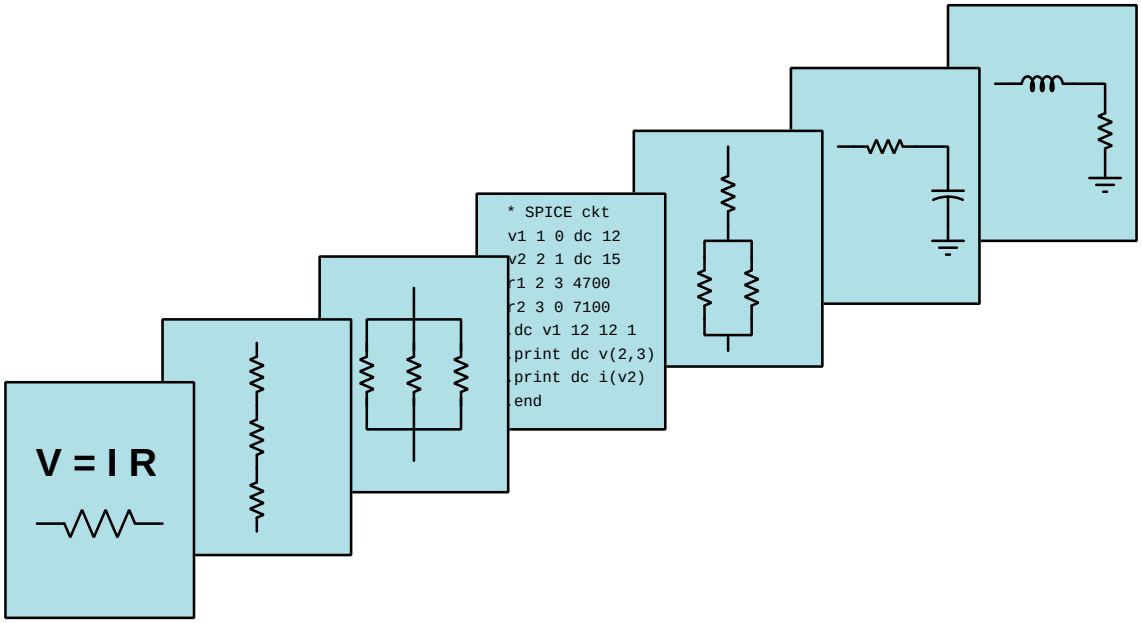
Modular Electronics Learning (ModEL) project



Signal Coupling and Noise

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Last update = 24 April 2025

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Chapter 1

Introduction

1.1 Recommendations for students

An often severe and generally poorly-understood problem in electronic circuits is unwanted *coupling* between conductors carrying different signals or power. The electric and/or magnetic fields generated by one conductor may induce voltages and/or currents on one or more adjacent conductors, that interference commonly called *noise*. This module explores the foundational concepts underlying these phenomena and also discusses practical mitigations.

Important concepts related to signal coupling and noise include **capacitance**, **inductance**, **electric fields**, **magnetic fields**, **rates of change**, **filter** networks, **Fourier's Theorem**, **harmonic** frequencies, **fundamental** frequency, **mutual inductance**, **analog** versus **digital** signaling, digital **logic levels**, **shielding**, **ground loops**, **common-mode** voltage, **differential** signaling, and magnetic **permeability**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to prove the existence of noise generated within a circuit component? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the effective shielding from electric fields? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the effective shielding from magnetic fields? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What is a *rate of change*?
- How do *rates of change* relate to the phenomena of capacitance and inductance?
- How do electric and magnetic fields differ from one another?

- What does it mean to say that a signal contains "harmonic" frequencies?
- What are some practical ways to minimize signal coupling between different conductors?
- What do the terms "victim" and "aggressor" refer to in this context?
- Which type of signal, analog or digital, typically tolerates greater noise?
- How may electric fields be re-directed away from conductors?
- How may magnetic fields be re-directed away from conductors?
- What purpose does the "shield" conductor serve in a cable?
- What role does frequency play in signal coupling between conductors?
- Where should a cable's shield conductor be bonded to Earth?
- How does Lenz's Law apply to electrical interference?
- What purpose do "twisted" pairs of conductors serve in a cable?
- What is "common-mode" voltage for a signal?
- How does differential signaling work to avoid noise problems?
- What are some of the different types of electrical noise intrinsic to components?

1.2 Challenging concepts related to signal coupling and noise

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Parasitic effects** – phenomena such as resistance, capacitance, and inductance don't just exist where we want them to, but in fact always exist everywhere they can. All conductors (except for superconductors) have some amount of resistance, all conductors separated by an insulating gap exhibit capacitance, and all conductors possess inductance by virtue of their ability to produce a magnetic field with the passage of current. Often these "parasitic" effects are small enough to ignore, but in some applications they may be severe.

- **Rates of change** – when learning the relationships between voltage and current for inductors and capacitors, one must think in terms of how fast a variable is changing. The amount of voltage induced across an inductor is proportional to how *quickly* the current through it changes, not how strong the current is. Likewise, the amount of current "through" a capacitor is proportional to how *quickly* the voltage across it changes. This is the first hurdle in calculus: to comprehend what a rate of change is, and it is far from obvious. A helpful strategy here is to relate electrical rates of change to other, more common, rates of change such as *speed* (i.e. the rate-of-change of physical position over time). Also helpful is to relate rates of change to the *slope* of a line or curve plotted on a graph, showing how *rise/fall over run* describes not only the phenomenon of a rate-of-change but also the appropriate units of measurement.

- **Current "through" a capacitance** – since a capacitor is by its very construction electrically *open*, having no conductive pathway from one terminal to the other, it seems extremely counter-intuitive to speak of capacitors as ever conducting a current "through" them. A key point about electrical opens prohibiting current is that this prohibition only holds for *continuous* currents. Momentary, transient currents are indeed possible for incomplete circuits, lightning being a spectacular natural example of this. For those who wish to dig deeper into the phenomenon, James Clerk Maxwell's notion of *displacement current* is a helpful start.

- **Twisted pairs** – twisting the two conductors of a cable pair in order to mitigate magnetic field interference can be challenging to comprehend, until you examine the effects of any external magnetic field as it attempts to induce a voltage on more than one of the "loops" formed along the length of that twisted-pair cable. There, you will find that every successive "loop" cancels out the one before it, resulting in a vastly-decreased amount of induced voltage compared to a non-twisted cable.

- **Phantom voltage measurements** – a phenomenon seen with high-impedance AC voltmeters is so-called *phantom* voltage readings where AC voltages may be measured between two wires that are known to not connect to any source. This is the result of coupling (usually capacitive) between powered conductors and the non-powered conductors the meter is connected to. Sketching the equivalent circuit showing parasitic capacitance between those conductors and with the voltmeter modeled as a high-valued resistor usually makes this concept easier to understand.

The *Case Tutorial* chapter has sections demonstrating electric versus magnetic coupling within

a length of multi-conductor cable, which makes for very good experiments proving the phenomena of signal coupling.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Evaluate a given system for signal separation

  Assessment – Examine a given electrical/electronic system and evaluate it for potential signal-coupling problems, as well as propose mitigations for that potential coupling; e.g. pose problems in the form of the "Compressor system wiring" Diagnostic Reasoning question.

- **Outcome** – Sketching equivalent circuits

  Assessment – Sketch a schematic diagram that is electrically equivalent to a signal-coupling scenario, showing the parasitic properties of capacitance and/or inductance modeled as real components.

- **Outcome** – Independent research

  Assessment – Read and summarize in your own words reliable source documents on the subject of signal integrity. Recommended readings include printed circuit board (PCB) layout guides discussing best practices for "high-speed" circuit designs.

Chapter 2

Case Tutorial

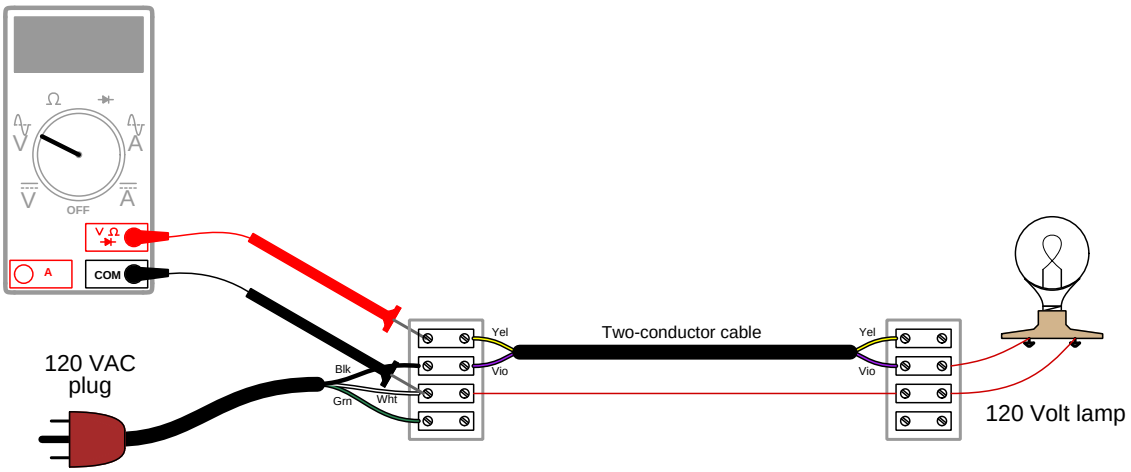
The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

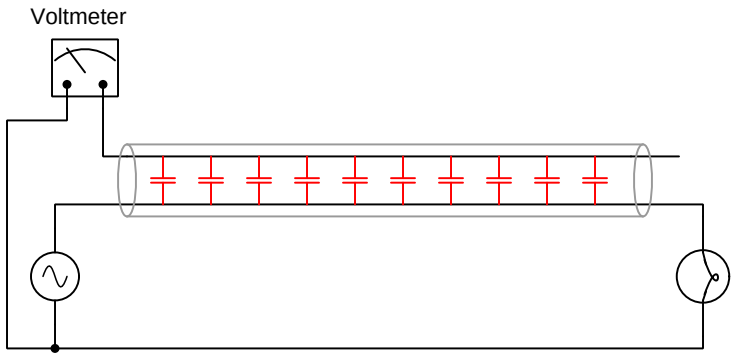
2.1 Example: capacitive coupling within cable

Capacitance varies inversely with separation distance, which means there will be more capacitance between two conductors spaced closer together, all other factors being equal. We may demonstrate this by using two conductors within a common cable, held in close proximity to each other within the confines of that cable.

Pictorial diagram:

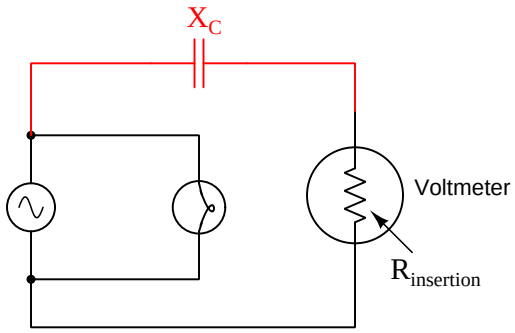


Equivalent schematic diagram:



Interference voltage indicated by voltmeter will increase as source voltage increases, and/or as source frequency increases. In this circuit, load current is irrelevant to the signal coupling, and so for a demonstration you are welcome to use any load suitable for the AC source. Electric fields are proportional to *voltage*, and so it is voltage that matters.

Re-drawn another way, we may look at the cable's parasitic capacitance as a capacitive *reactance* (X_C) forming an AC voltage divider with the voltmeter's internal resistance:

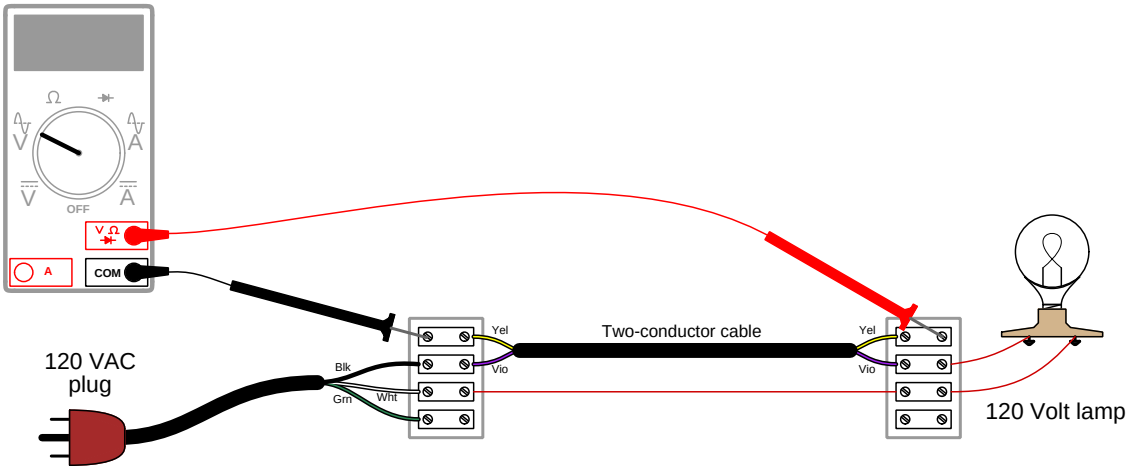


Since the internal ("insertion") resistance of a high-quality digital voltmeter is typically in the tens of millions of Ohms, even a very small amount of capacitance translating into a capacitive reactance in the millions of Ohms will present the voltmeter with a substantial percentage of the source's voltage.

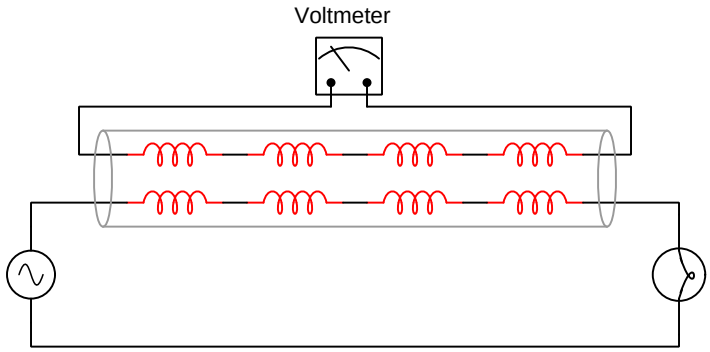
2.2 Example: inductive coupling within cable

Magnetic coupling varies inversely with separation distance, which means there will be more mutual inductance between two parallel conductors spaced closer together, all other factors being equal. We may demonstrate this by using two conductors within a common cable, held in close proximity to each other within the confines of that cable.

