.41
Helium (gas)	1.24
Nitrogen (gas)	0.249
Oxygen (gas)	0.219
Steam (gas)	0.476

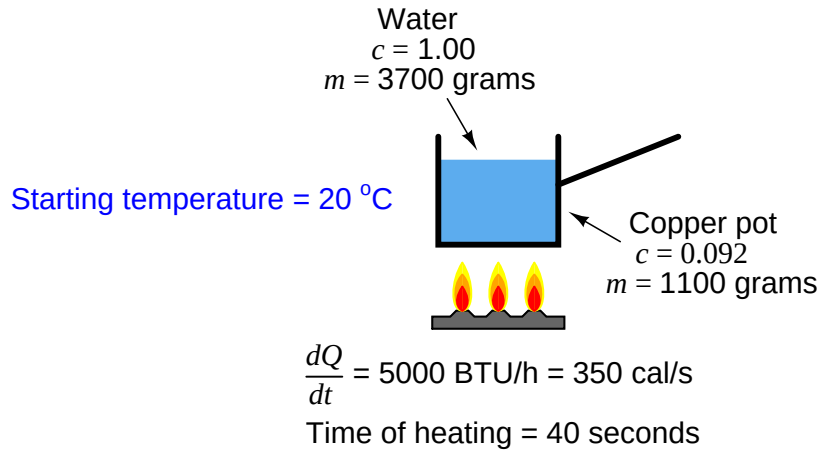
If a liquid or a gas is chosen for use as a coolant (a substance to efficiently convect heat away from an object), greater values of c are better. Water is one of the best liquid coolants with its relatively high c value of one: it has more capacity to absorb heat than other liquids, for the same rise in temperature. The ideal coolant would have an infinite c value, being able to absorb an infinite amount of heat without itself rising in temperature at all.

As you can see from the table, the light gases (hydrogen and helium) have extraordinarily high c values. Consequently, they function as excellent media for convective heat transfer. This is why large electric power generators often use hydrogen gas as a coolant: hydrogen has an amazing ability to absorb heat from the wire windings of a generator without rising much in temperature. In other words, hydrogen absorbs a lot of heat while still remaining "cool" (i.e. remains at a low temperature). Helium, although not quite as good a coolant as hydrogen, has the distinct advantage of being chemically inert (non-reactive), in stark contrast to hydrogen's extreme flammability. Some nuclear reactors use helium gas as a coolant rather than a liquid such as water or molten sodium metal.

Lead has an extraordinarily low c value, being a rather "easy" substance to heat up and cool down. Anyone who has ever cast their own lead bullets for a firearm knows how quickly a new lead bullet cools off after being released from the mold, especially if that same person has experience casting other metals such as aluminum.

[27] An important detail to note is that specific heat does *not* remain constant over wide temperature changes. This complicates calculations of heat required to change the temperature of a sample: instead of simply multiplying the temperature change by mass and specific heat ($Q = mc\Delta T$ or $Q = mc[T_2 - T_1]$), we must *integrate* specific heat over the range of temperature ($Q = m \int_{T_1}^{T_2} c\,dT$), summing up infinitesimal products of specific heat and temperature change ($c\,dT$) over the range starting from temperature T_1 through temperature T_2 then multiplying by the mass to calculate total heat required. So, the specific heat values given for substances at 25 ^{o}C only hold true for relatively small temperature changes deviating from 25 ^{o}C. To accurately calculate heat transfer over a large temperature change, one must incorporate values of c for that substance at different temperatures along the expected range.

Numerical examples are helpful to better understand specific heat. Consider a case where a copper pot filled with water receives heat from a small gas burner operating at an output of 5000 BTU per hour (350 calories per second):



A reasonable question to ask would be, "How much will the temperature of this water-filled pot rise after 40 seconds of heating?" With the burner's heat output of 350 calories per second and a heating time of 40 seconds, we may assume[28] the amount of heat absorbed by the water-filled pot will be the simple product of heat rate times time:

$$Q = \left(\frac{dQ}{dt}\right) t = \left(\frac{350 \text{ cal}}{\text{s}}\right) 40 \text{ s} = 14000 \text{ calories}$$

This amount of heat not only goes into raising the temperature of the water, but it also raises the temperature of the copper pot. Each substance (water, copper) has its own specific heat and mass values (c and m), but they will share the same temperature rise (ΔT), so we must sum their heats as follows:

$$Q_{total} = Q_{pot} + Q_{water}$$

$$Q_{total} = m_{pot} c_{pot} \Delta T + m_{water} c_{water} \Delta T$$

Since both the pot and the water start at the same temperature and end at the same temperature, ΔT is a common variable to both terms and may therefore be factored out:

$$Q_{total} = (m_{pot} c_{pot} + m_{water} c_{water}) \Delta T$$

[28] In reality, the amount of heat actually absorbed by the pot will be less than this, because there will be heat losses from the warm pot to the surrounding (cooler) air. However, for the sake of simplicity, we will assume *all* the burner's heat output goes into the pot and the water it holds.

Solving this equation for temperature rise, we get:

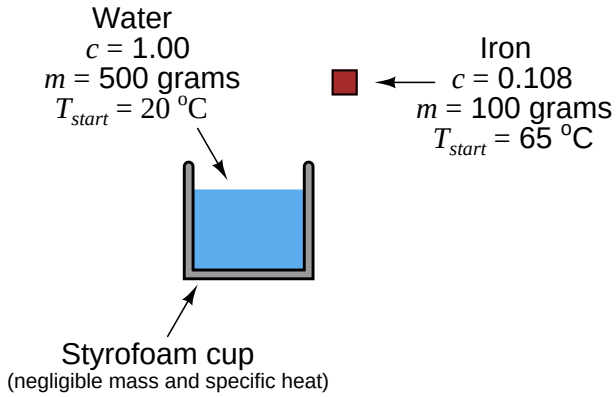
$$\Delta T = \frac{Q_{total}}{m_{pot}c_{pot} + m_{water}c_{water}}$$

$$\Delta T = \frac{14000 \text{ cal}}{(1100 \text{ g})(0.092 \frac{\text{cal}}{\text{g}^o\text{C}}) + (3700 \text{ g})(1 \frac{\text{cal}}{\text{g}^o\text{C}})}$$

$$\Delta T = 3.68\ ^o\text{C}$$

So, if the water and pot began at a temperature of 20 degrees Celsius, they will be at a temperature of 23.68 degrees Celsius after 40 seconds of heating over this small burner.

Another example involves the mixing of two substances at different temperatures. Suppose a heated mass of iron drops into a cool container[29] of water. Obviously, the iron will lose heat energy to the water, causing the iron to decrease in temperature while the water rises in temperature. Suppose the iron's mass is 100 grams, and its original temperature is 65 degrees Celsius. Suppose the water's mass is 500 grams, and its original temperature is 20 degrees Celsius:



[29] We will assume for the sake of this example that the container holding the water is of negligible mass, such as a Styrofoam cup. This way, we do not have to include the container's mass or its specific heat into the calculation.

What will the equilibrium temperature be after the iron falls into the water and both their temperatures equalize? We may solve this by setting two heat equations equal to each other[30]: the heat lost by the iron and the heat gained by the water, with the final equilibrium temperature being T:

$$Q_{iron} = Q_{water}$$

$$m_{iron}c_{iron}\Delta T_{iron} = m_{water}c_{water}\Delta T_{water}$$

$$m_{iron}c_{iron}(65\,^oC - T) = m_{water}c_{water}(T - 20\,^oC)$$

Note how the ΔT term is carefully set up for each side of the equation. In order to make the iron's heat loss a positive value and the water's heat gain a positive value, we must ensure the quantity within each set of parentheses is positive. For the iron, this means ΔT will be 65 degrees minus the final temperature. For the water, this means ΔT will be the final temperature minus its starting temperature of 20 degrees.

In order to solve for the final temperature (T), we must distribute the terms, collecting all T-containing terms to one side of the equation, then factor and isolate T:

$$m_{iron}c_{iron}(65) - m_{iron}c_{iron}T = m_{water}c_{water}T - m_{water}c_{water}(20)$$

$$m_{iron}c_{iron}(65) + m_{water}c_{water}(20) = m_{iron}c_{iron}T + m_{water}c_{water}T$$

$$m_{iron}c_{iron}(65) + m_{water}c_{water}(20) = T(m_{iron}c_{iron} + m_{water}c_{water})$$

$$T = \frac{m_{iron}c_{iron}(65) + m_{water}c_{water}(20)}{m_{iron}c_{iron} + m_{water}c_{water}}$$

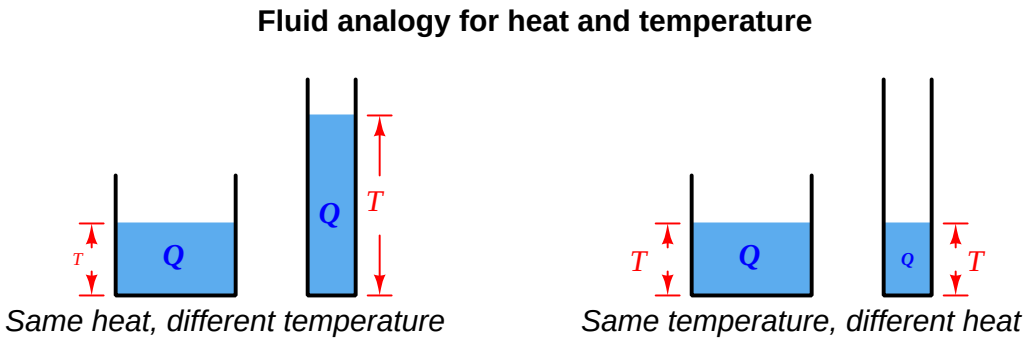
$$T = \frac{(100\text{ g})(0.108\text{ cal/g}^oC)(65^oC) + (500\text{ g})(1\text{ cal/g}^oC)(20^oC)}{(100\text{ g})(0.108\text{ cal/g}^oC) + (500\text{ g})(1\text{ cal/g}^oC)}$$

$$T = 20.95\,^oC$$

Thus, the iron's temperature falls from 65 degrees Celsius to 20.95 degrees Celsius, while the water's temperature rises from 20 degrees Celsius to 20.95 degrees Celsius. The water's tremendous specific heat value compared to the iron (nearly 10 times as much!), as well as its superior mass (5 times as much) results in a much larger temperature change for the iron than for the water.

[30] An alternative way to set up the problem would be to calculate ΔT for each term as $T_{final} - T_{start}$, making the iron's heat loss a negative quantity and the water's heat gain a positive quantity, in which case we would have to set up the equation as a zero-sum balance, with $Q_{iron} + Q_{water} = 0$. I find this approach less intuitive than simply saying the iron's heat loss will be equal to the water's heat gain, and setting up the equation as two positive values equal to each other.

An analogy to help grasp the concept of specific heat is to imagine heat as a fluid[31] that may be "poured" into vessels of different size, those vessels being objects or substances to be heated. The amount of liquid held by any vessel represents the total amount of thermal energy, while the *height* of the liquid inside any vessel represents its temperature:



The factor determining the relationship between liquid volume (heat) and liquid height (temperature) is of course the cross-sectional area of the vessel. The wider the vessel, the more heat will be required to "fill" it up to any given temperature. In this analogy, the area of the vessel is analogous to the term mc: the product of mass and specific heat. Objects with larger mass require more heat to raise their temperature to any specific point, specific heats being equal. Likewise, objects with large specific heat values require more heat to raise their temperature to any specific point, masses being equal.

In the first numerical calculation example where we determined the temperature of a pot of water after 40 seconds of heating, the analogous model would be to determine the height of liquid in a vessel after pouring liquid into it for 40 seconds at a fixed rate. A model for the second numerical example would be to calculate the equilibrium height (of liquid) after connecting two vessels together at their bottoms with a tube. Although the liquid heights of those vessels may be different at first, the levels will equalize after time by way of liquid passing through the tube from the higher-level vessel to the lower-level vessel.

[31] This is not far from the hypotheses of eighteenth-century science, where heat was thought to be an invisible fluid called *caloric*.

Many industrial processes use fluids to convectively transfer thermal energy from one object (or fluid) to another. In such applications, it is important to know how much thermal energy will be carried by a specific quantity of that fluid over a specified temperature drop. One common way to express this quantity is called *enthalpy*. Enthalpy is the amount of heat lost by a unit mass (one gram metric, or one pound British) of a substance as it cools from a given temperature all the way down to the freezing point of water (0 degrees Celsius, or 32 degrees Fahrenheit). In other words, enthalpy is a measure[32] of a substance's thermal energy using the freezing temperature of water as a reference. A sample of water at a temperature of 125 degrees Fahrenheit, for example, has an enthalpy of 93 BTU per pound (or 93 calories per gram), because 93 BTU of thermal energy would be lost if one pound of that water happened to cool from its given temperature (125 ^{o}F) down to 32 ^{o}F:

$$Q = mc\Delta T$$

$$Q = (1 \text{ lb}) \left(1 \frac{\text{BTU}}{\text{lb}^o\text{F}}\right) (125\ ^o\text{F} - 32\ ^o\text{F})$$

$$Q = 93 \text{ BTU}$$

Even if the substance in question does not cool down to the freezing temperature of water, enthalpy is a useful figure for comparing the thermal energy "content" of hot fluids (per unit mass). For example, if one were given the enthalpy values for a substance before and after heat transfer, it would be easy to calculate the amount of heat transfer that transpired simply by subtracting those enthalpy values[33]. If water at 125 ^{o}F has an enthalpy value of 93 BTU/lb and water at 170 ^{o}F has an enthalpy of value 138 BTU/lb, we may calculate the amount of heat needed to increase the temperature of a sample of water from 125 ^{o}F to 170 ^{o}F simply by subtracting 93 BTU/lb from 138 BTU/lb to arrive at 45 BTU/lb.

In this rather trivial example, it would have been just as easy for us to calculate the heat necessary to increase water's temperature from 125 ^{o}F to 170 ^{o}F by using the specific heat formula ($Q = mc\Delta T$)[34], and so it might appear as though the concept of enthalpy sheds no new light on the subject of heat transfer. However, the ability to calculate heat transfer based on a simple subtraction of enthalpy values proves quite useful in more complex scenarios where substances change phase, as we will see next.

[32] A useful analogy for enthalpy is the *maximum available balance* of a bank account. Suppose you have a bank account with a minimum balance requirement of $32 to maintain that account. Your maximum available balance at any time would be the total amount of money in that account minus $32, or to phrase this differently your maximum available balance is the most money you may spend from this account while still keeping that account open. Enthalpy is much the same: the amount of thermal energy a sample may "spend" (i.e. lose) before its temperature reaches 32 degrees Fahrenheit.

[33] Appealing to the *maximum available balance* analogy, if we compared the maximum available balance in your bank account before and after a transaction, we could determine how much money was deposited or withdrawn from your account simply by subtracting those two values.

[34] Following the formula $Q = mc\Delta T$, we may calculate the heat as $(1)(1)(170 - 125) = 45$ BTU. This is obviously the same result we obtained by subtracting enthalpy values for water at 170 ^{o}F and 125 ^{o}F.

3.4.6 Phase changes

Scientists often speak of four *phases* of matter: *solid*, *liquid*, *gas* (or *vapor*), and *plasma*. Of these four, the first three are common to everyday life. Plasma is a phase of matter where the atoms of a gas are excited (energized) to the point where they become electrically ionized, such as neon gas in an electric tube light, or the gas comprising stars in space.

Phase changes are very important in thermodynamics, principally because energy transfer (heat loss or heat gain) must occur for a substance to change states, often with negligible change in temperature. To cite an example, consider the case of water (a liquid) turning into steam (a vapor) at atmospheric pressure. At sea level, this phase change will occur at a temperature of 100 degrees Celsius, or 212 degrees Fahrenheit. The amount of energy required to increase the temperature of water from ambient up to its boiling point is a simple function of the sample's mass and its original temperature. For instance, a sample of water 70 grams in mass starting at 24 degrees Celsius will require 5320 calories of heat to reach the boiling point:

$$Q = mc\Delta T$$

$$Q = (70 \text{ g}) \left(1 \frac{\text{cal}}{\text{g}^o\text{C}} \right) (100\,^o\text{C} - 24\,^o\text{C})$$

$$Q = 5320 \text{ cal}$$

However, actually boiling the 70 grams of water into 70 grams of steam (both at 100 degrees Celsius) requires a comparatively enormous input of heat: *37734 calories* – over seven times as much heat to turn the water to steam as what is required to warm the water to its boiling point! Furthermore, this additional input of 37734 calories does not increase the temperature of the water at all: the resulting steam is still at a temperature of (only) 100 degrees Celsius. If further heat is added to the 70 gram steam sample, its temperature will rise, albeit at a rate proportional to the value of steam's specific heat (0.476 calories per gram degree Celsius, or BTU per pound degree Fahrenheit).

What we see here is a fundamentally different phenomenon than we saw with specific heat. Here, we are looking at the thermal energy required to transition a substance from one phase to another, not to change its temperature. We call this quantity *latent heat* rather than *specific heat*, because no temperature change occurs[35]. Conversely, if we allow the steam to condense back into liquid water, it must release the same 37734 calories of heat energy we invested in it turning the water into steam before it may cool at all below the boiling point (100 degrees Celsius).

Latent heat has the effect of greatly increasing a substance's enthalpy. Recall that "enthalpy" is the amount of heat lost by one pound (mass) of a substance if it happened to cool from its given temperature all the way down to the freezing temperature of water (0 ^{o}C, or 32 ^{o}F). Hot water has an enthalpy of 1 BTU/lb for every degree of temperature above freezing. Steam, however, possesses far greater enthalpy because of the latent heat released in the phase change from vapor to liquid before it releases heat as water cooling down to 32 ^{o}F.

[35] The word "latent" refers to something with potential that is not yet realized. Here, heat exchange takes place without there being any realized change in temperature. By contrast, heat resulting in a temperature change ($Q = mc\Delta T$) is called *sensible heat*.

As with specific heat, there is a formula relating mass, latent heat, and heat exchange:

$$Q = mL$$

Where,

Q = Heat of transition required to completely change the phase of a sample (metric calories or British BTU)

m = Mass of sample (metric grams or British pounds)

L = Latent heat of substance

Each substance has its own set of latent heat values, one[36] for each phase-to-phase transition. Water, for example, exhibits a latent heat of vaporization (boiling/condensing) of 539.1 calories per gram, or 970.3 BTU per pound, at atmospheric pressure (boiling point = 100 ^{o}C = 212 ^{o}F). Water also exhibits a latent heat of fusion (melting/freezing) of 79.7 calories per gram, or 143.5 BTU per pound. Both figures are enormous compared to water's specific heat value of 1 calorie per gram-degree Celsius (or 1 BTU per pound-degree Fahrenheit[37]): it takes only one calorie of heat to warm one gram of water one degree Celsius, but it takes *539.1 calories* of heat to boil that same gram of water into one gram of steam, and *79.7 calories* of heat to melt one gram of ice into one gram of water. The lesson here is simple: phase changes involve huge amounts of heat.

