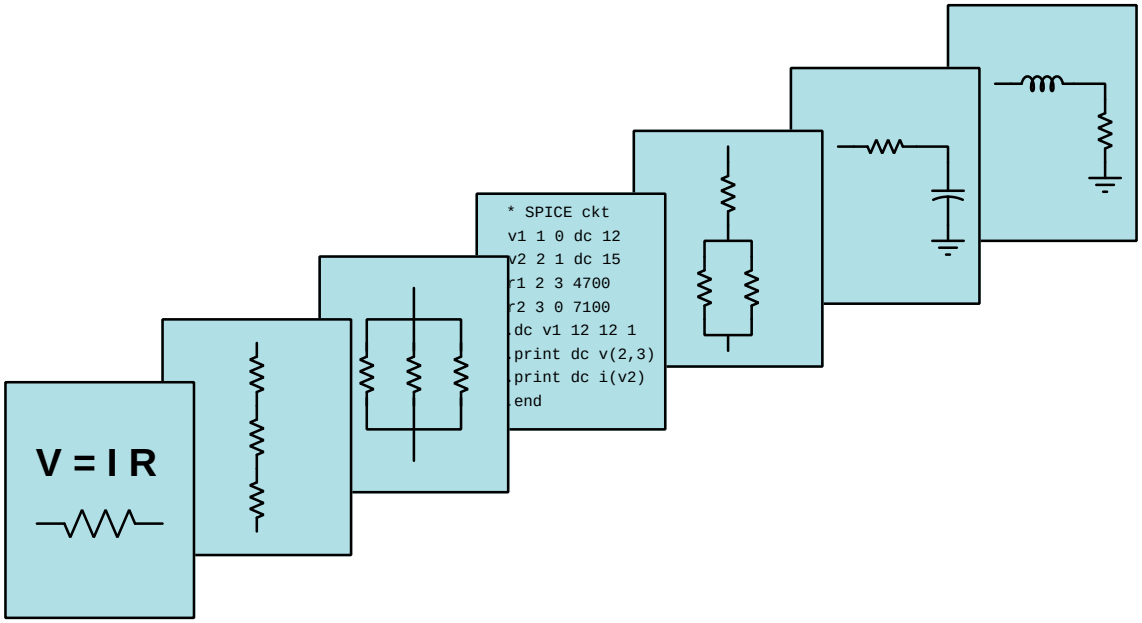
Modular Electronics Learning (ModEL) project



Mixers

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Chapter 1

Introduction

One of the critically important functions of a radio transmitter or receiver system is the *mixer*, sometimes implemented as a distinct sub-circuit and other times as one function among others implemented by a single circuit, the purpose of which being to create new frequencies from a non-linear combination of other frequencies. Mixing is at the heart of modulation (impressing information on an otherwise featureless carrier signal) as well as the frequency-shifting of AC signals.

Important concepts related to mixers include **modulation** and **demodulation**, frequency **conversion**, **sidebands**, **filtering**, **time-domain** versus **frequency domain**, **trigonometric identities**, **heterodyning**, FET **square law** behavior, **harmonic** frequencies, **diode** behavior, mathematical **series**, **artifacts**, **differential pairs**, **balanced** versus **unbalanced** mixing, signal **leakage**, port **isolation**, **passive** versus **active** devices, and **integrated circuits**.

Problem-solving techniques applied in the text include *thought experiments* and *limiting cases*. It is strongly recommended that the reader perform these mental activities themselves, rather than simply trust what the author writes. The goal here is not only to understand the concepts, but also to cultivate a set of powerful problem-solving tools that the reader may apply in many other contexts.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to measure the amount of signal leakage in a balanced mixer? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to test the effectiveness of a filter network intended for use with a mixer? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What are some practical applications of mixers?
- What does it mean to *modulate* a signal?
- What does it mean to *demodulate* a signal?

- What does it mean to *upconvert* or *downconvert* a signal?
- What is a *sideband*, and how is one evident on a frequency-domain (e.g. spectrum analyzer) plot?
- What is the purpose of a *local oscillator* circuit in a radio transmitter or receiver system?
- What fundamental characteristic is necessary to properly mix two or more AC signals?
- What determines the frequencies present in the output signal from a mixer?
- Why are filter networks often used within radio transmitter and receiver circuits?
- How is a FET able to properly mix signals, being that it does not strictly multiply two input signals together?
- How is a diode able to properly mix signals, being that it does not strictly multiply two input signals together?
- Why does a diode ring mixer exhibit a balanced mixing characteristic?
- Why does a Gilbert Cell mixer exhibit a balanced mixing characteristic?
- What does it mean for a mixer circuit to be balanced, versus unbalanced?
- What is an *artifact* in the context of signal mixing?
- What does the term *leakage* mean in the context of signal mixing?
- What does the term *isolation* mean in the context of signal mixing?
- Why are RF mixers often able to use square-wave LO signals rather than sinusoidal?
- How are we to interpret the block-diagram filter symbols shown in this tutorial, with the wavy lines?
- What does it mean when two or more signals *heterodyne* together?
- What function(s) are provided by a radio *converter* circuit?

1.1 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

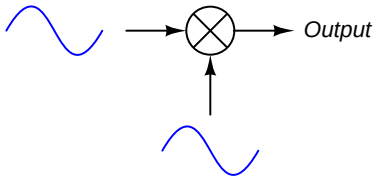
These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: balanced mixer frequencies

A *balanced mixer* is a circuit designed to multiply the instantaneous values of two input signals together, resulting in an output signal containing the *sum* and *difference* frequencies of those input signals. In an ideal world, such a mixer's output would *only* contain those sum and difference frequencies. In the real world there will always be some *leakage* of the original input frequencies into the output, as well as leakage of one input signal into the other input port, and even artifacts such as harmonics of those input frequencies owing to internal signal distortion within an imperfect mixer.

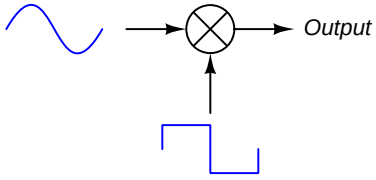
For the following mixer examples we will assume perfectly balanced mixing with total isolation between channels, meaning no leakage of signals between ports on the mixer whatsoever. Such a perfect mixer should only produce sum and difference frequencies at its output port and nothing else.

Consider the following cases of two sinusoidal signals input to a balanced mixer:



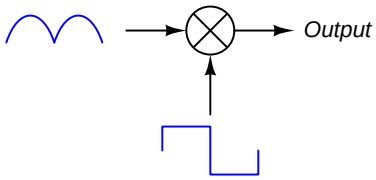
- **Input sinusoids:** 40 kHz and 55 kHz – **Output signal contains:** 15 kHz and 95 kHz
- **Input sinusoids:** 3 kHz and 70 kHz – **Output signal contains:** 67 kHz and 73 kHz
- **Input sinusoids:** 2 MHz and 30 kHz – **Output signal contains:** 1.97 MHz and 2.03 MHz

Consider the following cases of a sinusoidal signal mixed with a square wave:



- **Input sinusoid:** 10 kHz ; **Input square:** 25 kHz – **Output signal contains:** 15 kHz, 35 kHz, 65 kHz, 85 kHz, 115 kHz, 135 kHz, etc.
- **Input sinusoid:** 2 kHz ; **Input square:** 10 kHz – **Output signal contains:** 8 kHz, 12 kHz, 28 kHz, 32 kHz, 48 kHz, 52 kHz, etc.
- **Input sinusoid:** 1 MHz ; **Input square:** 5 kHz – **Output signal contains:** 0.995 MHz, 1.005 MHz, 0.985 MHz, 1.015 MHz, 0.975 MHz, 1.025 MHz, etc.

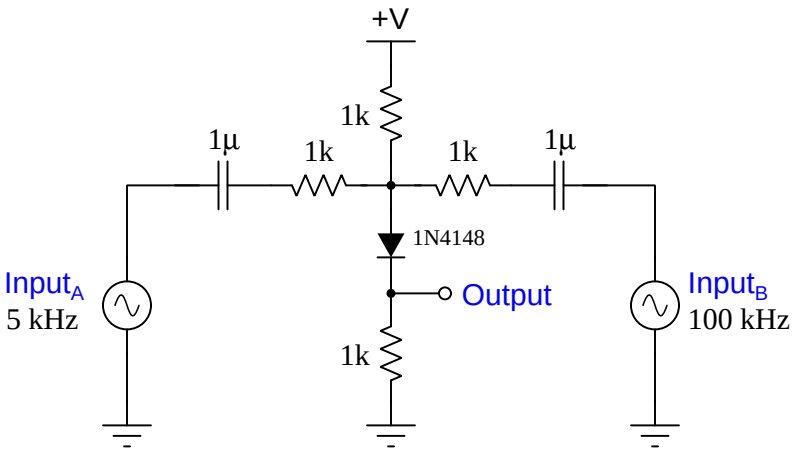
Consider the following cases of two non-sinusoidal signals mixed together – a non-symmetrical waveform containing both odd and even harmonics, mixed with a square wave which of course only contains odd harmonics:



- **Input non-symm:** 8 kHz ; **Input square:** 1 kHz – **Output signal contains:** 7 kHz, 9 kHz, 5 kHz, 11 kHz, 3 kHz and 13 kHz, etc. ; 15 kHz, 17 kHz, 13 kHz, 19 kHz, 11 kHz, 21 kHz, etc. ; 23 kHz, 25 kHz, 21 kHz, 27 kHz, 19 kHz, 29 kHz, etc.
- **Input non-symm:** 40 kHz ; **Input square:** 2 kHz – **Output signal contains:** 38 kHz, 42 kHz, 34 kHz, 46 kHz, 30 kHz, 50 kHz, etc. ; 78 kHz, 82 kHz, 74 kHz, 86 kHz, 70 kHz, 90 kHz, etc. ; 118 kHz, 122 kHz, 114 kHz, 126 kHz, 110 kHz, 130 kHz, etc.

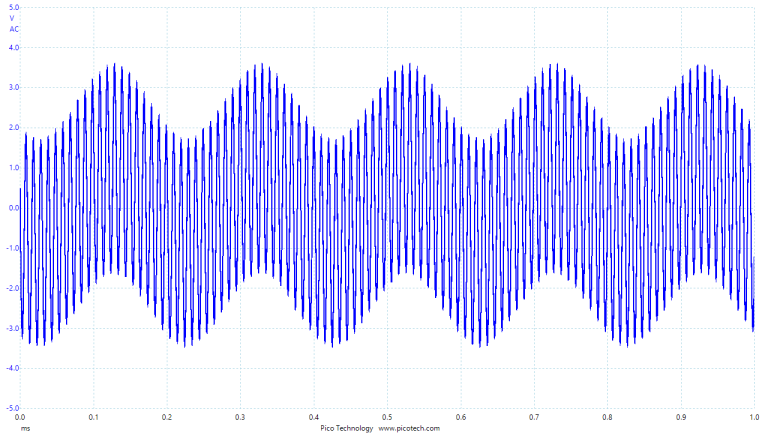
2.2 Example: simple diode mixer circuit

In this circuit two AC voltage signals (5 kHz and 100 kHz) are mixed using a single model 1N4148 fast-switching diode as the nonlinear element. An adjustable DC voltage source (+V) provides biasing for the diode, allowing us to explore the mixing behavior for different DC operating points:

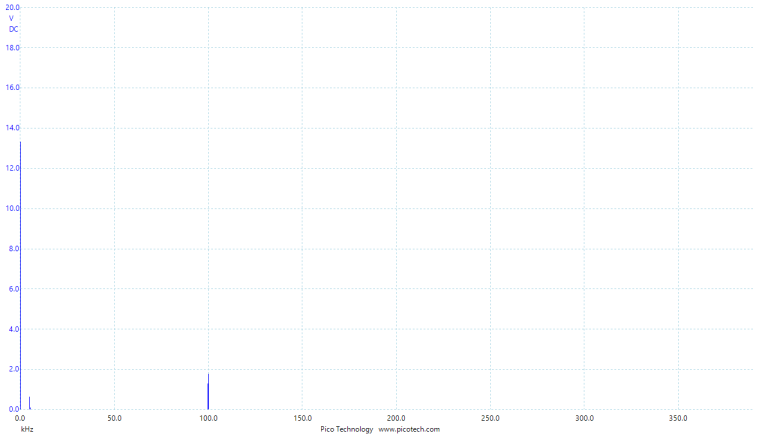


With a strong DC bias – strong enough to keep the diode continuously conducting – the signal we see at the output terminal is not "mixed" properly, but is merely the sum of the two waveforms:

Time-domain view



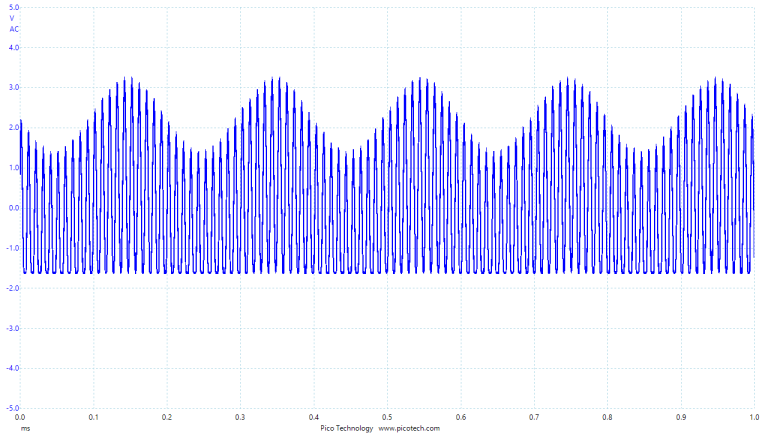
Frequency-domain view



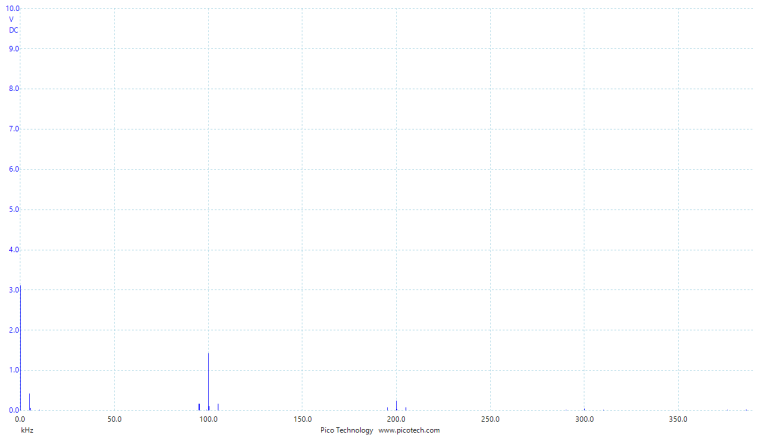
The spectrum display shows a single peak at 100 kHz (signal "B") and a single peak at 5 kHz (signal "A"), with no real mixing. A DC peak also appears at the far-left end of the spectrum, representing the DC bias voltage dropped across the output resistor.