Pictorial diagram:



Equivalent schematic diagram:

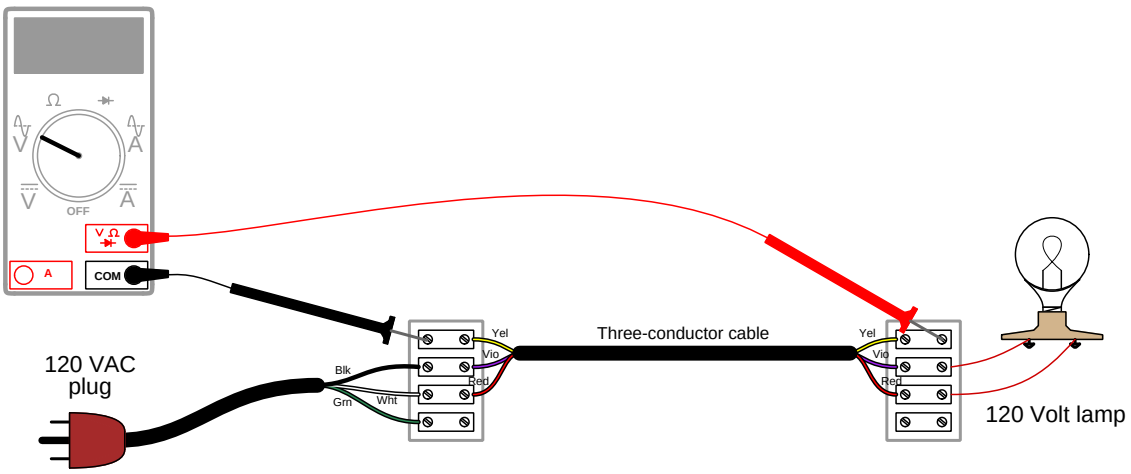


Interference voltage indicated by voltmeter will increase as source current increases, and/or as source frequency increases. In order to generate a strong enough interference signal to register on an AC voltmeter, I suggest using a power cable (e.g. SO-type power cord) of substantial length, as well as a load drawing at least several Amperes of AC current. Magnetic fields are proportional to *current* and so it is load current that matters in this demonstration. In one instance a student built this circuit using 2-conductor SO cable 50 feet in length, and a hot-air gun drawing in excess of 12

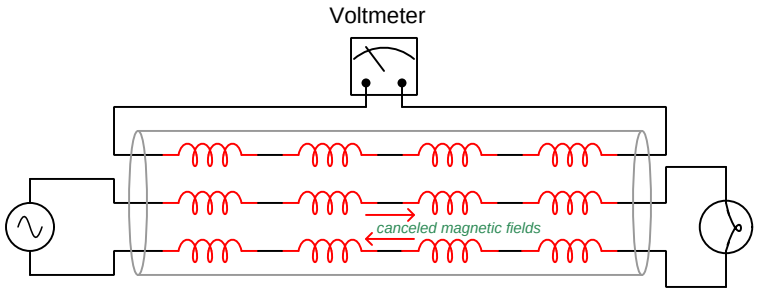
Amperes from a 120 VAC source, and was able to measure approximately 0.7 Volts RMS along the length of the conductor alongside the power conductor.

If we run both load conductors through the same cable, however, their magnetic fields tend to cancel out because at any given moment in time the directions of current through them will be equal in magnitude and opposite in direction. This means at any given time the individual wires' magnetic fields will be equally strong but directly opposed to each other. This results in a net magnetic field that is nearly zero, and therefore little or no coupled signal detectable by the voltmeter:

Pictorial diagram:



Equivalent schematic diagram:



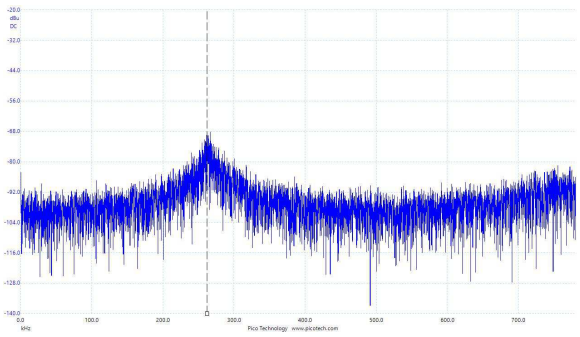
2.3 Example: field probing inside a PC

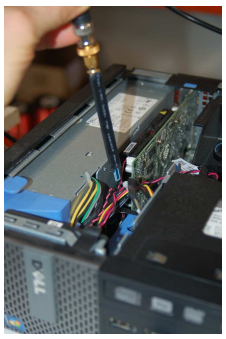
AC electric and magnetic fields may be qualitatively measured on printed circuit boards (PCBs) using special probes called *near-field probes*. These probes usually have BNC- or SMA-style coaxial cable connectors suitable for connection to the input of an oscilloscope or a spectrum analyzer.

E-field probes sense electric fields with respect to ground, and consist of a metallic electrode coated in plastic (to avoid direct contact with circuit conductors) with a connector appropriate to the input of an oscilloscope or a spectrum analyzer. E-field probes tend to be omnidirectional in their sensing.

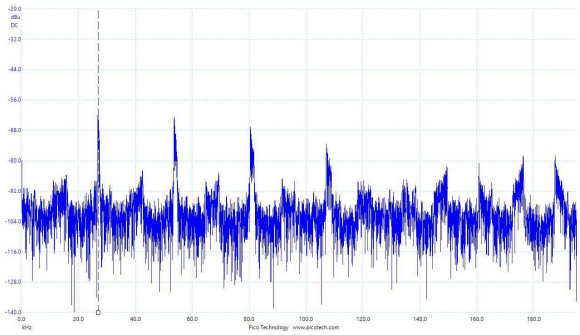
H-field probes sense magnetic fields created by current-carrying conductors, and consist of conductive loops (also insulated to avoid direct contact with circuit conductors) generating small AC voltages when exposed to AC magnetic fields. H-field probes tend to be planar in their sensing, which means they are maximally sensitive to magnetic fields caused by current through conductors parallel to the plane of the probe's loop(s) and minimally sensitive to magnetic fields perpendicular to that.

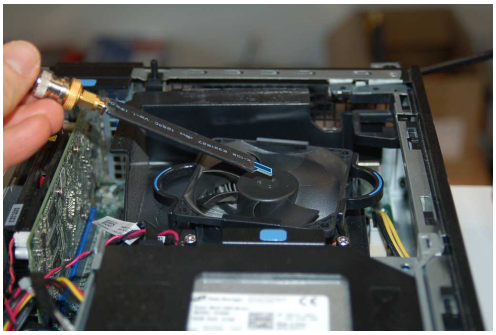
This first set of images shows the signal spectrum (left) and photograph (right) of an E-field probe held near a pair of wires connecting to a video processing PCB inside of a personal computer, picking up a spread of frequencies centered around 260 kHz:



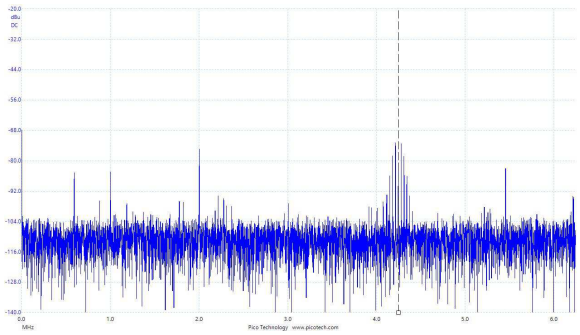


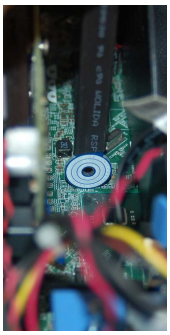
Next we see the same E-field probe held over the hub of a cooling fan motor, intercepting the electric field from a square-wave voltage signal with a fundamental frequency of approximately 27 kHz as well as its associated odd harmonics:



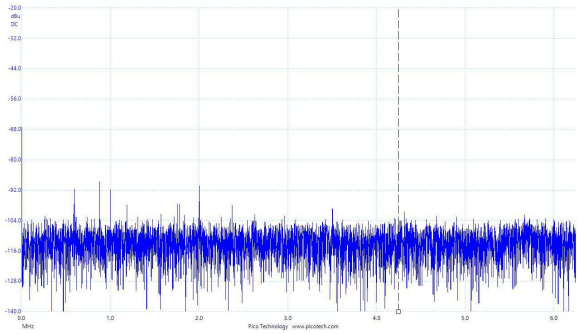


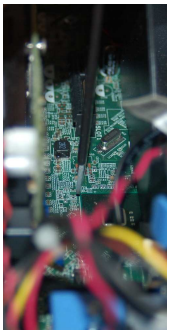
Switching to an H-field probe, the following two examples illustrate the polarization of this probe. In the first set of images we see the spectrum detected by placing the probe with its loop parallel to the current-carrying trace on the PC's motherboard, detecting a cluster of signals centered around 4.25 MHz with maximum sensitivity because parallel conductors ensure mutually perpendicularity with the magnetic field lines:





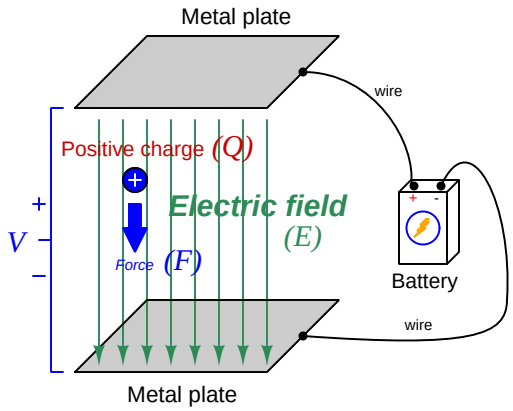
In the second image set we see the cluster of frequencies all but gone with the probe loop rotated to be perpendicular with the offending PCB trace. The pulsed current still flows through the PCB trace, but because the probe's loop is now perpendicular to that trace it means the magnetic field lines are now parallel to the probe loop, and therefore induction (from that signal) no longer occurs:





2.4 Example: high-voltage corona

Electric fields (E) are fundamentally defined by the amount of force (F) exerted on an electric charge (Q) placed within that field:



$$\vec{F} = Q\vec{E}$$

Where,
$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{E}$ = Electric field (Newtons per Coulomb)

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

Where,
$\vec{E}$ = Electric field (Newtons per Coulomb, or Volts per meter)
V = Voltage between two points (Volts)
d = Distance between those same two points (meters)

Relating electric fields to voltage and distance helps us compare electric potential with gravitational potential energy, the latter being the potential energy possessed by any mass (m) having the potential to fall some distance d while exposed to a gravitational field (a, the "acceleration" of gravity). The force exerted by a gravitational field on any mass is given by the formula $F = ma$, while the amount of potential energy (E_p, not to be confused with electric field $\vec{E}$) possessed by that mass is $E_p = mad$ or $E_p = Fd$. The farther a mass is lifted against the force of gravity, the more potential energy it has; the farther an electric charge is lifted against the force of an electric field, the more potential energy it has.

[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

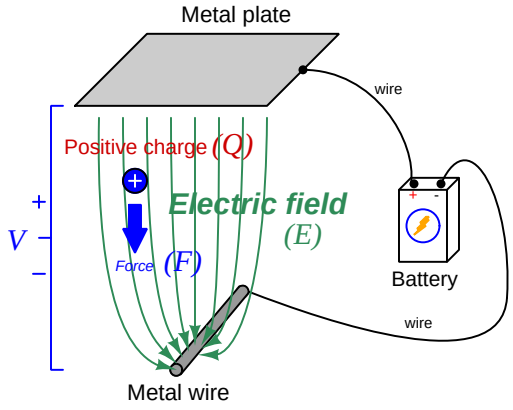
Where,

$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)

Φ_E = Electric flux (Newton-meter squared per Coulomb)

$\vec{A}$ = Area over which flux is distributed (square meters)

For two separated plates of infinite area[2], the electric flux lines will all be parallel to each other, and the electric field (or flux density) will be constant at any location between those plates. However, if we replace one plate with a wire we see the electric flux lines concentrating around that wire in contrast to their spreading over the plate's area:



This concentration of electric flux lines near the thin wire mean the electric flux density is strongest at the surface of that wire, and therefore that any electric charge will experience a greater force exerted on it as it approaches that wire. Stated in terms of voltage and distance, there are literally more Volts per meter of electric field strength near the wire than there are near the flat plate.

When atoms experience an electric field, that field applies opposing forces to the atoms' negative electrons and positive protons. If the field is concentrated enough, these forces will become sufficient to rip the electrons away from the atoms' positive nuclei, a process called *ionization*. When this happens to any substance that is normally an electrical insulator, the presence of these negative ions and positive ions constitute mobile charge carriers, and their resulting motion with the applied field is an electric current. In other words, when an electric field is strong enough, any insulator may *break down* and become electrically conductive. This *dielectric breakdown* limit is usually expressed in *Volts per meter*, which makes perfect sense because those are the units of electric field measurement,

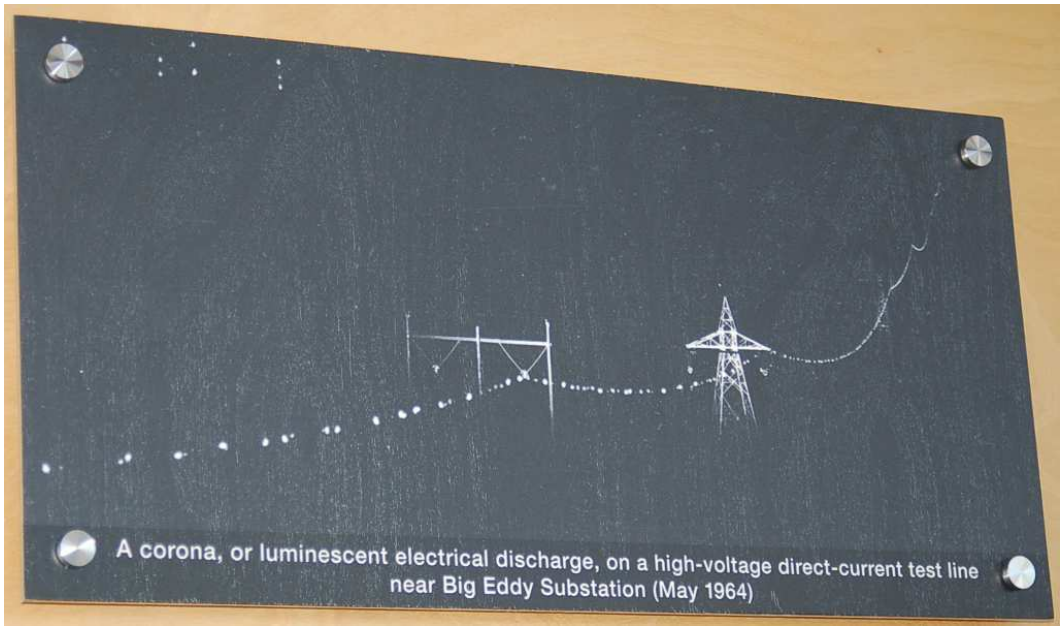
[2]For plates of finite area, electric flux lines tend to "bulge" at the outer edges of the plate, but even for realistically-sized plates the flux lines are nearly parallel to each other in the middle of the plates and therefore electric flux density is relatively constant at any location between the plates centered in the plates' areas.

and directly predict the amount of force applied to the atoms' individual electric charges (protons and electrons).