A table showing various latent heats of vaporization (all at room temperature, 70 degrees Fahrenheit) for common industrial fluids appears here, contrasted against their specific heat values (as liquids). In each case you will note how much larger L is than c:

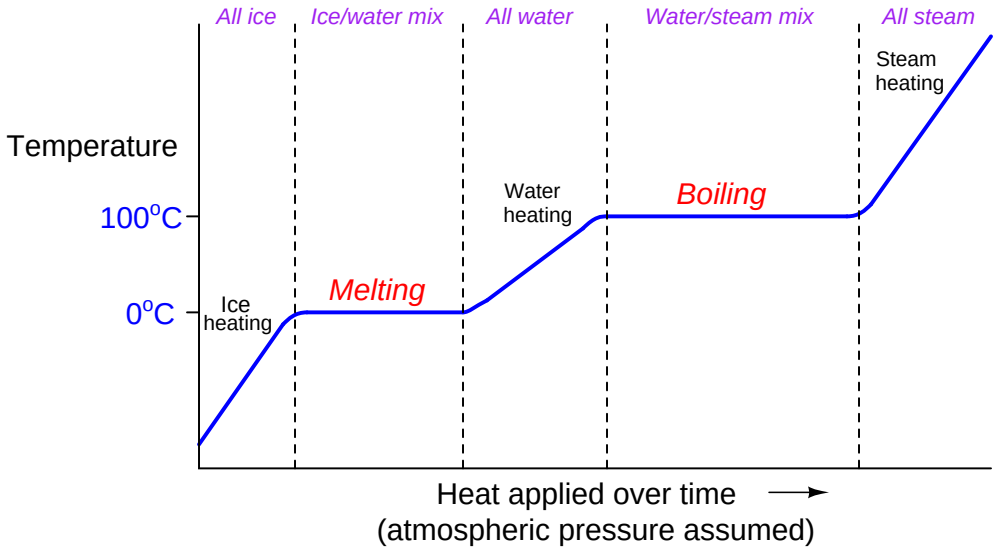
Fluid (@ 70 ^{o}F)	$L_{vaporization}$, BTU/lb	$L_{vaporization}$, cal/g	c_{liquid}
Water	970.3	539.1	1
Ammonia	508.6	282.6	1.1
Carbon dioxide	63.7	35.4	0.66
Butane	157.5	87.5	0.56
Propane	149.5	83.06	0.6

One of the most important, and also non-intuitive, consequences of latent heat is the relative stability of temperature during the phase-change process. Referencing the table of latent heats of vaporization, we see how much more heat is needed to boil a liquid into a vapor than is needed to warm that same liquid by one degree of temperature. During the process of boiling, all heat input to the liquid goes into the task of phase change (latent heat) and none of it goes into increased temperature. In fact, until all the liquid has been vaporized, the liquid's temperature *cannot* rise above its boiling point! The requirement of heat input to vaporize a liquid forces temperature to stabilize (not rise further) until *all* the liquid has evaporated from the sample.

[36] Latent heat of vaporization also varies with pressure, as different amounts of heat are required to vaporize a liquid depending on the pressure that liquid is subject to. Generally, increased pressure (increased boiling temperature) results in less latent heat of vaporization.

[37] The reason specific heat values are identical between metric and British units, while latent heat values are not, is because latent heat does not involve temperature change, and therefore there is one less unit conversion taking place between metric and British when translating latent heats. Specific heat in both metric and British units is *defined* in such a way that the three different units for heat, mass, and temperature all cancel each other out. With latent heat, we are only dealing with mass and heat, and so we have a proportional conversion of $\frac{5}{9}$ or $\frac{9}{5}$ left over, just the same as if we were converting between degrees Celsius and Fahrenheit alone.

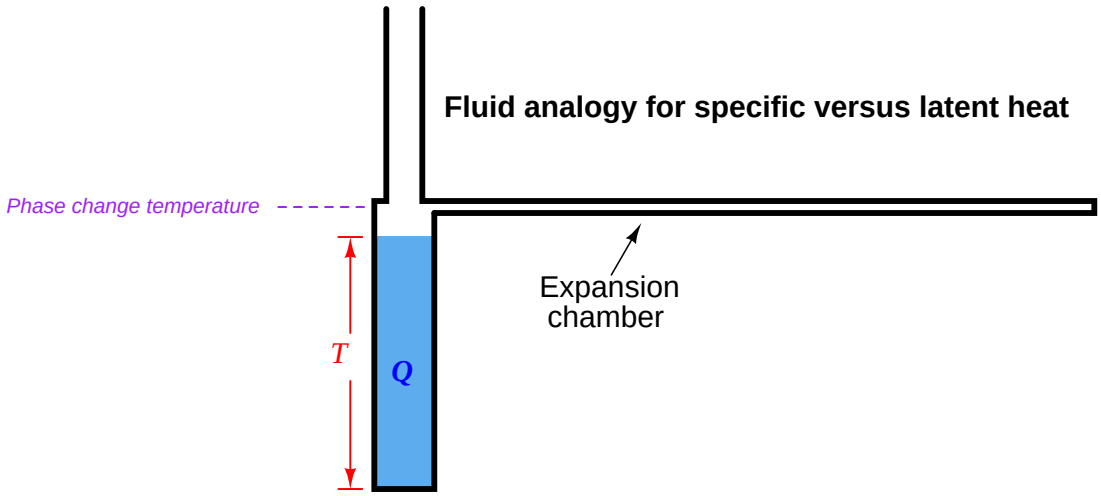
If we take a sample of ice and add heat to it over time until it melts, warms, boils, and then becomes steam, we will notice a temperature profile that looks something like this:



The flat areas of the graph during the melting and boiling periods represents times where the sample's temperature does not change at all, but where all heat input goes into the work of changing the sample's phase. Only where we see the curve rising does the temperature change. So long as there is a *mixture* of different phases, the temperature remains "locked" at one value. Only when there is a single phase of material is the temperature "allowed" to rise or fall.

The sloped areas of the graph reveal the specific heat of the substance in each particular phase. Note how the liquid (water) portion of the graph has a relatively shallow slope, due to the specific heat value (c) of water being equal to 1. Both the ice and the steam portions of the graph have steeper slopes because both of those phases possess smaller values of specific heat ($c = 0.5$ and $c = 0.476$, respectively). The smaller the value of c, the more a sample's temperature will rise for any given input of thermal energy. For any given rate of heat transfer, smaller c values result in more rapid temperature changes.

We may employ our liquid-filled vessel analogy to the task of explaining latent heat. Any point of phase change is analogous to a point along the vessel's height equipped with a large expansion chamber, so that the vessel "acts" as if its area were much larger at one point, requiring *much* more fluid volume (heat) to change height (temperature) past that one point:



Liquid poured into this vessel will fill it at a rate proportional to the volume added and inversely proportional to the vessel's cross-sectional area at the current liquid height. As soon as the liquid level reaches the expansion chamber, a great deal more liquid must be added to cause level to increase, since this chamber must completely fill before the liquid level may rise above it. Once that happens, the liquid level rises at a different rate with addition introduced volume, because now the phase is different (with a different specific heat value).

Remember that the filling of a vessel with liquid is merely an analogy for heat and temperature, intended to provide an easily visualized process mimicking another process not so easily visualized. The important concept to realize with latent heat and phase change is that it constitutes a discontinuity in the temperature/heat function for any given substance.

A vivid demonstration of this phenomenon is to take a paper[38] cup filled with water and place it in the middle of a roaring fire[39]. "Common sense" might tell you the paper will burn through with the fire's heat, so that the water runs out of the cup through the burn-hole. This does not happen, however. Instead, the water in the cup will rise in temperature until it boils, and there it will maintain that temperature no matter how hot the fire burns. The boiling point of water happens to be substantially below the burning point of paper, and so the boiling water keeps the paper cup too cool to burn. As a result, the paper cup remains intact so long as water remains in the cup. The *rim* of the cup above the water line will burn up because the steam does not have the same temperature-stabilizing effect as the water, leaving a rimless cup that grows shorter as the water boils away.

[38] Styrofoam and plastic cups work as well, but paper exhibits the furthest separation between the boiling point of water and the burning point of the cup material, and it is usually thin enough to ensure good heat transfer from the outside (impinging flame) to the inside (water).

[39] This is a lot of fun to do while camping!

The point at which a pure substances changes phase not only relates to temperature, but to pressure as well. We may speak casually about the boiling point of water being 100 degrees Celsius (212 degrees Fahrenheit), but that is only if we assume the water and steam are at atmospheric pressure (at sea level). If we reduce the ambient air pressure[40], water will boil at a lesser temperature. Anyone familiar with cooking at high altitudes knows you must generally cook for longer periods of time at altitude, because the decreased boiling temperature of water is not as effective for cooking. Conversely, anyone familiar with *pressure cooking* (where the cooking takes place inside a vessel pressurized by steam) knows how comparatively little cooking time is required because the pressure raises water's boiling temperature. In either of these scenarios, where pressure influences[41] boiling temperature, the latent heat of water acts to hold the boiling water's temperature stable until all the water has boiled away. The only difference is the temperature at which the water begins to boil (or when the steam begins to condense).

Many industrial processes use boiling liquids to convectively transfer heat from one object (or fluid) to another. In such applications, it is important to know how much heat will be carried by a specific quantity of the vapor as it condenses into liquid over a specified temperature drop. The quantity of *enthalpy* (heat content) used for rating the heat-carrying capacity of liquids applies to condensing vapors as well. Enthalpy is the amount of heat lost by a unit mass (one gram metric, or one pound British) of the fluid as it cools from a given temperature all the way down to the freezing point of water (0 degrees Celsius, or 32 degrees Fahrenheit)[42]. When the fluid's initial state is vapor, and it condenses into liquid as it cools down to the reference temperature (32 ^{o}F), the heat content (enthalpy) is not just a function of specific heat, but also of latent heat.

Water at its atmospheric boiling point has an enthalpy of approximately 180 BTU per pound. Steam at atmospheric pressure and 212 ^{o}F, however, has an enthalpy of about *1150 BTU per pound*: more than six times as much heat as water at the same temperature. 970 of that 1150 BTU/lb is due to the phase change from steam to water, while the rest is due to water's specific heat as it cools from 212 ^{o}F to 32 ^{o}F.

Many technical reference books contain a set of data known as a *steam table* showing various properties of steam at different temperatures and pressures. Enthalpy is one of the most important parameters given in a steam table, showing how much available energy resides in steam under different pressure and temperature conditions. For this reason, enthalpy is sometimes referred to as *total heat* (h_g). Steam tables also show saturation temperature (the condensing temperature for steam at that pressure) and steam density.

[40] This may be done in a vacuum jar, or by traveling to a region of high altitude where the ambient air pressure is less than at sea level.

[41] The mechanism of this influence may be understood by considering what it means to boil a liquid into a vapor. Molecules in a liquid reside close enough to each other that they cohere, whereas molecules in a vapor or gas are relatively far apart and act as independent objects. The process of boiling requires that cohesion between liquid molecules to be broken, so the molecules may drift apart. Increased pressure encourages cohesion in liquid form by helping to hold the molecules together, while decreased pressure encourages the separation of molecules into a vapor/gas.

[42] As mentioned previously, a useful analogy for enthalpy is the *maximum available balance* for a bank account with a $32 minimum balance requirement: that is, how much money may be spent from that account without closing it out.

If the vapor in question is at a temperature greater than its boiling point at that pressure, the vapor is said to be *superheated*. The enthalpy of superheated vapor comes from three different heat-loss mechanisms:

- Cooling the vapor down to its condensing temperature (specific heat of vapor)
- Phase-changing from vapor to liquid (latent heat of phase change)
- Cooling the liquid down to the reference temperature (specific heat of liquid)

Using steam as the example once more, a sample of superheated steam at 500 ^{o}F and atmospheric pressure (boiling point = 212 ^{o}F) has an enthalpy of approximately 1287 BTU per pound. We may calculate the heat lost by one pound of this superheated steam as it cools from 500 ^{o}F to 32 ^{o}F in each of the three steps previously described. Here, we will assume a specific heat for steam of 0.476, a specific heat for water of 1, and a latent heat of vaporization for water of 970:

Heat loss mechanism	**Formula**	**Quantity**
Cooling vapor	$Q = mc\Delta T$	$(1)(0.476)(500 - 212) = 137$ BTU
Phase change	$Q = mL$	$(1)(970) = 970$ BTU
Cooling liquid	$Q = mc\Delta T$	$(1)(1)(212 - 32) = 180$ BTU
TOTAL		1287 BTU

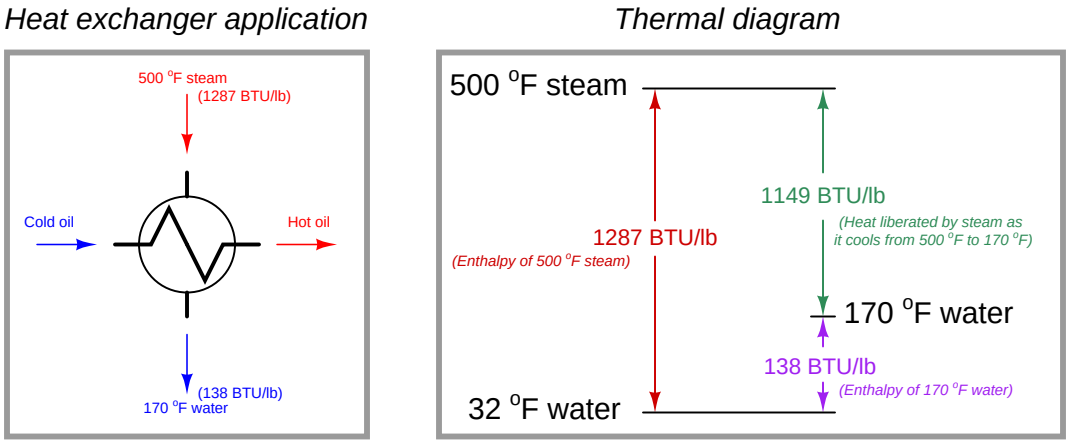
Enthalpy values are very useful[43] in steam engineering to predict the amount of thermal energy delivered to a load given the steam's initial temperature, its final (cooled) temperature, and the mass flow rate. Although the definition of enthalpy – where we calculate heat value by supposing the vapor cools all the way down to the *freezing point* of water – might seem a bit strange and impractical (how common is it for steam to lose so much heat to a process that it reaches freezing temperature?), it is not difficult to shift the enthalpy value to reflect a more practical final temperature. Since we know the specific heat of liquid water is very nearly one, all we have to do is offset the enthalpy value by the amount that the final temperature differs from freezing in order to calculate how much heat the steam will lose (per pound) to its load[44].

Furthermore, the *rate* at which heat is delivered to a substance by steam (or conversely, the rate at which heat is required to boil water into steam) may be easily calculated if we take this heat value in units of BTU per pound and multiply it by the mass flow rate in pounds per minute: as the unit of "pound" cancels in the multiplication, we arrive at a result for heat transfer rate in units of BTU per minute.

[43] At first it may seem as though the enthalpy of steam is so easy to calculate it almost renders steam tables useless. If the specific heats of water and steam were constant, and the latent heat of vaporization for water likewise constant, this would be the case. However, both these values (c and L) are not constant, but rather change with pressure and with temperature. Thus, steam tables end up being quite valuable to engineers, allowing them to quickly reference heat content of steam across a broad range of pressures and temperatures without having to account for changing c and L values (performing integral calculus in the form of $Q = m \int_{T_1}^{T_2} c\, dT$ for specific heat) in their heat calculations.

[44] This is not unlike calculating the voltage dropped across an electrical load by measuring the voltage at each of the load's two terminals with respect to *ground*, then subtracting those two measured voltage values. In this analogy, electrical "ground" is the equivalent of water at freezing temperature: a common reference point for energy level.

For example, suppose we were to employ the same 500 ^{o}F superheated steam used in the previous example to heat a flow of oil through a heat exchanger, with the steam condensing to water and then cooling down to 170 degrees Fahrenheit as it delivers heat to the flowing oil. Here, the steam's enthalpy value of 1287 BTU per pound may simply be offset by 138 (170 degrees minus 32 degrees) to calculate how much heat (per pound) this steam will deliver to the oil: 1287 − 138 = 1149 BTU per pound:



Here we see how 500 ^{o}F steam has an enthalpy (total heat) of 1287 BTU/lb, but since the steam does not in fact cool all the way down to 32 ^{o}F in the act of heating oil in the heat exchanger, we must subtract the enthalpy of the 170 ^{o}F water (138 BTU/lb) to determine[45] the amount of heat actually delivered to the oil by the steam (1149 BTU/lb). Calculating heat transfer rate is a simple matter of multiplying this heat per pound of steam by the steam's mass flow rate: for example, if the mass flow rate of this steam happened to be 2 pounds per minute, the heat transfer rate would be 2298 BTU per minute.

If we happen to be dealing with a situation where steam gives up some heat energy to a process fluid but not enough to cool to the point of condensation, all we need to do to calculate the amount of heat liberated by the superheated steam as it cools is subtract the enthalpy values between its hot and cool(er) states.

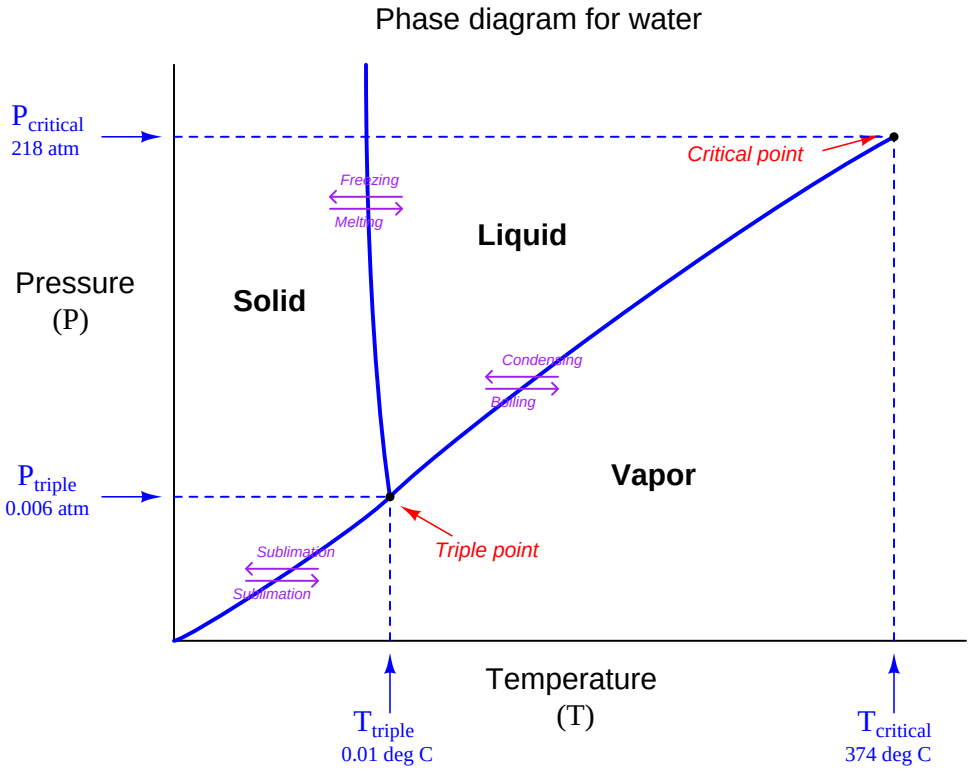
For example, suppose we have a heat-exchange process where superheated steam enters at 105 PSIG and 600 ^{o}F, exiting at 75 PSIG and 360 ^{o}F. The enthalpy of the steam under those two sets of conditions as given by a superheated steam table are 1328 BTU/lb and 1208 BTU/lb, respectively. Thus, the heat lost by the steam as it goes through this heat exchanger is the difference in enthalpy values: 1328 BTU/lb − 1208 BTU/lb = 120 BTU/lb. Once again, calculating heat transfer rate is a simple matter of multiplication: if the mass flow rate of this steam happened to be 80 pounds per hour, the heat transfer rate would be 120 BTU/lb × 80 lb/hr = 9600 BTU/hr.

[45] Applying the *maximum available balance* analogy to this scenario, it would be as if your bank account began with a maximum available balance of $1287 and then finished with a maximum available balance of $138 after an expenditure: the amount of money you spent is the different between the initial and final maximum available balances ($1287 − $138 = $1149).