With a weaker DC bias the diode stops conducting during portions of the cycle, the result being a "clipped" time-domain waveform that actually exhibits variations in amplitude (i.e. amplitude-modulation). The resulting spectrum shows sidebands at 95 kHz and 105 kHz in addition to the original signal peaks at 5 kHz and 100 kHz:

Time-domain view



Frequency-domain view

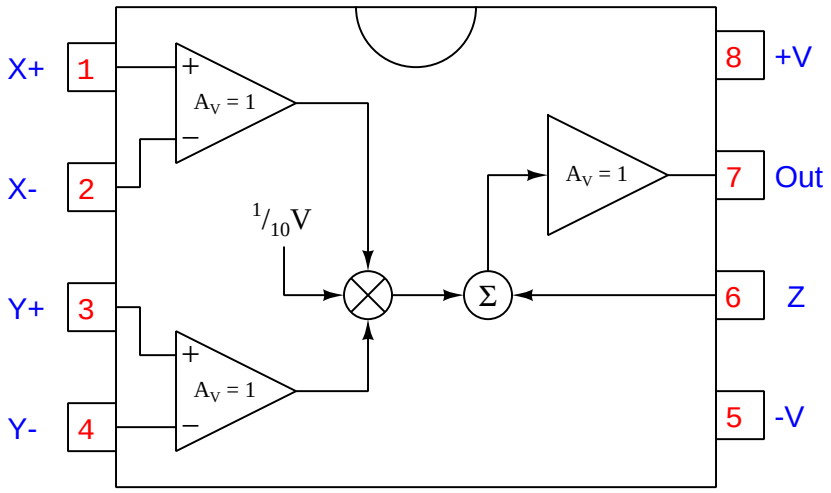


Now we see the original DC, 5 kHz, and 100 kHz peaks as before, but with the addition of sidebands surrounding the 100 kHz carrier peak: one at 95 kHz and another at 105 kHz. Interestingly, we also see smaller peaks at 195 kHz, 200 kHz, and 205 kHz. The 200 kHz peak represents the second harmonic of the 100 kHz carrier signal, generated by the diode's clipping of that sinusoidal signal (making it asymmetrical, therefore containing even-numbered harmonics). This second harmonic frequency of 200 kHz also gets mixed with the 5 kHz signal to produce 195 kHz and 205 kHz sidebands. Such distortion of an input signal and subsequent mixing of the distortion products is

generally considered undesirable behavior for any mixer, and demonstrates just how crude of a mixer a single-diode circuit like this really is.

2.3 Example: AD633 as a balanced mixer

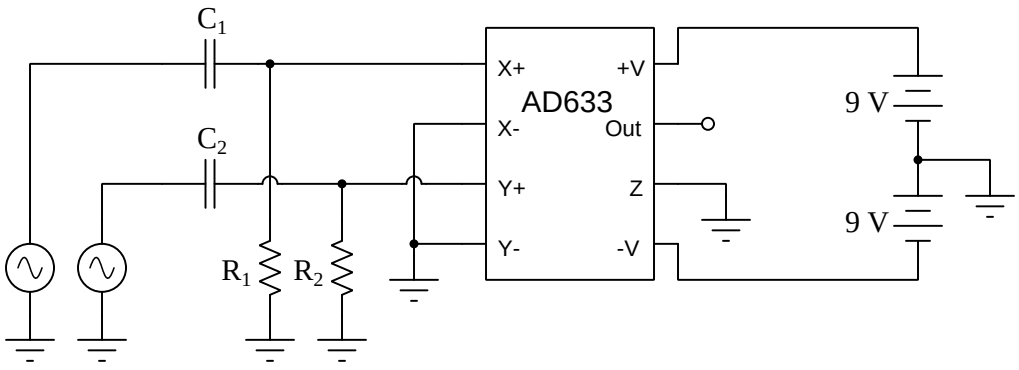
The Analog Devices AD633 analog multiplier IC works well as a balanced mixer for audio frequencies. An internal block diagram is as follows:



Two unity-gain amplifiers receive the analog signals to be multiplied (in our case, the baseband signal X and carrier signal Y we intend to mix), pass it to the multiplying cell, and then an optional offset is added to that product signal and buffered for the output. Its transfer function is:

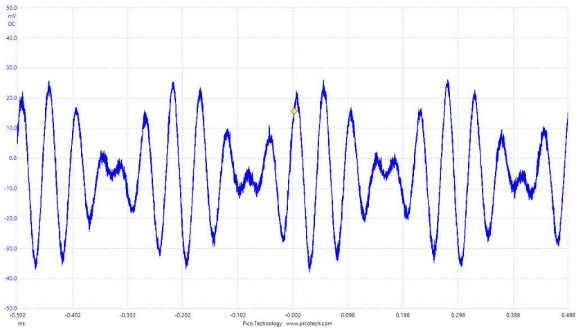
$$Out = \frac{(X_+ - X_-)(Y_+ - Y_-)}{10 \text{ Volts}} + Z$$

Here we will use the AD633 to amplitude-modulate (AM) a 20 kHz carrier signal with a 2 kHz baseband signal, using a pair of 9-Volt batteries to form a split DC power supply, and coupling capacitors to ensure no DC offset (bias) voltage from either signal source reaches the multiplier:

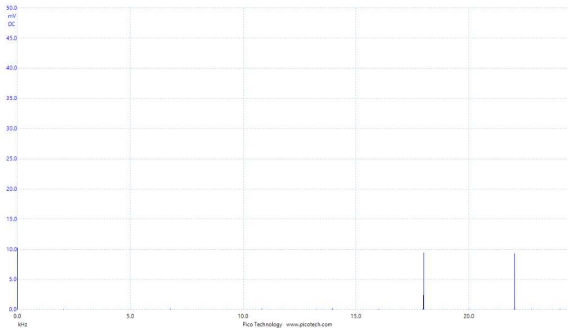


The output signal is then measured in both the time and frequency domains, with the 2 kHz baseband signal and the 20 kHz carrier:

Time-domain (oscilloscope) plot:

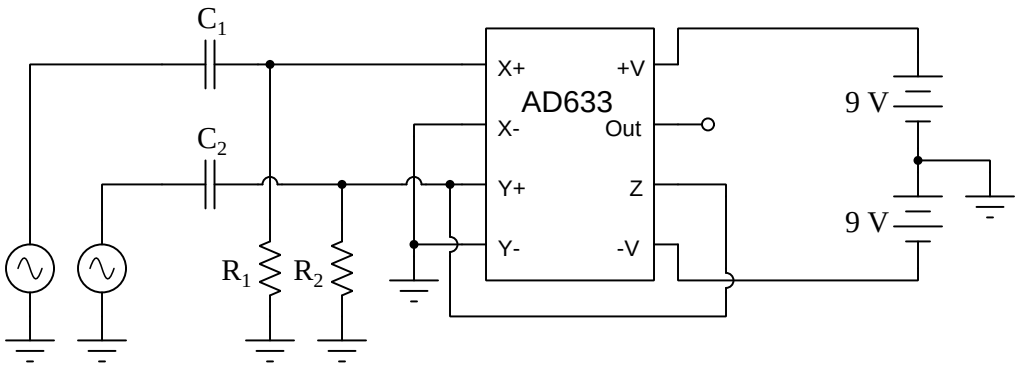


Frequency-domain (spectrum analyzer) plot:

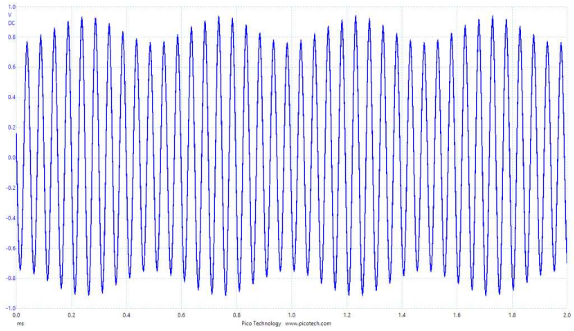


What we see here is what we'd expect for a balanced mixer circuit: just two sidebands with no trace of the original baseband or carrier signals in the output spectrum.

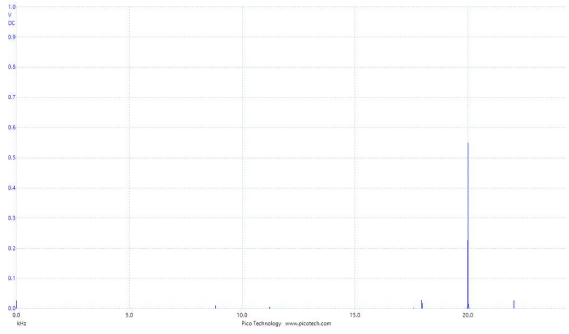
If we make a small modification to the circuit, connecting the carrier signal (Y) to the offset input (Z) so that the carrier will intentionally "leak" into the output spectrum, we see a more traditional AM result:



Time-domain (oscilloscope) plot:

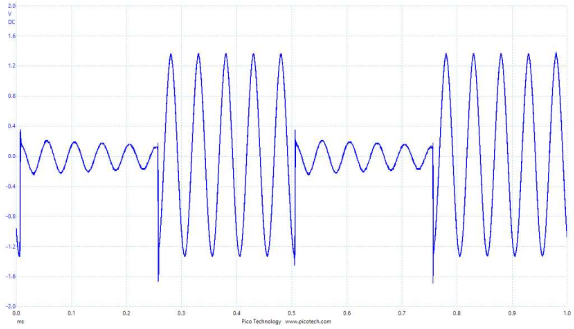


Frequency-domain (spectrum analyzer) plot:

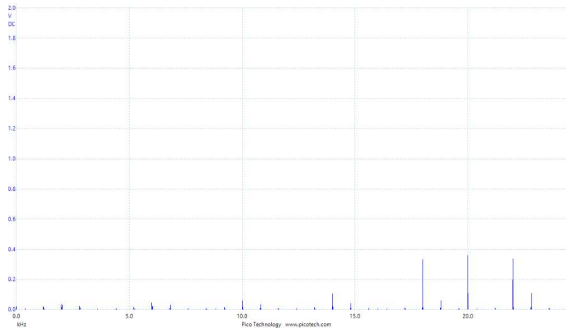


Lastly, we will substitute a 2 kHz square-wave as the baseband signal, while retaining the 20 kHz sine-wave as the carrier:

Time-domain (oscilloscope) plot:



Frequency-domain (spectrum analyzer) plot:



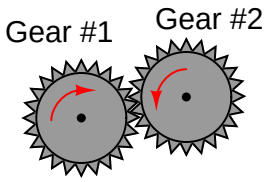
Note how many more sidebands we see due to the harmonics intrinsic to the square wave.

2.4 Example: frequency-mixing in gear systems

Rotating machinery produces vibrations whose frequencies are a function of the rotating components' speeds. Rotary speed is customarily expressed in units of *revolutions per minute* or *RPM*, though frequency is more commonly expressed in *cycles per second* or *Hertz*.

If a rotating component is off-balance, such that it produces a vibration oscillating once per rotation, the component's rotating speed divided by 60 yields the fundamental frequency of that vibration, since there is one cycle of vibration per rotation and 60 seconds within every minute of time. For example, an off-balance shaft rotating at 1800 RPM generates a fundamental vibration at 30 Hz.

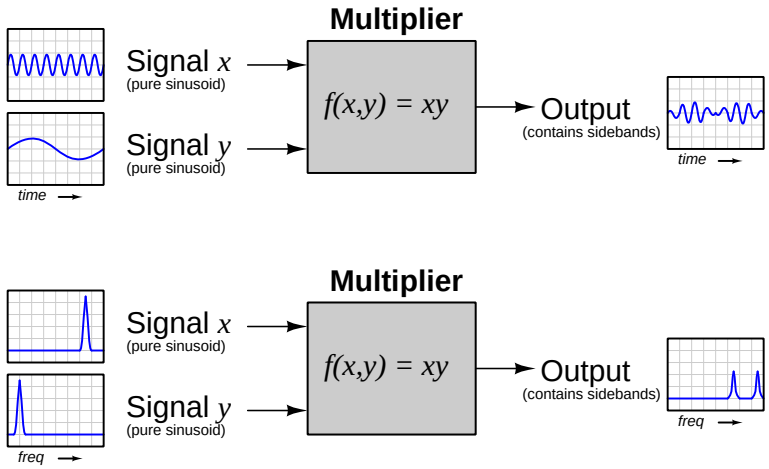
When rotating *gears* mesh together, the repeated contact between those gears' teeth produces vibrations as well, except these vibrations' fundamental frequency is equal to the shaft rotational speed multiplied by the number of teeth on the gear. Consider the gear-set shown in the illustration below, having 24 teeth each. If they both rotate at 1800 RPM, then the frequency of their teeth must be 24 times 30 Hz, or 720 Hz:



Something very interesting occurs if the shaft supporting one of these gears is slightly bent so that the gears mesh tightly during parts of their rotation but loosely during others. This will have the effect of modulating the intensity (amplitude) of the tooth vibration once per shaft revolution, so that the high-frequency vibrations produced by the teeth grow louder and softer at a frequency equal to the shaft speed. One can think of the bent shaft as imposing a sinusoidal *multiplying* factor to the tooth vibrations, which is mathematically the same as *mixing* applied to electronic signals.

Like all multiplicative mixing processes, a pair of gears suffering a bent shaft will produce vibrations at the shaft frequency (f_{shaft}), at the tooth frequency (f_{tooth}), and *sideband* frequencies of $f_{tooth} - f_{shaft}$ and $f_{tooth} + f_{shaft}$. The intensity of these sidebands as viewed on a frequency-domain plot (i.e. spectrum analyzer display) indicate the depth of the amplitude-modulation and therefore the severity of the shaft's bend.

Instead of this frequency-mixing occuring within an electronic multiplier circuit, it occurs naturally between the gears as a mechanical phenomenon. If we consider the "multiplier" in the following block diagrams to represent this natural function of meshing gear teeth, with signal x being the tooth noise and signal y being the bent shaft's oscillation, we may visualize the effects of this mixing in both the time domain (upper) and frequency domain (lower):



Machinery health experts know to look for sidebands and other such effects when analyzing the vibrations of complex mechanisms, and realize their diagnostic value in locating causes of abnormal vibration signatures.

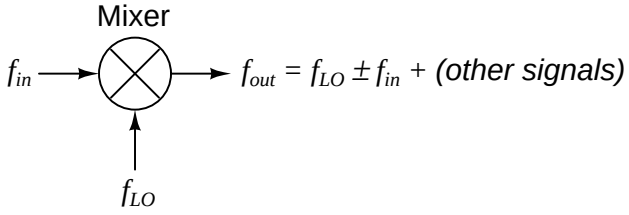
Chapter 3

Tutorial

3.1 Mixer applications

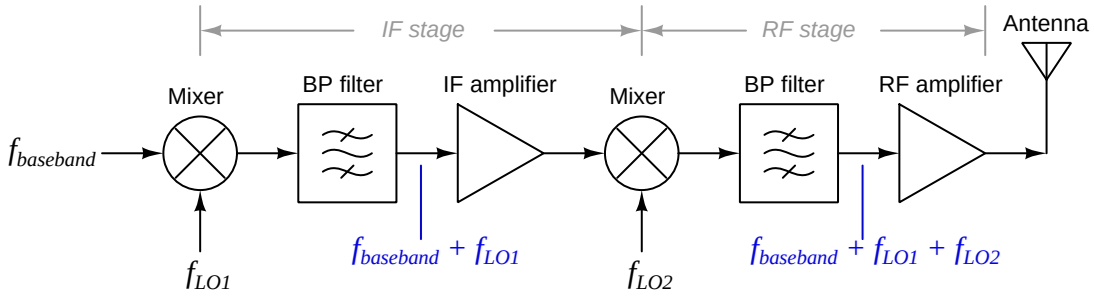
One of the most important building-blocks of any radio system is the *mixer*, the purpose of which being to combine two or more AC signals in a nonlinear fashion. This is done for a variety of reasons, including to *modulate* a carrier signal with information, to *demodulate* a received signal to extract the original information, and also to *convert* one band of frequencies into either a higher- or lower-frequency band.