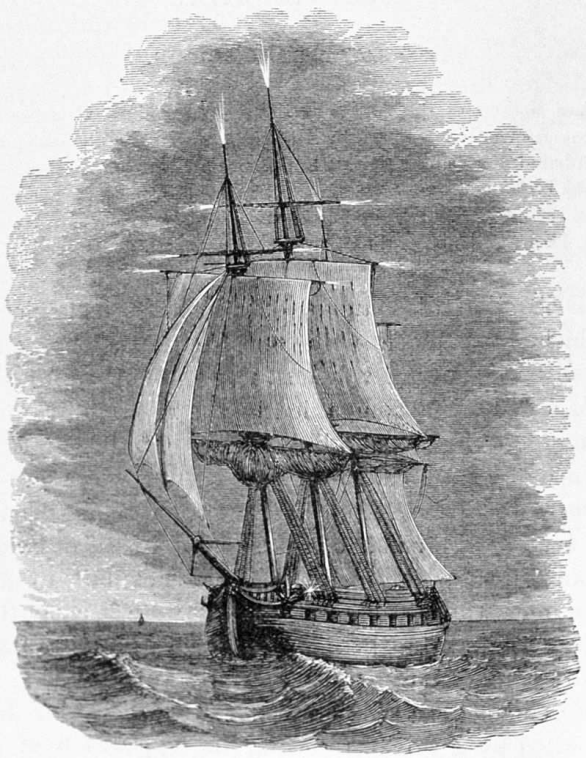
When dielectric breakdown occurs in open air, the phenomenon is called *corona discharge*. The ionization results in an emission of blue-white light, making corona discharge a spectacular sight to behold when it is intense enough. At atmospheric pressure this breakdown limit for air is approximately 30 kV/cm, or 3 MegaVolts per meter.

Corona discharge is problematic for high-voltage power lines, responsible for creating the "crackling" or "buzzing" sound audible near AC power lines with potentials in the range of hundreds of thousands of Volts. Corona discharge prompts oxygen atoms to join together in triplets to form *ozone* (O_3), as well as prompting atoms of nitrogen to join with atoms of oxygen to form various oxides of nitrogen (NO_x). Both ozone and NO_x gases are unhealthy for humans to breathe in large quantity, and they are also a causal factor in metallic corrosion which over time degrades the metal wires and fastener assemblies used in high voltage power lines.

The following image shows a picture taken in 1964 of a high-voltage DC power line undergoing corona tests at night. Glowing points of light along one of the conductors mark where corona discharge occurs:



Corona discharge sometimes occurs in nature during electrical storms, and this is historically known as *Saint Elmo's Fire*. This eerie glow occurs at the tips of tall, pointed structures such as antenna towers and sailing ship masts when the storm's electric field grows strong enough. The image below[3] is a historical representation of this phenomenon as observed by sailors:

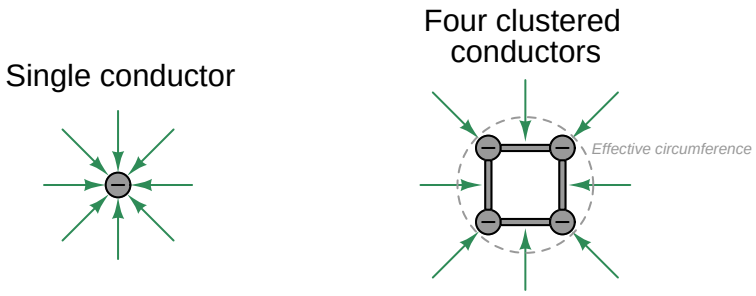


Note how the corona discharge occurs at each tip of the wooden mast structure (e.g. at the lift-beam tips as well as the mast tips on top), where the wood is wet enough (especially during a storm!) to sufficiently conduct electricity. Other pointed structures on the ship lower to the water's surface, such as the bowstrip, are not shown with any such "fire" in this image. Likely, the electric flux lines spread back out as they near the water's surface to decrease the electric flux density. Only when far away from the water's surface and sufficiently pointed (such as the mast tips and lift-beam ends) will the flux lines concentrate enough to ionize the air.

[3]Public domain, courtesy of Saibo.

Sharp edges and pointed structures tend to concentrate electric flux lines and therefore invite corona discharge in high-voltage electrical conductors. Therefore, care is taken to avoid such shapes on high-voltage electrical connections. Note the round metallic structures intentionally placed at the ends of these high-voltage electrical terminals found in a 500 kiloVolt electrical substation:

Another corona-mitigating design feature seen in these photographs is the clustering of multiple electrical conductors into bundles of three or four wires – all electrically common to one another – for the purpose of simulating the profile of a much larger-diameter electrical conductor. In other words, the overlapping electric field profiles of the individual metal cables spreads out the electric flux lines to keep the E field strength below the corona limit:



2.5 Example: measuring signal rates of change

Capacitors and inductors relate voltage to current by rates of change. For a capacitor, the amount of current is proportional to how quickly voltage across that capacitor either rises or falls over time ($I = C\frac{dV}{dt}$). For an inductor, the amount of voltage is proportional to how quickly current through that inductor rises or falls over time ($V = L\frac{dI}{dt}$).

For example, a 330 microFarad capacitor experiencing a voltage increasing at a rate of 45 Volts per second will pass 14.85 milliAmperes. If the voltage happens to decrease at an equivalent rate (i.e. $\frac{dV}{dt} = -45$ Volts per second) then the 14.85 milliAmpere current will reverse direction through the capacitor compared to how it flowed with the increasing voltage.

Similarly, a 100 milliHenry inductor experiencing a current increasing at a rate of 5 Amperes per second will induce a voltage of 500 milliVolts. If the current happens to decrease by an equivalent rate (i.e. $\frac{dI}{dt} = -5$ Amperes per second) then the 500 milliVolt voltage induced across the inductor will reverse polarity from what it was during the period of increasing current.

Not only are rates-of-change important in determining how energy-storing devices such as capacitors and inductors will respond in circuits, but rates-of-change are also important for determining how *parasitic* capacitances and inductances will affect intended circuit behavior. Parasitic capacitance exists between any two conducting surfaces separated by an electrically insulating medium, and parasitic inductance exists along any length of conductor. This means any rate-of-change of voltage over time between two separated conductors will cause some amount of current to "pass"[4] between them, and that any rate-of-change of current over time through any single conductor will cause some amount of voltage to drop across its length. In many circuits these parasitic effects are negligible, but in circuits experiencing extremely fast rates of change for voltage and/or current the effects can be significant or even severe.

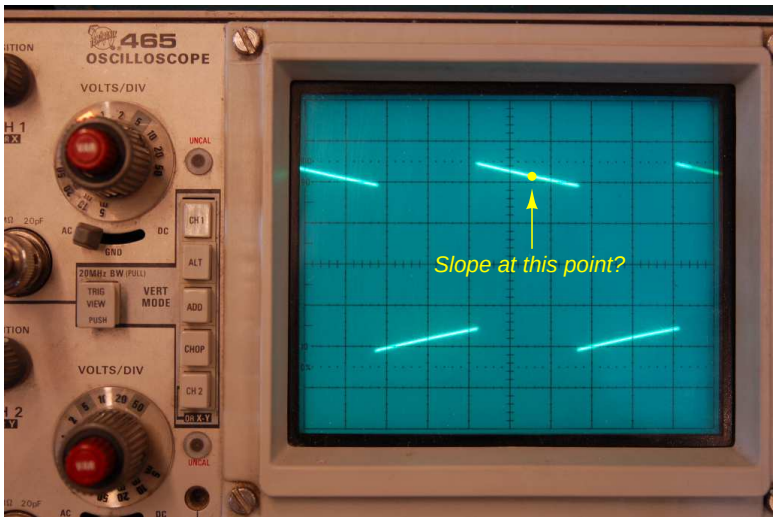
Oscilloscopes are ideal for performing empirical measurements of voltage rates-of-change, and of current rates-of-change given the proper accessories[5]. Some skill is required to do this, though, and here we will explore practical examples to show how it is done.

For any signal plotted in the time domain, where the horizontal axis of the plot is expressed in units of seconds, milliseconds, microseconds, etc., the signal's rate of change at any given point will be the *slope* or *pitch* of the waveform, mathematically defined as its *rise over run*. A great aid to discerning slope at any location on a waveform is to sketch a straight line visually matching the wave's slope at that point, then use locations along that straight line to more easily discern how far it rises (or falls) over some "run" of time. We call this straight line a *tangent line*.

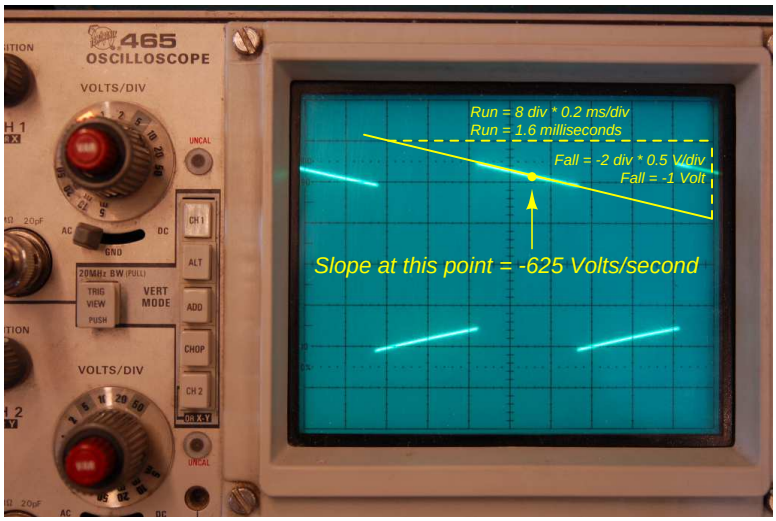
[4] The Scottish physicist James Clerk Maxwell referred to this current as *displacement current*.

[5] For example, a *current probe* converting a sensed current into a voltage the oscilloscope may directly sense, or a *shunt resistor* placed in the circuit developing an oscilloscope-measurable voltage drop for any current passing through.

Here we see an example of a waveform with sloping sections. In the first image we see a specific location on the waveform where we wish to measure voltage rate-of-change ($\frac{dV}{dt}$):



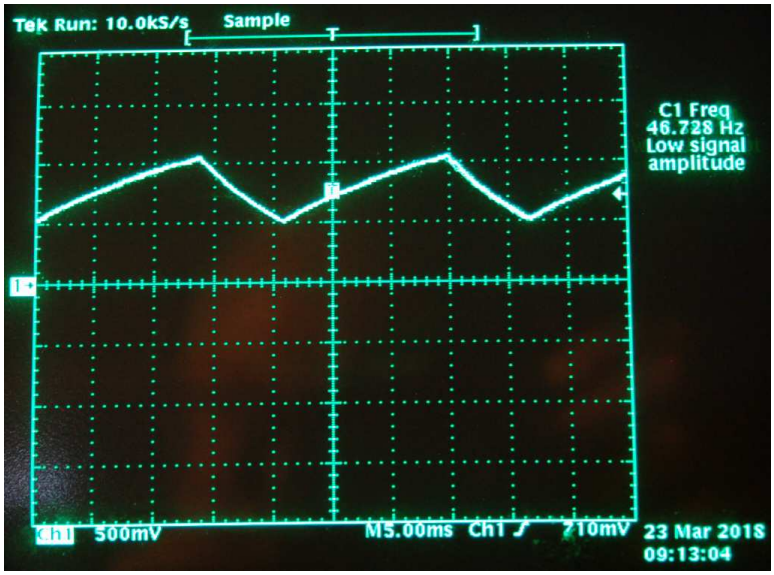
Next we see a tangent line drawn to match the slope of the waveform at the specified location, with convenient points for fall/run measurements taken on that line against the oscilloscope grid's major divisions. In this example, the oscilloscope's vertical sensitivity has been set for 0.5 Volts per division, and the horizontal timebase for 0.2 milliseconds per division:



As we can see, the tangent line falls 2 vertical divisions (-1 Volt) over a timespan of 8 horizontal divisions (1.6 milliseconds), yielding a $\frac{dV}{dt}$ quotient of -625 Volts per second, which may also be expressed as -0.625 Volts per millisecond. The negative sign is important, as it distinguishes this particular rate-of-change as *falling* rather than rising over time.

Such rate-of-change measurements are necessarily approximate, as they require us to visually gauge where a tangent line may be overlaid on the waveform's oscillograph, and also to visually assess the slope of that tangent line using the grid provided on the instrument's display screen. However, in most applications extremely precise rate-of-changes are not necessary, and such techniques suffice quite well.

Below is another example of a waveform with sloping sections, the oscilloscope configured for 500 milliVolts per division on the vertical axis and 5 milliseconds per division on the horizontal:

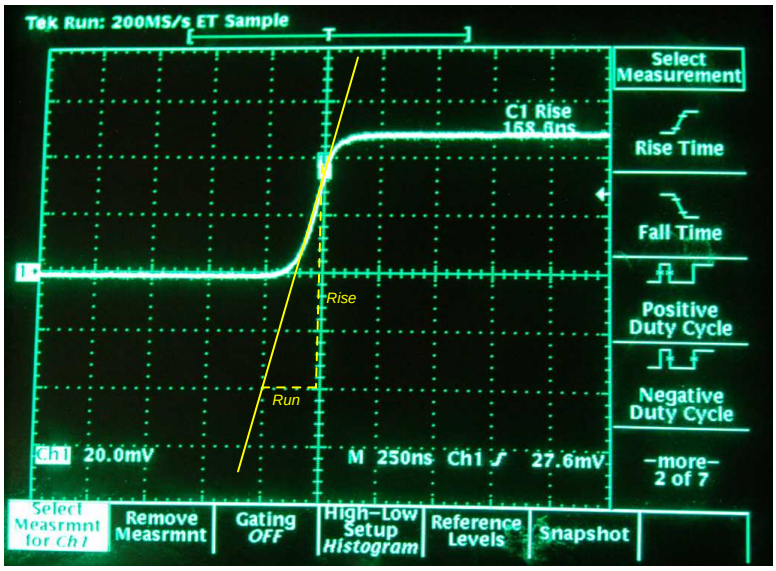


Approximating the slope for each rising section of this wave, we count one division of rise over 2.8 divisions of run, or 500 milliVolts rise over 14 milliseconds of run. This is a rate-of-rise of +35.7 Volts per second.

Approximating the slope for each falling section of this wave, we count one division of fall over 1.4 divisions of run, or −500 milliVolts fall over 7 milliseconds of run. This is a rate-of-fall of −71.4 Volts per second.