By encompassing both specific heat and latent heat into one quantity, enthalpy figures given in steam tables greatly simplify heat transfer calculations, as compared to evaluating specific heat and latent heat formulae ($Q = mc\Delta T$ and $Q = mL$, respectively) for water. Calculations based on steam tables are also more accurate than those derived from the specific and/or latent heat formulae, because steam tables take into account the changing values of c and L over wide temperature and pressure ranges. This is the power of empirical data: steam tables were developed by taking actual calorimetric measurements of steam under those temperature and pressure conditions, and as such are a *record* of water's true behavior rather than a *prediction* of water's theoretical behavior.

3.4.7 Phase diagrams and critical points

A comprehensive way of describing the relationship between pressure, temperature, and substance phase is with something called a *phase diagram*. With pressure shown on one axis, and temperature on the other, a phase diagram describes the various phases of a substance in possible equilibrium at certain pressure/temperature combinations.



This phase diagram (for water) illustrates some of the features common to all phase diagrams: curved lines define the boundaries between solid, liquid, and vapor phases; the point of intersection of these three curves is where the substance may exist in all three phases simultaneously (called the *triple point*[46]) and points where a curve simply ends within the span of the graph indicate critical points, where the certain phases cease to exist.

The curved line from the triple point up and to the right defines the boundary between liquid water and water vapor. Each point on that line represents a set of unique pressure and temperature conditions for boiling (changing phase from liquid to vapor) or for condensation (changing phase from vapor to liquid). As you can see, increased pressure results in an increased boiling point (i.e. at higher pressures, water must be heated to greater temperatures before boiling may take place).

[46] When H_2O is at its triple point, vapor (steam), liquid (water), and solid (ice) of water will co-exist in the same space. One way to visualize the triple point is to consider it the pressure at which the boiling and freezing temperatures of a substance become the same.

In fact, the whole concept of a singular boiling *point* for water becomes quaint in the light of a phase diagram: boiling is seen to occur over a wide range of temperatures[47], the exact temperature varying with pressure.

Something interesting happens when the temperature is raised above a certain value called the *critical temperature*. At this value (approximately 374 degrees Celsius for water), no amount of pressure will maintain it in a liquid state. Water, once heated beyond 374 degrees Celsius, is no longer a liquid and may only exist in a stable condition as a vapor. The *critical pressure* of any substance is the pressure value at the liquid/vapor boundary at the point of critical temperature.

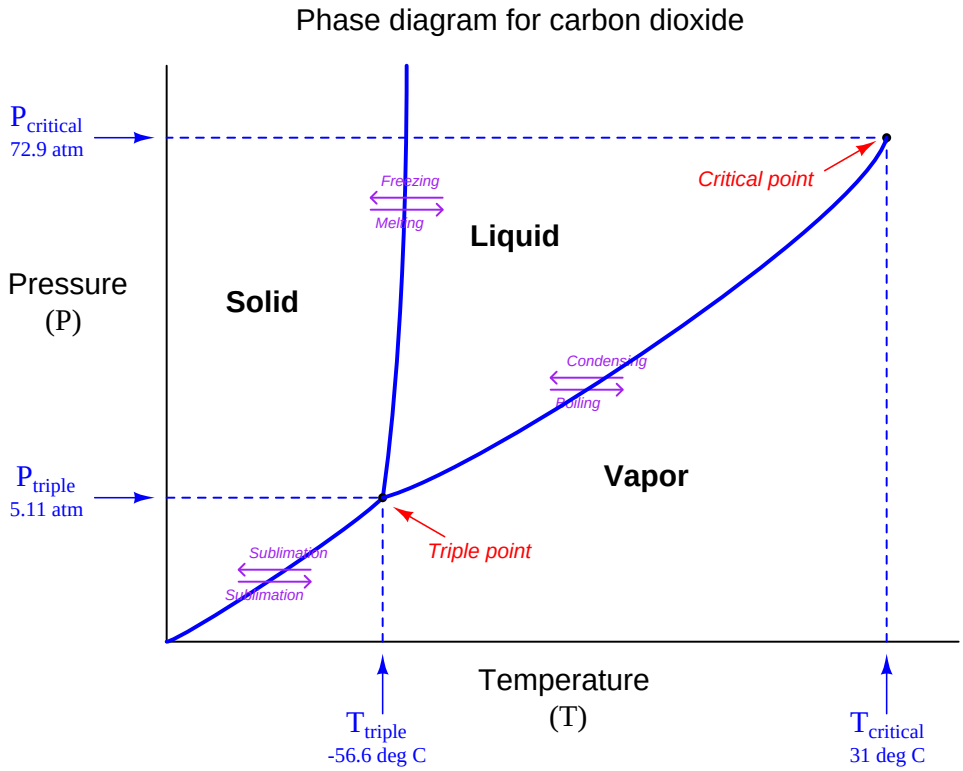
A vivid example of critical temperature is this photograph of an ultra-high pressure storage vessel for oxygen gas, at a rocket engine testing facility:

The critical temperature for oxygen is 154.58 Kelvin, which is equal to −118.57 degrees Celsius or −181.43 degrees Fahrenheit. Since this pressure vessel is completely uninsulated, we know the temperature of the oxygen inside of it will be the same (or nearly the same) as ambient temperature, which is obviously much warmer than −118.57 ^{o}C. Since the oxygen's temperature is well above the critical temperature for the element oxygen, we may safely conclude that the oxygen inside this vessel must exist as a gas. Even at the extremely high pressure this vessel is capable of holding (15000 PSIG), the oxygen cannot liquefy.

The slightly curved line from the triple point up and to the left defines the boundary between solid ice and liquid water. As you can see, the near-vertical pitch of this curve suggests the freezing temperature of water is quite stable over a wide pressure range.

[47] Anywhere between the triple-point temperature and the critical temperature, to be exact.

Carbon dioxide exhibits a different set of curves than water on its phase diagram, with its own unique critical temperature and pressure values:



Note how the triple-point pressure of carbon dioxide is well above ambient conditions on Earth. This means carbon dioxide is not stable in its liquid state unless put under substantial pressure (at least 60.4 PSIG). This is why solid carbon dioxide is referred to as *dry ice*: it does not liquefy with the application of heat, rather it *sublimates* directly into its vapor phase.

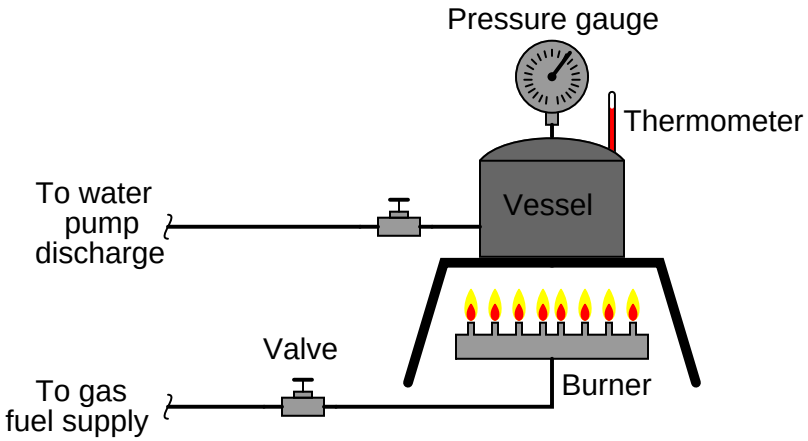
Another interesting difference between the carbon dioxide and water phase diagrams is the slope of the solid/liquid boundary line. With water, this boundary drifts to the left (lower temperature) as pressure increases. With carbon dioxide, this boundary drifts to the right (higher temperature) as pressure increases. Whether the fusion temperature increases or decreases with increasing pressure marks whether that substance contracts or expands as it transitions from liquid to solid. Carbon dioxide, like most pure substances, contracts to a smaller volume when it goes from liquid to solid, and its fusion curve drifts to the right as pressure increases. Water is unusual in this regard, expanding to a larger volume when freezing, and its fusion curve drifts to the left as pressure increases.

3.4.8 Thermodynamic degrees of freedom

If we look at the areas bounded by phase transition curves in a phase diagram (solid area, liquid area, and vapor area), we see that both pressure and temperature may change independent of one another. A vessel filled with liquid water, for instance, may be at 30 degrees Celsius and 2 atmospheres, or at 50 degrees Celsius and 2 atmospheres, or at 50 degrees Celsius and 1 atmosphere, all equally stable. A more technical way to state this is to say the liquid water has *two degrees of freedom*. Here, the word "degree" has a completely different meaning than it does when used to denote a unit of temperature or angle. In this context, "degree" may be thought of loosely as "dimension." A cube has three physical dimensions, a square has two and a line has one. A point within a cube has three degrees of freedom (motion), while a point within a square only has two, and a point along a line only has one. Here, we use the word "degree" to denote the number of independent ways a system may change. For areas bounded by phase transition curves in a phase diagram, *pressure* and *temperature* are the two "free" variables, because within those bounded areas we may freely alter pressure without altering temperature, and vice-versa.

Such is not the case at any point lying along one of the phase transition curves. Any point on a curve is geometrically defined by a pair of coordinates, which means that for a two-phase mixture in equilibrium there will be exactly one temperature value valid for each unique pressure value. At any point along a phase transition curve, pressure and temperature are not independent variable, but rather are *related*. This means that for any single substance, there is only one degree of freedom along any point of a phase transition curve.

To illustrate this concept, suppose we equip a closed vessel containing water with both a thermometer and a pressure gauge. The thermometer measures the temperature of this water, while the pressure gauge measures the pressure of the water. A burner beneath the vessel adds heat to alter the water's temperature, and a pump adds water to the vessel to alter the pressure inside:



So long as the water is all liquid (one phase), we may adjust its pressure and temperature independently. In this state, the system has two thermodynamic degrees of freedom.

However, if the water becomes hot enough to boil, creating a system of two phases in direct contact with each other (equilibrium), we will find that pressure and temperature become linked: one cannot alter one without altering the other. For a steam boiler, operation at a given steam

pressure thus *defines* the temperature of the water, and vice-versa. In a single-component, two-phase system, there is only one degree of thermodynamic freedom.

Our freedom to alter pressure and temperature becomes even more restricted if we ever reach the *triple point*[48] of the substance. For water, this occurs (only) at a pressure of −14.61 PSIG (0.006 atmospheres) and a temperature of 0.01 degrees Celsius: the coordinates where all three phase transition curves intersect on the phase diagram. In this state, where solid (ice), liquid (water), and vapor (steam) coexist, there are zero degrees of thermodynamic freedom. Both the temperature and pressure are *locked* at these values until one or more of the phases disappears.

The relationship between degrees of freedom and phases is expressed neatly by *Gibbs' Phase Rule* – the sum of phases and degrees of freedom equals the number of substances ("components") plus two:

$$N_{freedom} + N_{phase} = N_{substance} + 2$$

We may simplify Gibbs' rule for systems of just one substance (1 "component") by saying the number of degrees of freedom plus phases in direct contact with each other is always equal to three. So, a vessel filled with nothing but liquid water (one component, one phase) will have two thermodynamic degrees of freedom: we may change pressure or temperature independently of one another. A vessel containing nothing but boiling water (two phases – water and steam, but still only one component) has just one thermodynamic degree of freedom: we may change pressure and temperature, but just not independently of one another. A vessel containing water at its triple point (three phases, one component) has no thermodynamic freedom at all: both temperature and pressure are fixed[49] so long as all three phases coexist in equilibrium.

3.4.9 Applications of phase changes

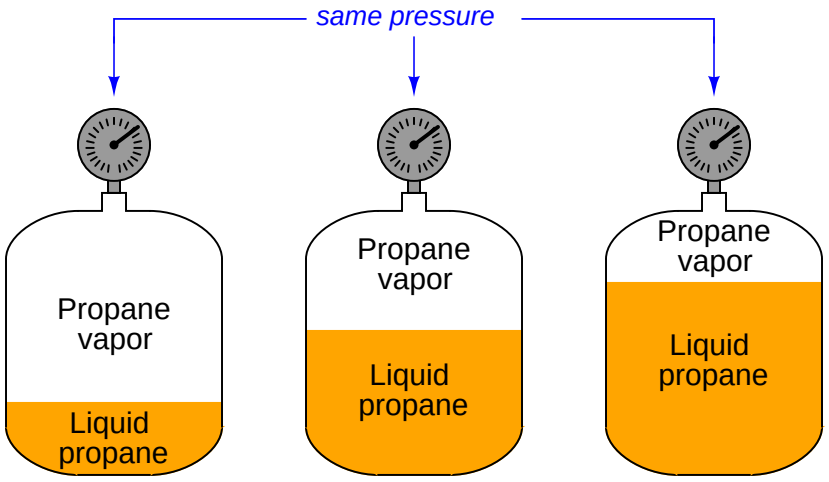
Applications of phase changes abound in industrial and commercial processes. Some of these applications exploit phase changes for certain production goals, such as the storage and transport of energy. Others merely serve to illustrate certain phenomena such as latent heat and degrees of thermodynamic freedom. This subsection will highlight several different processes for your learning benefit.

[48] The triple point for any substance is the pressure at which the boiling and freezing temperatures become one and the same.

[49] The non-freedom of both pressure and temperature for a pure substance at its triple point means we may exploit different substances' triple points as *calibration standards* for both pressure and temperature. Using suitable laboratory equipment and samples of sufficient purity, anyone in the world may force a substance to its triple point and calibrate pressure and/or temperature instruments against that sample.

Propane storage tanks

A common example of a saturated liquid/vapor (two-phase) system is the internal environment of a propane storage tank, such as the kind commonly used to store propane fuel for portable stoves and gas cooking grills. If multiple propane storage tanks holding different volumes of liquid propane are set side by side, pressure gauges attached to each tank will all register the exact same pressure:



This is counter-intuitive, as most people tend to think the fullest tank should register the highest pressure (having the least space for the vapor to occupy). However, since the interior of each tank is a liquid/vapor system in equilibrium, the pressure is defined by the point on the liquid/vapor transition curve on the phase diagram for pure propane matching the tanks' *temperature*. Thus, the pressure gauge on each tank actually functions as a *thermometer*[50], since pressure is a direct function of temperature for a saturated liquid/vapor system and therefore cannot change without a corresponding change in temperature. This is a thermodynamic system with just *one* degree of freedom.

Storage tanks containing liquid/vapor mixtures in equilibrium present unique safety hazards. If ever a rupture were to occur in such a vessel, the resulting decrease in pressure causes the liquid to spontaneously boil, halting any further decrease in pressure. Thus, a punctured propane tank does not lose pressure in the same manner than a punctured compressed air tank loses pressure. This gives the escaping vapor more "power" to worsen the rupture, as its pressure does not fall off over time the way it would in a simple compressed-gas application. As a result, relatively small punctures can and often do grow into catastrophic ruptures, where all liquid previously held inside the tank escapes and flashes into vapor, generating a vapor cloud of surprisingly large volume[51].

Compounding the problem of catastrophic tank rupture is the fact that propane happens to be highly flammable. The thermodynamic properties of a boiling liquid combined with the chemical

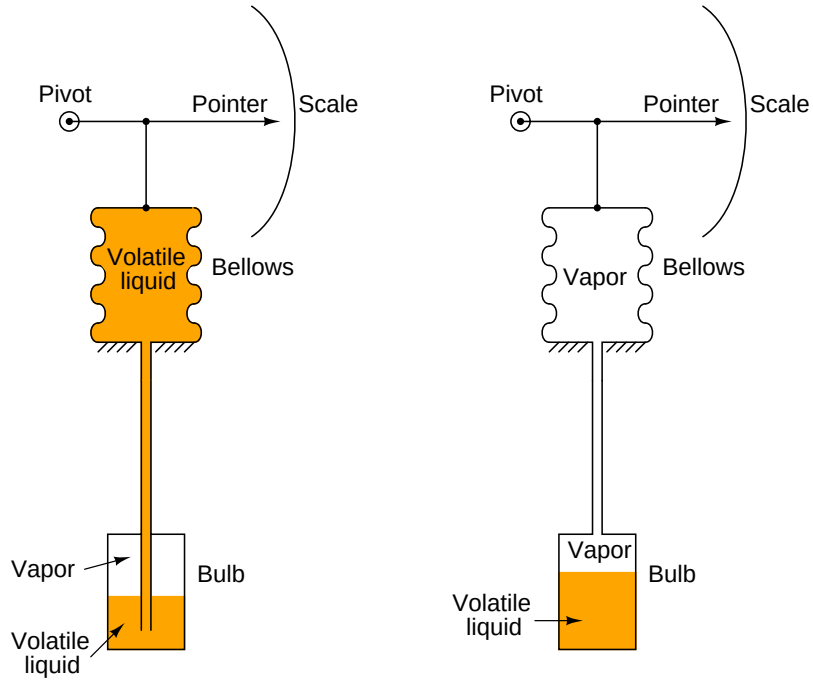
[50] To be more precise, a propane tank acts like a *Class II filled-bulb* thermometer, with liquid and vapor coexisting in equilibrium.

[51] Steam boilers exhibit this same explosive tendency. The expansion ratio of water to steam is on the order of a thousand to one (1000:1), making steam boiler ruptures very violent even at relatively low operating pressures.

property of flammability in air makes propane tank explosions particularly violent. Fire fighters often refer to this as a *BLEVE*: a *Boiling Liquid Expanding Vapor Explosion*.

Class II Filled-bulb thermometers

This same pressure-temperature interdependence finds application in a type of temperature measurement instrument called a *Class II filled-bulb*, where a metal bulb, tube, and pressure-sensing element are all filled with a saturated liquid/vapor mixture:



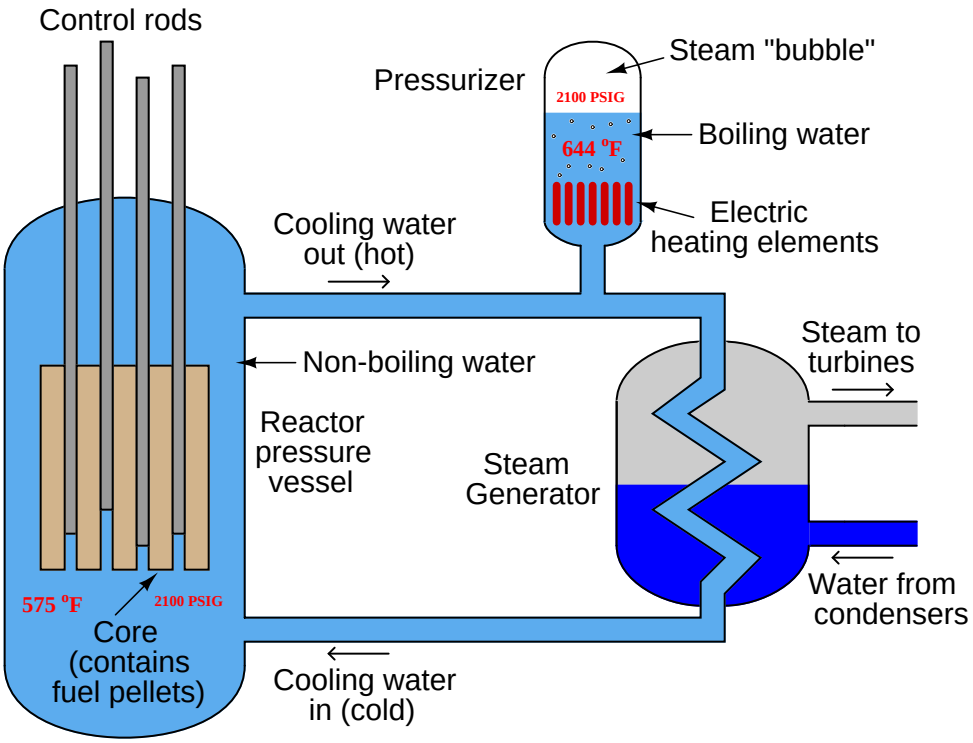
Heat applied to the bulb literally "boils" the liquid inside until its pressure reaches the equilibrium point with temperature. As the bulb's temperature increases, so does the pressure throughout the sealed system, indicating at the operator display where a bellows (or some other pressure-sensing element) moves a pointer across a calibrated scale.