Mixers are represented in block-diagram form by a circle with an "X" symbol inside. Generally, they take in a signal of one frequency and nonlinearly combine it with a signal provided by a "local oscillator" (LO) circuit to produce an output signal containing both the sum and difference frequencies of those two input signals, and usually some additional frequencies as well. Each input and output on a mixer is commonly referred to as a *port*:

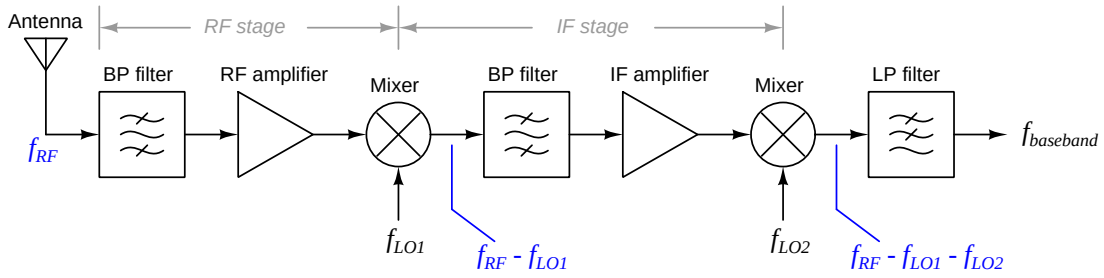


Some mixer circuits output *only* the sum and difference frequencies (called *sidebands*), while others require filter stages to remove the additional signals from the output.

In a radio transmitter system where some low-frequency "baseband" signal must be impressed upon a radio-frequency (RF) "carrier" signal suitable for broadcast via an antenna, mixers may be used to shift, or *upconvert*, the low-frequency baseband signal into frequencies high enough to effectively radiate as electromagnetic waves (i.e. radio waves). This upconversion is often performed in multiple stages, shifting the baseband signal's frequency up and filtering[1] it into an Intermediate-Frequency (IF) band, then upconverting and filtering again into a Radio-Frequency (RF) band suitable for radiation at an antenna:

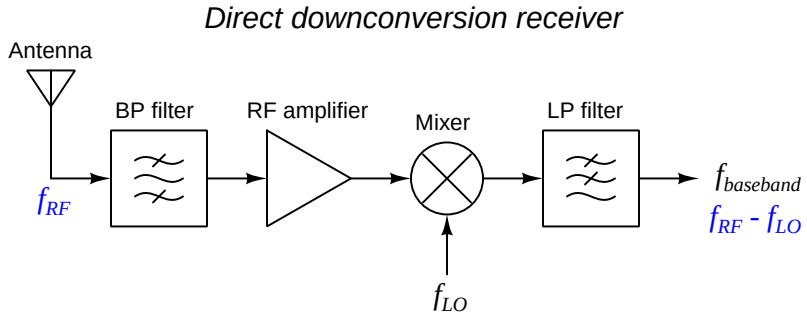


A radio receiver system must *downconvert* the modulated RF signal received by its antenna into "baseband" information understandable to the human or machine processing that communicated data, and once again mixers are useful for this purpose. Often we may use the exact same mixer circuits to perform the upconversion (transmitter) or downconversion (receiver) functions:



[1] Note the intuitive symbols used in these block diagrams for each filter: a set of three waves represent high-frequency (top), low-frequency (bottom), and intermediate frequency (middle) respectively, with single-slash marks denoting which of those frequency ranges is blocked by the filter. Thus, a filter symbol with the middle wave unblocked but the top and bottom blocked is a band-pass filter; a filter symbol with the bottom wave unblocked but the middle and top waves blocked is a low-pass filter, etc.

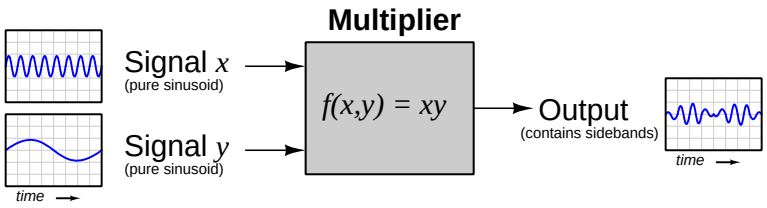
Downconversion is also possible in a single step, and this is commonly referred to as *direct downconversion* or *zero-IF* or *homodyne* where the local oscillator signal is set precisely equal to the carrier frequency of interest:



Direct downconversion results in the RF signal being stripped of all high frequencies, leaving only the information (baseband) conveyed by that signal in the mixer's output.

3.2 Multiplicative mixing

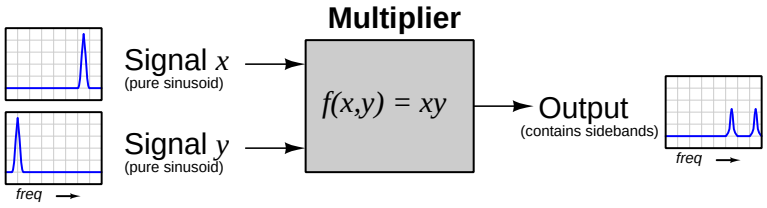
At root, a mixer circuit must *multiply* the instantaneous magnitudes of its input signals in order to modulate, demodulate, and/or shift the frequencies. The following block-diagram example shows how an analog multiplier circuit processes two sinusoidal input signals, one having a much greater frequency value than the other:



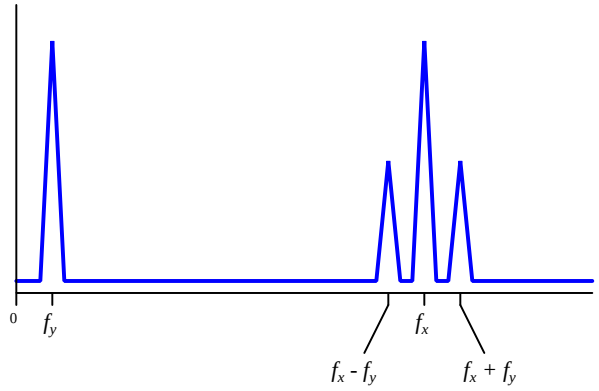
We may see the effect of this multiplication by examination of the time-domain displays, as shown by the blue-colored waveforms in the above illustration. Note how the peak-to-peak amplitude of the output signal is greatest at those points in time where signal y reaches its peak values, and lowest when y crosses zero. This is because the value of y serves as a multiplying coefficient for the value of x. Thus, while signal x has a constant magnitude (i.e. its "envelope" is level) the output signal's envelope mimics the shape of signal y.

What is far less intuitive to grasp is how these signals appear in the *frequency domain*, viewed as independent sine (or cosine) waves at different frequencies superimposed upon each other to comprise complex wave-shapes. When two sinusoidal signals are multiplied together, the result is the production of new frequencies called *sidebands*. The lower sideband signal's frequency is the difference between the two input signal frequencies, and the upper sideband signal's frequency is the sum of the two input signal frequencies.

Viewing the same signals in the frequency domain rather than in the time domain allows us to clearly see these sidebands:



Superimposing the three signals together on the same frequency-domain plot:



The signal having a frequency value of $f_x - f_y$ is called the *lower sideband* while the signal having a frequency value of $f_x + f_y$ is called the *upper sideband*. In order to understand the mathematical origins of these sideband frequencies, we must review some trigonometry.

Let us represent the two input signals as cosine functions, each with its own frequency (ω_x and ω_y). Since the value passed to a cosine function must be an angle and not a frequency, we will represent the two input signals as $\cos \omega_x t$ and $\cos \omega_y t$, respectively[2]. We may express the output signal as a time-based function, as follows:

$$f(x, y) = (\cos \omega_x t)(\cos \omega_y t)$$

Among the various mathematical identities in trigonometry is this one, relating the product of two cosine functions with cosines of the angular sum and difference:

$$\cos x \cos y = \frac{\cos(x - y) + \cos(x + y)}{2}$$

Substituting $\omega_x t$ for x and $\omega_y t$ for y yields the following result:

$$\cos(\omega_x t) \cos(\omega_y t) = \frac{\cos(\omega_x t - \omega_y t) + \cos(\omega_x t + \omega_y t)}{2}$$

Factoring out t in the angle terms on the right-hand side of this equation:

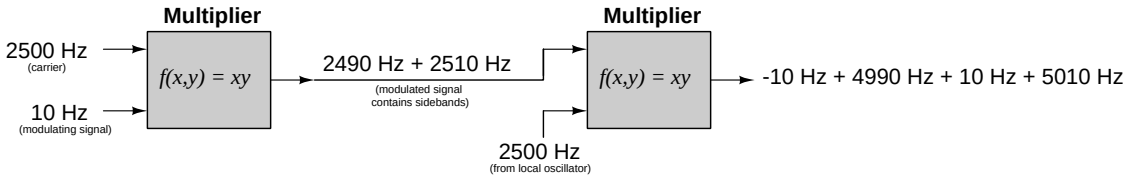
$$\cos(\omega_x t) \cos(\omega_y t) = \frac{\cos[(\omega_x - \omega_y)t] + \cos[(\omega_x + \omega_y)t]}{2}$$

This proves the multiplier's output signal consists of two sidebands. Given two sinusoidal input signals of differing frequency ($\cos(\omega_x t)$) and ($\cos(\omega_y t)$), the result is a *lower sideband* sinusoid having a frequency equal to the difference of the two input frequencies ($\cos[(\omega_x - \omega_y)t]$) and an *upper sideband* sinusoid having a frequency equal to the sum of the two input frequencies ($\cos[(\omega_x + \omega_y)t]$). The

[2] ω represents frequency in angular units per second, typically radians per second. This is an alternative to frequency measured in cycles per second (Hz), denoted as f. Time (t), of course, is typically measured in seconds. The product of radians per second and seconds is simply radians: an angle.

divisor of 2 merely means these sidebands will be weaker (i.e. less amplitude) than the input signals, and is irrelevant from the perspective of frequency.

For example, if signal x has a frequency of 2500 Hz and signal y has a frequency of 10 Hz, the two sidebands will have frequencies of 2490 Hz and 2510 Hz, respectively. This is what happens when a 2500 Hz *carrier* signal is amplitude-modulated by a 10 Hz *baseband* wave. If these modulated sideband frequencies happen to be fed into another multiplier along with a 2500 Hz *local oscillator* signal, the lower sideband wave of 2490 Hz will generate two sidebands of its own (−10 Hz and 4990 Hz) while the upper sideband wave will generate two sidebands of its own (10 Hz and 5010 Hz), as each of the modulated signal sideband frequencies gets mixed with the local oscillator's frequency in the second multiplier:



The two "difference" frequencies of 10 Hz and −10 Hz carry the information embodied by the original modulating signal. In fact, 10 Hz and −10 Hz are really identical to each other, since the cosine function is an *even*[3] mathematical function and therefore $\cos(-10t) = \cos(10t)$. This is why we generally consider the "difference" sideband frequency to be the *absolute value* of the difference. The two high "sum" frequencies of (4990 Hz and 5010 Hz) are irrelevant to our purposes here and may be easily filtered out using a low-pass filter, if they aren't already severely attenuated by the bandwidth limitations of the circuitry.

Thus, we see how a multiplying function may be used to *amplitude-modulate* a signal onto a higher-frequency carrier, and how the same type of function may be used to *demodulate* that modulated output to obtain the original signal's information. Another way of describing this is to say the first multiplier *upconverts* the 10 Hz signal into sidebands of much greater frequency, and then the second multiplier *downconverts* those sidebands to recover the original 10 Hz signal.

Note how the only mixing that takes place is between each frequency contained in one input signal and each frequency contained in the other. Multiple frequencies present within just one signal do *not* get mixed together by an ideal multiplier circuit. This is good, or else in our above example we would also end up with 20 Hz (2510 Hz − 2490 Hz) as well as 5000 Hz (2510 Hz + 2490 Hz) in the output spectrum, the 20 Hz frequency being easily mistaken for meaningful information in the original modulating signal! Just to be clear on this point, if we were to imagine a more complex set of signals entering into an ideal multiplier, one signal containing frequencies a and b and the other signal containing frequencies x and y, the multiplied output would consist solely of the following eight frequencies: $a + x$, $|a - x|$, $a + y$, $|a - y|$, $b + x$, $|b - x|$, $b + y$, and $|b - y|$.

[3] Any mathematical function $f(x)$ is considered *odd* if $f(-x) = -f(x)$, as is the case with polynomial functions having only odd degree terms (e.g. $2x^7 + 8x^5 - x^3 + 9x$). Conversely, any mathematical function is considered *even* if $f(-x) = f(x)$, as with polynomial functions having only even (or zero) degree terms (e.g. $x^6 - 6x^4 + 3x^2 - 8$). The sine function is considered odd while the cosine function is even: for example, $\sin(-10) = -\sin(10)$ while $\cos(-10) = \cos(10)$.

The use of a multiplier to "downconvert" the frequency of a signal by mixing it with another signal of differing frequency is called *heterodyning*. This is the same phenomenon you hear with two audio tones of nearly-identical pitch played simultaneously[4]: the result as heard by your ears is a low-frequency "beat" pattern representing the difference in frequency between the two original tones. Heterodyning was first applied to radio technology as a means to impart an audible tone to an inaudible (radio-frequency) signal, in order to construct a radio receiver circuit capable of indicating the presence or absence of a carrier-wave signal broadcast by a transmitter using a keyswitch to switch on and off.

[4]This is also the same phenomenon responsible for the perception of musical *harmony*: two or more tones played simultaneously that give rise to the impression of even more tones. This is why two or three tones of appropriate frequency played with each other sound so much richer than those same tones played one at a time: your auditory senses "mix" them together in such a way that you hear not only the original tones but also their respective sidebands. When mixed by a nonlinear system (in this case, your sense of hearing provides the nonlinearity necessary for true mixing to occur) the whole actually is greater than the sum of its parts.

3.3 Non-ideal mixing

While *multiplication* comprises the essential basis of signal mixing, few electronic devices naturally provide this function. However, this fact does not exclude common devices from being used as mixing elements. As it so happens, other device characteristics such as square-law and exponential response contain product terms, and so are able to mix signals together effectively.