If these rates-of-change appear suspiciously large compared to the actual amplitude of the waveform, which barely crests over +1 Volt on the oscillograph, bear in mind that we are calculating *rates of change* for voltage and not absolute values of voltage itself. This is analogous to the distinction between *speed* and *distance*: traveling at a rate of 30 kilometers per hour does not necessarily mean you will travel 30 kilometers, as the actual distance traveled depends on how long that speed is sustained. A voltage rising at a rate of 35.7 Volts per second would indeed rise 35.7 Volts *if given a full second to do so*, but since each rising/falling portion of this waveform is so short in duration the actual amount of rise or fall in each case is only one-half of one Volt. It is therefore perfectly appropriate to consider any $\frac{dV}{dt}$ value as being the *speed* at which a voltage increases or decreases over time, distinct from the actual value of that voltage at any particular moment in time.

Here we see another oscillograph, this one zoomed into the rising edge of a square wave. For this measurement the oscilloscope was configured for 20 milliVolts per division of vertical sensitivity and a timebase of 250 nanoseconds per division on the horizontal:



The tangent line overlaid on this screenshot for the purpose of measuring the pulse edge's rate-of-change rises approximately 4 vertical divisions over a run of 1 division, which is 80 milliVolts of rise over 250 nanoseconds of run. The pulse edge's rate-of-change, therefore, is approximately +320,000 Volts per second, or +320 Volts per millisecond, or +0.32 Volts per microsecond (all equivalent expressions of $\frac{dV}{dt}$).

Chapter 3

Tutorial

Simply defined, *noise* is any stimulus found in conjunction with a signal that itself carries no useful information and tends to obscure the signal of interest. In colloquial terms, noise is generally associated with audible sounds that make it more challenging to hear or interpret sounds we are interested in such as speech or music. However, in electrical circuit theory "noise" consists of spurious voltages and/or currents interfering with other voltages and/or currents bearing useful information. The general term "noise" applies to any circuit, even those unassociated with audio signals.

Frequency is a very important parameter of noise, and the fact that many noise sources are random rather than periodic in nature means the noise often spans a broad range of frequencies. Although it may seem strange at first, especially if your definition of "noise" is anchored to audible experience, but it is true that noise may take the form of an extremely low frequency or band of frequencies. If an adjacent AC power conductor causes an audio amplifier system to reproduce a constant "hum" due to the AC power circuit's frequency, we would certainly recognize that as a form of noise. However, another form of noise in this same audio amplifier system might be alterations in the circuit's gain (the amount it amplifies) caused by changes in room temperature, even though room temperature may require minutes or even hours of time to complete a single cycle. Literally *any* form of unwanted variation in a signal, regardless of how rapid or slow, is a form of noise.

Noise sources may be external to our circuit, or internal to it. External sources of noise include nearby circuits and environmental factors such as sound, temperature, radiation, etc. Internal sources consist of phenomena within the components themselves generating unwanted fluctuations in the signal of interest. External and internal noise sources will be treated separately in this Tutorial.

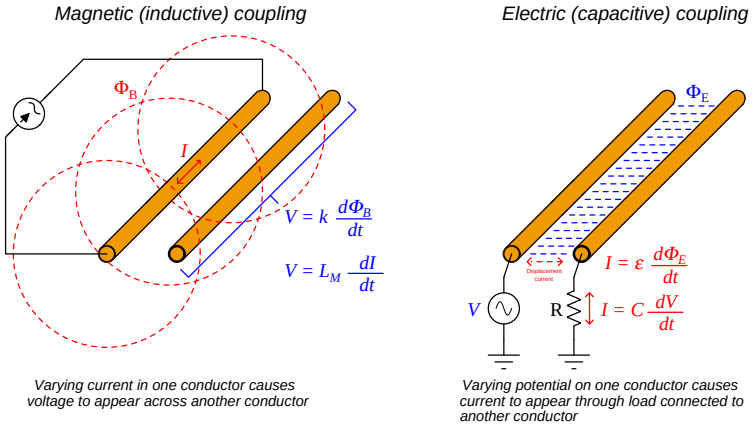
3.1 Signal coupling and cable separation

If sets of wires lie too close to one another, electrical signals may "couple" from one wire (or set of wires) to the other(s). This can be especially detrimental to signal integrity when the coupling occurs between AC power conductors and low-voltage signal wiring, but it also occurs between sets of signal wires. Terms commonly used to describe such "crosstalk" is to call the conductor generating the interference the *aggressor* and the conductor receiving the interference the *victim*.

Two mechanisms of electrical "coupling" exist: *inductive* and *capacitive*. Both are capable of "coupling" signals from one conductor to another, and they usually exist in tandem.

- Inductance is a property intrinsic to any conductor, whereby energy is stored in the magnetic field formed by current through the wire. Mutual inductance existing between parallel wires forms another "bridge" whereby an AC current through one wire is able to induce an AC voltage along the length of another wire. Specifically, the amount of voltage induced across the length of the victim conductor is proportional to the mutual inductance between the victim and aggressor conductors as well as the rate-of-change of current through the aggressor conductor.

- Capacitance is a property intrinsic to any pair of conductors separated by a dielectric (an insulating substance), whereby energy is stored in the electric field formed by voltage between the wires. The natural capacitance existing between mutually insulated wires forms a "bridge" for AC signals to cross between those wires. Specifically, the amount of current induced in the victim conductor is proportional to the capacitance and to the rate-of-change of voltage between the victim and aggressor conductors.



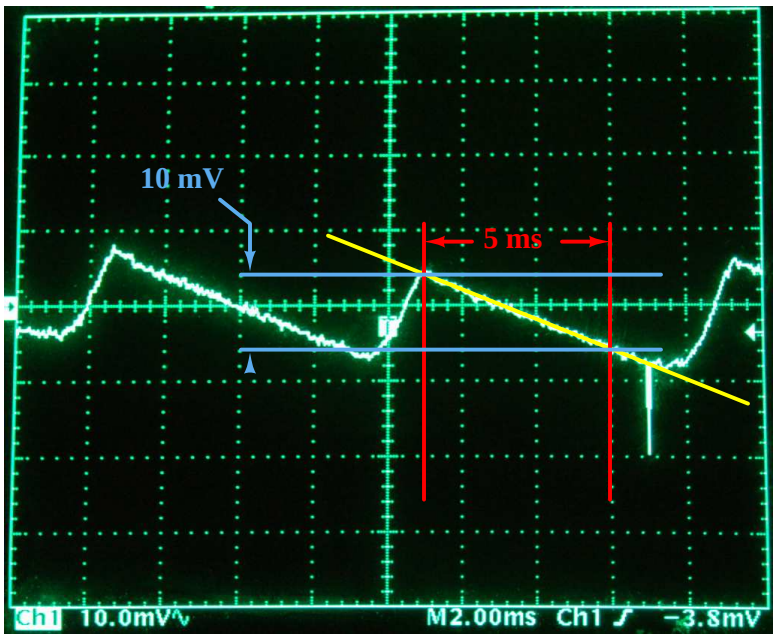
The fundamental "Ohm's Law" formulae for inductance[1] and capacitance is central to the phenomenon of signal coupling between conductors:

$$V = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

[1] For the particular case of coupling between conductors, the inductive "Ohm's Law" formula could be written more precisely as $V_{victim} = L_M \frac{dI_{aggressor}}{dt}$, with L_M representing *mutual* inductance.

These "Ohm's Law" formulae relate voltage and current to *rate-of-change* of the other. That is, voltage across an inductance is proportional to the *speed* at which current rises or falls; current through a capacitance is proportional to the *speed* at which voltage rises or falls. These rates of change, represented mathematically as $\frac{dI}{dt}$ and $\frac{dV}{dt}$, respectively, are distinct from the instantaneous values of current and voltage even though they are related. An analogy is the relationship between the speed of a vehicle and its distance from some starting point: a vehicle's speed in kilometers per hour is related to, but not the same as, its distance away from a starting point in kilometers.

To clarify the concept of a rate-of-change, we will examine a screenshot from an oscilloscope measuring a sawtooth-shaped voltage waveform. The annotations overlaid on the photograph after it was captured on camera from the display of an oscilloscope:



Here the yellow line follows the downward slope of the voltage waveform, while the red lines mark a time interval along that slope and the blue lines mark the voltage fall over that time interval. With the values shown here, -10 milliVolts of fall over 5 milliseconds of time, the estimated $\frac{dV}{dt}$ for this signal's downward slope is -2 Volts per second. This value of -2 Volts per second is the rate-of-change (or "speed") of the voltage over time, not the amount of voltage at some particular point in time. Note how the amplitude of this sawtooth waveform is only about 10 mV peak-to-peak, yet it falls at a *rate* of approximately -2000 milliVolts per second. The rising slope of this sawtooth waveform is steeper yet, somewhere in the neighborhood of $+10$ Volts per second[2].

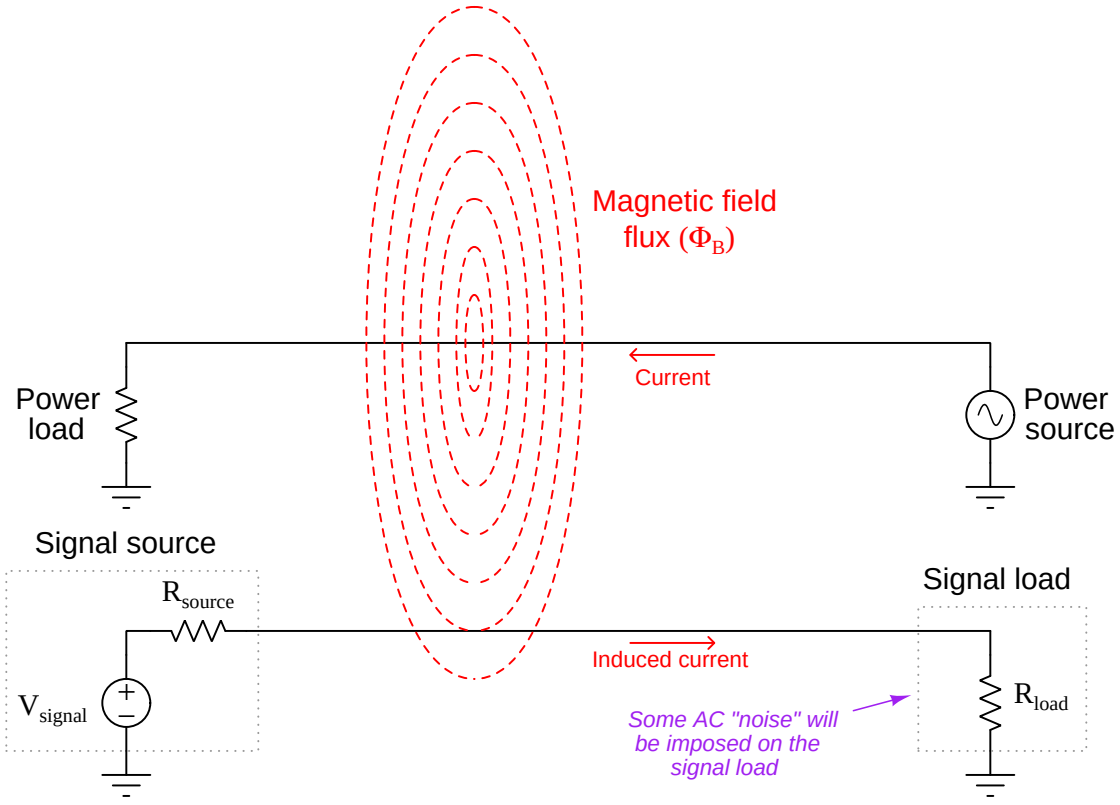
Interestingly, the downward "glitch" seen near the lower-right area of this oscillograph exhibits much larger rates-of-change than the relatively gentle slopes. Even though the vertical height of this transient is not very large, its extremely short timespan means its leading edge will have a large

[2]If you project the rising edge over one major division of time (2 ms) you see that it rises approximately 20 mV. 20 mV divided by 2 ms is 10 Volts per second.

negative slope ($-\frac{dV}{dt}$) and its trailing edge will have a large positive slope ($+\frac{dV}{dt}$), possibly hundreds or even *thousands of Volts per second* each.

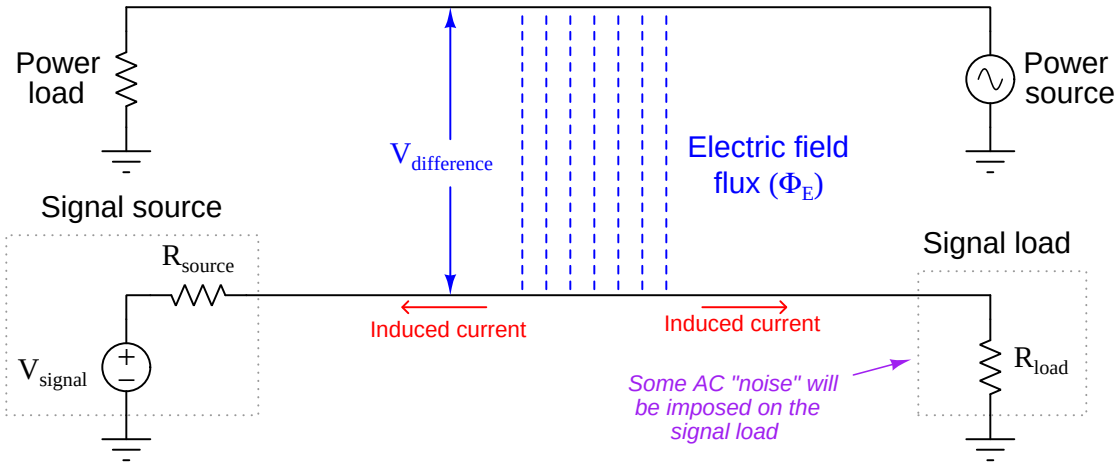
Since coupling between adjacent conductors happens due to shared inductance and/or capacitance, and the "Ohm's Law" formulae for both L and C predict coupling proportional to rates-of-change, the rates-of-change of current and of voltage are major factors for signal coupling. In other words, how quickly currents and voltages rise or fall over time is a more significant predictor of signal coupling than their absolute values at any given time.

Inductive (magnetic) coupling between an AC power conductor and a DC signal conductor is shown in the following diagram:



The magnitude of the noise induced within the "victim" (DC) signal circuit by the "aggressor" (AC) power circuit is proportional to the amount of mutual inductance (L_M) existing between the two conductors and also the "speed" that the aggressor current increases or decreases ($\frac{dI}{dt}$). This rate-of-change over time for any AC current is a function of both peak amplitude and frequency: higher peak value results in greater $\frac{dI}{dt}$ and therefore more coupled noise; higher frequency also results in greater $\frac{dI}{dt}$.