The only difference between the two filled-bulb thermometers shown in the illustration is which end of the instrument is warmer. The Class IIA system on the left (where liquid fills the pressure-indicating element) is warmer at the bulb than at the indicating end. The Class IIB system on the right (where vapor fills the indicating bellows) has a cooler bulb than the indicating bellows. The long length and small internal diameter of the connecting tube prevents any substantial heat transfer from one end of the system to the other, allowing the sensing bulb to easily be at a different temperature than the indicating bellows. Both types of Class II thermometers work the same[52], the indicated pressure being a strict function of the bulb's temperature where the liquid and vapor coexist in equilibrium.

[52] Class IIA systems do suffer from *elevation error* where the indicator may read a higher or lower temperature than it should due to hydrostatic pressure exerted by the column of liquid inside the tube connecting the indicator to the sensing bulb. Class IIB systems do not suffer from this problem, as the gas inside the tube exerts no pressure over an elevation.

Nuclear reactor pressurizers

Nuclear reactors using pressurized water as the moderating and heat-transfer medium must maintain the water coolant in liquid form despite the immense heat output of the reactor core, to avoid the formation of steam bubbles within the reactor core which could lead to destructive "hot spots" inside the reactor. The following diagram shows a simplified[53] pressurized water reactor (PWR) cooling system:



In order to maintain a liquid-only cooling environment for the reactor core, the water is held at a pressure too high for boiling to occur inside the reactor vessel. Typical operating conditions for a pressurized water reactor are 575 ^{o}F and 2100 PSIG. A steam table shows the boiling point of water at 2100 PSIG to be over 640 ^{o}F, which means the water inside the reactor cannot boil if the reactor only operates at 575 ^{o}F. Referencing the phase diagram for water, the operating point of the reactor core is maintained *above* the liquid/vapor phase transition line by an externally supplied pressure.

This excess pressure comes from a device in the primary coolant loop called a *pressurizer*. Inside the pressurizer is an array of immersion-style electric heater elements. The pressurizer is essentially an electric boiler, purposely boiling the water inside at a temperature greater[54] than that reached

[53] Circulation pumps and a multitude of accessory devices are omitted from this diagram for the sake of simplicity.

[54] This is another example of an important thermodynamic concept: the distinction between *heat* and *temperature*. While the temperature of the pressurizer heating elements exceeds that of the reactor core, the total heat output of course does not. Typical comparative values for pressurizer power versus reactor core power are 1800 kW versus

by the reactor core itself. For the example figure of 2100 PSIG, the pressurizer elements would have to operate at a temperature of approximately 644 ^{o}F to maintain a boiling condition inside the pressurizer.

By maintaining the water temperature inside the pressurizer greater than at the reactor core, the water flowing through the reactor core literally *cannot* boil. The water/vapor equilibrium inside the pressurizer is a system with one degree of freedom (pressure and temperature linked), while the water-only environment inside the reactor core has two degrees of freedom (temperature may vary to any amount below the pressurizer's temperature without water pressure changing at all). Thus, the pressurizer functions like the temperature-sensing bulb of a *gigantic* Class IIA filled-bulb thermometer, with a liquid/vapor equilibrium inside the pressurizer vessel and liquid only inside the reactor vessel and all other portions of the primary coolant loop. Reactor pressure is then controlled by the temperature inside the pressurizer, which is in turn controlled by the amount of power applied to the heating element array[55].

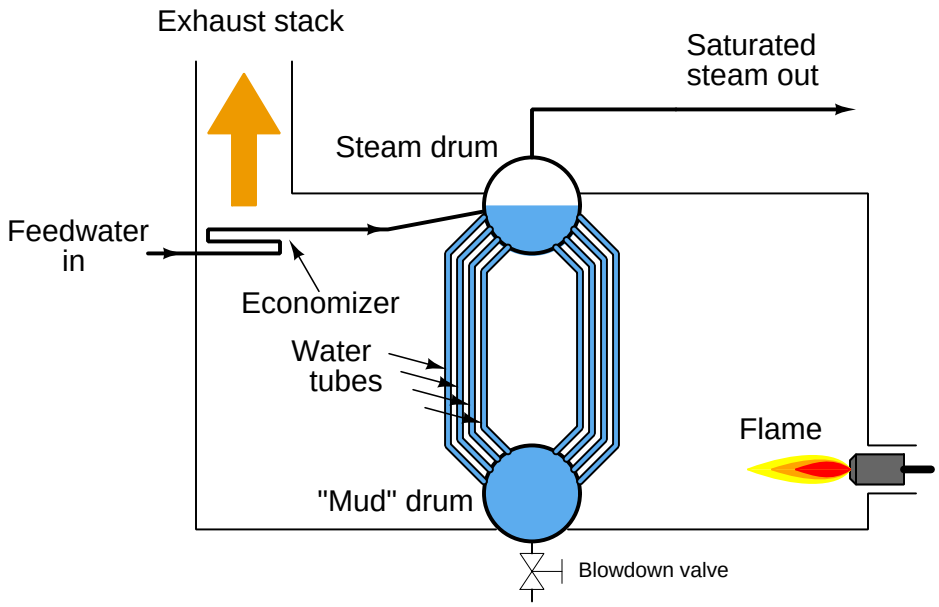
Steam boilers

Boilers in general (the nuclear reactor system previously described being just one example of a large "power" boiler) are outstanding examples of phase change applied to practical use. The purpose of a boiler is to convert water into steam, sometimes for heating purposes, sometimes as a means of producing mechanical power (through a steam engine), sometimes for chemical processes requiring pressurized steam as a reactant, sometimes for utility purposes (maintenance-related cleaning, process vessel purging, sanitary disinfection, fire suppression, etc.) or all of the above. Steam is a tremendously useful substance in many industries, so you will find boilers in use at almost every industrial facility.

3800 MW, respectively: a ratio exceeding three orders of magnitude. The pressurizer heating elements don't have to dissipate much power (compared to the reactor core) because the pressurizer is not being cooled by a forced convection of water like the reactor core is.

[55] In this application, the heaters are the *final control element* for the reactor pressure control system.

A simplified diagram of a *water tube* boiler appears here:



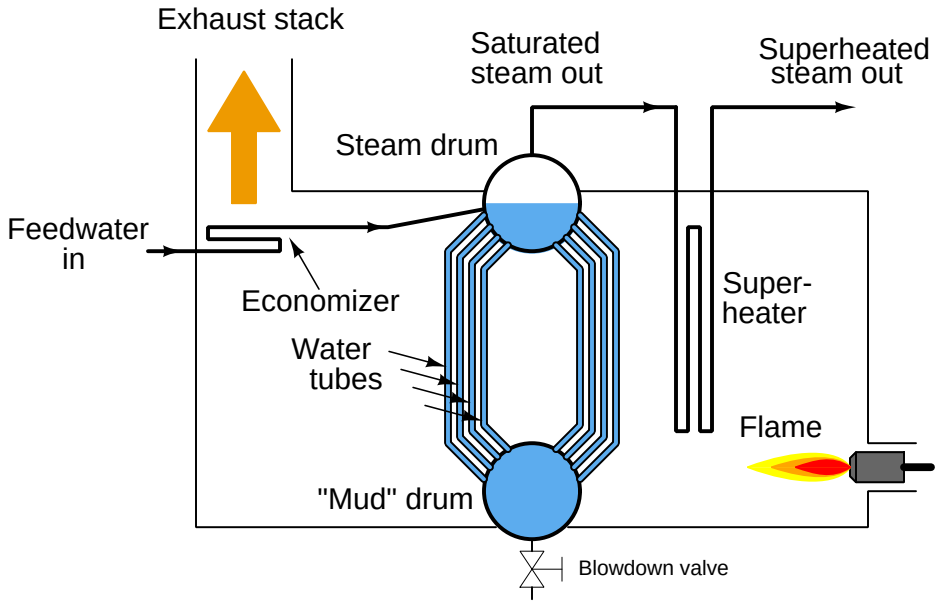
Water enters the boiler through a heat exchanger in the stack called an *economizer*. This allows cold water to be pre-heated by the warm exhaust gases before they exit the stack. After pre-heating in the economizer, the water enters the boiler itself, where water circulates by natural convection ("thermosiphon") through a set of tubes exposed to high-temperature fire. Steam collects in the "steam drum," where it is drawn off through a pipe at the top. Since this steam is in direct contact with the boiling water, it will be at the same temperature as the water, and the steam/water environment inside the steam drum represents a two-phase system with only one degree of freedom. With just a single degree of freedom, steam temperature and pressure are direct functions of each other – coordinates at a single point along the liquid/vapor phase transition line of water's phase diagram. One cannot change one variable without changing the other.

Consulting a steam table[56], you will find that the temperature required to boil water at a pressure of 120 PSIG is approximately 350 degrees Fahrenheit. Thus, the temperature of the steam drum will be fixed at 350 ^{o}F while generating steam pressure at 120 PSIG. The only way to increase pressure in that boiler is to increase its temperature, and vice-versa.

When steam is at the same temperature as the boiling water it came from, it is referred to as *saturated* steam. Steam in this form is very useful for heating and cleaning, but not as much for operating mechanical engines or for process chemistry. If saturated steam loses any temperature at all (by losing its latent heat), it immediately condenses back into water. Liquid water can cause major mechanical problems inside steam engines (although "wet" steam works wonderfully well as a cleaning agent!), and so steam must be made completely "dry" for some process applications.

[56] Since the relationship between saturated steam pressure and temperature does not follow a simple mathematical formula, it is more practical to consult published tables of pressure/temperature data for steam. A great many engineering manuals contain steam tables, and in fact entire books exist devoted to nothing but steam tables.

The way this is done is by a process known as *superheating*. If steam exiting the steam drum of a boiler is fed through another heat exchanger inside the firebox so it may receive more heat, its temperature will rise beyond the saturation point. This steam is now said to be *superheated*:



Superheated steam is absolutely dry, containing no liquid water at all. It is therefore safe to use as a fluid medium for engines (piston and turbine alike) and as a process reactant where liquid water is not tolerable. The difference in temperature between superheated steam and saturated steam at any given pressure is the amount of *superheat*. For example, if saturated steam at 350 degrees Fahrenheit and 120 PSI drawn from the top of the steam drum in a boiler is heated to a higher temperature of 380 degrees Fahrenheit (at the same pressure of 120 PSI), it is said to have 30 degrees (Fahrenheit) of superheat.

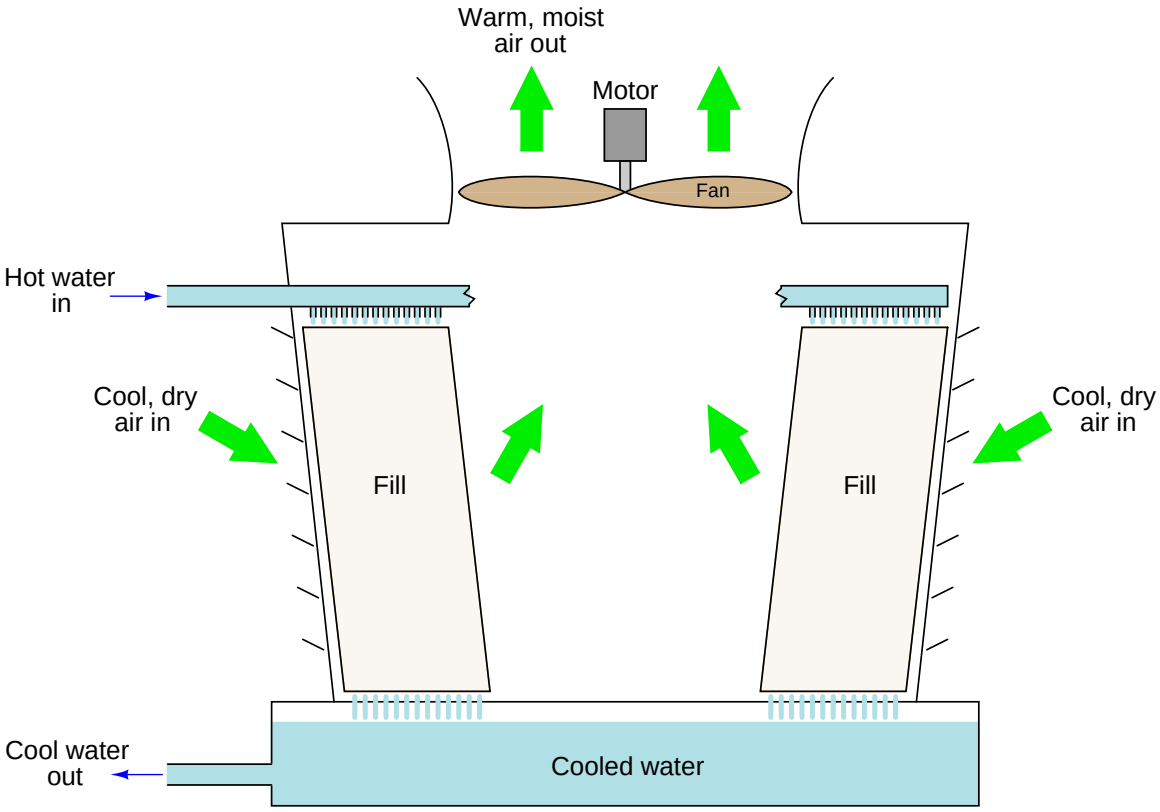
Fruit crop freeze protection

An interesting application of phase changes and latent heat occurs in agriculture. Fruit growers, needing to protect their budding crop from the damaging effects of a late frost, will spray water over the fruit trees to maintain the sensitive buds above freezing temperature. As cold air freezes the water, the water's latent heat of fusion prevents the temperature at the ice/water interface from dropping below 32 degrees Fahrenheit. So long as liquid water continues to spray over the trees, the buds' temperature *cannot* fall below freezing. Indeed, the buds cannot even freeze in this condition, because once they cool down to the freezing point, there will be no more temperature difference between the freezing water and the buds. With no difference of temperature, no heat will transfer out of the buds. With no heat loss, water inside the buds cannot change phase from liquid to solid (ice) even if held at the freezing point for long periods of time, thus preventing freeze damage[57]. Only if the buds are exposed to cold air (below the freezing point), or the water turns completely to ice and no longer holds stable at the freezing point, can the buds themselves ever freeze solid.

[57] An experiment illustrative of this point is to maintain an ice-water mixture in an open container, then to insert a sealed balloon containing liquid water into this mixture. The water inside the balloon will eventually equalize in temperature with the surrounding ice-water mix, but it will not itself freeze. Once the balloon's water reaches 0 degrees Celsius, it stops losing heat to the surrounding ice-water mix, and therefore cannot make the phase change to solid form.

Evaporative cooling towers

A very common use of a liquid-to-vapor phase change is for cooling purposes: taking hot water and mechanically forcing that hot water to evaporate in order to remove large quantities of heat energy from the water, thus cooling it to a lower temperature. Devices called *evaporative cooling towers* accomplish this task by causing ambient air to flow past droplets of water. As the rising air contacts the falling water droplets, some of the water is forced to evaporate, the heat required of this evaporation being provided by sensible heat within the remaining liquid water. As a result, the still-liquid water must cool in temperature as it gives up heat energy to the newly-formed water vapor.



Smaller evaporative cooling towers use fans to force air upward through the tower, employing inert "fill" material to provide large amounts of surface area for the liquid water and the air to contact each other. Some large evaporative cooling towers are self-drafting, the heat of the water providing enough convective force to the air that no fans are needed.

The following photograph shows a pair of induced-draft evaporative cooling towers used at a beer brewery:

This next photograph shows a forced-draft evaporative cooling tower used at a coal-fired electric power plant. Note the large air fans located around the circumference of the cooling tower, blowing cool air into the tower from outside. This fan placement eliminates problems arising from having the fan blades and motor located within the moist air stream:

3.5 Unit conversions and physical constants

Converting between disparate units of measurement is the bane of many science students. The problem is worse for students in the United States of America, who must work with British ("Customary") units such as the pound, the foot, the gallon, etc. World-wide adoption of the metric system would go a long way toward alleviating this problem, but until then it is important for students to master the art of unit conversions[58].

It is possible to convert from one unit of measurement to another by use of tables designed expressly for this purpose. Such tables usually have a column of units on the left-hand side and an identical row of units along the top, whereby one can look up the conversion factor to multiply by to convert from any listed unit to any other listed unit. While such tables are undeniably simple to use, they are practically impossible to memorize.

A better way to convert between different units is shown in the next subsection.

[58] An interesting point to make here is the United States did get something right when they designed their monetary system of dollars and cents. This is essentially a *metric* system of measurement, with 100 cents per dollar. The founders of the USA wisely decided to avoid the utterly confusing denominations of the British, with their pounds, pence, farthings, shillings, etc. The denominations of penny, dime, dollar, and eagle ($10 gold coin) comprised a simple power-of-ten system for money. Credit goes to France for first adopting a metric system of general weights and measures as their national standard.

3.5.1 Unity fractions

An important principle in the physical sciences is to closely track all units of measurement when performing calculations of physical quantities. This practice is generally referred to as *dimensional analysis*. A brief example of dimensional analysis is shown here, used to analyze the simple formula $P = IV$ which describes the amount of power dissipated by an electrical load (P) given its current (I) and voltage drop (V):

$$P = IV$$

Substituting units of measurement for each variable in this formula (i.e. Watts for power, Amperes for current, and Volts for voltage), using bracket symbols to denote these as unit abbreviations rather than variables, we get this result:

$$[\text{Watts}] = [\text{Amperes}] \times [\text{Volts}] \qquad \text{or} \qquad [\text{W}] = [\text{A}][\text{V}]$$

If we happen to know that "Watts" is equivalent to Joules of energy dissipated per second, and that "Amperes" is equivalent to Coulombs of charge motion per second, and that "Volts" is equivalent to Joules of energy per Coulomb of electrical charge, we may substitute these units of measurement into the formula and see that the unit of "Coulomb" cancels just like identical variables in the numerator and denominator of multiplied fractions:

$$\left[\frac{\text{Joules}}{\text{Seconds}}\right] = \left[\frac{\cancel{\text{Coulombs}}}{\text{Seconds}}\right] \times \left[\frac{\text{Joules}}{\cancel{\text{Coulombs}}}\right] \qquad \text{or} \qquad \left[\frac{\text{J}}{\text{s}}\right] = \left[\frac{\cancel{\text{C}}}{\text{s}}\right] \left[\frac{\text{J}}{\cancel{\text{C}}}\right]$$

As it so happens, dimensional analysis may be employed in a similar manner to convert between different units of measurement via a technique I like to call *unity fractions*.

This technique involves setting up the original quantity as a fraction, then multiplying by a series of fractions having *physical* values of unity (1) so that by multiplication the original value does not change, but the units do. Let's take for example the conversion of quarts into gallons, an example of a fluid volume conversion:

$$35 \text{ qt} = ??? \text{ gal}$$

Now, most people know there are four quarts in one gallon, and so it is tempting to simply divide the number 35 by four to arrive at the proper number of gallons. However, the purpose of this example is to show you how the technique of unity fractions works, not to get an answer to a problem.