Take for example the *square-law* characteristic of certain electronic devices such as field-effect transistors. Recall the characteristic equation for a FET, where drain current is proportional to the square of applied gate-source voltage:

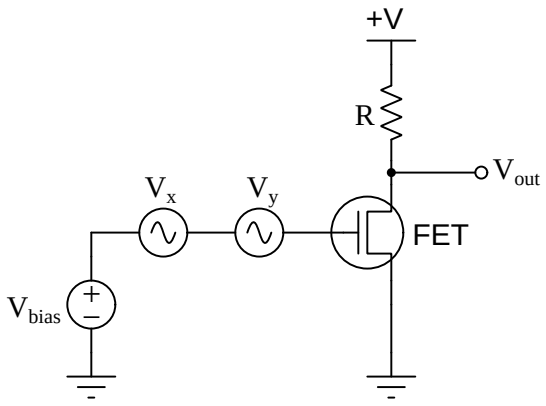
$$I_D = k(V_G - V_{TO})^2$$

Where,
I_D = Drain current, Amperes
k = Constant of proportionality
V_G = Gate-to-source voltage, Volts
V_{TO} = Threshold ("turn-on") voltage, Volts

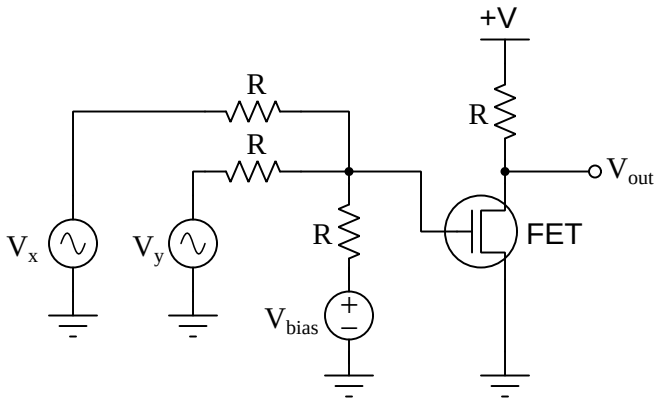
For the sake of simplification, we may consider a FET to exhibit the following proportional relationship between applied gate-source voltage and drain current:

$$I_D \propto V_G^2$$

We may exploit the inherently quadratic nature of the FET in order to mix two sinusoidal signals of differing frequency by electrically *adding* those signals together and sending that superposition into the FET as a single input signal voltage between gate and emitter:



Addition of input voltages V_x and V_y is achieved simply by virtue of their series connection, since voltages always add in series. Other possibilities exist for summing the two AC signals at the FET's input, such as the following resistor network allowing for ground-referenced voltage sources:



The fact that a network of resistors can only *average* the two input signals rather than *sum* them is irrelevant to our purpose. What does matter is that somehow we end up with a mathematical *product* of the two input signals, regardless of how or if they may be attenuated.

Now we must mathematically explore how it might be possible to extract the product of two signals from their squared sum. First, we will represent our two input signal voltages as cosine waveforms with different frequencies (ω_x and ω_y). The superposition of these two signals will comprise the FET's gate voltage, and its output (i.e. drain current) will be proportional to the square of that superposition:

$$I_D \propto [\cos(\omega_x t) + \cos(\omega_y t)]^2$$

Algebraic expansion of this equation is as simple as expanding any squared binomial, for example $(a + b)^2$ may be expanded to $a^2 + 2ab + b^2$:

$$I_D \propto \cos^2(\omega_x t) + 2\cos(\omega_x t)\cos(\omega_y t) + \cos^2(\omega_y t)$$

The middle term of this expanded equation clearly shows the two cosine-based signals multiplied by one another, which is precisely the mathematical relationship we require for mixing (i.e. amplitude modulation). Included with this product-of-signals term are two additional terms, each one being the square of its respective sinusoidal signal.

At this point the trigonometric identity $\cos^2 x = \frac{1+\cos(2x)}{2}$ proves useful to explore these additional terms. This identity tells us that squaring a cosine function results in another cosine function with a doubled angle. In the case of AC signals represented by cosines of continuously-changing angles, this doubling of the angle value is equivalent to a doubling of frequency. In other words, these cosine-squared terms are actually *second harmonics* of their respective x and y signal frequencies. This tells us the mixing will be impure, containing additional frequencies a pure multiplying mixer would never produce. With proper filtering after the mixer circuit stage, these harmonics may not pose any serious problem, but it is important to realize they will be present as an artifact of the FET's square-law nonlinearity.

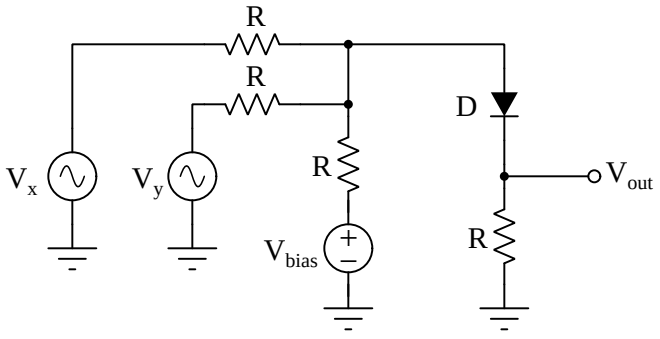
Another semiconductor device property exploitable for mixing purposes is the *exponential response*, notably found in the *Shockley diode equation* describing the current/voltage characteristics of a PN junction:

$$I = I_S \left(e^{\frac{qV}{nKT}} - 1 \right)$$

Where,

I = Forward-bias current through the diode, Amperes
I_S = Reverse-bias saturation current[5] through the diode, Amperes
e = Euler's constant (≈ 2.71828)
V = Voltage applied to the PN junction externally, Volts
q = Elementary charge of an electron (1.602×10^{-19} Coulombs)
n = Ideality factor (1 for a perfect junction)
k = Boltzmann's constant ($1.3806504 \times 10^{-23}$ J / K)
T = Absolute temperature (Kelvin), 273.15 more than degrees Celsius

Replacing the FET in the previous circuit with a simple diode, we see that V_{out} will depend[6] on this exponential equation:



Like current mirror circuits, diode-based mixers exploit the Shockley diode equation and therefore it is no longer a valid simplification to assume the diode's voltage drop will be constant (≈ 0.7 Volts). Instead, we must consider the nonlinear relationship between voltage and current for PN junctions – e.g. increasing voltage across a PN junction results in a disproportionately increased current. For the sake of simplification, we may consider the diode's current (and therefore V_{out}) to be proportional to the *exponential* of the average of the applied signal voltages:

$$I_D \propto e^{(V_x + V_y)} - 1$$

[5] A very small amount of current will still flow in the reverse-biased condition, due to so-called *minority carriers* in the P and N halves of the diode. This tiny current, usually in the range of *nano-Amperes*, is referred to as the *reverse saturation current* because its value does not increase appreciably with greater reverse-bias voltage but rather "saturates" or "plateaus" at a constant value. This saturation current, while fairly independent of applied voltage, varies greatly with changes in device *temperature*.

[6] If the value of the resistor connecting the V_{out} terminal to ground is relatively low in resistance, the voltage dropped by the diode will be approximately proportional to the average of the input voltages.

Some mathematical functions are equivalent to an infinite series of terms, and for the basic exponential function (e^x) this series is as follows:

$$e^x = 1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Re-writing this series without the factorial[7] quantities in the denominators:

$$e^x = 1 + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

As we can see, successive terms in this series become vanishingly small given the factorial progression of the denominators. This means we may ignore most of the higher-order terms for the sake of simplicity:

$$e^x \approx 1 + \frac{x^2}{2}$$

From this equivalence we see that the input voltages to the Shockley diode equation effectively become *squared* through the very nature of the exponential function:

$$I_D \propto e^{[\cos(\omega_x t) + \cos(\omega_y t)]} - 1$$

$$I_D \approx 1 + \frac{[\cos(\omega_x t) + \cos(\omega_y t)]^2}{2} - 1$$

$$I_D \approx \frac{[\cos(\omega_x t) + \cos(\omega_y t)]^2}{2}$$

Expanding this binomial leads us directly to the same $2\cos(\omega_x t)\cos(\omega_y t)$ term we saw with the FET example (as well as the additional, unwanted terms we got as a result of mixing the signals through a square-law device). Therefore, we may successfully mix signals together using a diode because the requisite product function is "hidden" within the diode's exponential characteristic.

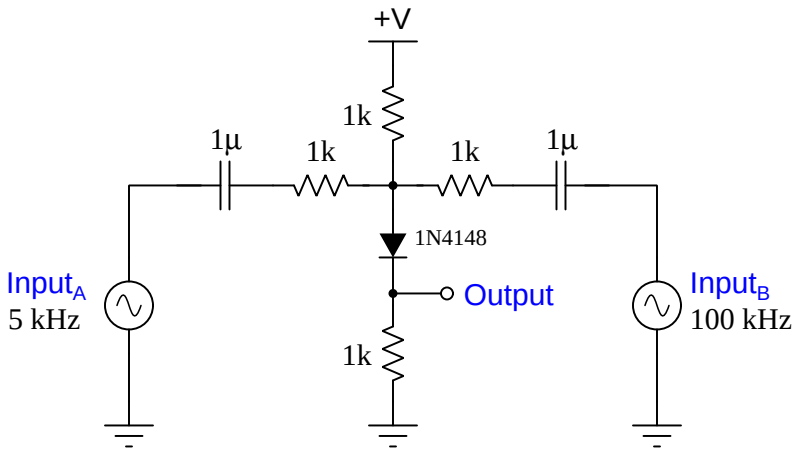
Unlike purely multiplicative mixing which takes in two signals of differing frequency and outputs only the two resulting sideband frequencies, square-law and exponential mixing outputs a range of additional frequencies along with the sidebands. These additional frequencies usually include the original input signals as well as certain harmonics of the original input signals, and is the price we pay for using primitive mixing circuitry (e.g. single FETs and diodes). In many applications these additional frequencies pose little problem, but in other applications it makes sense to use a more complex mixer circuit design with the aim of directly attenuating these additional frequencies. In particular, the suppression of the original signals (x and y inputs) in a mixer's output signal is a common design goal. *Leakage* is the term used to describe a signal at one port of a mixer appearing at another port, and *isolation* is the term used to describe a mixer's ability to attenuate leakage.

[7] For those unfamiliar with factorials, the factorial of a number is the product of that whole number as well as all whole numbers counting down to 1. So, $2! = 2 \times 1$ and $3! = 3 \times 2 \times 1$ and $4! = 4 \times 3 \times 2 \times 1$ and so on.

3.4 Passive mixers

A *passive* electronic device is one that cannot amplify a signal. Resistors, capacitors, inductors, and diodes are all examples of passive devices. In order to properly mix signals, the device in question must have a nonlinear characteristic, and of the passive devices previously listed only one of them does[8]: the diode. Therefore, passive mixers use diodes as their nonlinear elements.

We have already seen an example of a single-diode passive mixer circuit in the previous section, but another version of that same circuit will be shown here complete with coupling capacitors to prevent any DC component of the two AC sources from interfering with the biasing of the diode:

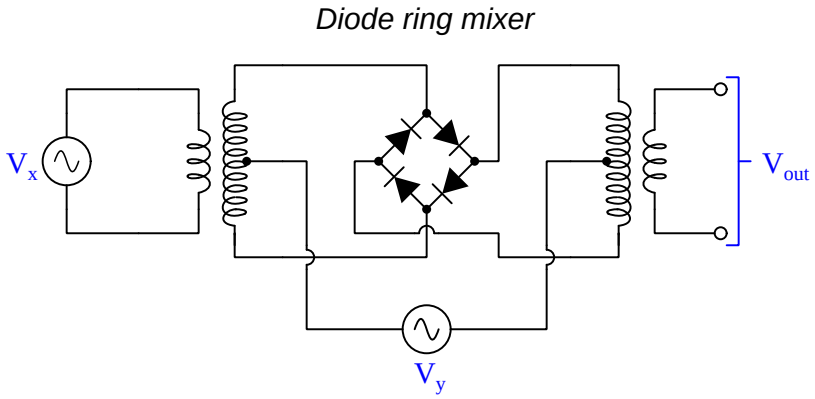


Given the frequencies of the two AC input signals, we would expect to see sidebands at 95 kHz and 105 kHz, as well as both of the original signals (5 kHz and 100 kHz), present in the output spectrum. Additionally, we would expect to see some *artifacts*[9] such as second harmonics present in the output spectrum due to the diode's exponential function.

[8] Iron-core inductors are slightly nonlinear, especially when the magnetic flux density approaches saturation for the iron core. However, these are generally impractical for high-frequency mixing applications due to the high parasitic energy losses inherent to iron (especially iron operating at near-saturation levels of flux!). It is worth mentioning here because audio transformers use ferromagnetic cores, and detectable mixing (called *intermodulation*) does indeed occur within these devices – an example of *unintended* mixing that results in slight distortion of the sound.

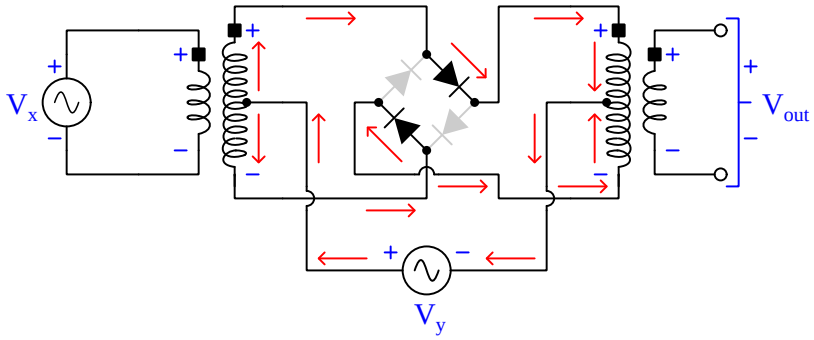
[9] An "artifact" is any error resulting from the technique or technology used to process the information.

A more sophisticated diode-based mixer design is the *diode ring* circuit, often referred to as a *ring modulator* because of its widespread use as an amplitude-modulator (AM). This particular design is called *double-balanced*:

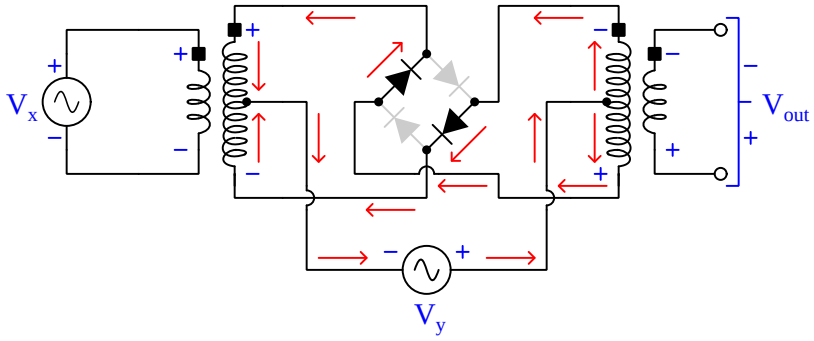


The four-diode network in the center of this circuit may resemble a bridge rectifier at first glance, but it is definitely different! Note how all four diodes point in the same circular direction (in this example, clockwise, but know that counter-clockwise is also acceptable). A good way to analyze the behavior of this circuit is to trace currents from one source at a time, one polarity at a time, where you will find no output signal generated with just one source active at any given time. This simple thought experiment demonstrates the *isolation* inherent to this double-balanced style of mixer circuit, in that its output signal will be devoid of either original input signal and in fact is equal to zero when either input is zero (as one would expect from any circuit that *multiplies* two voltage signals together: the product should be zero when either input is zero).