Capacitive (electric) coupling between an AC power conductor and a DC signal conductor is shown in the following diagram:



The magnitude of the noise induced within the "victim" (DC) signal circuit by the "aggressor" (AC) power circuit is proportional to the amount of capacitance (C) existing between the two conductors and also the "speed" that the aggressor voltage increases or decreases between the two ($\frac{dV}{dt}$). This rate-of-change over time for any AC voltage is a function of both peak amplitude and frequency: higher peak value results in greater $\frac{dV}{dt}$ and therefore more coupled noise; higher frequency also results in greater $\frac{dV}{dt}$.

Since both types of coupling favor higher-frequency aggressor voltages/currents, we may think of these natural phenomena as functioning like *high-pass filters*[3] between the aggressor and victim circuits. Aggressor circuits carrying non-sinusoidal and pulse waveforms are particularly problematic because these embody a wide range of frequencies as described by Fourier analysis[4]: the *fundamental* frequency and multiple *harmonics* whose frequencies are multiples of the fundamental.

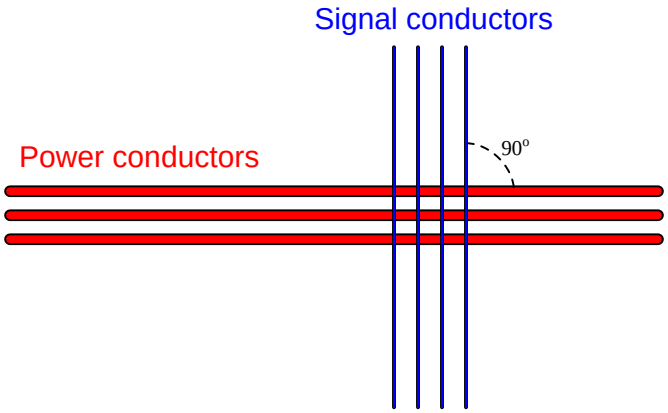
A practical example of this is the greater interference from AC power conductors carrying voltages and currents rich in harmonics, as opposed to purely sinusoidal AC with just one (fundamental) frequency. Electronic power-control devices such as variable-frequency motor drives (VFDs) are a good case-in-point because they function by rapidly pulsing power to loads, and as such they output power having a broad spectrum of frequencies. The higher of these frequencies couple well to nearby signal conductors, causing interference problems if steps are not taken to mitigate the coupling. 50 Hz or 60 Hz fundamental noise simply does not "couple" as effectively to victim circuits as the 11th harmonic (550 Hz or 660 Hz), the 12th harmonic (600 Hz or 720 Hz), or other harmonics due to their higher frequencies.

[3] A *filter* is an electrical network favoring the passage of some AC frequencies more than others. A "high-pass" filter is a network more easily passing signals of high frequency than signals of low frequency.

[4] The basic concept here is that any waveshape, however complex, is mathematically equivalent to a series of sinusoids at different frequencies and amplitudes.

A good way to minimize signal coupling is to simply separate conductors carrying incompatible signals. This is one reason why electrical power conductors and signal cables are almost never found in the same raceway (e.g. conduit, electrical ductwork) together. Separation decreases capacitance between the conductors because $C = \frac{A\epsilon}{d}$ where d is the distance between the conductive surfaces. Separation also decreases the coupling coefficient between inductors, which in turn decreases mutual inductance (recall that $L_M = k\sqrt{L_1 L_2}$ where k is the coupling coefficient and M is the mutual inductance between two inductances L_1 and L_2). In control panel wiring, it is customary to route AC power wires in such a way that they do not lay parallel to low-level signal wires, so that both forms of coupling may be reduced.

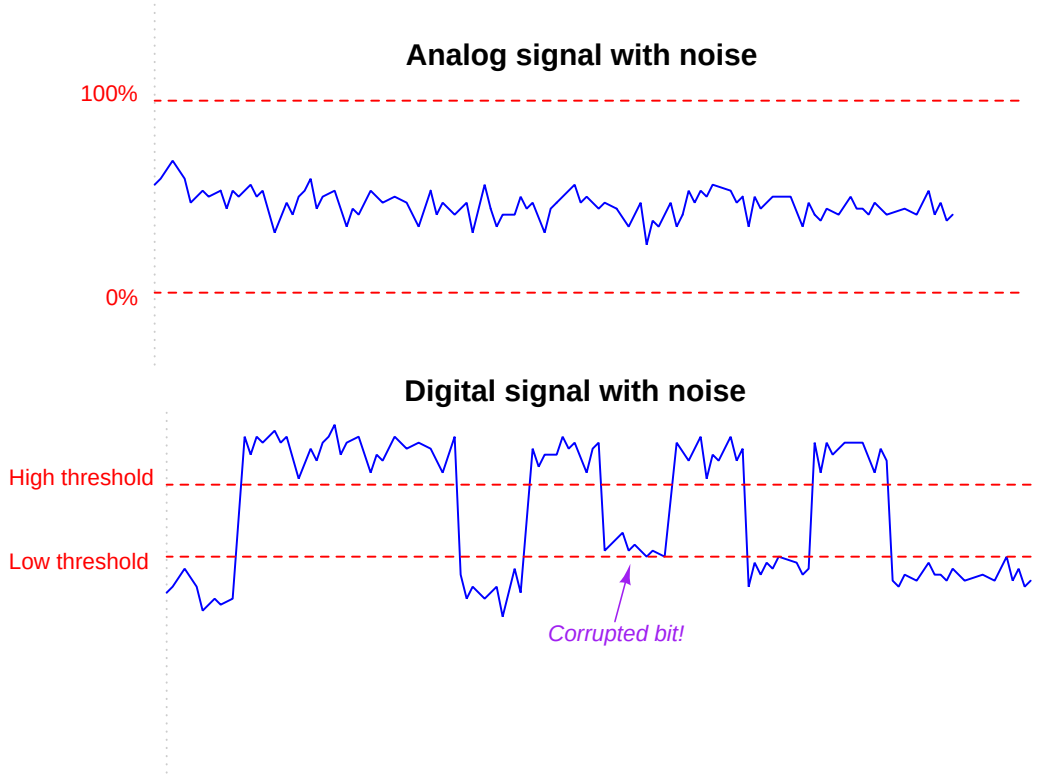
If conductors carrying incompatible signals *must* cross paths, it is advisable to orient the conductors perpendicular to each other rather than parallel, like this:



Perpendicular conductor orientation reduces both inter-conductor capacitance *and* mutual inductance by two mechanisms. Capacitance between conductors is reduced by means of minimizing overlapping area (A) resulting from the perpendicular crossing. Mutual inductance is reduced by decreasing the coupling coefficient (k) to nearly zero since the magnetic field generated perpendicular to the aggressor conductor will be *parallel* and not perpendicular to the victim conductor. Since the vector for induced voltage is perpendicular to the magnetic field (i.e. parallel with the aggressor current vector) there will be no voltage induced along the length of the victim conductor.

The problem of power-to-signal line coupling is most severe when the signal in question is *analog* rather than *digital*. In analog signaling, even the smallest amount of coupled "noise" corrupts the signal. A digital signal, by comparison, will become corrupted only if the coupled noise is so great that it pushes the signal level above or below a detection threshold it should not cross. This disparity is best described through illustration.

Two signals are shown here, coupled with equal amounts of noise voltage:



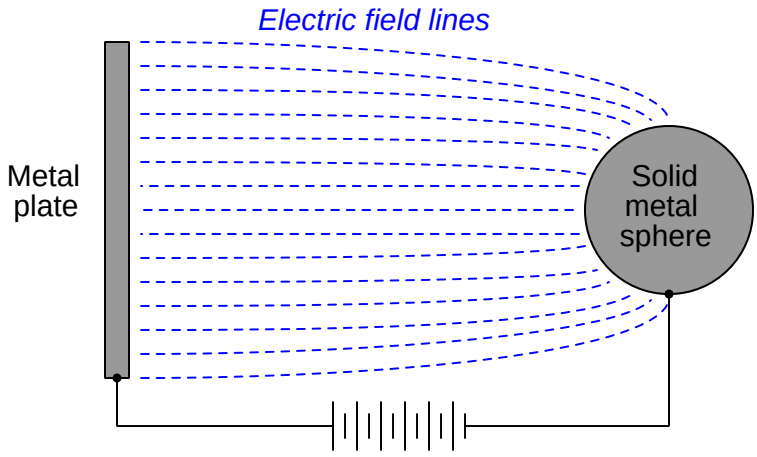
The peak-to-peak amplitude of the noise on the analog signal is almost 20% of the entire signal range (the distance between the 0% and 100% values), representing a substantial degradation of signal integrity. Analog signals have infinite resolution, which means *any* change in signal amplitude has meaning. Therefore, any noise whatsoever introduced into an analog signal will be interpreted as variations in the quantity that signal is supposed to represent.

That same amount of noise imposed on a digital signal, however, causes no degradation of the signal except for one point in time where the signal attempts to reach a "low" state but fails to cross the threshold due to the noise. Other than that one incident represented in the pulse waveform, the rest of the signal is completely unaffected by the noise, because digital signals only have meaning above the "high" state threshold and below the "low" state threshold. Changes in signal voltage level caused by induced noise will not affect the meaning of digital data unless and until the amplitude of that noise becomes severe enough to prevent the signal's crossing through a threshold (when it should cross), or causes the signal to cross a threshold (when it should not).

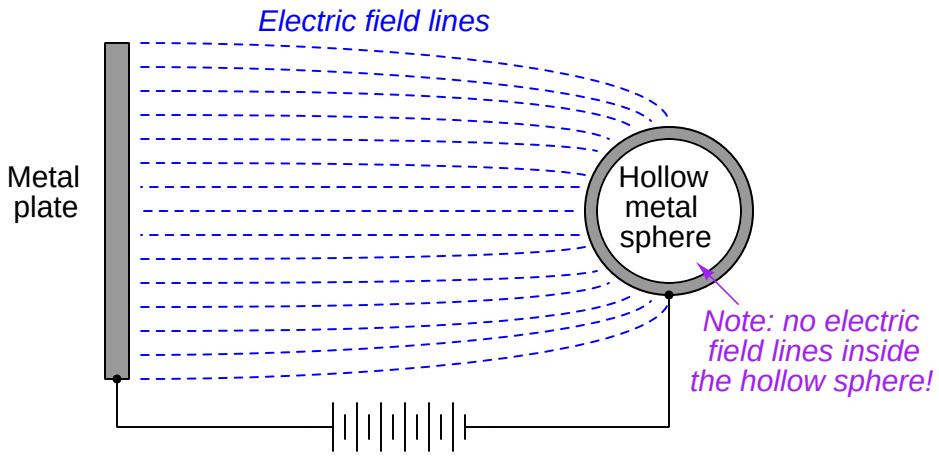
From what we have seen here, digital signals are far more tolerant of induced noise than analog signals, all other factors being equal.

3.2 Electric field (capacitive) de-coupling

The fundamental principle invoked in *shielding* signal conductor(s) from external electric fields is that no substantial electric field can exist within a solid conductor. Electric fields exist due to imbalances of electric charge. If such an imbalance of charge ever were to exist within a conductor, charge carriers (typically electrons) in that conductor would quickly move to equalize the imbalance, thus eliminating the electric field. Another way of saying this is to state that *electric fields only exist between points of different potential, and therefore cannot exist between equipotential points.* Thus, electric flux lines may be found only in the dielectric (insulating media) between conductors, not within a solid conductor:



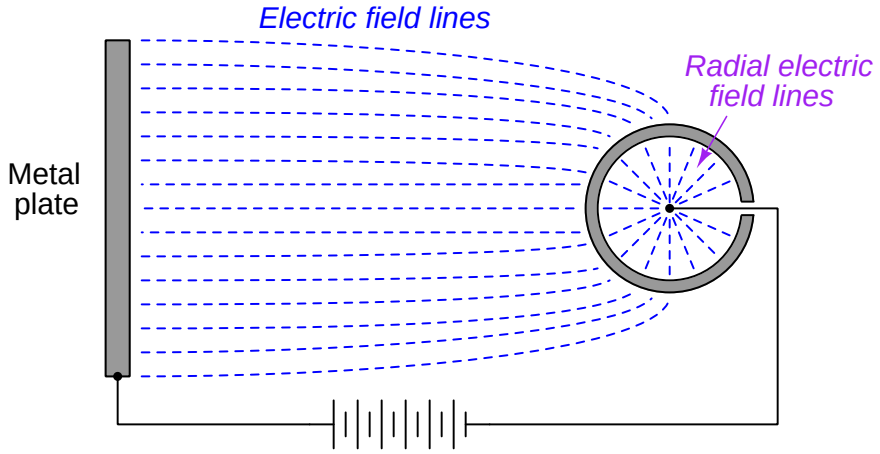
This also means electric flux lines cannot span the diameter of a hollow conductor:



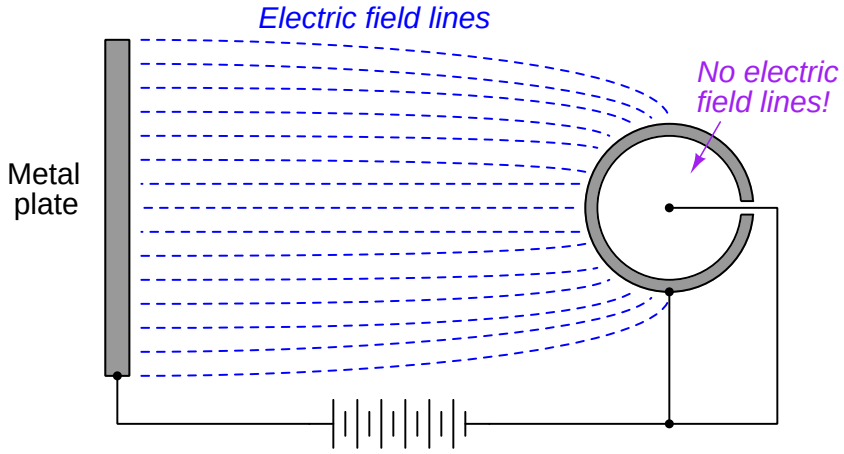
The electrical conductivity of the hollow sphere's wall ensures that all points on the circumference of the sphere are equipotential to each other. This in turn prohibits the formation of any electric

flux lines within the interior air space of the hollow sphere. Thus, all points within the hollow sphere are *shielded* from any electric fields originating outside of the sphere.