To demonstrate the unity fraction technique, we will first write the original quantity as a fraction, in this case a fraction with 1 as the denominator:

$$\frac{35 \text{ qt}}{1}$$

Next, we will multiply this fraction by another fraction having a *physical* value of unity (1) so that we do not alter[59] the quantity. This means a fraction comprised of equal measures in the numerator and denominator, but having different units of measurement. This "unity" fraction must be arranged in such a way that the undesired unit cancels out and leaves only the desired unit(s) in the product. In this particular example, we wish to cancel out quarts and end up with gallons, so we must arrange a fraction consisting of quarts and gallons having equal quantities in numerator and denominator, such that quarts will cancel and gallons will remain:

$$\left(\frac{35 \text{ qt}}{1}\right)\left(\frac{1 \text{ gal}}{4 \text{ qt}}\right)$$

Now we see how the unit of "quarts" cancels from the numerator of the first fraction and the denominator of the second ("unity") fraction, with "gallons" being the only unit remaining:

$$\left(\frac{35 \text{ } \cancel{\text{qt}}}{1}\right)\left(\frac{1 \text{ gal}}{4 \text{ } \cancel{\text{qt}}}\right) = 8.75 \text{ gal}$$

One reason this conversion technique is so powerful is it allows one to perform the largest range of unit conversions while memorizing the smallest possible set of conversion factors. Another reason is that when written out like this, showing canceled units, anyone examining this work will be able to tell how and why this conversion is correct. In other words, the "unity fraction" technique literally *teaches* how to convert between units using a bare minimum of memorized equivalent measures.

[59] A basic mathematical identity is that multiplication of any quantity by 1 does not change the value of that original quantity. If we multiply some quantity by a fraction having a physical value of 1, no matter how strange-looking that fraction may appear, the value of the original quantity will be left intact. The goal here is to judiciously choose a fraction with a physical value of 1 but with its units of measurement so arranged that we cancel out the original quantity's unit(s) and replace them with the units we desire.

Here is a set of six equal volumes, each one expressed in a different unit of measurement:

1 gallon (gal) = 231.0 cubic inches (in^3) = 4 quarts (qt) = 8 pints (pt) = 128 fluid ounces (fl. oz.) = 3.7854 liters (l)

Since all six of these quantities are physically equal, it is possible to build a "unity fraction" out of any two, to use in converting any of the represented volume units into any of the other represented volume units. Shown here are a few different volume unit conversion problems, using unity fractions built only from these factors (all canceled units shown using strike-out lines):

40 gallons converted into fluid ounces (using *128 fl. oz. = 1 gal* in the unity fraction):

$$\left(\frac{40 \cancel{\text{gal}}}{1}\right)\left(\frac{128 \text{ fl. oz.}}{1 \cancel{\text{gal}}}\right) = 5120 \text{ fl. oz}$$

5.5 pints converted into cubic inches (using *231 in^3 = 8 pt* in the unity fraction):

$$\left(\frac{5.5 \cancel{\text{pt}}}{1}\right)\left(\frac{231 \text{ in}^3}{8 \cancel{\text{pt}}}\right) = 158.8 \text{ in}^3$$

1170 liters converted into quarts:

$$\left(\frac{1170 \cancel{\text{l}}}{1}\right)\left(\frac{4 \text{ qt}}{3.7854 \cancel{\text{l}}}\right) = 1236 \text{ qt}$$

By contrast, if we were to try to memorize a 6×6 table giving conversion factors between *any two* of six volume units, we would have to commit 30 different conversion factors to memory! Clearly, the ability to set up "unity fractions" is a much more memory-efficient and practical approach.

This economy of conversion factors is very useful, and may also be extended to cases where linear units are raised to powers to represent two- or three-dimensional quantities. To illustrate, suppose we wished to convert 5.5 pints into cubic *feet* instead of cubic *inches*: with no conversion equivalence between pints and cubic feet included in our string of six equalities, what do we do?

We should know the equality between inches and feet: there are exactly 12 inches in 1 foot. This simple fact may be applied by incorporating *another* unity fraction in the original problem to convert cubic inches into cubic feet. We will begin by including another unity fraction comprised of 12 inches and 1 foot,just to see how this might work:

5.5 pints converted into cubic feet (*our first attempt!*):

$$\left(\frac{5.5 \cancel{\text{pt}}}{1}\right)\left(\frac{231 \text{ in}^3}{8 \cancel{\text{pt}}}\right)\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right) = 13.23 \text{ in}^2 \cdot \text{ft}$$

Unfortunately, this yields a non-sensical unit of square inch-feet. Even though $\frac{1 \text{ ft}}{12 \text{ in}}$ is a valid unity fraction, it does not *completely* cancel out the unit of cubic inches in the numerator of the first unity fraction. Instead, the unit of "inches" in the denominator of the unity fraction merely cancels out one of the "inches" in the "cubic inches" of the previous fraction's numerator, leaving square inches (in^2). What we need for full cancellation of cubic inches is a unity fraction relating *cubic* feet to *cubic* inches. We can get this, though, simply by *cubing* the $\frac{1 \text{ ft}}{12 \text{ in}}$ unity fraction:

5.5 pints converted into cubic feet (*our second attempt!*):

$$\left(\frac{5.5 \text{ pt}}{1}\right)\left(\frac{231 \text{ in}^3}{8 \text{ pt}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^3$$

Distributing the third power to the interior terms of the last unity fraction:

$$\left(\frac{5.5 \text{ pt}}{1}\right)\left(\frac{231 \text{ in}^3}{8 \text{ pt}}\right)\left(\frac{1^3 \text{ ft}^3}{12^3 \text{ in}^3}\right)$$

Calculating the values of 1^3 and 12^3 inside the last unity fraction, then canceling units and solving:

$$\left(\frac{5.5 \cancel{\text{pt}}}{1}\right)\left(\frac{231 \cancel{\text{in}}^3}{8 \cancel{\text{pt}}}\right)\left(\frac{1 \text{ ft}^3}{1728 \cancel{\text{in}}^3}\right) = 0.0919 \text{ ft}^3$$

Now the answer makes sense: a volume expressed in units of cubic feet.

Once again, this unit conversion technique shows its power by minimizing the number of conversion factors we must memorize. We need not memorize how many cubic inches are in a cubic foot, or how many square inches are in a square foot, if we know how many linear inches are in a linear foot and we simply let the fractions "tell" us whether a power is needed for unit cancellation.

Unity fractions are also useful when we need to convert more than one unit in a given quantity. For example, suppose a flowmeter at a wastewater treatment facility gave us a flow measurement of 205 cubic feet per minute but we needed to convert this expression of water flow into units of cubic yards per day. Observe the following unit-fraction conversion to see how unity fractions serve the purpose of converting cubic feet into cubic yards, and minutes into days (by way of minutes to hours, and hours to days):

$$\left(\frac{205\ \cancel{\text{ft}}^3}{\cancel{\text{min}}}\right)\left(\frac{1^3\ \text{yd}^3}{3^3\ \cancel{\text{ft}}^3}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}}\right)\left(\frac{24\ \cancel{\text{hr}}}{1\ \text{day}}\right) = 10933.3\ \text{yd}^3/\text{day}$$

Note how the only units left un-canceled on the left-hand side of the "equals" symbol are cubic yards (yd^3) and days, which therefore become the units of measurement for the final result.

A major caveat to this method of converting units is that the units must be *directly proportional* to one another, since this multiplicative conversion method is really nothing more than an exercise in mathematical proportions. Here are some examples (but not an exhaustive list!) of conversions that *cannot* be performed using the "unity fraction" method:

- Absolute / Gauge pressures, because one scale is *offset* from the other by 14.7 PSI (atmospheric pressure).
- Celsius / Fahrenheit, because one scale is *offset* from the other by 32 degrees.
- Wire diameter / gauge number, because gauge numbers grow smaller as wire diameter grows larger (inverse proportion rather than direct) and because there is no proportion relating the two.
- Power / decibels, because the relationship is logarithmic rather than proportional.

The following subsections give sets of physically equal quantities, which may be used to create unity fractions for unit conversion problems. Note that only those quantities shown in the same line (separated by = symbols) are truly equal to each other, not quantities appearing in different lines!

3.5.2 Conversion formulae for temperature

Note: all of the conversion factors given for temperature are *exact*, not approximations.

^{o}F = (^{o}C)(9/5) + 32
^{o}C = (^{o}F − 32)(5/9)
^{o}R = ^{o}F + 459.67
K = ^{o}C + 273.15

3.5.3 Conversion factors for distance

Note: all of the conversion factors given for distance are *exact*, not approximations.

1 inch (in) = **2.54 centimeters** (cm)
1 foot (ft) = **12 inches** (in)
1 yard (yd) = **3 feet** (ft)
1 mile (mi) = **5280 feet** (ft)

3.5.4 Conversion factors for volume

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 gallon (gal) = 231.0 cubic inches (in^3) = **4 quarts** (qt) = **8 pints** (pt) = **16 cups** = **128 fluid ounces** (fl. oz.) = 3.7854 liters (l)

1 milliliter (ml) = **1 cubic centimeter** (cm^3)

3.5.5 Conversion factors for velocity

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 mile per hour (mi/h) = **88 feet per minute** (ft/m) = 1.46667 feet per second (ft/s) = 1.60934 kilometer per hour (km/h) = 0.44704 meter per second (m/s) = 0.868976 knot (knot – international)

3.5.6 Conversion factors for mass

1 pound-mass (lbm) = 0.4535924 kilogram (kg) = 0.031081 slugs

3.5.7 Conversion factors for force

1 pound-force (lbf) = 4.448222 newtons (N)

1 kilogram-force (kgf) = **9.80665 newtons** (N)

3.5.8 Conversion factors for area

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 acre = 43560 square feet (ft^2) = 4840 square yards (yd^2) = 4046.86 square meters (m^2)

3.5.9 Conversion factors for pressure (either all gauge or all absolute)

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 pounds per square inch (PSI) = 2.03602 inches of mercury at 0 ^{o}C (in. Hg) = 27.6799 inches of water at 4 ^{o}C (in. W.C.) = 6.894757 kilopascals (kPa) = 0.06894757 bar

1 bar = 100 kilopascals (kPa) = 14.504 pounds per square inch (PSI)

1 meter of water at 4 ^{o}C (m W.C.) = **9.80665 kilopascals** (kPa)

3.5.10 Conversion factors for pressure (absolute pressure units only)

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 standard atmosphere (Atm) = 14.7 pounds per square inch absolute (PSIA) = **101.325 kilopascals absolute** (kPaA) = **1.01325 bar absolute** = 760 millimeters of mercury absolute (mmHgA) = 760 torr (torr)

3.5.11 Conversion factors for energy or work

1 British thermal unit (Btu – "International Table") = 251.996 calories (cal – "International Table") = 1055.06 joules (J) = 1055.06 watt-seconds (W-s) = 0.293071 watt-hour (W-hr) = 1.05506 x 10^{10} ergs (erg) = 778.169 foot-pound-force (ft-lbf)

3.5.12 Conversion factors for power

Note: all conversion factors shown in **bold** type are *exact*, not approximations.

1 horsepower = 550 foot-pounds per second (ft-lbf/s) = 745.7 watts (W) = 2544.43 British thermal units per hour (Btu/h) = 0.0760181 boiler horsepower (hp – boiler)

Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Dimensional analysis

A powerful technique called *dimensional analysis* helps us check for errors when evaluating formulae related to physical systems. This entails writing units of measurement in place of variables in physical formulae, to see how units cancel and combine to form the appropriate unit(s) of measurement for the result.

For example, let's take the familiar power formula used to calculate power in a simple DC electric circuit:

$$P = IV$$

Where,
P = Power (Watts)
I = Current (Amperes)
V = Voltage (Volts)

Each of the units of measurement in the above formula (Watt, Ampere, Volt) are actually comprised of more fundamental physical units. One Watt of power is one Joule of energy exchanged per second of time. One Ampere of current is one Coulomb of electric charge moving per second. One Volt of potential is one Joule of energy per Coulomb of electric charge. When we write the equation showing these units in their proper orientations, we see that the result (power in Watts, or Joules per second) actually does agree with the units for Amperes and Volts because the unit of electric charge (Coulombs) cancels out. In dimensional analysis we customarily distinguish unit symbols from variables by using non-italicized letters and surrounding each one with square brackets:

$$P = IV$$

$$[\text{Watts}] = [\text{Amperes}] \times [\text{Volts}] \qquad \text{or} \qquad [\text{W}] = [\text{A}][\text{V}]$$

$$\left[\frac{\text{Joules}}{\text{seconds}}\right] = \left[\frac{\text{Coulombs}}{\text{seconds}}\right] \times \left[\frac{\text{Joules}}{\text{Coulombs}}\right] \qquad \text{or} \qquad \left[\frac{\text{J}}{\text{s}}\right] = \left[\frac{\text{C}}{\text{s}}\right] \left[\frac{\text{J}}{\text{C}}\right]$$

Dimensional analysis gives us a way to "check our work" when setting up new formulae for physics- and chemistry-type problems.

4.2 The International System of Units

The very purpose of physics is to quantitatively describe and explain the physical world in as few terms as possible. This principle extends to units of measurement as well, which is why we usually find different units used in science actually defined in terms of more fundamental units. The *watt*, for example, is one joule of energy transferred per second of time. The joule, in turn, is defined in terms of three base units, the kilogram, the meter, and the second:

$$[J] = \frac{[\text{kg}][\text{m}^2]}{[\text{s}^2]}$$

Within the metric system of measurements, an international standard exists for which units are considered fundamental and which are considered "derived" from the fundamental units. The modern standard is called *SI*, which stands for *Système International*. This standard recognizes seven fundamental, or *base* units, from which all others are derived[1]:

Physical quantity	**SI unit**	**SI symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

An older standard existed for base units, in which the *centimeter*, *gram*, and *second* comprised the first three base units. This standard is referred to as the *cgs* system, in contrast to the SI system[2]. You will still encounter some derived cgs units used, including the *poise* and the *stokes* (both used to express fluid viscosity). Then of course we have the *British engineering system* which uses such wonderful[3] units as feet, pounds, and (thankfully) seconds. Despite the fact that the majority of the world uses the metric (SI) system for weights and measures, the British system is sometimes referred to as the *Customary* system.

[1] The only exception to this rule being units of measurement for angles, over which there has not yet been full agreement whether the unit of the *radian* (and its solid counterpart, the *steradian*) is a base unit or a derived unit.

[2] The older name for the SI system was "MKS," representing meters, kilograms, and seconds.

[3] I'm noting my sarcasm here, just in case you are immune to my odd sense of humor.

4.3 Newton's Laws of Motion

These laws were formulated by the great mathematician and physicist Isaac Newton (1642-1727). Much of Newton's thought was inspired by the work of an individual who died the same year Newton was born, Galileo Galilei (1564-1642).

1. An object at rest tends to stay at rest; an object in motion tends to stay in motion
2. The acceleration of an object is directly proportional to the net force acting upon it and inversely proportional to the object's mass
3. Forces between objects always exist in equal and opposite pairs

Newton's first law may be thought of as the *law of inertia*, because it describes the property of inertia that all objects having mass exhibit: resistance to change in velocity. This law is quite counter-intuitive for many people, who tend to believe that objects require continual force to keep moving. While this is true for objects experiencing friction, it is not for ideal (frictionless) motion. This is why satellites and other objects in space continue to travel with no mode of propulsion: they simply "coast" indefinitely on their own inertia because there is no friction in space to dissipate their kinetic energy and slow them down.

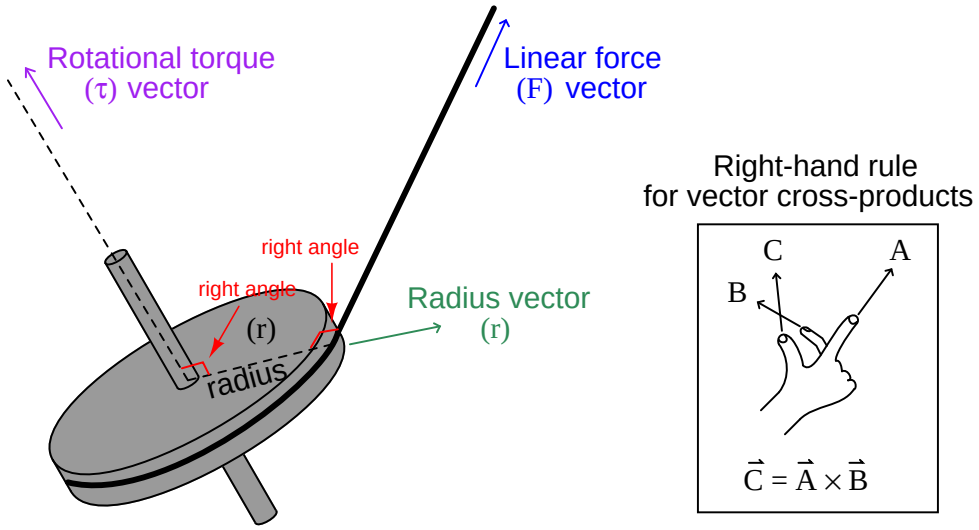
Newton's second law is the verbal equivalent of the force/mass/acceleration formula: $F = ma$. This law elaborates on the first, in that it mathematically relates force and motion in a very precise way. For a frictionless object, the *change in velocity* (i.e. its *acceleration*) is proportional to force. This is why a frictionless object may continue to move without any force applied: once moving, force would only be necessary for continued acceleration. If zero force is applied, the acceleration will likewise be zero, and the object will maintain its velocity indefinitely (again, assuming no friction at work).

Newton's third law describes how forces always exist in *pairs* between two objects. The rotating blades of a helicopter, for example, exert a downward force on the air (accelerating the air), but the air in turn exerts an upward force on the helicopter (suspending it in flight). A spider hanging on the end of a thread exerts a downward force (weight) on the thread, while the thread exerts an upward force of equal magnitude on the spider (tension). Force pairs are always equal in magnitude but opposite in direction.

4.4 Rotational motion

Rotational motion may be quantified in terms directly analogous to linear motion, using different symbols and units.

The rotational equivalent of linear *force* (F) is *torque* (τ). Linear force and rotational torque are both vector quantities, mathematically related to one another by the radial distance separating the force vector from the centerline of rotation. To illustrate with a string pulling on the circumference of a wheel:



This relationship may be expressed mathematically as a *vector cross-product*, where the vector directions are shown by the *right-hand rule* (the first vector $\vec{r}$ is the direction of the index finger, the second vector $\vec{F}$ is the direction of the middle finger, and the product vector $\vec{\tau}$ is the direction of the thumb, with all three vectors perpendicular to each other):

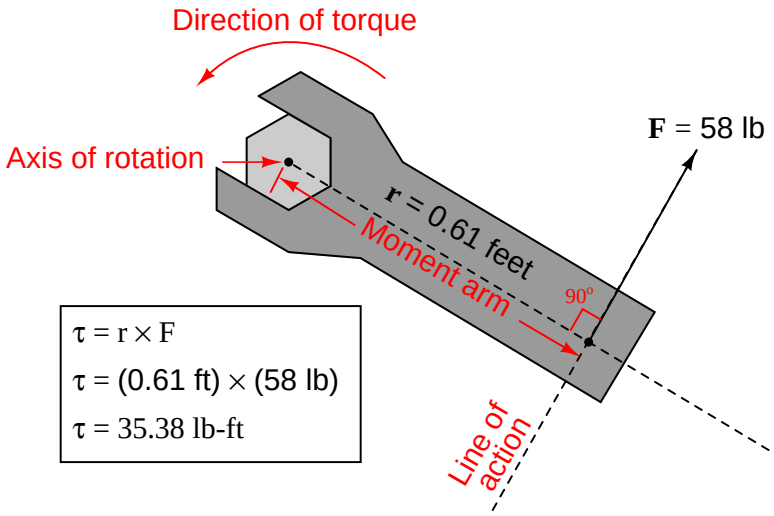
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Labeling force, radius, and torque as vectors is the formally correct way of noting the variables in a mechanical system such as this, and is the way college students studying physics typically learn the calculation of torque. In less academic settings, the force vector ($\vec{F}$) is typically labeled as a force along the *line of action*, and the radius vector ($\vec{r}$) is called the *moment arm*, with the line of action and moment arm always being perpendicular to each other.