If we continue our line of thought experiments to include cases where both source amplitudes are non-zero, we see how the action of one source "steers" the other source's signal to the output transformer along two different pathways. In the following diagrams the current arrows show only the currents from source V_y, alternately "turning on" one pair of diodes and "turning off" the other pair of diodes in the ring network so that any voltage from V_x gets directed to the output transformer either in non-inverting or inverting fashion:



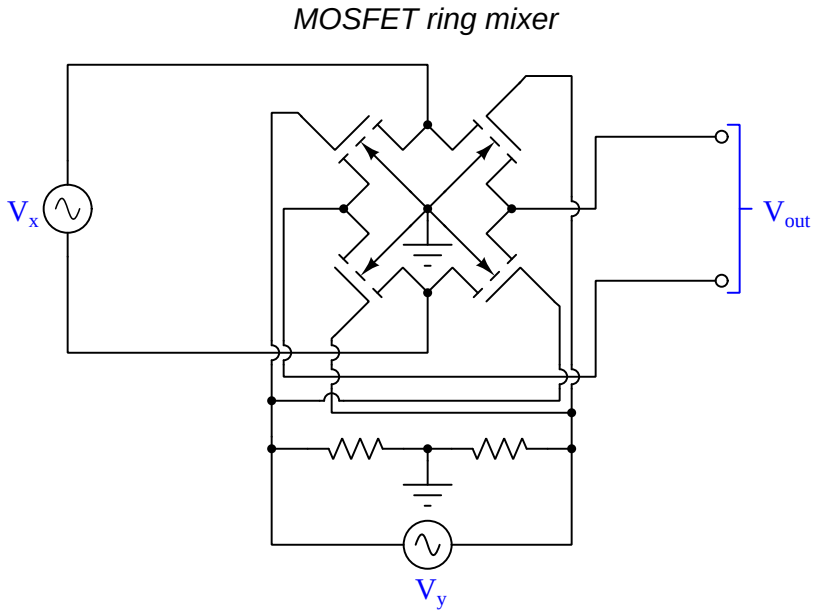
V_{out} is in-phase with V_x



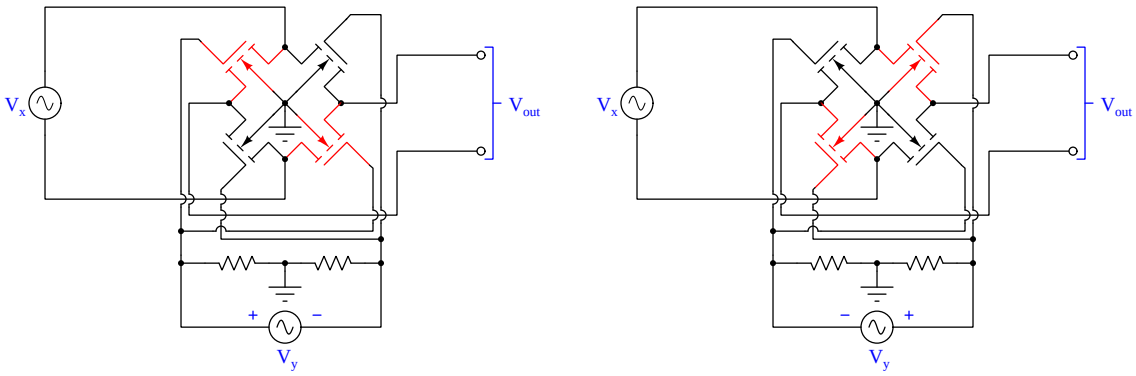
V_{out} is 180^o out of phase with V_x

This behavior also agrees with the model of a mixer as a *multiplier*: the sign of V_y has the ability to alter the output signal sign for any given sign of V_x, just as multiplying any number by a positive coefficient preserves that number's sign while multiplying by a negative coefficient reverses the sign.

A variation on the design theme of a diode ring is a ring comprised of MOSFETs:



Like the diode ring mixer, the MOSFET ring mixer alternately "turns on" opposite pairs of elements in the ring network and thus "steers" the V_x signal to the V_{out} terminals so as to be either in-phase or out-of-phase with the original V_x signal:



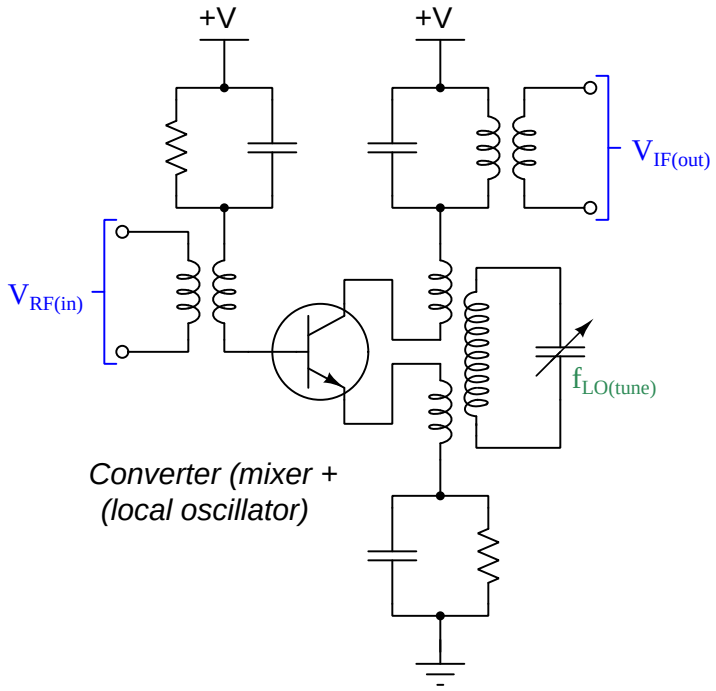
Considering the diodes or transistors to be on/off switching elements is not just a simplification strategy for analyzing the circuit's behavior, but indeed may be a real mode of operation. In modern digital RF upconversion and downconversion circuits the local oscillator signal is often a *square wave* rather than a sinusoid. From a frequency perspective the fact that square waves are rich in harmonics is of little consequence since those may generally be filtered out after the mixer.

3.5 Active mixers

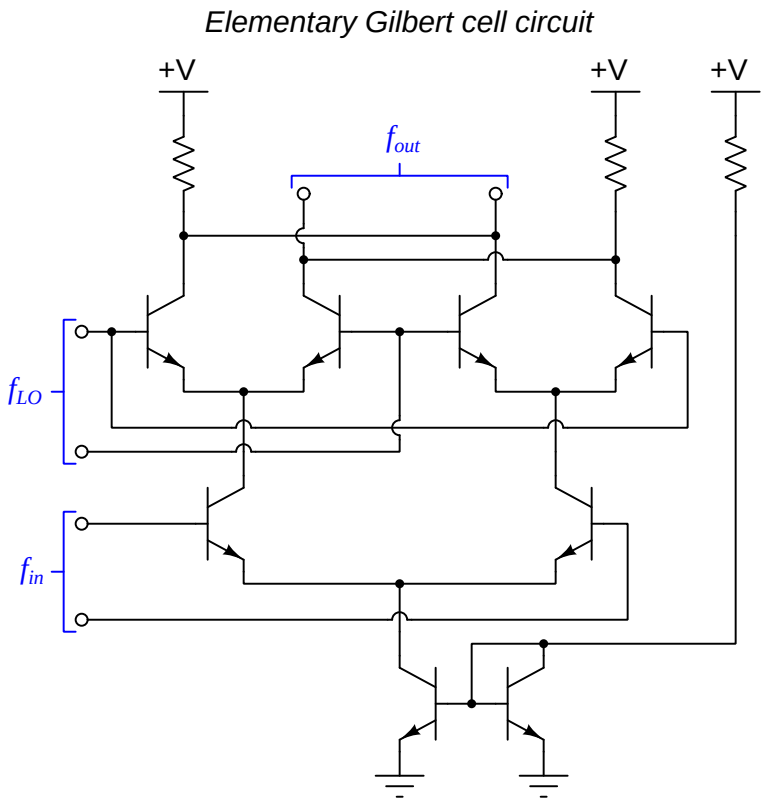
An *active* electronic device is one that amplifies a signal. Transistors and vacuum tubes are both examples of devices capable of operating in an active mode. In the previous section we saw a passive mixer circuit using field-effect transistors rather than diodes, but that was still considered passive rather than active because the transistors were not actually being used to *amplify*, but rather to merely *switch* polarity of the other signal. In the mixer circuits we are about to explore, the transistors or vacuum tubes connect to sources of DC power which enable them to boost the power of the mixed signals, unlike the passive mixer circuits of the previous section which often require substantial amounts of input signal power to properly function.

Legacy AM (amplitude-modulation) transmitter circuits often combined modulation with other functions in an effort to reduce parts count and thereby conserve the circuit's complexity. Older radio transmitter designs often mixed the audio signal with the RF signal in the power amplifier stage, and older receivers were known for using circuits called *converters* that combined mixer and local oscillator functions in one tube or transistor.

An example of a single-transistor converter circuit appears below, as might be found within a legacy radio receiver circuit. The variable capacitor and its connected inductor form an LC "tank circuit" dictating the self-oscillating (local) frequency of the converter. The RF input signal is transformer-coupled to the transistor's base, and the inherent nonlinearities of the transistor are sufficient to mix the RF input and local oscillator signals together. A second LC "tank circuit" filters the downconverted spectrum from all other signal frequencies and couples it to the output terminals, where it is the Intermediate Frequency (IF) signal:



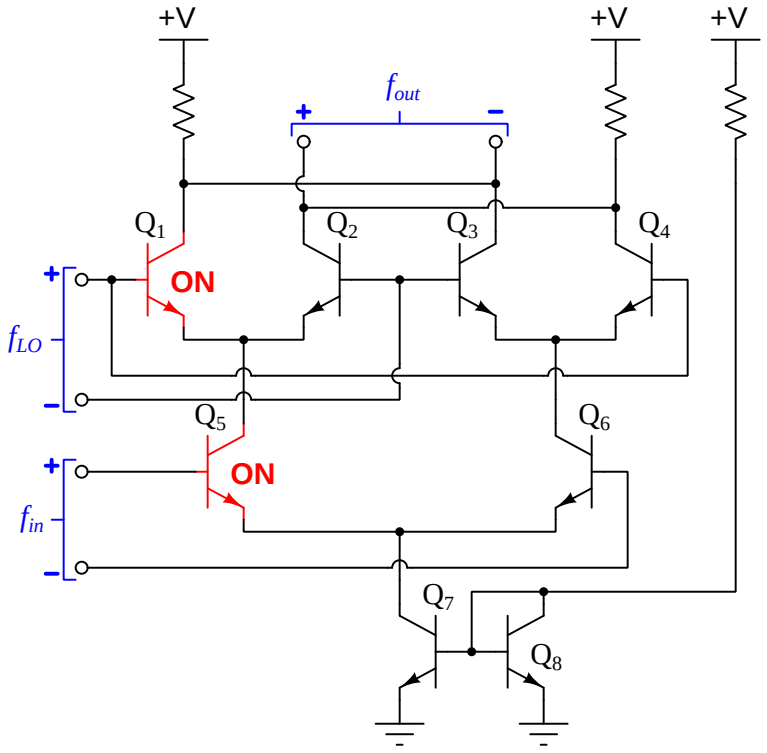
A differential amplifier circuit commonly used for RF signal mixing in modern radio circuits is the so-called *Gilbert cell*, named after Barrie Gilbert. Its most basic configuration is a six-transistor network of differential amplifiers with a current source[10], as shown in the following schematic diagram:



Even in its simplified form the Gilbert cell circuit seems daunting at first observation. However, we may approach its analysis by simplifying the types of signals we send to its two differential inputs (f_{LO} and f_{in}): using "square-wave" signals rather than sinusoidal. This will be an example of the *limiting cases* problem-solving strategy in action, considering only the "limit" cases of peak AC signal values rather than their full analog range in order to simplify the circuit's action.

[10] The bottom two transistors function as a *current mirror*, passing a constant amount of current as the six transistors above turn on and off to varying degrees. This is an amplifier design principle known as *active loading*.

First we will consider the action of the Gilbert cell circuit with both input terminal pairs positive on top and negative on bottom. Transistor states will be assessed as either "on" or "off" in this limiting-case analysis except for transistor Q_7 which functions as a constant-current regulator and is therefore partially "on" at all times:



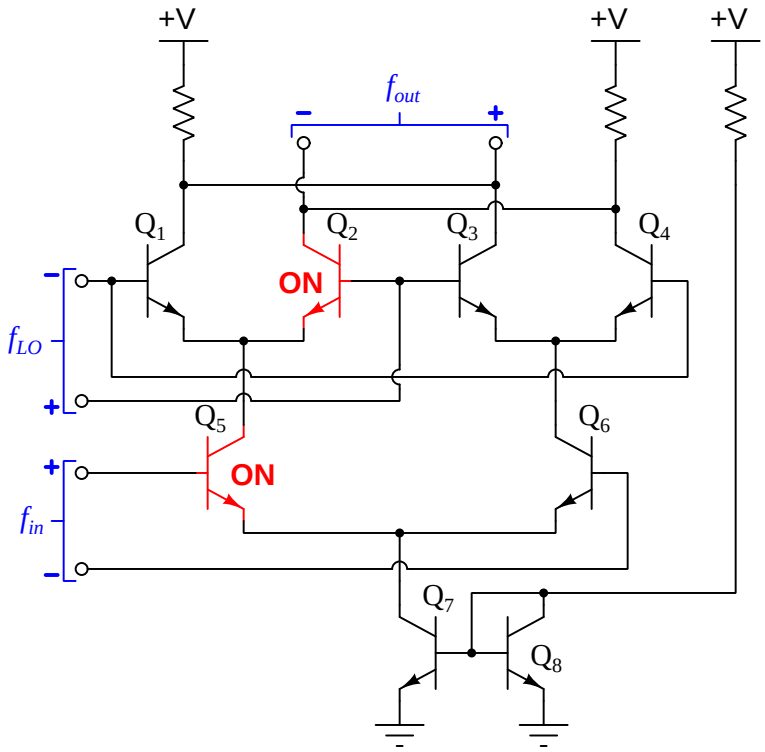
f_{in}'s polarity turns on Q_5 more than Q_6, allowing differential pair Q_1/Q_2 to function while shutting off differential pair Q_3/Q_4. f_{LO}'s polarity turns on Q_1, drawing down the electrical potential at the right-hand terminal of f_{out} while the left-hand terminal of f_{out} rises to full +V potential as a result of both Q_2 and Q_4 being non-conducting.