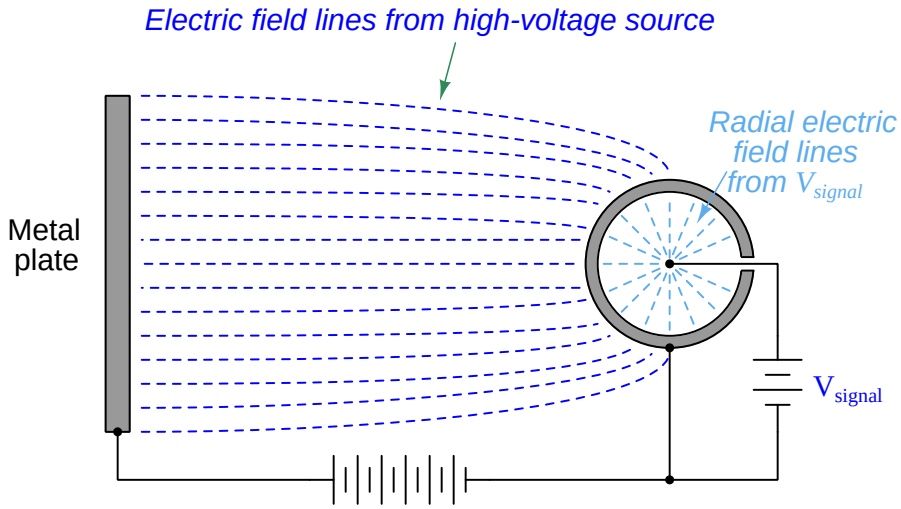
The only way to allow an external electric field to penetrate a hollow conductor from the outside is if that conductive shell is left "floating" with respect to another conductor placed within the shell. In this case the lines of electric flux do not exist between different points on the conductive sphere, but rather between the shell of the sphere and the conductor at the center of the sphere because those are the points between which a potential difference (voltage) exists. To illustrate:



However, if we make the hollow shell electrically common to the negative side of the high-voltage source, the flux lines inside the sphere vanish, since there is no potential difference between the internal conductor and the conductive shell:



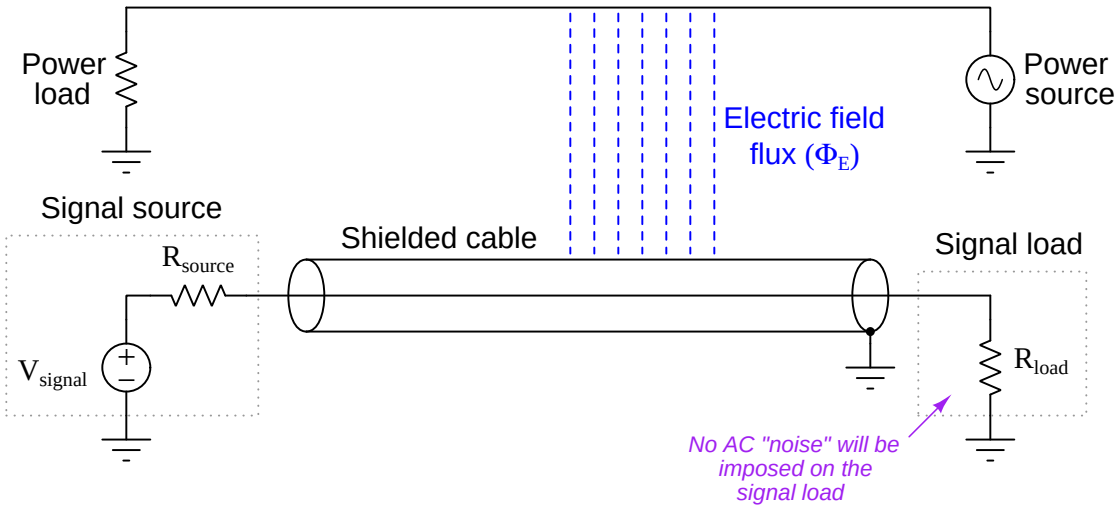
If the conductor within the hollow sphere is elevated to a potential different from that of the high-voltage source's negative terminal, electric flux lines will once again exist inside the sphere, but they will reflect this second potential and not the potential of the original high-voltage source. In other words, an electric field will exist inside the hollow sphere, but it will be completely isolated from the electric field outside the sphere. Once again, the conductor inside is *shielded* from external electrostatic interference:



If conductors located inside the hollow shell are thus shielded from external electric fields, it means there cannot exist any capacitance between external conductors and internal (shielded) conductors. If there is no capacitance between conductors, there will never be capacitive coupling of signals between those conductors, which is what we want for signal cables to protect those signals from external interference[5].

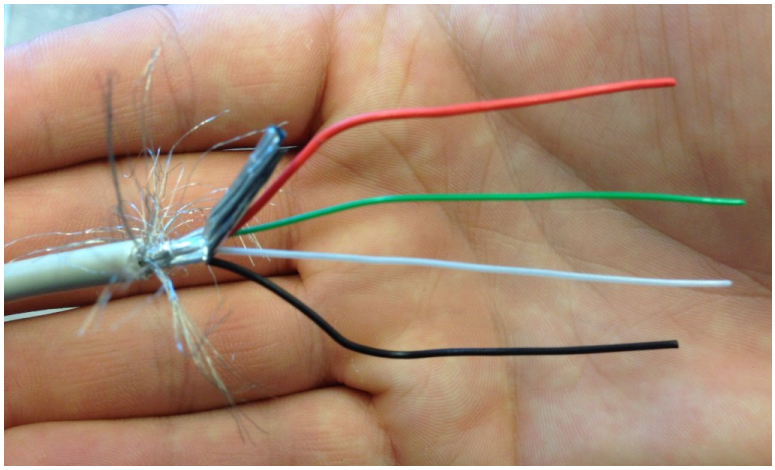
[5] Incidentally, cable shielding likewise guards against strong electric fields *within* the cable from capacitively coupling with conductors outside the cable. This means we may elect to shield "noisy" power cables instead of (or in addition to) shielding low-level signal cables. Either way, good shielding will prevent capacitive coupling between conductors on either side of a shield.

All this discussion of hollow metal spheres is just an introduction to a discussion of *shielded cable*, where electrical cables are constructed with a conductive metal foil wrapping or conductive metal braid surrounding the interior conductors. Thus, the foil or braid creates a conductive *tube* which may be connected to ground potential (the "common" point between external and internal voltage sources) to prevent capacitive coupling between any external voltage sources and the conductors within the cable:



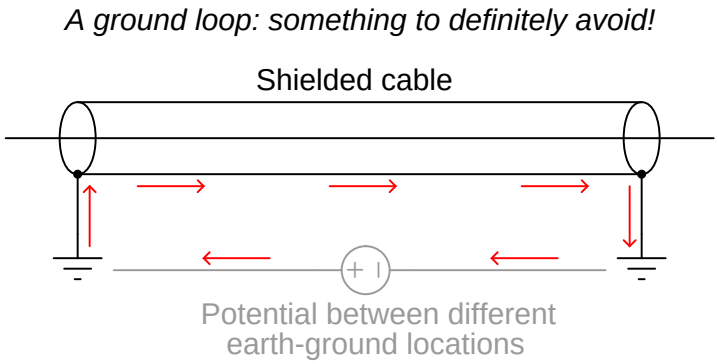
The following photograph shows a set of signal cables with braided shield conductors all connected to a common copper "ground bus." This particular application happens to be in the control panel of a 500 kV circuit breaker, located at a large electrical power substation where strong electric fields abound:

This next photograph shows a four-conductor USB cable stripped at one end, revealing a metal-foil shield as well as silver-colored wire strands in direct contact with the foil, all wrapped around the four colored power and signal conductors:

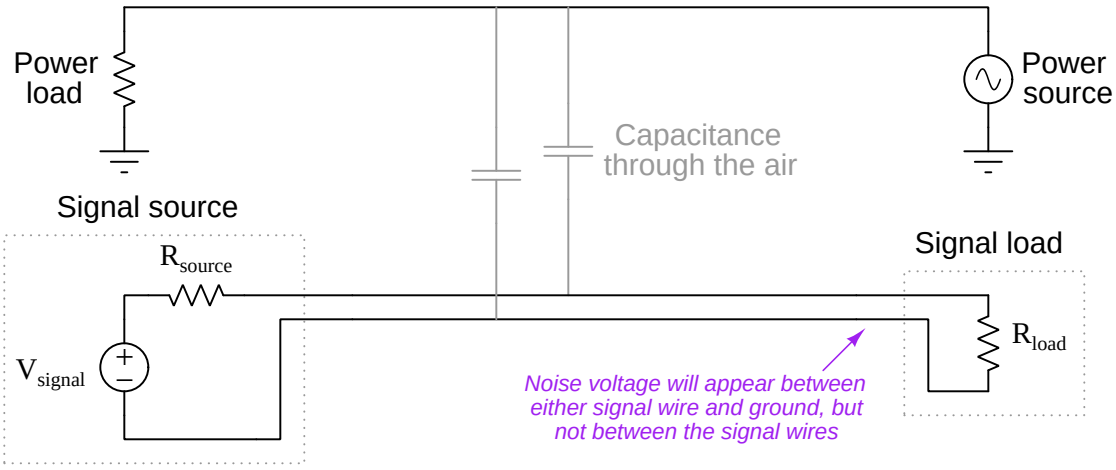


At the terminating end we typically twist the loose shield conductor strands together to form a wire which is then attached to a ground point to fix the cable's shield at Earth potential.

It is very important to ground *only one end* of a cable's shield, or else you will create the possibility for a *ground loop*: a path for current to flow through the cable's shield resulting from differences in Earth potential at the cable ends. Not only can ground loops induce noise in a cable's conductor(s), but in severe cases it can even overheat the cable and thus present a fire hazard:



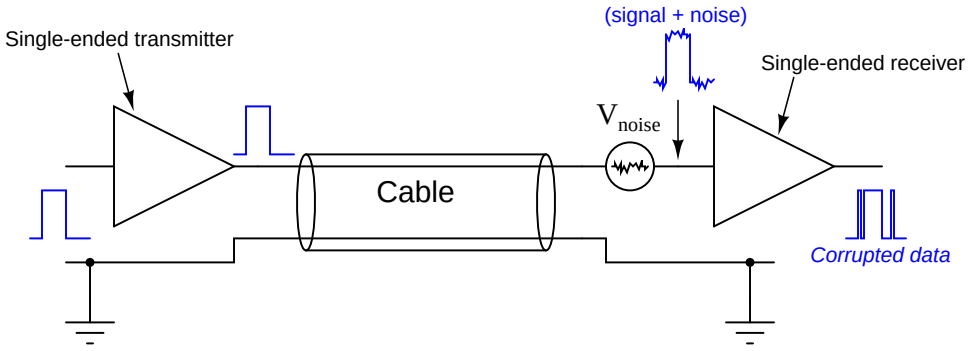
An important characteristic of capacitively-coupled noise voltage is that it is *common-mode* in nature: the noise appears equally on every conductor within a cable because those conductors lie so close to each other (i.e. because the amount of capacitance existing between each conductor and the noise source is the same). One way we may exploit this characteristic in order to help escape the unwanted effects of capacitive coupling is to use *differential signaling*. Instead of referencing our signal voltage to ground, we let the signal voltage "float." The following schematic diagram illustrates how this works:



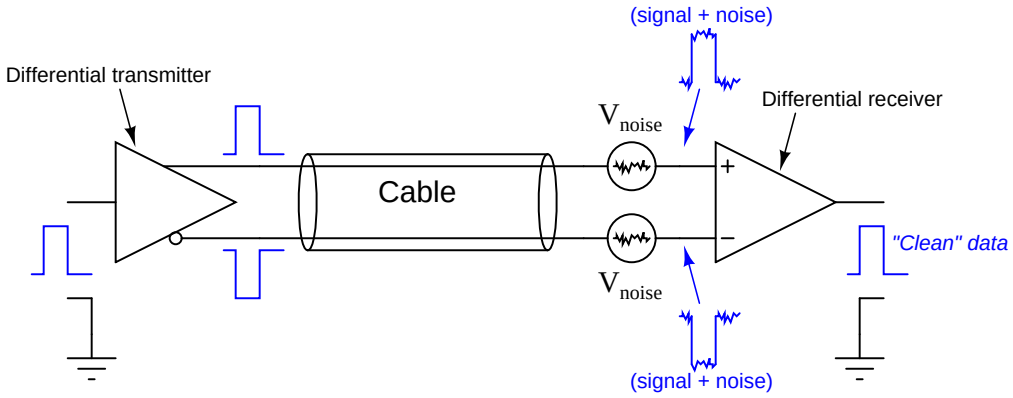
The lack of a ground connection in the DC signal circuit prevents capacitive coupling with the AC voltage from corrupting the measurement signal "seen" by the instrument. Noise voltage *will* still appear between either signal wire and ground as a common-mode voltage, but noise voltage will not appear *between* the two signal wires where our signal of interest exists. In other words, we

side-step the problem of common-mode noise voltage by making common-mode voltage irrelevant to the sensor and to the signal receiver.

Multiple digital communication standards employ differential signaling for its noise immunity, and this may be easily understood by graphical comparison. Noise is modeled below as a voltage source in series along the ungrounded conductor, near the receiving end. In reality, it is more likely to be distributed along the bulk of the cable length:



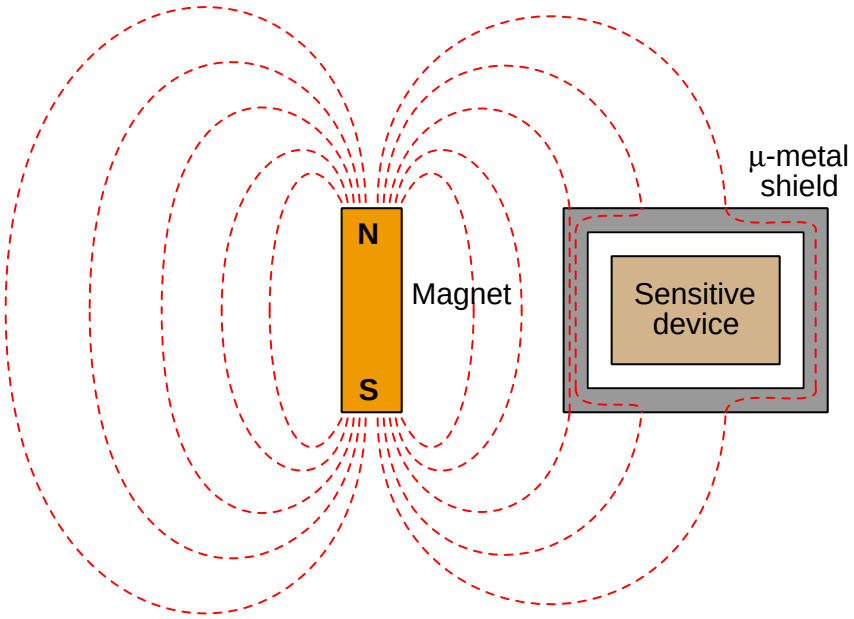
By contrast, any noise superimposed on ungrounded conductors in a differential signaling circuit cancel at the receiver, because the close proximity of those two conductors ensures any induced noise will be the same. Since the receiver responds only to *differential* voltage between its two inputs, this common-mode noise cancels, revealing a "clean" data signal at the end:



Some digital data communications standards such as EIA/TIA-485 (RS-485), Ethernet, and Universal Serial Bus (USB) use differential signaling to minimize the corrupting effects of electrical noise. Other standards such as EIA/TIA-232 (RS-232) use single-ended signaling, and must overpower noise by using much higher-amplitude signal voltages.