The proper unit of measurement for torque is the product of the force unit and distance unit. In the metric system, this is customarily the *Newton-meter* (N-m). In the British system, this is customarily the *foot-pound* (ft-lb) or alternatively the *pound-foot* (lb-ft). Note that while these are the exact same *units* as those used to express work, they are not the same types of *quantities*. Torque is a vector cross-product, while work is a *dot-product* ($W = \vec{F} \cdot \vec{x}$). The cross-product of two vectors

is always another vector[4], while the dot-product of two vectors is always a scalar (direction-less) quantity. Thus, torque always has a direction, whereas work or energy does not.

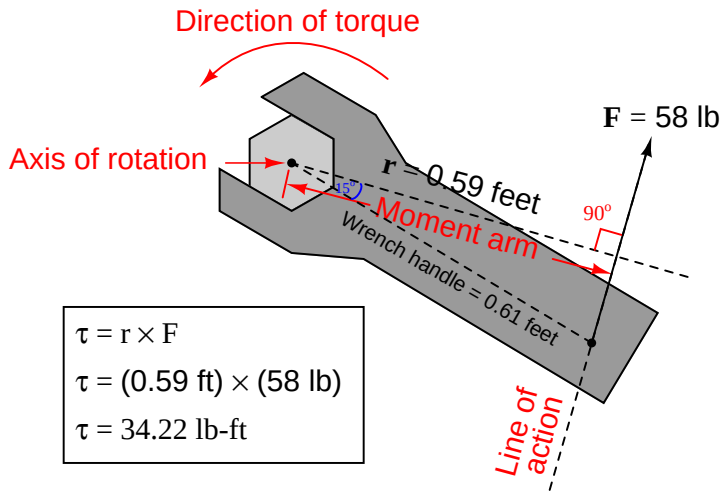
An example calculation applied to a hand wrench turning a bolt appears here:



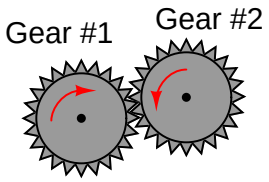
With the radius and force vectors at right angles to each other, torque is simply the product of both. In many non-academic settings, torque is calculated this way as a scalar quantity, with the direction of rotation determined by observation rather than by strict adherence to the right-hand rule of vector cross products. In this example, we see the magnitude of torque as the simple product of 58 pounds force and 0.61 feet of moment arm (35.38 lb-ft of torque), with the torque direction obviously counter-clockwise as viewed from the head of the bolt.

[4]Technically, it is a *pseudovector*, because it does not exhibit all the same properties of a true vector, but this is a mathematical abstraction far beyond the scope of this book!

If we apply the same force to the wrench handle at a different angle (not perpendicular to the handle), the resulting torque will be less. The radius vector (moment arm), however, will still remain perpendicular to the force vector (line of action) – it just decreases in length. To determine the placement of the radius vector, all one must do is draw a line straight from the axis of rotation perpendicular to the line of action, then use trigonometry to calculate its magnitude:



A very practical example of torque is in the action of meshing gears, transferring mechanical power from one gear to another. Each gear effectively acts as a wheel, the point of contact between gear teeth acting to transfer force perpendicular to the radius of each gear (wheel). Thus, torque applied to one gear becomes a linear force at the meshing teeth, which translates into another torque at the second gear:



The ratio of torques between two meshing gears is equal to the ratio of gear teeth:

$$\frac{\tau_1}{\tau_2} = \frac{n_1}{n_2}$$

Where,

τ_1 = Torque of first gear
τ_2 = Torque of second gear
n_1 = Number of teeth on first gear
n_2 = Number of teeth on second gear

For example, if a small gear having 28 teeth meshes with a larger gear having 75 teeth, the torque multiplication factor from the small gear to the large gear will be 75:28, or 2.679 to 1. A torque of 40 lb-ft applied to the small gear will result in a torque of 107.1 lb-ft or torque generated at the large gear. This ratio of gear teeth is called the *gear ratio*.

As gears multiply torque (τ), they divide rotational speed (ω). Thus, the 75:28 tooth gear set creates a multiplication of torque from the small gear to the large gear, and an identical *reduction ratio* of speed from the small gear to the large gear. Given this ratio, the small gear will have to be turned 2.679 revolutions in order to make the large gear turn just one revolution.

We may express gear speeds as another ratio of gear teeth, reciprocated in relation to torque:

$$\frac{\omega_1}{\omega_2} = \frac{n_2}{n_1}$$

Where,

ω_1 = Rotational speed of first gear
ω_2 = Rotational speed of second gear
n_1 = Number of teeth on first gear
n_2 = Number of teeth on second gear

In a set of meshed gears, the smaller gear will have the least torque and the greatest speed; the larger gear will have the greatest torque and the least speed.

This is precisely how gear sets are used in industry: to transform torque and speed in mechanical power systems. The complementary effects of a gear set on torque and speed is analogous to the complementary effects that a transformer has on AC voltage and current: a *step-up transformer* (having more turns of wire in the secondary coil than in the primary coil) will multiply voltage but reduce (divide) current, both by the same turns ratio.

Every quantity of force and motion which may be expressed in linear form has a rotational equivalent. As we have seen, torque (τ) is the rotational equivalent of force (F). The following table contrasts equivalent quantities for linear and rotational motion (all units are metric, shown in *italic* font):

Linear quantity, symbol, and unit	**Rotational quantity, symbol, and unit**
Force (F) *N*	Torque (τ) *N-m*
Linear displacement (x) *m*	Angular displacement (θ) *radian*
Linear velocity (v) *m/s*	Angular velocity (ω) *rad/s*
Linear acceleration (a) m/s^2	Angular acceleration (α) rad/s^2
Mass (m) *kg*	Moment of Inertia (I) $kg\text{-}m^2$

Familiar equations for linear motion have rotational equivalents as well. For example, Newton's Second Law of motion states, "The acceleration of an object is directly proportional to the net force acting upon it and inversely proportional to the object's mass." We may modify this law for rotational motion by saying, "The angular acceleration of an object is directly proportional to the net torque acting upon it and inversely proportional to the object's moment of inertia." The mathematical expressions of both forms of Newton's Second Law are as follows:

$$F = ma \qquad\qquad \tau = I\alpha$$

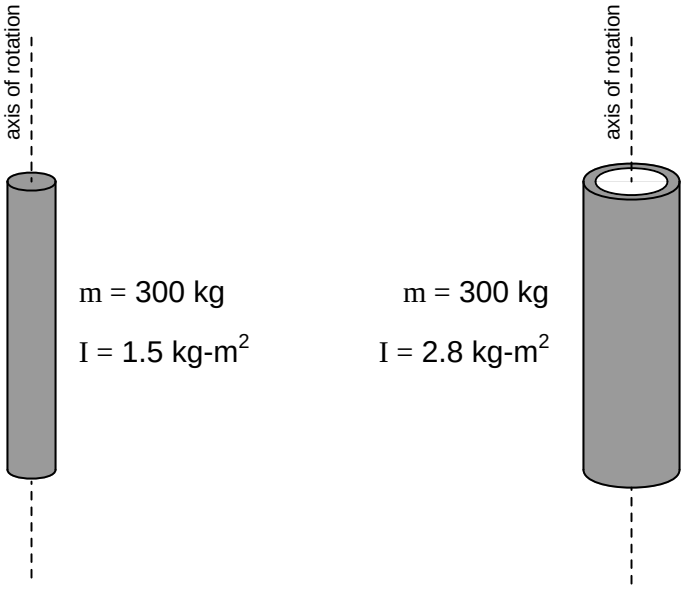
The calculus-based relationships between displacement (x), velocity (v), and acceleration (a) find parallels in the world of angular motion as well. Consider the following formula pairs, linear motion on the left and angular motion on the right:

$$v = \frac{dx}{dt} \quad \text{(Velocity as the time-derivative of displacement)} \quad \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} \quad \text{(Acceleration as the time-derivative of velocity)} \quad \alpha = \frac{d\omega}{dt}$$

$$a = \frac{d^2x}{dt^2} \quad \text{(Acceleration as the second time-derivative of displacement)} \quad \alpha = \frac{d^2\theta}{dt^2}$$

An object's "moment of inertia" represents its angular inertia (opposition to changes in rotational velocity), and is proportional to the object's mass and to the square of its radius. Two objects having the same mass will have different moments of inertia if there is a difference in the distribution of their mass relative to radius. Thus, a hollow tube will have a greater moment of inertia than a solid rod of equal mass, assuming an axis of rotation in the center of the tube/rod length:



This is why *flywheels*[5] are designed to be as wide as possible, to maximize their moment of inertia with a minimum of total mass.

The formula describing the amount of work done by a torque acting over an angular displacement is remarkably similar to the formula describing the amount of work done by a force acting over a linear displacement:

$$W = Fx \qquad\qquad W = \tau\theta$$

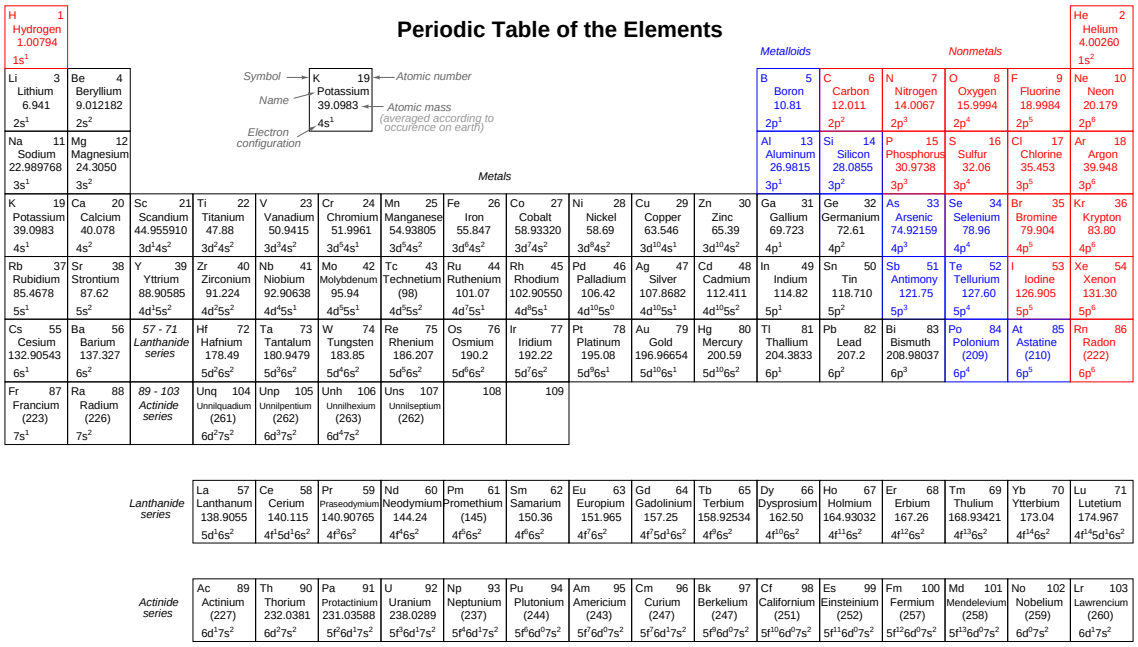
The formula describing the amount of kinetic energy possessed by a spinning object is also similar to the formula describing the amount of energy possessed by a linearly-traveling object:

$$E_k = \frac{1}{2}mv^2 \qquad\qquad E_k = \frac{1}{2}I\omega^2$$

[5] A "flywheel" is a disk on a shaft, designed to maintain rotary motion in the absence of a motivating torque for the function of machines such as piston engines. The rotational kinetic energy stored by an engine's flywheel is necessary to give the pistons energy to compress the gas prior to the power stroke, during the times the other pistons are not producing power.

4.5 Periodic Table of the Elements

All substances are comprised of various elements in various combinations and proportions. Elements may thus be thought of as the building-blocks of matter. A Periodic Table of the Elements is a table listing the known elements in order of their atomic numbers.



Multiple attributes appear for each element in the table. Two of these attributes – atomic number and atomic mass – are directly related to the number of particles in the nucleus of each atom. We will examine the table's entry for the element *potassium* (K) to explore these concepts.

Potassium has an *atomic number* (number of protons in the nucleus of each potassium atom) of 19. This number defines the element. If we were somehow to add or subtract protons from the nucleus of a potassium atom[6], it would cease being potassium and *transmutate* into a different element. Note how *every* element in the table has its own unique atomic number, and how each of these numbers is whole (no fractions or decimals).

The *atomic mass* or *atomic weight* shown for potassium is 39.0983. This quantity is the sum of protons and neutrons found in the nucleus of each potassium atom. Like the atomic number (19),

[6]The amount of energy required to rearrange particles in the nucleus for even just a single atom is *tremendous*, lying well outside the energy ranges of chemical reactions. Such energy levels are the exclusive domain of *nuclear* reactions and high-energy radiation (subatomic particles traveling at high velocity). The extremely large energy "investment" required to alter an atom's nucleus is why atomic identities are so stable. This is precisely why alchemists of antiquity utterly failed to turn lead into gold: no materials, processes, or techniques they had at their disposal were capable of the targeted energy necessary to dislodge three protons from a nucleus of lead ($_{82}$Pb) to that it would turn into a nucleus of gold ($_{79}$Au). That, and the fact the alchemists had no clue about atomic structure to begin with, made their endeavor fruitless.

we would logically expect the atomic mass to be a whole number as well, since protons and neutrons only come in whole quantities. The primary reason we see a non-whole number for potassium's atomic mass is that this table reflects the *average* atomic mass of potassium atoms as found in nature. Some potassium atoms have atomic masses greater than 39, and some have atomic masses less than 39. We know that the number of protons in every potassium atom is fixed (which is what gives potassium its elemental identity), which means the only quantity that may cause the atomic mass to vary is the number of *neutrons* in the nucleus. The most common form of potassium (^{39}K) atom possesses 19 protons and 20 neutrons in its nucleus, giving it an atomic mass of 39 (19 + 20). The next most common form of potassium found on Earth is (^{41}K), possessing 19 protons and 22 neutrons.

We refer to atoms of the same element with differing atomic masses as *isotopes*. From a chemical perspective, isotopes are identical. That is to say, they engage in the exact same chemical reactions in the exact same manner. To use potassium as an example, an atom of ^{39}K will join with a chlorine atom (Cl) to form the compound *potassium chloride* (KCl) just as readily as an atom of ^{41}K will join with a chlorine atom to form the same compound. The three isotopes of hydrogen (^{1}H, ^{2}H, and ^{3}H: hydrogen, deuterium, and tritium, respectively) are all chemically identical: all are highly flammable, combining with oxygen to create water (H_2O). However, deuterium (^{2}H) has twice the density of normal hydrogen (^{1}H), while tritium (^{3}H) has three times the density of normal hydrogen and is highly radioactive! Isotopes only differ in their mass and in their nuclear properties (such as *radioactivity*: the tendency for a nucleus to spontaneously decay, usually resulting in a loss or gain of protons that subsequently alters the identity of the decayed atom).

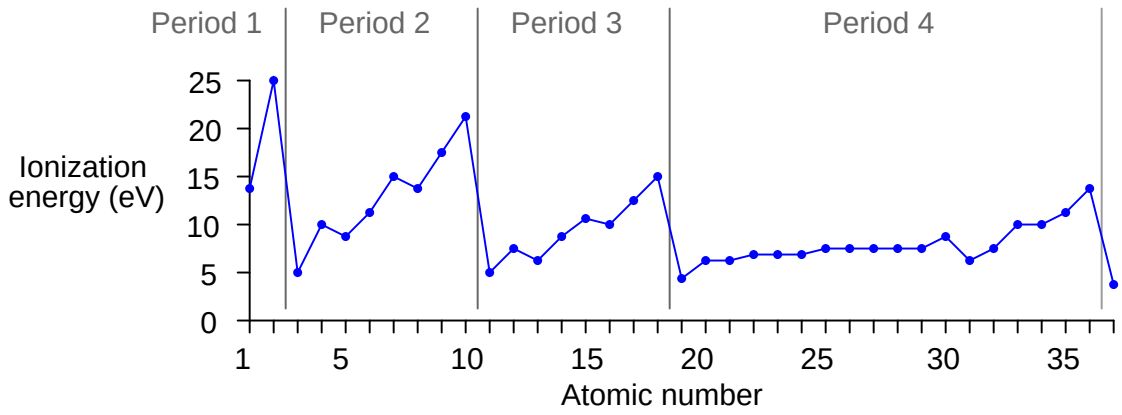
The Periodic Table is called "periodic" because its configuration reveals a repeating pattern of chemical behaviors approximately following atomic number. Horizontal rows in the table are called *periods*, while vertical columns are called *groups*. Elements in the same group (vertical column) share similar chemical reactivities – that is, they tend to engage in the same types of chemical reactions – despite having different masses and physical properties such as melting point, boiling point, etc. This *periodicity* is a function of how electrons are arranged around the nucleus of each atom, a subject we will explore in more detail later in this chapter. As mentioned previously, chemistry is the study of how atoms bond together to form molecules, and this bonding takes place through the interaction of the electrons surrounding the atoms' nuclei. It makes perfect sense, then, that the configuration of those electrons determine the chemical (bonding) properties of atoms.

Some periodic tables show the *first ionization energy* value for each element – the amount of energy required to force the first electron of an electrically balanced atom to separate from that atom – in addition to other attributes such as atomic number and atomic mass. If we note the ionization energies of the elements, reading each element in turn from left-to-right, starting with period 1 (hydrogen and helium) and progressing to subsequent periods, we see an interesting pattern:

Element	**Period**	**First ionization energy** (measured in "electron-Volts")
Hydrogen (H)	1	13.5984
Helium (He)	1	24.5874
Lithium (Li)	2	5.3917
Beryllium (Be)	2	9.3227
Boron (B)	2	8.2980
Carbon (C)	2	11.2603
Nitrogen (N)	2	14.5341
Oxygen (O)	2	13.6181
Fluorine (F)	2	17.4228
Neon (Ne)	2	21.5645
Sodium (Na)	3	5.1391
Magnesium (Mg)	3	7.6462
Aluminum (Al)	3	5.9858
Silicon (Si)	3	8.1517
Phosphorus (P)	3	10.4867
Sulfur (S)	3	10.3600
Chlorine (Cl)	3	12.9676
Argon (Ar)	3	15.7596
Potassium (K)	4	4.3407

First ionization energy represents the relative stability of the last electron balancing the electrical charge of an atom. We see from this table that 24.5874 electron-Volts of energy is needed to remove one electron from an electrically-balanced atom of helium (changing He into He^{1+}), while only 13.5984 electron-Volts of energy is required to do the same to an atom of hydrogen. This tells us the electron configuration of helium is at a lower energy (and therefore more stable) than that of hydrogen.

The ionization energies increase with increasing atomic number (with an occasional down-step) until the last column of the period is reached, and then there is a comparatively enormous down-step in energy at the first column of a new period. This pattern is clearly evident when the first ionization energies are plotted against atomic number:



This periodicity suggests that as atoms grow in atomic number, the additional electrons do not simply pile on in random fashion or in a plain and simple progression from inner orbits to outer orbits. Rather, they "fill in" a structured energy pattern, with major changes in structure at the start of each new period. More details of this structured pattern will be explored later in this chapter.