For the sake of analyzing this circuit as a *mixer* (ideally, a *multiplier*), we will arbitrarily consider all three signals (f_{LO}, f_{in}, and f_{out}) to be numerically "positive" in the polarities shown, and represent this state of affairs as the product of a positive quantity and another positive quantity:

$$(f_{LO}) \times (f_{in}) = (f_{out})$$

$$(+) \times (+) = (+)$$

Next we will reverse the polarity of the local oscillator signal (f_{LO}) and re-analyze the circuit in terms of "on" transistor states:

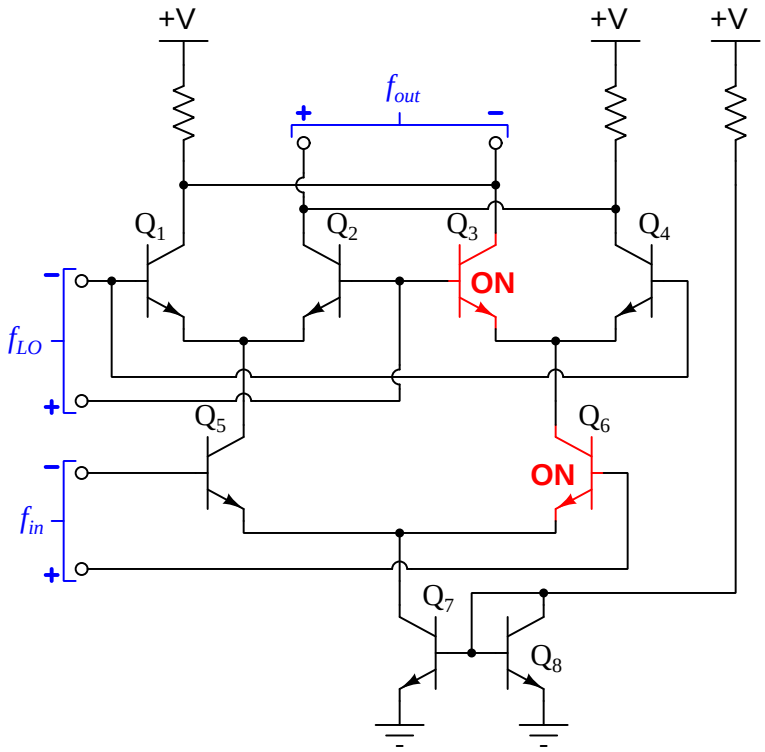


f_{in}'s polarity still has Q_5 on more than Q_6, allowing differential pair Q_1/Q_2 to function while shutting off differential pair Q_3/Q_4. The reversed polarity of f_{LO} turns on Q_2, drawing down the electrical potential at the left-hand terminal of f_{out} while the right-hand terminal of f_{out} rises to full +V potential. Represented mathematically:

$$(f_{LO}) \times (f_{in}) = (f_{out})$$

$$(-) \times (+) = (-)$$

Next we will reverse the polarity of the input signal (f_{in}) and re-analyze the circuit in terms of "on" transistor states:

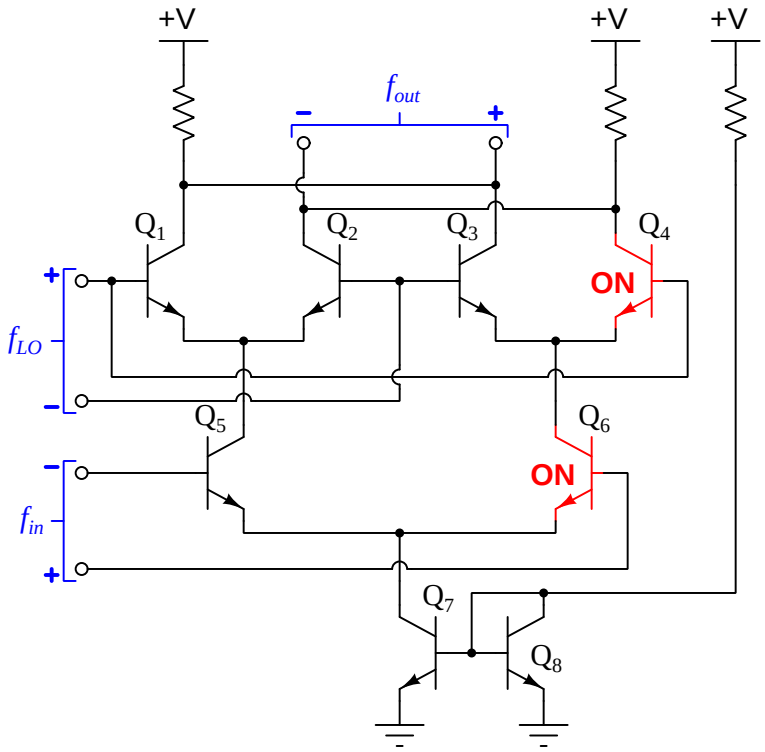


f_{in}'s polarity now turns on Q_6 more than Q_5, de-energizing differential pair Q_1/Q_2 while allowing differential pair Q_3/Q_4 to function. The polarity of f_{LO} turns on Q_3, drawing down the electrical potential at the right-hand terminal of f_{out} while the left-hand terminal of f_{out} rises to full +V potential. Represented mathematically:

$$(f_{LO}) \times (f_{in}) = (f_{out})$$

$$(-) \times (-) = (+)$$

Lastly we will reverse the polarity of the local oscillator again (f_{LO}) and re-analyze the circuit in terms of "on" transistor states:

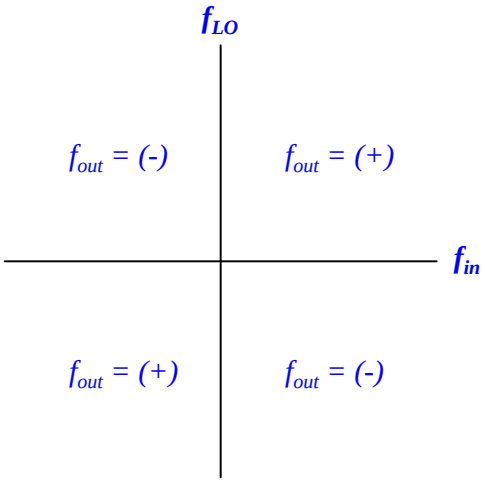


f_{in}'s polarity still holds Q_6 in the "on" state more than Q_5, maintaining differential pair Q_1/Q_2 in an inactive state while allowing differential pair Q_3/Q_4 to function. The reversed polarity of f_{LO} turns on Q_4, drawing down the electrical potential at the left-hand terminal of f_{out} while the right-hand terminal of f_{out} rises to full +V potential. Represented mathematically:

$$(f_{LO}) \times (f_{in}) = (f_{out})$$

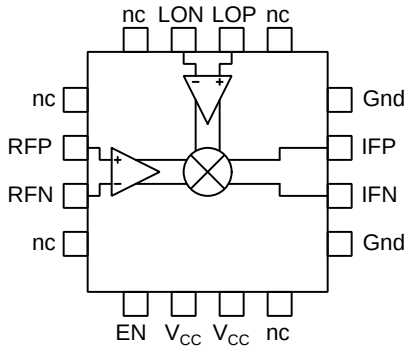
$$(+) \times (-) = (-)$$

As we can see from these four limiting-case analyses, the Gilbert cell functions as a *four-quadrant multiplier*. This phrase simply means the circuit is able to effectively multiply all four combinations of mathematical signs for its two inputs. Shown on an x-y coordinate graph, each "quadrant" of the graph represents a different combination of input signal polarity and local oscillator polarity:



Multiplication is the fundamental property necessary for signal mixing, and a mixer circuit balanced such that it multiplies equally well in all four quadrants will function with minimal artifact frequencies present in its output. Gilbert cell mixers lend themselves to construction in integrated-circuit (IC) form where the transistor parameters may be tightly controlled and well-matched to each other.

A typical IC downconversion mixer pinout will resemble something like this:



Being an active device, it requires DC power to function. Being designed to interface with digital circuitry, the manufacturer will often provide an "enable" input (EN) that may be used to strobe the device to enable and disable its output on command. Pins marked "nc" have "no connection" and are unused.

When operated as RF mixers, the local oscillator signal (f_{LO}) is often a square-wave or an

over-driven sinusoid rather than a pure sinusoid. Such signals are easier to generate on demand, and when the upper portion of the Gilbert cell is driven as such, those four transistors need not amplify linearly in order for the circuit to perform its task as a mixer. Only the lower transistors (operating with the "live load" current mirror) need function linearly in order to be able to process non-square-wave signals at f_{in}.

Chapter 4

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

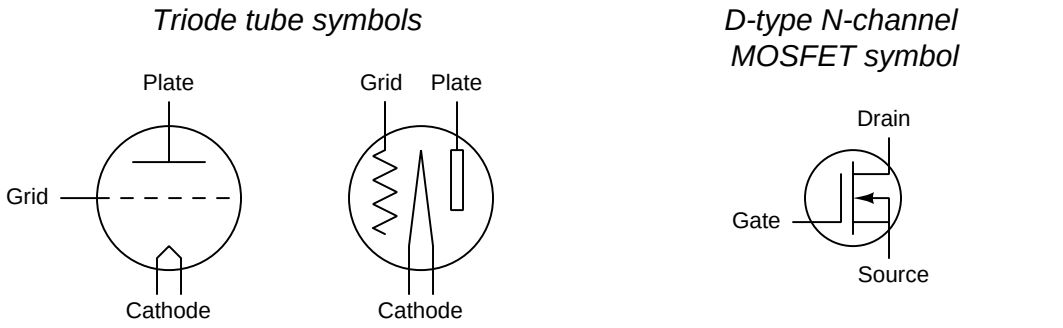
Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

4.1 Trunked telephony system

A United States patent (number 1,835,031) granted to Lloyd Espenschied and Herman Affel on 8 December 1931 documents in some detail a telephony system using frequency-division multiplexing to convey multiple audio conversations along a common coaxial cable. The patent's claims centered on the coaxial cable which was a novelty[1] at that time, however the rest of the system as described in this patent gives us a fascinating look into communications technology of that era, and specifically to the use of mixing (heterodyning) as a signal-processing technique.

State-of-the-art electronics technology in the 1930's was all based on vacuum tubes as active elements, as the transistor had not yet been invented. Specifically, the type of vacuum tube used in this telephone system's mixer (modulator/demodulator) units was called a *triode*, the characteristics of which being most similar to a modern D-type N-channel MOSFET.

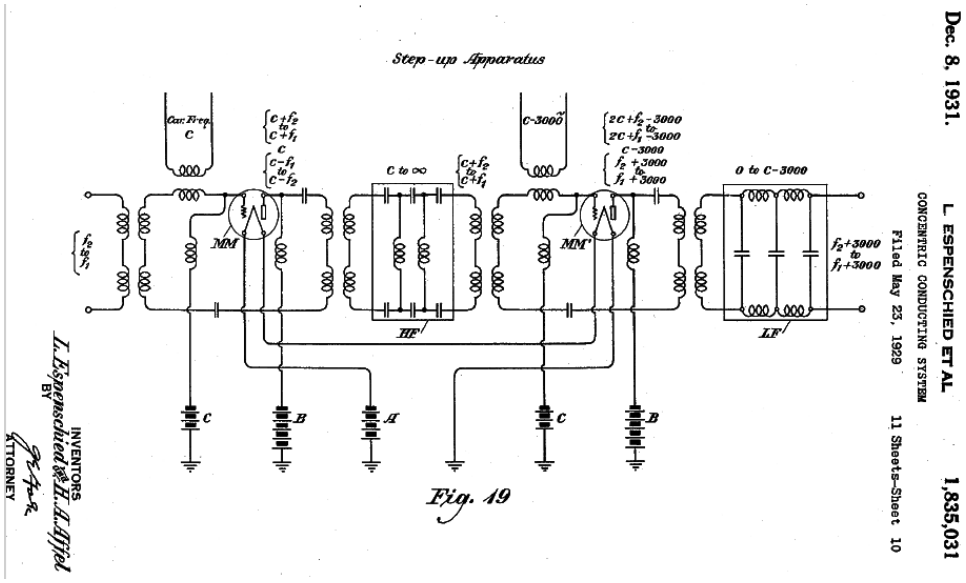
Triode tubes have three internal metallic elements (hence the name *tri*ode): a *cathode*, a *grid*, and a *plate*. The cathode is a metal filament electrically heated to a dull-red glowing temperature, which causes electrons to be thermally liberated from the surface of the metal. These electrons may travel through the high vacuum inside the tube to the positively-charged plate but not in the other direction, giving the tube a diode (one-way) characteristic. The grid was a wire mesh positioned between the negative cathode and the positive plate, any voltage impressed between the grid and cathode serving to modulate the flow of electrons from cathode to plate:



Like MOSFETs, triode tubes exhibited a square-law characteristic which made them useful as mixing elements. If two signals of differing frequency were superimposed at the tube's input (i.e. grid), the resulting output signal (i.e. at the plate) would contain the original signals as well as sidebands formed by those signals' intermodulation.

[1] The distinct advantage of coaxial cabling over open-wire transmission lines was less loss due to electromagnetic radiation, as well as significantly improved shielding and signal integrity when such lines were positioned alongside other communication and power lines. These improved performance characteristics allowed for a much greater range of signal frequencies to be conveyed over long distances via coaxial cable, and so much of the patent focuses on techniques to exploit that enhanced bandwidth.

One of the possible multiplexing schemes taking advantage of the high bandwidth of the inventors' new coaxial cable relied upon cascaded frequency-shifting units which they called "step-up" and "step down" apparatus. We would refer to such circuits now as *upconverting* and *downconverting* mixers, respectively. Figure 19 in the patent shows the schematic diagram for one of these upconverters, its purpose being to shift a range of input frequencies up by 3000 Hz:



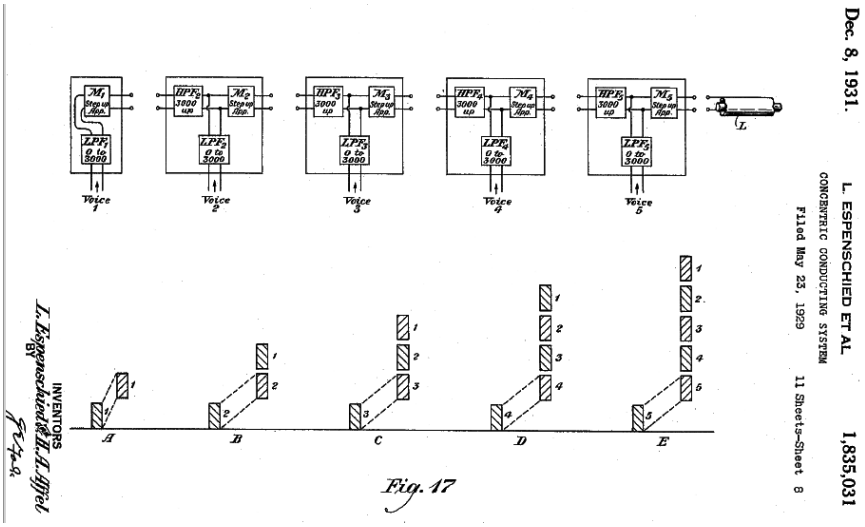
A "carrier" signal c (i.e. local oscillator signal) is mixed with the incoming band of frequencies (from f_1 to f_2), the resulting output at the plate of the first triode being the original carrier (c) along with an upper sideband ($c + f_1$ to $c + f_2$) and a lower sideband ($c - f_1$ to $c - f_2$). This spectrum of frequencies then enters a high-pass filter network with a cutoff frequency of c, so that the original carrier and lower sideband are practically eliminated. The upper sideband then gets mixed again with another carrier (local oscillator) signal having a frequency 3000 Hz less than the first ($c - 3000$). The output signal at the plate of the second tube contains this new carrier along with another upper and lower sideband pair:

$$\text{USB } = (c - 3000) + (c + f_1 \text{ to } c + f_2) = 2c + f_1 - 3000 \text{ to } 2c + f_2 - 3000$$

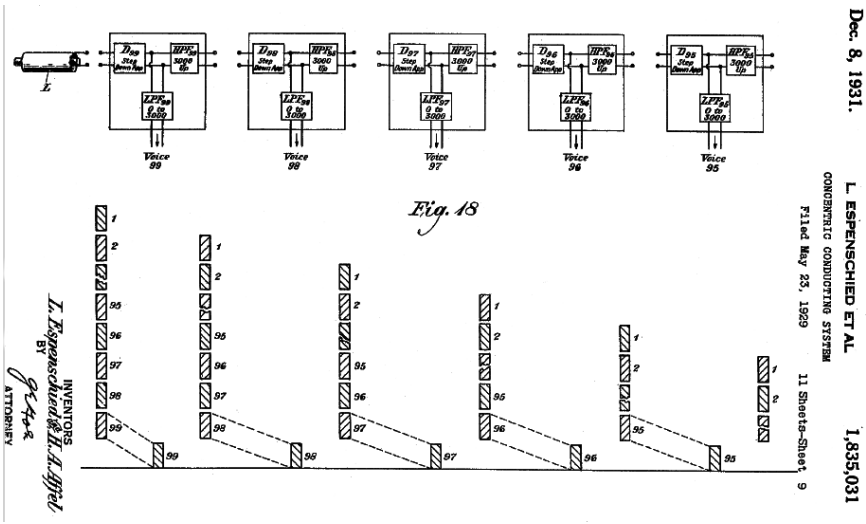
$$\text{LSB } = (c - 3000) - (c + f_1 \text{ to } c + f_2) = f_1 + 3000 \text{ to } f_2 + 3000$$

This output signal enters a low-pass filter network with a cutoff frequency of $c - 3000$ so that only the lower sideband passes through. Thus, the input signal band (f_1 to f_2) becomes shifted upward in its entirety to $f_1 + 3000$ to $f_2 + 3000$.