3.3 Magnetic field (inductive) de-coupling

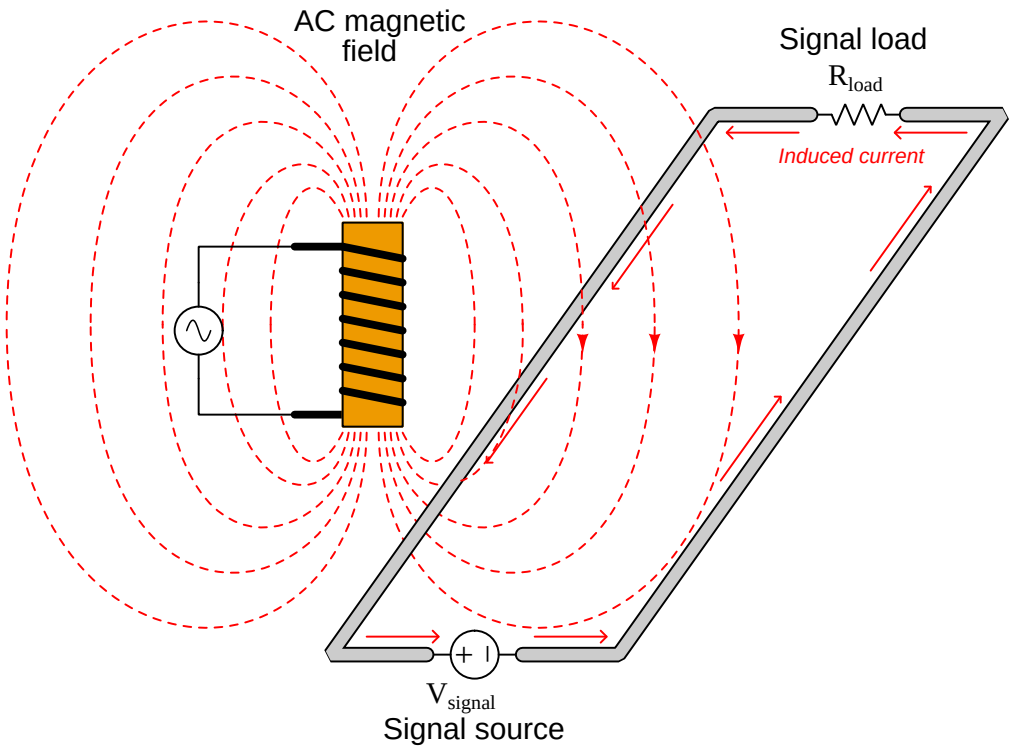
Magnetic fields, unlike electric fields, are exceedingly difficult to completely shield. Magnetic flux lines do not terminate, but rather *loop*. Thus, one cannot "stop" a magnetic field, only re-direct its path. A common method for magnetically shielding a sensitive instrument is to encapsulate it in an enclosure made of some material having an extremely high magnetic permeability (μ): a shell offering much easier passage of magnetic flux lines than air. A material often used for this application is *mu-metal*, or *μ-metal*, so named for its excellent magnetic permeability:



This sort of shielding is impractical for protecting signal cables from inductive coupling, as mu-metal is rather expensive and must be layered relatively thick in order to provide a sufficiently low-reluctance path to shunt most of the external magnetic flux lines.

The most practical method of granting magnetic field immunity to a signal cable follows the differential signaling method discussed in the electric field de-coupling section, with a twist (literally). If we *twist* a pair of wires rather than allow them to lie along parallel straight lines, the effects of electromagnetic induction are vastly minimized.

The reason this works is best illustrated by drawing a differential signal circuit with two thick wires, drawn first with no twist at all. Suppose the magnetic field shown here (with three flux lines entering the wire loop) happens to be *increasing* in strength at the moment in time captured by the illustration:



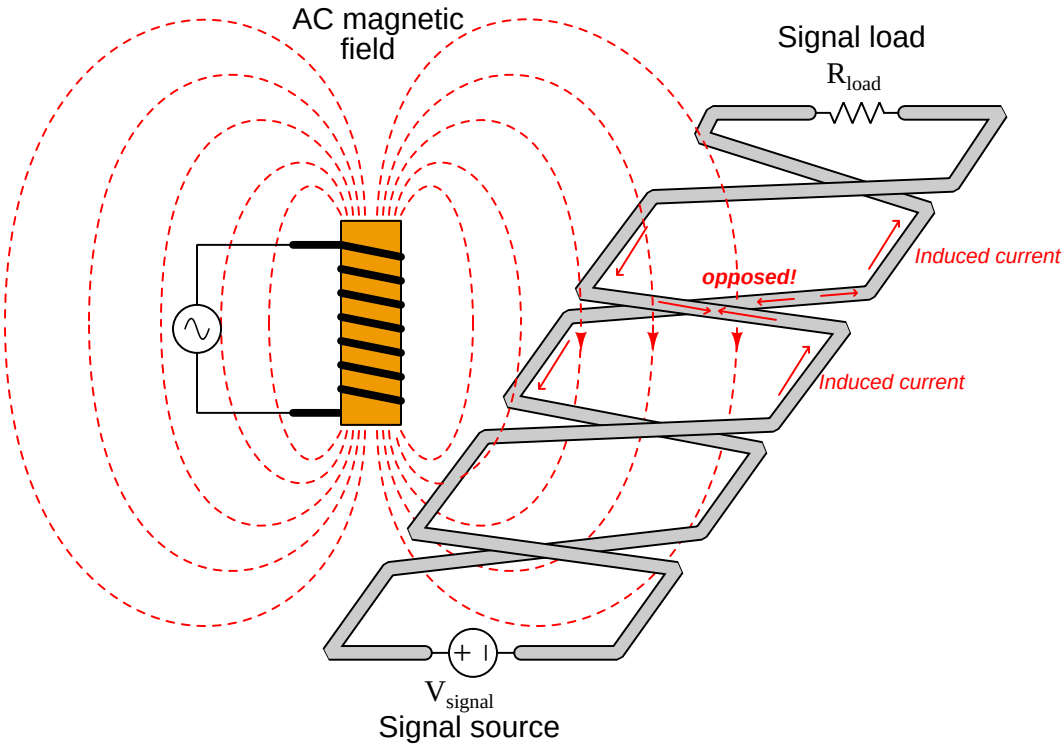
According to Lenz's Law, a current will be induced in the wire loop in such a polarity as to oppose the increase in external field strength. In other words, the induced current tries to "fight" the imposed field to maintain zero net change. According to the right-hand rule of electromagnetism (tracing current in conventional flow notation), the induced current must travel in a counter-clockwise direction as viewed from above the wire loop in order to generate a magnetic field opposing the rise of the external magnetic field. This induced current works against the DC current produced by the sensor, detracting from the signal received at the instrument.

When the external magnetic field strength diminishes, then builds in the opposite direction, the induced current will reverse. Thus, as the AC magnetic field oscillates, the induced current will also oscillate in the circuit, causing AC "noise" voltage to appear at the measuring instrument. This is precisely the effect we wish to mitigate.

Immediately we see a remarkable difference between noise voltage induced by a magnetic field versus noise voltage induced by an electric field: whereas capacitively-coupled noise is always common-mode, here we see inductively-coupled noise as *differential*[6].

[6]This is not to say magnetic fields cannot induce common-mode noise voltage: on the contrary, magnetic fields are

If we twist the wires so as to create a series of loops instead of one large loop, we will see that the inductive effects of the external magnetic field tend to cancel:



Not all the lines of flux go through the same loop. Each loop represents a reversal of direction for current in the instrument signal circuit, and so the direction of magnetically-induced current in one loop directly opposes the direction of magnetically-induced current in the next. So long as the loops are sufficient in number and spaced close together, the net effect will be complete and total opposition between all induced currents, with the result of no net induced current and therefore no AC "noise" voltage appearing at the instrument.

In order to enjoy the benefits of magnetic *and* electric field rejection, instrument cables are generally manufactured as *twisted, shielded pairs*. The twists guard against magnetic (inductive) interference, while the grounded shield guards against electric (capacitive) interference. If multiple wire pairs are twisted within the same cable, the twist rates of each pair may be made different so as to avoid magnetic coupling from pair to pair[7].

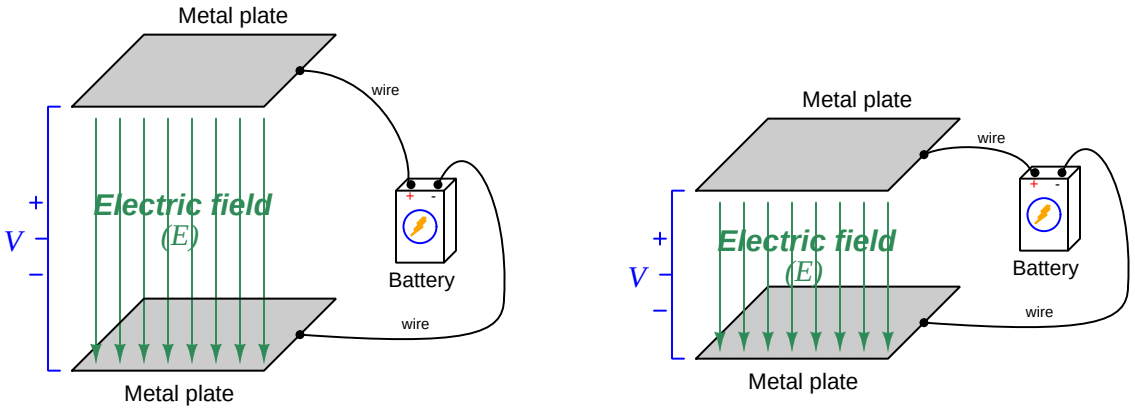
capable of inducing voltage in any electrically-conductive loop. For this reason, both differential and ground-referenced signals are susceptible to interference by magnetic fields.

[7] An example of this is the UTP (Unshielded, Twisted Pair) cabling used for Ethernet digital networks, where four pairs of wires having different twist rates are enclosed within the same cable sheath.

3.4 Electric field mitigation

An obvious strategy for reducing signal coupling is to mitigate fields at their sources rather than trying to shield them at the point of impact. Though this is not always possible in circuit design, it is worth exploring for those cases where we do have a choice in the matter.

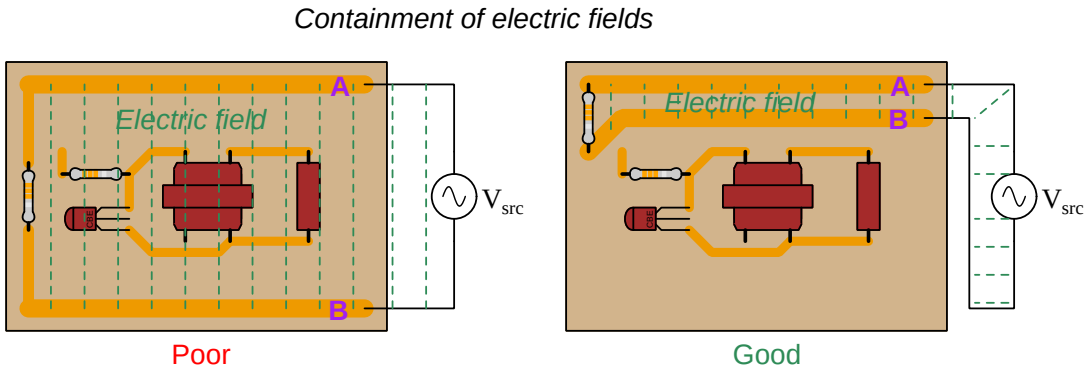
Electric fields are a function of *voltage*, and like voltage always span between two locations. One way to reduce the physical span of an electric field between two conductors of differing electric potential is to simply decrease the distance between those two conductors:



Between parallel conductive surfaces such as these metal plates, the intensity of the electric field (E) – otherwise known as *electric flux density* – is equal to the voltage divided by the separation distance in meters, and may be expressed in units of Volts per meter. From this we can see that while decreasing the separation distance certainly reduces the physical span of the electric field, it also increases the intensity of that field for any given applied voltage. The intensified electric field means that the capacitance[8] existing between those conductive surfaces increases with decreased separation distance, and this increased capacitance may have adverse effects to the circuit.

[8] Capacitance is the storage of energy within an electric field.

Consider these two printed circuit board (PCB) layouts, each with a pair of traces labeled **A** and **B** energized by an AC source:



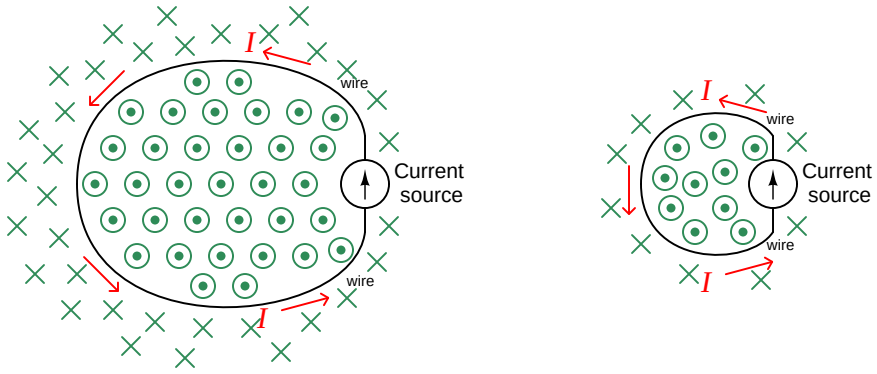
Of course, locating the sensitive components as far away from the offending (aggressor) conductors is also recommended.

This same strategy applies to free conductors as well. If we have a pair of free conductors with relatively high voltage in-between, the electric field stretching from one conductor to the other could impact other devices or circuits located between the conductors. However, if we use a two-conductor *cable* instead of two free conductors, the electric field will be constrained to the small gap between those conductors and will be far less likely to impact surrounding circuits. *Shielded cables* are the ultimate realization of this principle, and are even more useful as a means to mitigate electric field emissions from source (aggressor) conductors than they are as protection for victim conductors.

3.5 Magnetic field mitigation

An obvious strategy for reducing signal coupling is to mitigate fields at their sources rather than trying to shield them at the point of impact. Though this is not always possible in circuit design, it is worth exploring for those cases where we do have a choice in the matter.