The low ionization energy values for all the "Group 1" elements (far left-hand column) suggest they are relatively easy to positively ionize, and indeed we find this to be the case through experimentation. Hydrogen, lithium, sodium, potassium, and the rest all readily become positively-charged ions upon interaction with other atoms, since their low ionization energy values means they may easily lose an electron.

The high ionization energy values for all the "Group 18" elements (far right-hand column) suggest they possess a very stable electron structure, which is also verified by experiment. These are the *noble* elements, possessing very little reactive potential[7].

Looking at the "Group 17" column, just to the left of the noble elements, we notice that they are all just one electron shy of the stable electron structure enjoyed by the noble atoms when in their electrically-balanced states. This suggests it might be easy to *add* one more electron to atoms of these elements, which (once again!) is a principle validated by experiment. Fluorine, chlorine, bromine, iodine, and even astatine[8] all readily ionize negatively, readily accepting an extra electron from surrounding atoms. As one might expect from this tendency, these elements readily bond through electrostatic attraction with the "Group 1" elements (hydrogen, lithium, sodium, potassium, etc.), each "Group 17" atom accepting an extra electron from each "Group 1" atom which readily provides it. Ordinary table salt (sodium chloride, or NaCl) is an example of a compound formed by this sort of bond.

[7] It used to be believed that these elements were completely *inert*: incapable of forming molecular bonds with other atoms. However, this is not precisely true, as some compounds are now known to integrate noble elements.

[8] All isotopes of astatine (At) are radioactive with very short half-lives, making this element difficult to isolate and study.

Thus, Group 1 and Group 17 elements are both highly reactive in a chemical sense, but in different ways. Group 1 elements easily form bonds with Group 17 elements, but Group 1 elements do not generally bond (solely) with other Group 1 elements, and likewise Group 17 elements do not generally bond (solely) with other Group 17 elements. It is the structure of the electrons surrounding each atom's nucleus that determines how those atoms will bond with other atoms.

4.6 Molecular quantities

Sample sizes of chemical substances are often measured in *moles*. One mole of a substance is defined as a sample having 6.022×10^{23} (*Avogadro's number*) molecules[9]. This number is not arbitrary – it was chosen[10] such that 1 mole of carbon-12 (6.022×10^{23} individual ^{12}C atoms together in one sample) would have a mass of exactly 12 grams. In other words, Avogadro's number is a proportionality between an element's atomic mass (measured in *amu* or *Daltons*) and the mass of a sample (measured in *grams*).

With Avogadro's number defined as such, we may interpret any element's atomic mass value as a conversion factor relating moles to grams. For example, if we look up the element *potassium* in a periodic table, we see that it has an average atomic mass of 39.0983 amu (39.0983 Daltons) as found in nature. This means 1 mole of naturally-occurring potassium atoms equals 39.0983 grams of mass. Likewise, 5 moles of potassium atoms will have a mass of 195.4915 grams. Note the use of the equivalence 1 mol potassium = 39.0983 g as a "unity fraction" in the following calculation, used to cancel the given unit of moles to yield an answer in the unit of grams:

$$\left(\frac{5 \text{ mol potassium}}{1}\right)\left(\frac{39.0983 \text{ g}}{1 \text{ mol potassium}}\right) = 195.4915 \text{ g}$$

Molar quantities make it convenient to relate macroscopic samples of elements and compounds with each other. We know, for instance, that one mole of naturally occurring iron (Fe) atoms will have a mass of 55.8 grams, and that one mole of naturally occurring oxygen (O) atoms will have a mass of 16.0 grams, because the average atomic mass of naturally occurring iron is 55.8 amu, and the average atomic mass of naturally occurring oxygen is 16.0 amu. One mole of naturally occurring oxygen *molecules* (O_2) will have a mass of 32.0 grams, since each molecule is a *pair* of oxygen atoms at 16 amu each, and "moles" counts the number of discrete entities which in the case of molecular oxygen is the number of O_2 *molecules* rather than the number of O *atoms*. Applying the same reasoning, one mole of ozone (O_3) molecules will have a mass of 48.0 grams.

The same mathematical proportions apply to compounds as they do to elements, since compounds are nothing more than different elements bound together in whole-number ratios, and the Conservation of Mass tells us a molecule cannot have a mass greater or less than the sum total of the constituent elements' masses. To illustrate this principle, we may calculate the mass of one mole of iron oxide (Fe_2O_3), the principal component of *rust*: $55.8 \times 2 + 16.0 \times 3 = 159.6$ grams. Likewise, we may calculate the mass of five moles of pure glucose ($C_6H_{12}O_6$): $5 \times (12.01 \times 6 + 1.01 \times 12 + 16.0 \times 6) = 900.9$ grams. The sum of the atomic masses of a molecule's constituent atoms is called the *molecular weight* or *formula weight* for that molecule. In the case of iron oxide, the molecular

[9] Truth be told, a "mole" is 602,200,000,000,000,000,000,000 counts of literally *any* discrete entities. Moles do not represent mass, or volume, or length, or area, but rather a *quantity of individual units*. There is nothing wrong with measuring the amount of eggs in the world using the unit of the mole, or the number of grains of sand in moles, or the number of bits in a collection of digital data. Think of "mole" as nothing more than a *really* big dozen, or more precisely, a really big *half*-dozen!

[10] Another way to define one mole is that it is the number of individual nucleons (i.e. protons and/or neutrons) necessary to comprise one gram of mass. Since protons and neutrons comprise the vast majority of an atom's mass, we may essentially ignore the mass of an atom's electrons when tabulating its mass and pay attention only to the nucleus. This is why one mole of Hydrogen atoms, each atom having just one lone proton in its nucleus, will have a combined mass of one gram. By extension, one mole of Carbon-12 atoms, each atom with 6 protons and 6 neutrons, will have a combined mass of twelve grams.

weight is 159.6 (typically rounded up to 160 grams per mole). In the case of glucose, the molecular weight is 180.18 (typically rounded down to 180 grams per mole).

When referring to liquid solutions, the concentration of a solute is often expressed as a *molarity*, defined as the number of moles of solute per liter of solution. Molarity is usually symbolized by an italicized capital letter *M*. It is important to bear in mind that the volume used to calculate molarity is that of the total solution (solute plus solvent) and not the solvent alone.

Suppose we had a solution of salt-water, comprised of 33.1 grams of table salt thoroughly mixed with pure water to make a total volume of 1.39 liters. In order to calculate the molarity of this solution, we first need to determine the equivalence between moles of salt and grams of salt. Since table salt is sodium chloride (NaCl), and we know the atomic masses of both sodium (23.0 amu) and chlorine (35.5 amu), we may easily calculate the mass of one mole of table salt:

$$1 \text{ mole of NaCl} = 23.0 \text{ g} + 35.5 \text{ g} = 58.5 \text{ g}$$

Another way to state this is to say that table salt (sodium chloride, or NaCl) has a molecular weight of 58.5 amu (58.5 grams of mass per mole).

We may use this equivalence as a unity fraction to help us convert the number of grams of table salt per unit volume of solution into a molarity (moles of table salt molecules per liter):

$$\left(\frac{33.1 \text{ g}}{1.39 \text{ l}}\right)\left(\frac{1 \text{ mol NaCl}}{58.5 \text{ g}}\right) = 0.407\frac{\text{mol NaCl}}{\text{l}} = 0.407\ M \text{ (NaCl)}$$

Another common expression for the concentration of a solute in either a liquid or a gas solution is related to the concept of *percent*, expressing the presence of the solute as a ratio of how many "parts" of solute exist per "parts" of solution. Earth's atmosphere, for example, contains approximately 20.9% oxygen gas by volume. This means that for every 100 molecules found in a sample of air, approximately 21 of those are oxygen molecules. When the concentration of a solute is very small, however, percent becomes an awkward unit of measurement. In such cases it is common to see low concentrations of solute expressed as *parts per million* (ppm) or even *parts per billion* (ppb). The volumetric concentration of methane gas in Earth's atmosphere is a good example where parts-per-million is a more appropriate expression than percent: for every million molecules found in a sample of air, approximately 2 of them are methane molecules (i.e. methane has an atmospheric concentration of 2 ppm). As a percentage, this equates to only 0.0002%.

We may use parts-per-unit concentration values as unity fractions just like molecular weights and just like molarity values, to relate total solution quantity to solute quantity. For example, if we need to calculate the total mass of hydrogen gas in a compressed air cylinder storing 47000 standard cubic feet of air, we could multiply the total volume of that air sample (47000 SCF) by the volumetric concentration of hydrogen naturally found in Earth's atmosphere which is 0.5 ppm:

$$\left(\frac{47000 \text{ SCF air}}{1}\right)\left(\frac{0.5 \text{ parts hydrogen}}{1000000 \text{ parts air}}\right) = 0.0235 \text{ SCF hydrogen}$$

Note how the units of "parts" and "air" cancel out to leave "SCF hydrogen".

An important caveat when using percent, ppm, or ppb is that we must clearly define the "parts" proportion as either being volume or mass. The concentration of hydrogen gas in the atmosphere (0.5 ppm) was specified as a *volumetric* concentration, and so it is appropriate to use this 0.5 ppm figure to calculate a proportion of the 47000 SCF total *volume*. If, however, we were given a ppm *mass* concentration for hydrogen, we could only use that figure in conjunction with a total *mass* quantity for the air sample.

The following photograph illustrates this concept, showing the label on a *calibration gas bottle* containing a certified mixture of gases used to check the accuracy of air-safety monitoring instruments. Note the concentrations of each gas type within the mixture – some expressed in percent, others in ppm – and how the label states "MOLE %" in the upper-right corner using large bold print to let you know the concentration values refer to molar quantities (e.g. 18% oxygen means 18% of the *molecules* contained in this bottle are oxygen molecules), which for gases closely corresponds to volumetric quantity rather than mass:

4.7 Saturated steam table

A saturated steam table shows temperatures and pressures for water at the liquid/vapor transition (i.e. points lying along the liquid/vapor interface shown in a phase change diagram), as well as enthalpy values for the water and steam under those conditions. The sensible heat of water is the amount of thermal energy per pound necessary to raise water's temperature from the freezing point to the boiling point. The latent heat of vapor is the amount of energy per pound necessary to convert water (liquid) into steam (vapor). The total heat is the enthalpy of steam (thermal energy per pound) between the listed condition in the table and the freezing temperature of water.

By definition a saturated steam table does *not* describe steam at temperatures greater than the boiling point. For such purposes, a *superheated steam table* is necessary.

Data for this saturated steam table was taken from *Thermal Properties of Saturated and Superheated Steam* by Lionel Marks and Harvey Davis, published in 1920 by Longmans, Green, and Company.

Saturated Steam Table

Temperature (Deg F)	**Pressure (PSIA)**	**Sensible heat of liquid (BTU/lb)**	**Latent heat of vapor (BTU/lb)**	**Total heat (BTU/lb)**
32	0.0886	0.00	1073.4	1073.4
40	0.1217	8.05	1068.9	1076.9
50	0.1780	18.08	1063.3	1081.4
60	0.2562	28.08	1057.8	1085.9
70	0.3626	38.06	1052.3	1090.3
80	0.505	48.03	1046.7	1094.8
50	0.696	58.00	1041.2	1099.2
60	0.2562	28.08	1057.8	1085.9
70	0.3626	38.06	1052.3	1090.3
80	0.505	48.03	1046.7	1094.8
90	0.696	58.00	1041.2	1099.2
100	0.946	67.97	1035.6	1103.6
110	1.271	77.94	1030.0	1108.0
120	1.689	87.91	1024.4	1112.3
130	2.219	97.89	1018.8	1116.7
140	2.885	107.87	1013.1	1121.0
150	3.714	117.86	1007.4	1125.3
160	4.737	127.86	1001.6	1129.5
170	5.992	137.87	995.8	1133.7
180	7.51	147.88	989.9	1137.8
190	9.34	157.91	983.9	1141.8
200	11.52	167.94	977.8	1145.8
210	14.13	177.99	971.6	1149.6
212	14.70	180.00	970.4	1150.4

Saturated Steam Table (continued)

Temperature (Deg F)	**Pressure (PSIA)**	**Sensible heat of liquid (BTU/lb)**	**Latent heat of vapor (BTU/lb)**	**Total heat (BTU/lb)**
220	17.19	188.1	965.2	1153.3
230	20.77	198.2	958.7	1156.9
240	24.97	208.3	952.1	1160.4
250	29.82	218.5	945.3	1163.8
260	35.42	228.6	938.4	1167.0
270	41.85	238.8	931.4	1170.2
280	49.18	249.0	924.3	1173.3
290	57.55	259.3	916.9	1176.2
300	67.00	269.6	909.5	1179.1
310	77.67	279.9	901.9	1181.8
320	89.63	290.2	894.2	1184.4
330	103.0	300.6	886.3	1186.9
340	118.0	311.0	878.3	1189.3
350	134.6	321.4	870.1	1191.5
360	153.0	331.9	861.8	1193.7
370	173.3	342.4	853.4	1195.8
380	195.6	352.9	844.8	1197.7
390	220.2	363.5	836.1	1199.6
400	247.1	374.1	827.2	1201.3
410	276.4	384.7	818.2	1202.9
420	308.4	395.4	809.0	1204.4
430	343.2	406.2	799.6	1205.8
440	380.8	417.0	790.1	1207.1

4.8 Weight densities of common materials

All density figures approximate for samples at standard temperature and pressure[11].

4.8.1 Common liquid densities

- Acetone: $\gamma = 49.4$ lb/ft^3
- Alcohol, ethyl (ethanol): $\gamma = 49.4$ lb/ft^3
- Alcohol, methyl (methanol): $\gamma = 50.5$ lb/ft^3
- Benzene: $\gamma = 56.1$ lb/ft^3
- Butane (liquid): $\gamma = 36.1$ lb/ft^3
- Carbon disulfide: $\gamma = 80.7$ lb/ft^3
- Carbon tetrachloride: $\gamma = 99.6$ lb/ft^3
- Chloroform: $\gamma = 93$ lb/ft^3
- Ethylene glycol (ethanediol): $\gamma = 69.22$ lb/ft^3
- Gasoline: $\gamma = 41$ lb/ft^3 to 43 lb/ft^3
- Glycerin: $\gamma = 78.6$ lb/ft^3
- Isobutane (liquid): $\gamma = 34.8$ lb/ft^3
- Kerosene: $\gamma = 51.2$ lb/ft^3
- Mercury: $\gamma = 849$ lb/ft^3
- Methanol (methyl alcohol): $\gamma = 50.5$ lb/ft^3
- Milk: $\gamma = 64.2$ lb/ft^3 to 64.6 lb/ft^3
- Naphtha, petroleum: $\gamma = 41.5$ lb/ft^3
- Oil, castor: $\gamma = 60.5$ lb/ft^3
- Oil, coconut: $\gamma = 57.7$ lb/ft^3
- Oil, linseed (boiled): $\gamma = 58.8$ lb/ft^3
- Oil, olive: $\gamma = 57.3$ lb/ft^3
- Propane (liquid): $\gamma = 31.2$ lb/ft^3
- Toluene: $\gamma = 54.1$ lb/ft^3

[11] Density figures taken or derived from tables in the *CRC Handbook of Chemistry and Physics*, 64th Edition. Most liquid densities taken from table on page F-3 and solid densities taken from table on page F-1. Some liquid densities taken from tables on pages E-27 through E-31. All temperatures at or near 20 ^{o}C.

- Turpentine: $\gamma = 54.3$ lb/ft^3
- Water, heavy: $\gamma = 68.97$ lb/ft^3
- Water, light (normal): $\gamma = 62.4$ lb/ft^3
- Water, sea: $\gamma = 63.99$ lb/ft^3

4.8.2 Common solid densities

- Beryllium: $\gamma = 115.37$ lb/ft^3
- Brass: $\gamma = 524.4$ lb/ft^3
- Calcium: $\gamma = 96.763$ lb/ft^3
- Carbon (diamond): $\gamma = 196.65$ lb/ft^3 to 220.37 lb/ft^3
- Cement (set): $\gamma = 170$ lb/ft^3 to 190 lb/ft^3
- Chromium: $\gamma = 448.86$ lb/ft^3
- Copper: $\gamma = 559.36$ lb/ft^3
- Cork: $\gamma = 14$ lb/ft^3 to 16 lb/ft^3
- Gold: $\gamma = 1178.6$ lb/ft^3
- Ice: $\gamma = 57.2$ lb/ft^3
- Iron: $\gamma = 490.68$ lb/ft^3
- Ivory: $\gamma = 114$ lb/ft^3 to 120 lb/ft^3
- Lead: $\gamma = 708.56$ lb/ft^3
- Leather: $\gamma = 54$ lb/ft^3
- Magnesium: $\gamma = 108.50$ lb/ft^3
- Molybdenum: $\gamma = 638.01$ lb/ft^3
- Quartz: $\gamma = 165$ lb/ft^3
- Rubber (soft): $\gamma = 69$ lb/ft^3
- Rubber (hard): $\gamma = 74$ lb/ft^3
- Salt, rock: $\gamma = 136$ lb/ft^3
- Sugar: $\gamma = 99$ lb/ft^3
- Tar: $\gamma = 66$ lb/ft^3
- Wood, balsa: $\gamma = 7$ lb/ft^3 to 9 lb/ft^3
- Wood, maple: $\gamma = 39$ lb/ft^3 to 47 lb/ft^3

Chapter 5

Programming References

A powerful tool for mathematical modeling is text-based *computer programming*. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, *C++* and *Python*.