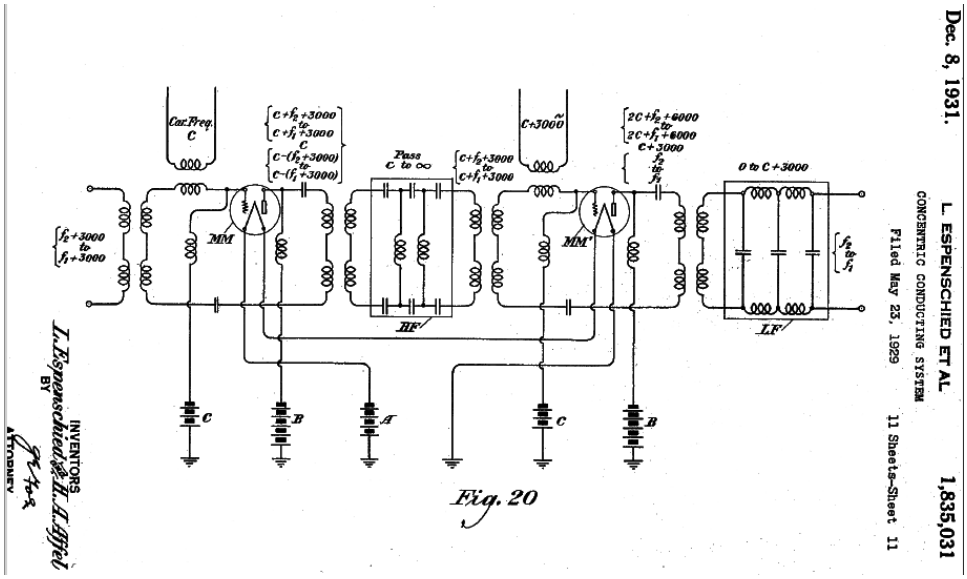
If a series of these upconverters (called "step-up apparatus" by the inventors) were cascaded and provided with the necessary local oscillator frequencies they would have the effect of shifting the narrow-band audio signals of each telephone microphone up by 3000 Hz and "stacking" those bands on top of each other in the frequency spectrum, as shown by Figure 17 from the patent. High-pass filters (HPF) and low-pass filters (LPF) help direct the baseband audio conversations and up-converted signals to their proper places:



On the receiving end of the trunked telephone line another series of mixers (downconverters, called "step-down apparatus") by the inventors performed the same frequency-shifting function in reverse. Figure 18 from the patent shows how such a cascade could "disassemble" the composite spectrum into its constituent telephone conversations, distributing each one to its respective telephone speaker:



A schematic diagram shown in Figure 20 of the patent shows how each downconverter worked:



An incoming frequency band ranging from $f_1 + 3000$ to $f_2 + 3000$ is first mixed with a local oscillator frequency c, resulting in an output at the first triode's plate consisting of that carrier frequency in addition to two sidebands:

$$\text{USB } = c + (f_1 + 3000 \text{ to } f_2 + 3000) = c + f_1 + 3000 \text{ to } c + f_2 + 3000$$

$$\text{LSB } = c - (f_1 + 3000 \text{ to } f_2 + 3000) = c - (f_1 + 3000) \text{ to } c - (f_2 + 3000)$$

A high-pass filter network screens out everything but the upper of these sidebands, which is then mixed with another local oscillator signal $(c + 3000)$. This second stage of mixing outputs that carrier signal plus two sidebands:

$$\text{USB } = (c + 3000) + (c + f_1 + 3000 \text{ to } c + f_2 + 3000) = 2c + f_1 + 6000 \text{ to } 2c + f_2 + 6000$$

$$\text{LSB } = (c + 3000) - (c + f_1 + 3000 \text{ to } c + f_2 + 3000) = f_1 \text{ to } f_2$$

The final filter stage is a low-pass network with a cutoff of $c + 3000$, which leaves only the lower sideband present at the output terminals.

It is truly impressive to consider how the concept of heterodyning (mixing) could be implemented using relatively primitive electronic devices to multiplex numerous telephone conversations over a single cable nearly one hundred years ago! This system is also a reminder of the enduring value of *first principles*: technologies come and go, but fundamental principles remain ever true.

Chapter 5

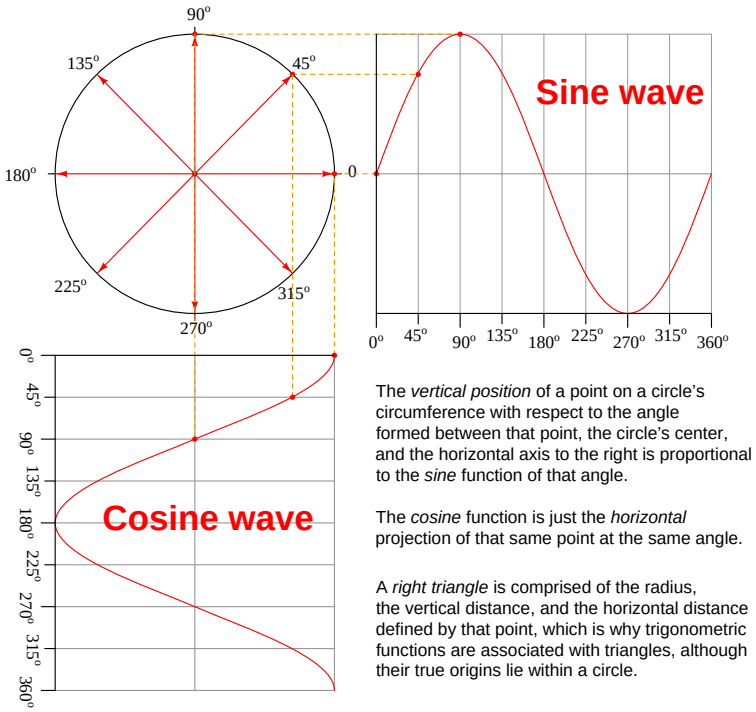
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Trigonometry Reference

Trigonometry is a very useful mathematical tool for understanding modulation techniques operating on sinusoidal (sine-wave-shaped) signals. This reference lists a number of trigonometric identities and relations involving only sine and cosine functions. The variables x and y are used to denote angles. In AC circuits the angle of an oscillating signal is constantly changing, and so instead of using x or y to denote a constant angle, we often use the product of angular velocity and time (ωt) to denote the angle as a function of time[1].

5.1.1 The Unit Circle



5.1.2 Pythagorean identity for sine and cosine

$$\sin^2 x + \cos^2 x = 1$$

[1] Since ω is expressed in angular units per second (e.g. radians per second, or degrees per second) and t is expressed in seconds, the product of ω and t must cancel the unit of time and leave us with just an angle.

5.1.3 Odd and even functions

Any mathematical function $f(x)$ is considered *odd* if $f(-x) = -f(x)$, as is the case with polynomial functions having only odd degree terms (e.g. $2x^7 + 8x^5 - x^3 + 9x$). Conversely, any mathematical function is considered *even* if $f(-x) = f(x)$, as with polynomial functions having only even (or zero) degree terms (e.g. $x^6 - 6x^4 + 3x^2 - 8$). The sine function is considered odd while the cosine function is even: for example, $\sin(-10) = -\sin(10)$ while $\cos(-10) = \cos(10)$.

$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$$

5.1.4 Sums and differences of angles

$$\sin(x \pm y) = (\sin x \cos y) \pm (\cos x \sin y) \qquad \cos(x \pm y) = (\cos x \cos y) \mp (\sin x \sin y)$$

5.1.5 Products and sums of sine and cosine functions

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2} \qquad \cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$$

$$\sin x \cos y = \frac{\sin(x+y) + \sin(x-y)}{2} \qquad \cos x \sin y = \frac{\sin(x+y) - \sin(x-y)}{2}$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

5.1.6 Squares of sine and cosine functions

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

5.1.7 Doubled angles

$$\sin(2x) = 2\sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

5.1.8 Halved angles

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Thought experiments as a problem-solving strategy

Limiting cases as a problem-solving strategy

Annotating diagrams as a problem-solving strategy

Algebraic substitution

Sinusoidal decomposition (i.e. Fourier's Theorem)

Time domain

Frequency domain

Fundamental frequency

Harmonic frequency

Linearity

Modulation

Demodulation

Baseband

Carrier

Mixing

Vacuum tube

Heterodyning

Local oscillator

Up/downconversion

Sideband

Filter

Amplification

Active versus Passive devices

Radio frequency (RF)

Intermediate frequency (IF)

Diode

Shockley diode equation

Tank circuit

Oscillator

BJT principles

FET principles

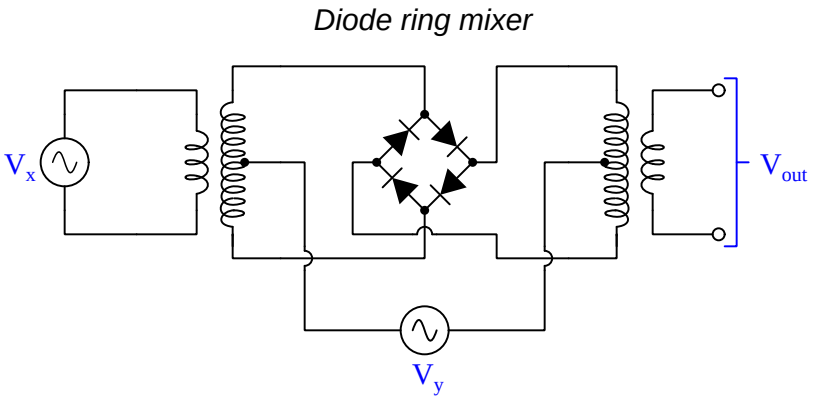
Balanced mixing

Artifact

Differential pair

6.1.3 Diode ring mixer limit cases

Prove that zero output signal will develop from this diode ring mixer circuit in each of the following conditions, each condition representing a "limit case" where one source is zero while the other source reaches its peak voltage value:



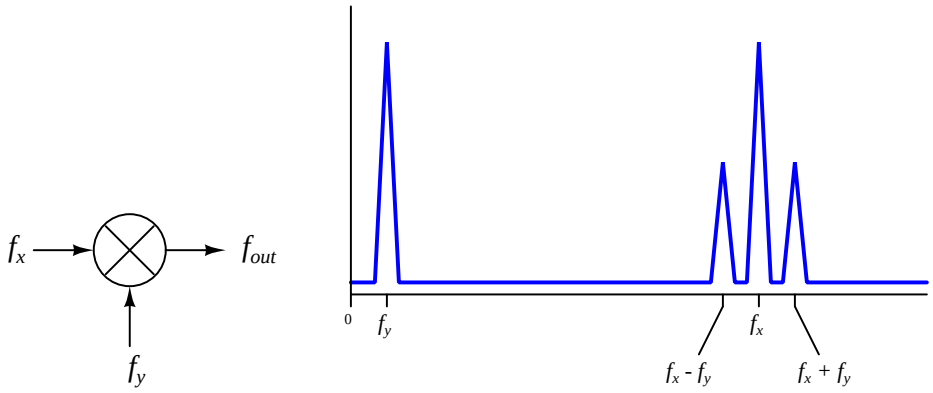
- V_x = pulsing + on top and − on bottom
- V_x = pulsing − on top and + on bottom
- V_y = pulsing + on left and − on right
- V_y = pulsing − on left and + on right

Challenges

- Why is this mixer circuit classified as *passive*?

6.1.4 Frequency adjustment on a mixer input

Identify the effect(s) of raising frequency f_y while holding f_x constant in the output spectrum of a mixer:



Now, identify the effect(s) of raising frequency f_x while holding f_y constant.

Challenges

- Does this output signal spectrum represent the behavior of a balanced or an unbalanced mixer?

6.1.5 Mixing of one signal's harmonics?

A student learning asks their instructor, "If one or more non-sinusoidal signals is input to a fully balanced mixer, will the various harmonics of each non-sinusoidal signal mix with each other to produce their own sum and difference frequencies, or will the output's sum and difference frequencies only be *between* the two input signals' frequencies?". In answer to this question, the instructor poses the following thought experiment: "What would happen if we input a square wave to one input of a multiplier circuit (i.e a fully balanced mixer) and input a plain DC signal to the other input?"

Explain how this thought experiment should definitively answer the student's question.

Challenges

- Suppose the square wave frequency in the thought experiment is 4 kHz and the DC voltage is 1 Volt. Predict the output signal frequencies if mixer would indeed mix harmonics of the square wave signal together.
- Suppose the square wave frequency in the thought experiment is 5 kHz and the DC voltage is -3 Volts. Predict the output signal frequencies if mixer would *not* mix harmonics of the square wave signal together.

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell `C1`, and the denominator term (z) in cell `C2`. This makes the two final formulae (in cells `B1` and `B2`) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Output frequency predictions

Predict the frequencies present in the output signal of an ideal mixer given the following input signal frequencies:

- f_{RF} = 150 MHz sinusoid and f_{LO} = 120 MHz sinusoid
- f_{RF} = 2.5 GHz sinusoid and f_{LO} = 2.8 GHz sinusoid
- f_{IF} = 80 kHz sinusoid and f_{LO} = 70 kHz sinusoid
- f_{IF} = 120 kHz sinusoid and f_{LO} = 115 kHz sinusoid
- f_{IF} = 50 kHz square wave and f_{LO} = 1 kHz triangle wave

Predict the frequencies present in the output signal of an unbalanced mixer given the following input signal frequencies:

- f_{RF} = 260 MHz sinusoid and f_{LO} = 200 MHz sinusoid
- f_{RF} = 1.8 GHz sinusoid and f_{LO} = 1.7 GHz sinusoid
- f_{IF} = 55 kHz sinusoid and f_{LO} = 62 kHz sinusoid
- f_{IF} = 300 kHz sinusoid and f_{LO} = 291 kHz sinusoid
- f_{IF} = 20 MHz sawtooth wave and f_{LO} = 5 kHz sinusoid

Challenges

- Identify a simple way you could test a mixer circuit for isolation between either input port and the output port, using just a signal generator and an oscilloscope.