5.1 Programming in C++

One of the more popular text-based computer programming languages is called *C++*. This is a *compiled* language, which means you must create a plain-text file containing C++ code using a program called a *text editor*, then execute a software application called a *compiler* to translate your "source code" into instructions directly understandable to the computer. Here is an example of "source code" for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer's console:

```
#include <iostream>
using namespace std;

int main (void)
{
  float x, y;

  x = 200;
  y = -560.5;

  cout << "This simple program performs basic arithmetic on" << endl;
  cout << "the two numbers " << x << " and " << y << " and then" << endl;
  cout << "displays the results on the computer's console." << endl;

  cout << endl;

  cout << "Sum = " << x + y << endl;
  cout << "Difference = " << x - y << endl;
  cout << "Product = " << x * y << endl;
  cout << "Quotient of " << x / y << endl;

  return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other "whitespace" is largely irrelevant in C++ code, and is included only to make the code more pleasing[1] to view.

[1] Although not included in this example, *comments* preceded by double-forward slash characters (`//`) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.

Let's examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }, often referred to as "curly-braces").

- `int main (void)` labels the "Main" function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an *integer* number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program's completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called *arguments* to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer's memory for two *floating-point* variables, in this case the variables' names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line's end is necessary because C++ ignores whitespace on the page and doesn't "know" otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the "standard output" stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent *toward* the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer's console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.

After saving this *source code* text to a file with its own name (e.g. `myprogram.cpp`), you would then *compile* the source code into an *executable* file which the computer may then run. If you are using a console-based compiler such as *GCC* (very popular within variants of the Unix operating system[2], such as Linux and Apple's OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the `GCC` compiler to take your source code (`myprogram.cpp`) and create with it an executable file named `myprogram.exe`. Simply typing `./myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio[3], you may simply create a new console application "project" using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on
the two numbers 200 and -560.5 and then
displays the results on the computer's console.

Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

[2] A very functional option for users of Microsoft Windows is called *Cygwin*, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

[3] Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the "Hello World" `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile ("Build") and run your new program. Upon execution a console window will appear showing the output of your program.

learn computer programming by closely examining others' example programs and modifying them than it is to write your own programs starting from a blank screen.

5.2 Programming in Python

Another text-based computer programming language called *Python* allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an *interpreted* language: a software application called an *interpreter* reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3`[4] and pressing the Enter key:

```
Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>>
```

The `>>>` symbols represent the prompt within the Python interpreter "shell", signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

[4] Using version 3 of Python, which is the latest at the time of this writing.

More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which "imports" these functions from Python's math *library* (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in *radians* rather than *degrees*, and how Python provides convenient functions for translating between the two. Logarithms assume a base of e unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person's first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.

Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using *phasors*[5] as shown in the following example. Here we see Python's interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor's impedance (`zc`) as $X_C \angle -90^o$ with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values ($0 - jX_c$ Ω and $0 + jX_l$ Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω $\angle$ -25.102^o). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in *radians*; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python's interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

[5] A "phasor" is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.

assignment of variables as well as a convenient text record[6] of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter's shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

```
x = 200
y = -560.5

print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
```

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace *is* significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console's prompt, at which point it would display the following result:

```
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
```

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

[6] Like many command-line computing environments, Python's interpreter supports "up-arrow" recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.

If you are interested in learning more about computer programming in *any* language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.

5.3 Modeling vehicle drive power using C++

```
#include <iostream>
#include <cmath>
using namespace std;

int main(void)
{
  float weight, rr, cda, grade, v_i, v_f, v, t, power, ratio, dia;
  float f_weight, f_roll, f_air, f_accel, rpm_wheel, rpm_driveshaft;
  int defaults;

  cout << "Use default F-150 parameters (1/0)?" << endl;
  cin >> defaults;

  if (defaults == 1)
  {
    weight = 4000.0;  // Curb weight of F-150 pickup truck
    cda = 11.36;      // Coefficient of drag-area for F-150 Lightning truck
    ratio = 3.73;     // Axle ratio of F-150
    dia = 30.0;       // Tire diameter of F-150, inches
  }

  else
  {
    cout << "F-150 shortbed = 4000(?) lb" << endl;
    cout << "Enter curb weight (lb): ";
    cin >> weight;

    cout << "CdA for F-150 Lightning = 11.36 ft^2" << endl;
    cout << "Enter coefficient of drag * frontal area (ft^2): ";
    cin >> cda;

    cout << "F-150 = 3.73:1 ratio" << endl;
    cout << "Enter differential gear ratio (x:1): ";
    cin >> ratio;

    cout << "Enter tire outside diameter (inches): ";
    cin >> dia;
  }

  cout << "Enter initial driving speed (MPH): ";
  cin >> v_i;
```

```
  cout << "Enter final driving speed (MPH): ";
  cin >> v_f;

  cout << "Enter acceleration time (seconds): ";
  cin >> t;

  cout << "Enter road grade (%): ";
  cin >> grade;
  cout << "Rolling resistance coefficient on asphalt = 0.01" << endl;
  cout << "Rolling resistance coefficient on loose sand = 0.4" << endl;
  cout << "Enter rolling resistance coefficient (lb/ton): ";
  cin >> rr;

  v_i = v_i * 5280 / 3600;  // Converting from MPH to feet per second
  v_f = v_f * 5280 / 3600;  // Converting from MPH to feet per second
  f_weight = weight * grade / 100;
  f_roll = weight / 2000 * rr;

  cout << "Speed (MPH) , Motor power (HP), Driveshaft (RPM)" << endl;
  for (v = v_i ; v <= v_f ; v = v + (1.0 * 5280 / 3600))
  {
    f_air = 0.5 * 0.07647 * v * v; // Air density = 0.07647 slugs per ft^3
    f_accel = (weight / 32.174) * ((v_f - v_i) / t); // F = ma w/ mass in slugs
    power = (v * (f_weight + f_roll + f_air + f_accel)) / 550;
    rpm_wheel = v * 12 * 60 / (dia * 3.14159);
    rpm_driveshaft = rpm_wheel * ratio;
    cout << v * 3600 / 5280 << " , " << power << " , " << rpm_driveshaft << endl;
  }

  f_air = 0.5 * 0.07647 * v_f * v_f; // Air density = 0.07647 slugs per ft^3
  f_accel = 0.0;  // No acceleration after reaching final speed
  power = (v * (f_weight + f_roll + f_air + f_accel)) / 550;
  rpm_wheel = v * 12 * 60 / (dia * 3.14159);
  rpm_driveshaft = rpm_wheel * ratio;
  cout << v * 3600 / 5280 << " , " << power << " , " << rpm_driveshaft << endl;

  return 0;
}
```

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's Simplified Tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Mass

Weight

Conservation of Mass

Charge

Conservation of Electric Charge

Energy

Conservation of Energy

Electrons

Protons

Neutrons

Ion

Electricity

Work

Potential energy

Kinetic energy

Isaac Newton's First Law of Motion

Isaac Newton's Second Law of Motion

Power

6.1.3 Residential electric billing

When a customer of electricity pays their electric bill every month, they are paying for a quantity of energy used during that period of time.

First, identify the various forms that electrical energy becomes converted into while it is used.

Next, if the Law of Energy Conservation is true, why do customers have to keep paying for their electricity? If energy can never be destroyed, why not just pay for electricity *once* and then keep using that same purchased electricity forever?

Electricity bills are often expressed in units of kiloWatt-hours (kW-h). How many Joules are represented by one kW-h?

Challenges

- Why do you suppose kiloWatt-hours are used in electric billing statements instead of Joules?

6.1.4 Roller-coaster ride

A *roller-coaster* ride at an amusement park works by hoisting a car full of passengers to a high location, then letting that car coast by gravity down the track. Consider this mechanism, and then answer the following questions:

- At which point(s) in the roller-coaster track is the car's potential energy greatest?
- At which point(s) in the roller-coaster track is the car's kinetic energy greatest?
- Can the car ever reach a height greater than the point it was initially hoisted before let loose to coast?
- What happens to the kinetic energy of the car when it comes to a stop at the end of the ride?

Challenges

- If energy is conserved, how come a roller-coaster cannot coast forever?

6.1.5 Potential or kinetic?

Each of the following is an example of energy in a particular form. Classify each one as being either *potential* or *kinetic* energy (or both):

- Compressed spring =
- Stretched spring =
- Falling stone =
- Speeding train =
- Water in an elevated reservoir =
- Hot coal =
- Charged battery =
- Tank full of gasoline fuel =
- Satellite in orbit around Earth =

Challenges

- For each case, identify how to convert that form of energy into the other form.

6.1.6 BB guns and arrows

As a child I learned to shoot a "BB gun" style of air rifle, using compressed air to propel a copper-clad ball projectile. The gun was "cocked" using a lever mechanism that compressed a long spring, which when released would drive a piston to produce compressed air which directly propelled the BB projectile.

Comment on the various forms of energy expressed in this mechanism.

Archery is another example of an energy-transfer system. Similarly comment on the various forms taken by energy in a bow-and-arrow system.

Challenges

- Suppose we wished to double the speed of a given BB (or arrow) with fixed mass. How much more energy would we need to put into the weapon to achieve this goal?
- Identify a way we double the speed of the projectile without "charging" the weapon with additional energy.

6.1.7 Applying foundational concepts to various systems

Identify which foundational concept(s) apply to each of the declarations shown below. If a declaration is true, then identify it as such and note which concept supports that declaration; if a declaration is false, then identify it as such and note which concept is violated by that declaration:

- If a whole atom becomes *ionized* and then later rejoins with the stripped electrons, it will once again have zero net electric charge
- An elevator requires a source of energy to lift a weight vertically up
- An elevator requires a means of shedding energy to slowly descend a weight
- More energy is required to hoist a weight rapidly than to hoist a weight slowly (assuming the same lifting height)
- Boiling a kilogram of water releases a kilogram of steam
- Boiling a cubic meter of water releases a cubic meter of steam
- An automobile crashing into a stationary barrier at 60 km/hr will suffer twice the damage of an automobile crashing at 30 km/hr, all other factors being equal
- A crate accidently dropped from a height of 6 meters will suffer twice the damage of a crate dropped from a height of 3 meters, all other factors being equal

Here is a list of foundational concepts for your reference: **Conservation of Energy**, **Conservation of Mass**, **Conservation of Electric Charge**, **Conservation of Momentum**, **definition of work** ($W = Fx\cos\theta$), **definition of power** (work done per unit time), **definition of potential energy** ($E_p = Fx$), **definition of kinetic energy** ($E_k = \frac{1}{2}mv^2$). More than one of these concepts may apply to a declaration, and some concepts may not apply to any listed declaration at all. Also, feel free to include foundational concepts not listed here.

Challenges

- Explain why conservation laws, properly understood, are valuable tools.

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = 1.25663706212(19) $\times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = 8.8541878128(13) $\times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = 6.67430(15) $\times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

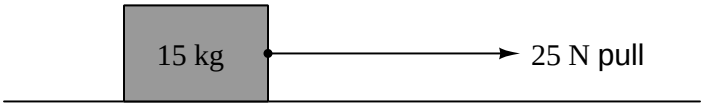
Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Moving weights

A 50 Newton weight is lifted 4 meters vertically against the pull of gravity. How much work is done?

A 15 kg mass is dragged along a level surface with a steady pull of 25 Newtons for 10 meters. How much work is done?



Challenges

- If the 50 Newton weight were dropped after being hoisted up 4 meters above ground level, how much energy would it have just before striking the ground?
- Work represents a transfer of energy from one system to another, or from one location to another. In each of these cases, where is energy being transferred to and from?

6.2.4 Stopping distance

Speed kills, or so the slogan says. When a car is driven faster than normal, it requires a longer distance to slow down (brake). This much is intuitively obvious, but *how much longer* will it take for a speeding automobile to slow to a stop assuming it is traveling at *twice* its normal speed?

Challenges

- Identify some of the assumptions implicit in your answer.

6.2.5 Potatoes and bullets

Calculate the amount of potential energy stored in the mass of an elevator plus its cargo of potatoes (570 pounds) as it lifts up to a height of 10 feet. Express this potential energy in both British and metric units.

Next, calculate the kinetic energy of a bullet with a mass of 150 grains (9.7198 grams) traveling at a velocity of 2820 feet per second, expressing your answer in both British and metric units.

Challenges

- Demonstrate how to *estimate* numerical answers for this problem without using a calculator.
- Describe the various energy-transfer operations taking place when a crane lifts a heavy weight and then sets it down in a different location.
- If a crane lifts a weight 10 feet above the ground, then sets that weight down on top of an object 6 feet above the ground, is energy still conserved? Explain in detail.
- Which has a greater effect on a moving object's kinetic energy: an increase in mass or an increase in velocity?

6.2.6 Antenna dimensions

A dipole antenna designed to resonate at a fundamental frequency of 1.8 GHz will have a total length of approximately 0.083275 meters, based on the fact that dipole antennas operate at half the wavelength ($\frac{\lambda}{2}$) of the RF energy transmitted and/or received.

If we wished to calculate this same length in a different unit of measurement such as *centimeters*, we could do so using a simple fraction-based technique whereby we set up the original quantity as a fraction and then multiply by other fractions consisting of *physically equal* numerator and denominator quantities such that the old unit is replaced by a new unit:

$$\left(\frac{0.083275 \text{ m}}{1}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = 8.3275 \text{ cm}$$

We may continue with this same unit-conversion strategy to compute an answer in inches, knowing there are exactly 2.54 centimeters to one inch:

$$\left(\frac{0.083275 \text{ m}}{1}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right) = 3.2785 \text{ in}$$

. . . and again, to compute an answer in feet:

$$\left(\frac{0.083275 \text{ m}}{1}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 0.27321 \text{ ft}$$

This method of unit-conversion offers several advantages over other methods. First, one need only commit a few conversion ratios to memory in order to convert back and forth between many combinations of units rather than memorize (or research) conversion ratios for every combination of units encountered. Second, the accuracy of the answer is assured by careful inspection of the canceled units, which is important because this tends to be the most common way in which unit-conversions go astray: by accidently dividing when multiplication is necessary, or vice-versa.

Knowing this, convert the length of a 0.089223 meter antenna into centimeters, feet, and inches.

Challenges

- Convert 40 miles into kilometers using nothing but the following ratios: 2.54 cm to 1 inch, 5280 feet to 1 mile, 12 inches to 1 foot, and of course metric prefixes (centi = 100, kilo = 1000, etc.).

6.2.7 Analog-digital converter signals

An *analog-to-digital converter* (ADC) is an electronic circuit designed to take in an analog signal – usually a DC voltage – and convert that signal's magnitude into a digital "count" value. For example, an 8-bit ADC with an input range of 0 to 5 Volts DC will output a count value of 0 (decimal) or `0b00000000` (binary) when sensing 0 Volts DC and a count value of 255 (decimal) or `0b11111111` (binary) when sensing 5 Volts DC.

Explain how the equivalence of *5 Volts DC = 255 counts* could be used as a unity fraction for converting any input voltage value into its corresponding count value.

Next, convert between the following V_{in} and output count values for this same ADC:

- V_{in} = 2.33 Volts ; Output = _____ counts
- V_{in} = 1.92 Volts ; Output = _____ counts
- V_{in} = 4.36 Volts ; Output = _____ counts
- Output count = 101 ; V_{in} = _____ Volts
- Output count = 28 ; V_{in} = _____ Volts
- Output count = 209 ; V_{in} = _____ Volts

Challenges

- Convert each of the decimal count values into binary notation.

6.2.8 Pressure unit conversions

Use the mathematical technique of "unity fractions" to convert between the following units of fluid pressure, using the following pressure equivalencies:

1.000 pound per square inch (PSI) = 2.036 inches of mercury (in. Hg) = 27.68 inches of water (in. W.C.) = 6.895 kilo-pascals (kPa) = 0.06895 bar

$$\left(\frac{25\text{ PSI}}{1}\right)\left(\text{——————}\right) = \text{??? kPa}$$

$$\left(\frac{40\text{ "WC}}{1}\right)\left(\text{——————}\right) = \text{??? PSI}$$

$$\left(\frac{5.6\text{ bar}}{1}\right)\left(\text{——————}\right) = \text{??? PSI}$$

$$\left(\frac{1200\text{ "Hg}}{1}\right)\left(\text{——————}\right) = \text{??? "WC}$$

$$\left(\frac{12\text{ "WC}}{1}\right)\left(\text{——————}\right) = \text{??? bar}$$

$$\left(\frac{110\text{ kPa}}{1}\right)\left(\text{——————}\right) = \text{??? "WC}$$

$$\left(\frac{982\text{ "Hg}}{1}\right)\left(\text{——————}\right) = \text{??? kPa}$$

$$\left(\frac{50\text{ PSI}}{1}\right)\left(\text{——————}\right) = \text{??? bar}$$

$$\left(\frac{250\text{ kPa}}{1}\right)\left(\text{——————}\right) = \text{??? "Hg}$$

$$\left(\frac{31\text{ bar}}{1}\right)\left(\text{——————}\right) = \text{??? "Hg}$$

Challenges

- Demonstrate how to *estimate* numerical answers for these conversion problems without using a calculator.

6.2.9 Flow rate conversion

Suppose an ecological survey team installs an electronic water-flow sensing apparatus on a small stream, to measure its water flow rate over a period of several months. After collecting the data, the team finds the flowmeter registered a maximum water flow of 3.622 cubic feet per second. Convert this into gallons per minute, showing your work (using "unity fractions").

Challenges

- From your work, derive a single factor converting CFS into GPM (or vice versa), so that anyone else needing to do this conversion (who isn't familiar with unity fractions) may easily convert from one to the other.

6.2.10 Frequency conversions

Convert between the following units of measurement for frequency:

- 500 Hz to rad/sec =
- 2400 rad/sec to Hertz (Hz) =
- 3 Hz to cycles per minute (cpm) =
- 5 rad/sec to cycles per hour (cph) =

Challenges

- What purpose does *natural frequency* (ω) expressed in radians per second serve?

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 A motor that runs without energy?

An inventor claims to have built an electric motor-generator that runs continuously on its own power. He demonstrates this machine running, using two meters to measure power received from the grid (the wall receptacle) versus power returned to the grid. The two meters show the same measurement of electrical power in and out, while the motor drives a large fan to blow air.

Furthermore, he claims energy companies "don't want you to know about this" because it would ruin their business models which are based on fossil fuels and other limited energy sources.

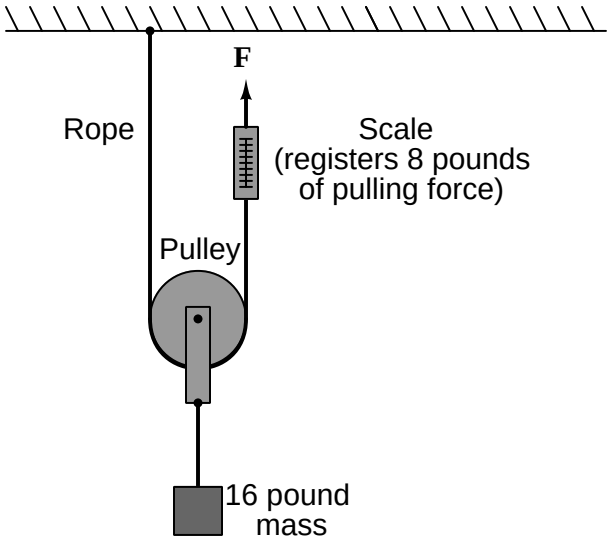
Explain what makes this claim suspicious at best, and how we could truly put it to the test just in case the inventor has indeed made a revolutionary discovery.

Challenges

- Identify some similar claims you have heard, and then identify ways you could comprehensively test those claims.
- Interestingly, you never encounter people claiming to have made machines that output more *mass* than they take in. Why not?

6.3.2 Pulley and rope system

The pulley-and-rope mechanism shown below allows the hoisting of a heavy weight with a pulling force on the rope that is only *half* that of the weight (e.g. lifting a 16 pound object with only 8 pounds of rope tension.



Suppose someone tells you this machine *multiplies your energy.* Identify the conceptual error at work here, and correct it.

Challenges

- Address this misconception in the converse: how would this machine actually behave if it *did* multiply your energy?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

5 July 2025 – added a Programming References chapter with a section in it showing a C++ program used to calculate drive power requirements for a vehicle given a range of speeds, curb weight, CdA, etc.

26 August 2024 – minor edits made to some of the section titles.

23 December 2023 – added a new Quantitative Reasoning question for frequency unit conversions.

29 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

6-9 March 2022 – added a Full Tutorial section on unit conversions, as well as some Quantitative Reasoning problems.

18 July 2021 – added an "Applying foundational concepts..." question to the Conceptual Reasoning section of the Questions chapter.

8 May 2021 – commented out or deleted empty chapters.

22 March 2021 – added more digits to Boltzmann's constant, and clarified that Kelvin is 273.15 more than Celsius.

18 March 2021 – corrected multiple instances of "volts" that should have been capitalized "Volts".

24 September 2020 – added more content to the Introduction chapter, including references to the "Reading outline and reflections" and "Foundational concepts" subsections. This is intended for helping students new to inverted instruction adapt to the expectations of courses based on these

modules, where daily reading of these texts is fundamental to learning. Many students enter college unfamiliar with how to outline texts, and so they need guidance on how to do so.

29 August 2020 – minor edits to the Tutorial regarding Electric Charge Conservation.

25 August 2020 – some additions to the Introduction chapter.

23 August 2020 – expanded Introduction chapter, and made a minor edit to the Full Tutorial for typesetting purposes.

19 August 2020 – updated Foundational concepts list.

9 March 2020 – added more content to Simplified Tutorial.

8 March 2020 – document first created.

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