6.2.4 Choose an LO frequency

Choose an appropriate local oscillator frequency to mix with the given input frequency to produce the desired output:

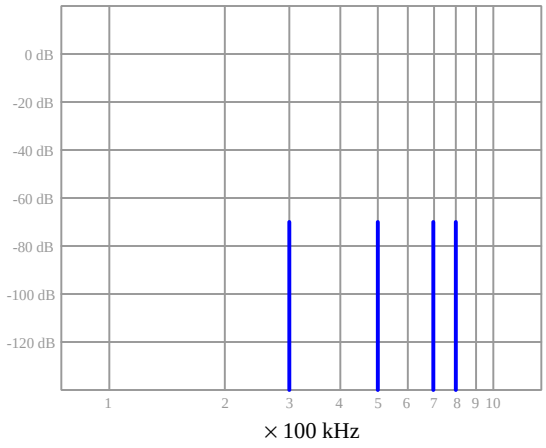
- Downconvert 140 MHz to 5 MHz
- Downconvert 5 GHz to 400 MHz
- Upconvert 80 MHz to 100 MHz
- Upconvert 370 kHz to 500 kHz

Challenges

- Why are there two possible answers for each of these scenarios?

6.2.5 Mixing a set of signals

Predict the output of a balanced mixer with the following set of signals at one input and a local oscillator input of 5 kHz:



Challenges

- Calculate the bandwidth of the original signal spectrum, as well as the bandwidth of the mixer's output spectrum.

6.2.6 Mixing two non-sinusoidal signals

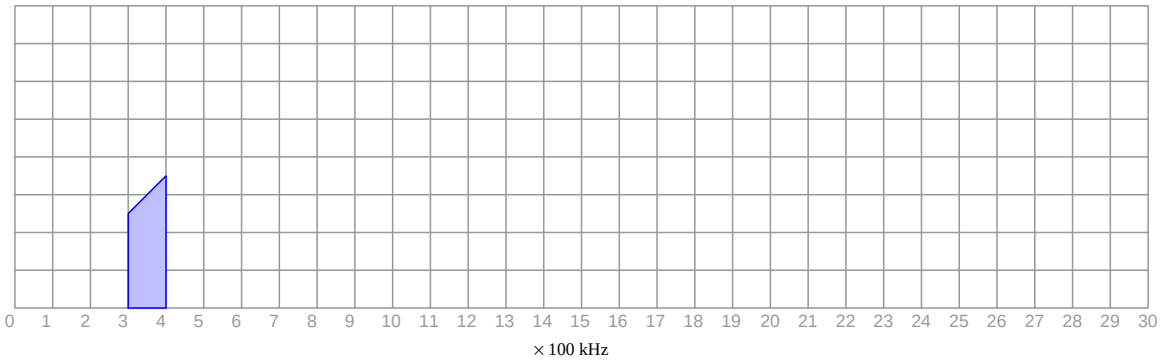
Suppose an ideal mixer takes in one non-sinusoidal 400 Hz signal consisting of a fundamental, second harmonic, and third harmonic, and mixes that signal with a different non-sinusoidal 30 Hz signal consisting of a fundamental and third harmonic only. In the output from this mixer, what distinct frequencies should we expect to see in the spectrum?

Challenges

- What would happen if we mixed the first (400 Hz) signal with plain DC at the other input?

6.2.7 Mixing a band of signals

Sketch the output spectrum for a balanced mixer with the 300-400 kHz band of signals (shown) going into one input and a local oscillator input of 2000 kHz:



Challenges

- Calculate the bandwidth of the original signal spectrum, as well as the bandwidth of the mixer's output spectrum.

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

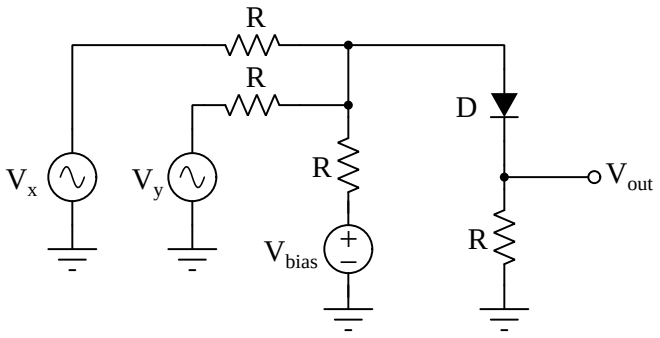
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 Faults in a simple diode mixer

Identify the effect(s) on the output signal resulting from the following component faults:



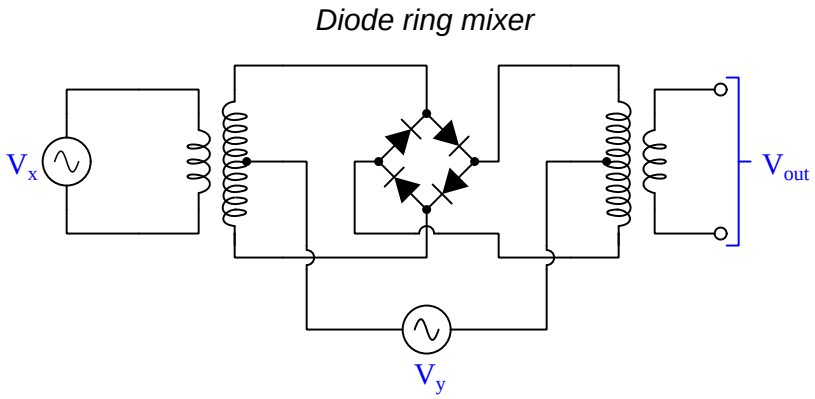
- Resistor below diode fails open
- Bias source resistor fails shorted
- X signal source resistor fails shorted
- X signal source resistor fails open
- Bias source polarity reversed

Challenges

- Redesign this circuit so that it will function with the diode in the reverse orientation from what is shown.

6.3.2 Faults in a diode ring mixer

Identify the effect(s) on the output signal resulting from the following component faults:



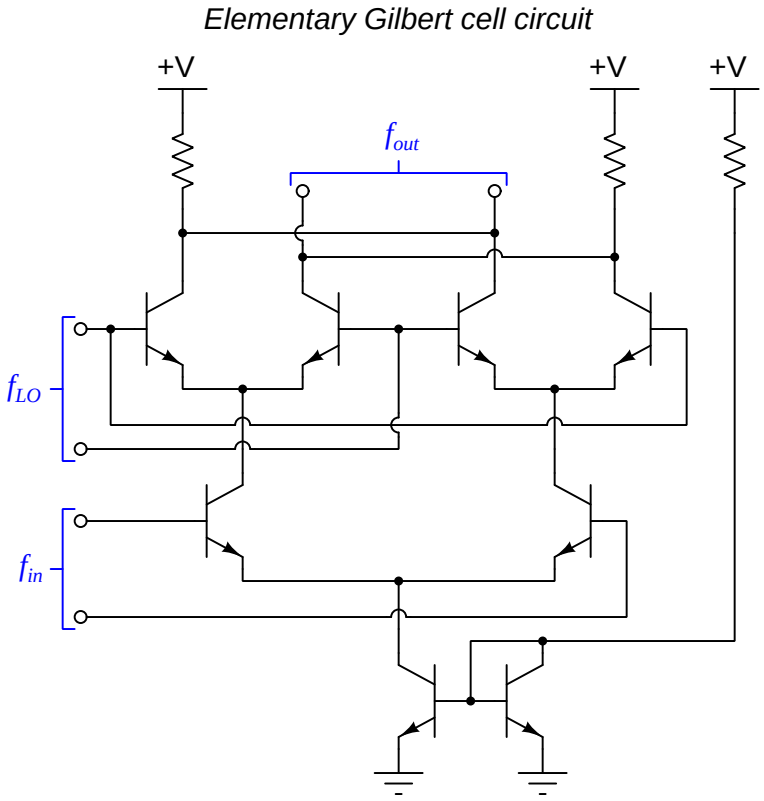
- Secondary winding open above center-tap for V_x source
- Any single diode fails shorted
- Any single diode fails open
- V_x primary winding fails open

Challenges

- Identify any manufacturing imperfections which may impair the mixing performance of this circuit.

6.3.3 Faults in a Gilbert cell mixer

Identify the effect(s) on the output signal resulting from the current mirror programming resistor failing open:



Challenges

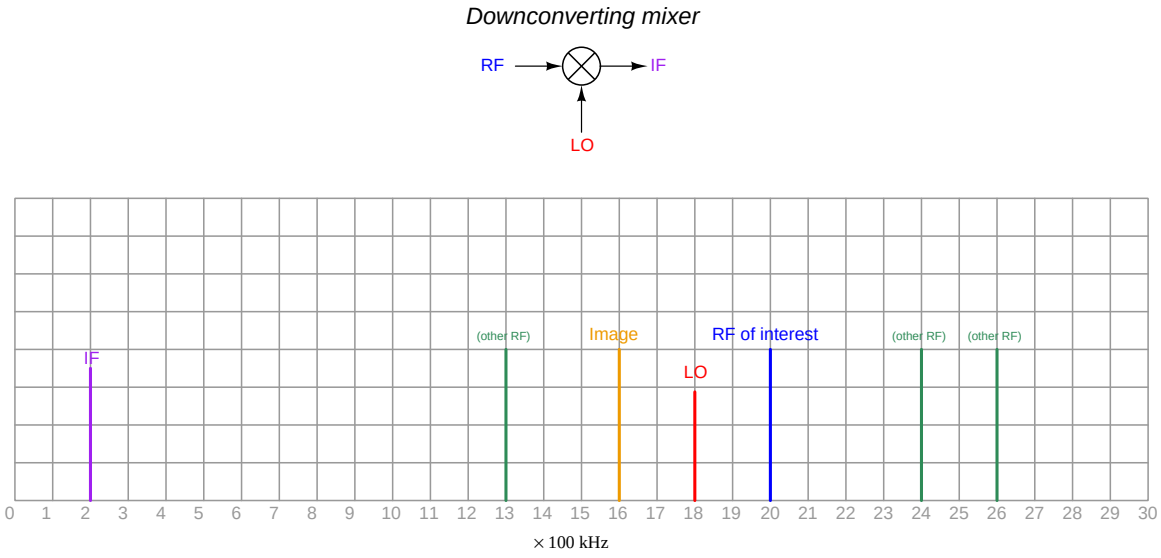
- Modify this circuit to use PNP transistors.

6.3.4 The image problem

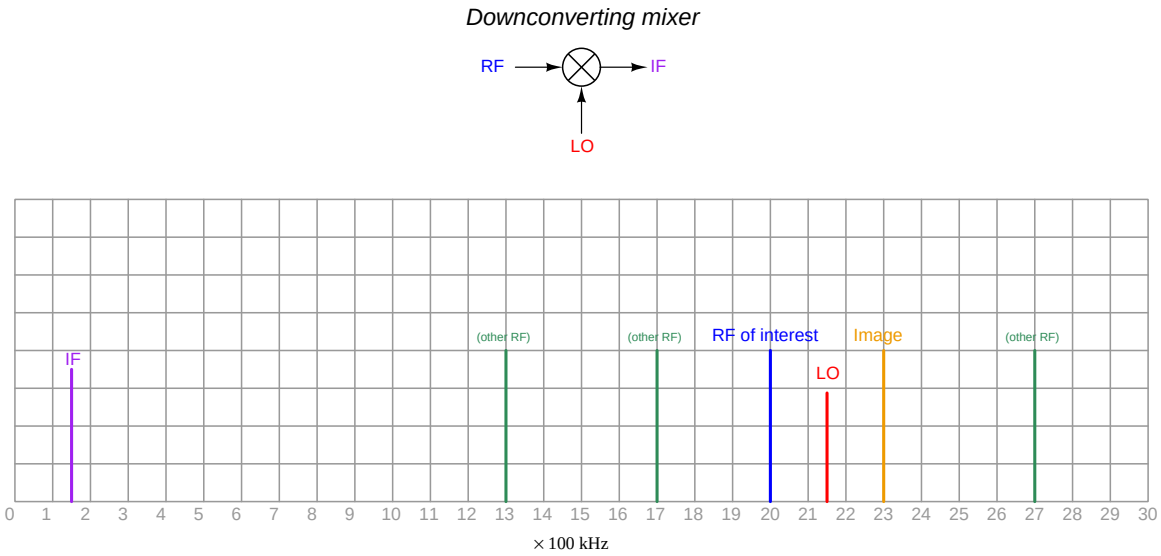
Using mixers to downconvert a received RF signal into an IF signal is extremely useful, but not without its limitations. One such limitation is the existence of *image* frequencies that can interfere with the signal of interest.

Simply defined, an *image* is any unwanted signal having a frequency that is equally distant from the local oscillator's frequency as the frequency of interest. The following spectrum displays show multiple signals presented to the input ports of a mixer circuit, one of which is our signal of interest, one of which is the local oscillator frequency used by the mixer, and one of which is the image frequency.

Example #1



Example #2



Explain why these unwanted "image" signals pose a problem in either downconversion scheme. Identify a means to overcome this problem.

Challenges

- Which of the presented examples uses low-side injection and which uses high-side injection?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

26 March 2025 – added comments on port isolation and corrected potential misleading statements where "balanced" mixer circuit designs were defined in terms of performance rather than topology.

7 March 2024 – added more instructor notes.

27 February 2024 – added more questions to the Introduction chapter, as well as made minor typo corrections to some instructor notes.

12 February 2024 – minor edit to the "Output frequency predictions" Quantitative Reasoning question, where I substituted the word "sawtooth" for "ramp" to describe one of the waveforms.

20 January 2024 – added more sub-questions and Challenge questions to the "Output frequency predictions" Quantitative Reasoning question, as well as added some comments to the "Example: simple diode mixer circuit" Case Tutorial section.

9 January 2024 – added comments to the Tutorial about direct downconversion, and clarified the concept of frequency-shifting in general. This includes edits made to image_4694, image_4695, and the inclusion of a new illustration image_7035. Also corrected some typographical errors.

8 January 2024 – clarified the concept that only frequencies entering a mixer at different input ports get multiplied in the time domain (i.e. produce sum and difference sidebands). If a summation of two or more frequencies enter into one port of an ideal mixer, those do not produce sidebands with each other! Also added another Quantitative Reasoning question related to this same topic, whereby two non-sinusoidal signals are mixed together.

13 May 2023 – added Case Tutorial section on on frequency-mixing within gear mechanisms.

29 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

31 May 2022 – clarified some details about diode-based mixer networks.

8 December 2021 – corrected an embarrassing number of errors and oversights in the Tutorial today: incorrect and missing polarity marks on output terminals of image_4689 and image_4690, improved text formatting showing the expansion of algebraic binomials, eliminated a sentence from an earlier version of the text that no longer matched its corresponding image, elaborated on the function of current-mirror transistor Q_7 in the Gilbert Cell analysis, clarified footnote on ω versus f as a variable for frequency, and finally added a footnote describing the wavy-line block-diagram symbols used in the beginning to represent filters. *Oy vey.*

10 May 2021 – commented out or deleted empty chapters.

27 April 2021 – minor additions to the Introduction.

22 April 2021 – added new Case Tutorial section showing how to use an AD633 analog multiplier as a balanced mixer.

22 March 2021 – added more digits to Boltzmann's constant, and clarified that Kelvin is 273.15 more than Celsius.

8 December 2020 – minor edits to the Tutorial.

23-26 November 2020 – added content to Tutorial, Introduction, Historical References, and Questions.

22 November 2020 – document first created.

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