Magnetic fields are produced by electric charge carriers in motion, and therefore the amount of magnetism exhibited by a circuit is a direct function of its current (I). One way to reduce the total quantity of magnetism generated by any given amount of current is to minimize the area enclosed by the "loop" of that circuit. Below we see a comparison of the magnetic flux (shown as green-colored vector tips and tails) for circular wire loops of different size:



The formula calculating total magnetic flux enclosed by a current-carrying circular wire loop indicates the same relationship between loop size and flux:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

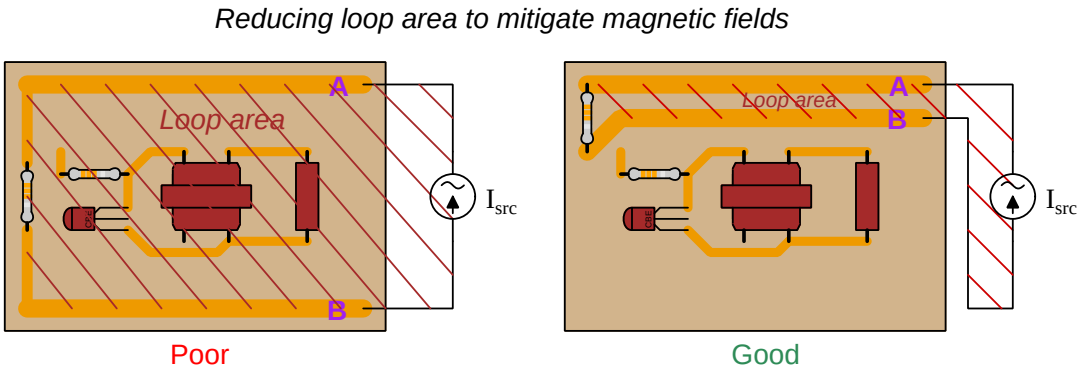
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

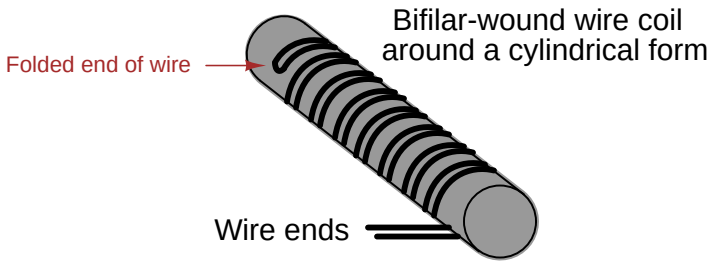
r = Radius of circular loop (meters)

Note how the loop's radius r appears in the numerator of the formula's fraction, mathematically indicating to us that the total amount of magnetic flux produced by a circuit increases proportionately with the physical size of the circuit's loop. The lesson to be learned here, whether mathematically or visually, is that large circuit loops are greater magnetic-field aggressors than small circuit loops, all other factors being equal.

Interestingly, a similar technique discussed in the previous section for containing electric fields works for mitigating magnetic fields too. Rather than intentionally placing conductors of differing potential close together to limit the span of an electric field, here we intentionally place conductors having opposite directions of current close together so that their magnetic fields will tend to cancel:



When forming devices from long lengths of wire, such as in the case of wire-wound resistors, we may use this same technique to make such devices non-magnetic simply by forming the wire coil with an equal number of "turns" carrying current in opposite directions. This is known as a *bifilar winding*, and is a common construction technique making wire-wound resistors having negligible inductance[9]. A simple way to wind a cylindrical coil in a such a manner that current passes through an equal number of clockwise and counter-clockwise turns is to first fold the wire in half and then wrap the folded wire around the cylindrical form as shown in this illustration:



Another technique for reducing magnetic field emission is to place a thick[10] layer of conductive material (e.g. a copper plane on an adjacent layer of a PCB) overlapping the current-carrying conductors. By the action of Lenz's Law, electric currents will be induced in this conductive plane in such a direction as to produce their own magnetic fields opposing the inducing fields.

One way to test for good magnetic field mitigation is to take measurements of the energized circuit's magnetic field emissions using a near-field probe, sometimes referred to as an *H-probe*.

[9] Inductance is the storage of energy within a magnetic field.

[10] The minimum practical thickness for this metal plane is that of the *skin effect depth*, the skin effect being a phenomenon where AC electric currents preferentially travel near the outer surface of a conductor as opposed to evenly throughout as is the case with DC. This depth becomes smaller with increased frequency.

Near-field probes don't typically yield good quantitative data on field strength, but they may be used for comparative tests to see which circuit layout produces the strongest or weakest fields.

If a fabricated circuit's layout needs to be tested for magnetic field mitigation while in an unpowered state, one technique is to connect an impedance analyzer to the conductive loop in question and measure that loop's inductance in Henrys. Smaller-area current loops and magnetic field cancellation techniques alike have the effect of reducing inductance, since inductance is fundamentally a measure of energy-storage ability via magnetic fields and such techniques directly reduce magnetic field production.

3.6 Relative electric/magnetic field severity

Electric field strength is proportional to potential difference (i.e. voltage), all other factors being equal. Magnetic field strength is proportional to current, all other factors being equal. In an AC or pulsing-DC circuit we will have differences of potential associated with electric fields and currents associated with magnetic fields, but how are we to know which of these phenomena poses the greatest threat and therefore which of these we should work more diligently to mitigate?

Circuits operating at high voltage and low current, should, based on the association of electric fields with voltage and magnetic fields with current, radiate stronger electric fields and weaker magnetic fields. Circuits operating at low voltage and high current will be opposite of this: radiating weaker electric fields but stronger magnetic fields. Thus, the relative magnitudes of AC voltage and current serve to indicate which type of field will dominate near the circuit, and therefore help determine which mitigation technique(s) we should prioritize. However, this is qualitative reasoning rather than quantitative, offering no guide as to the ratio of voltage to current that would delineate one type of field being stronger (and thus more deserving of mitigation) than the other. How may we predict whether an aggressor circuit's emissions will be predominantly electric-field or magnetic-field, or evenly balanced?

This question has a surprisingly simple answer: *a circuit impedance of 377 Ohms is the threshold*. Any circuit impedance less than this means stronger magnetic fields and weaker electric fields, suggesting we should work more to mitigate the magnetic fields. Any circuit impedance greater than this means stronger electric fields and weaker magnetic fields, suggesting we should prioritize mitigation of the electric fields.

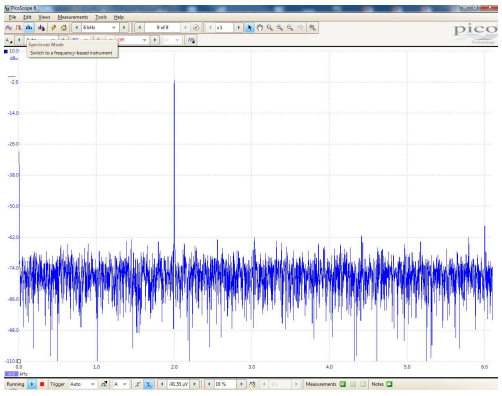
When AC electric and magnetic fields radiate out into empty space at the speed of light, they reinforce one another to produce an *electromagnetic wave*. These waves are the basis of radio waves, light, X-rays, gamma rays, and a host of other phenomena, differing from each other only in the frequency and wavelength of the oscillating fields. This mutual reinforcement results in the magnetic and electric field oscillations settling at a constant ratio between the two fields' peak amplitudes which we call the *characteristic impedance of free space* or Z_0. Determined by the electric permittivity of free space (ϵ_0) and the magnetic permeability of free space (μ_0), the value of Z_0 is equal to 120π, or approximately 377 Ohms. In this free-space scenario far away from the circuit radiating element(s) the electric and magnetic fields may be considered "equally strong".

However, if we measure electric and magnetic field strength very near to the aggressor circuit element(s) we find that the ratio between electric and magnetic field strengths follows the ratio of voltage to current in that circuit, which of course is the definition of impedance ($Z = \frac{V}{I}$). Magnetic field emissions dominate in circuits with impedance values less than 377 Ohms, whereas electric field emissions dominate in circuits having more than 377 Ohms of impedance. Any imbalance in the strengths of these field types will naturally resolve at the ratio of Z_0 ($\approx$ 377 Ohms) once they travel a distance of at least one wavelength away from the radiating circuit. If our purpose is to mitigate a strong field at its source, though, we don't care about this "far field" zone where the electric and magnetic fields settle into a fixed proportion of 377 Ohms, but rather the "near field" zone where our mitigation(s) are best located. The circuit's own impedance therefore dictates whether its electric and magnetic field emissions will be stronger, that threshold value being 377 Ohms.

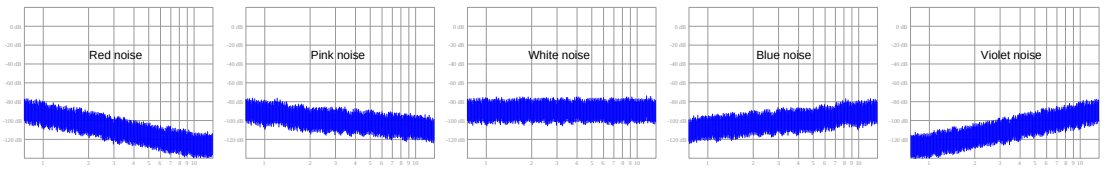
3.7 Intrinsic noise

Although noise may enter a circuit from some outside influence, such as nearby conductors belonging to another circuit, this is not the only source of noise in circuits. A variety of *intrinsic* (i.e. internal) noise sources exist as well. This is where the circuit's own components act to generate noise, even in perfect isolation from anything else. Intrinsic noise tends to be more random than extrinsic noise, and for this reason intrinsic noise often spans a range of frequencies. Recall that any periodic waveform is mathematically equivalent to a series of sinusoidal waves summed together, and that the frequencies of these summed sinusoids could be expressed as whole-number multiples of some fundamental frequency called *harmonics*. Noise that is random, however, is necessarily non-periodic, and so the relationship to definite frequencies is not so simple. Random noise generally manifests as continuous *bands* covering ranges of frequency.

A practical example of this phenomenon is the *noise floor* seen on the display of a spectrum analyzer. In the screenshot shown below, the strong "peak" represents a 2 kHz sinusoidal signal while the "fuzz" spanning the rest of the spectrum represents random noise coexisting with that pure 2 kHz signal:



Note how the noise floor appears to be more or less uniform across the entire width of the spectrum, from 0 kHz to 6 kHz. This is called *white noise* because the noise is equally spread across all frequencies just as white light is a combination of all colors. Not all noise is so evenly distributed across the frequency spectrum, though, and various color names describe noise with uneven spectra. *Red noise* (also called *Brownian noise*) and *pink noise* are both more heavily-weighted at the low-frequency end of the spectrum: that is to say, a "pink" or "red" noise source generates stronger signal variations at low frequency than at high frequency, like white noise after low-pass filtering. *Blue noise* and *violet noise* are both more heavily-weighted at the high-frequency end of the spectrum, like white noise following high-pass filtering. The disinction between pink/red and blue/violet is in the rolloff rate, as shown in the following spectra:



3.7.1 Thermal noise

All matter existing at temperatures greater than absolute zero possess motion on a molecular or atomic scale. In fact, temperature may be thought of in terms of this atomic or molecular motion, with absolute zero being that temperature where all motion ceases. These random motions of atoms and molecules is itself an intrinsic source of noise for electrical and electronic components, because electrical charge carriers are directly affected. As strange as it may seem, even a component such as a resistor which we typically consider a load, actually behaves as a *noise source* at all temperatures other than absolute zero:

The noise output by a resistor is random in all regards: varying in magnitude and polarity with no discernable pattern, all due to the influence of temperature on the electric charge carriers within. Its DC value is zero because all the random variations cancel out over time, but as an AC quantity it exists as an equal amount of voltage (open-circuit) or current (short-circuit) over equal ranges (i.e. bands) of frequency:

$$I_{noise} = \sqrt{\frac{4kT\Delta f}{R}} \qquad V_{noise} = \sqrt{4kT\Delta f R}$$

Where,

I_{noise} = Thermal noise current through short-circuited resistor (AC Amperes RMS)
V_{noise} = Thermal noise voltage across open-circuited resistor (AC Volts RMS)
k = Boltzmann's constant ($1.3806504 \times 10^{-23}$ J / K)
T = Absolute temperature (Kelvin), 273.15 more than degrees Celsius
Δf = Band-width of frequency over which noise is measured (Hertz)
R = Resistor value (Ohms)

All conductors exhibit thermal noise, and this includes the conductive channels of field-effect transistors (FETs).

Thermal noise is also known as *Johnson* noise, or *Johnson-Nyquist* noise after its discoverers John Johnson and Harry Nyquist at Bell Labs in 1926. Since it arises from thermally-motivated charge carriers passing through a resistance, it is fundamental and unavoidable, which is to say *all* resistances exhibit this phenomenon. It is also quite small, as one might conclude by the extremely small value of Boltzmann's constant. For example, the noise voltage produced by a 1000 Ohm resistor at 25 degrees Celcius (298.15 Kelvin) over a bandwidth of 15 kiloHertz (e.g. from 0 to 15 kHz, or from 40 kHz to 55 kHz) is only 0.496975 *micro*Volts.

The "color" of thermal noise is white, being constant in magnitude across all frequencies.

3.7.2 Shot noise

Another form of intrinsic and random noise is *shot noise*. Unlike thermal noise which depends only on temperature and resistance, shot noise is a function of current through any component where charge carriers traverse a gap, and it originates from the fact that electric current is not a continuous substance but rather a passage of discrete electrical charges. A good mental image of shot noise is that of dropping grains of sand onto a hard surface: each grain of sand makes an impact sound when it strikes the surface. Shot noise was investigated in vacuum tubes by Walter Schottky in 1918, arising from the effect of individual electrons reaching the tube's plate one at a time through the gap separating the plate from the tube's cathode. Semiconductor PN junctions (including the junctions of bipolar junction transistors and junction field-effect transistors) also exhibit shot noise, as do photoelectric devices where each incident photon liberates electron-hole pairs.

$$I_{noise} = \sqrt{2eI_{DC}\Delta f}$$

Where,

I_{noise} = Shot noise current through component (AC Amperes RMS)
e = Charge of a single electron ($1.602176487 \times 10^{-19}$ C)
I_{DC} = Constant current through component (DC Amperes)
Δf = Band-width of frequency over which noise is measured (Hertz)

Shot noise is also called *Poisson noise* in honor of its probability distribution. As the equation describes, shot noise is proportional to the intensity of the DC current passing through a component. That is to say, components generate more shot noise as more current passes through them. Shot noise is unrelated to device temperature and therefore is distinctly different from thermal (Johnson) noise which exists independent of circuit current.

The "color" of shot noise is white, being constant in magnitude across all frequencies.

3.7.3 Flicker noise

A type of intrinsic noise found in many types of electrical and electronic devices is *flicker noise*, which grows in proportion to the DC current through the device similar to shot noise. Unlike shot noise, though, flicker noise has a "pink" spectrum which means it is stronger at low frequencies than at high frequencies. The degree to which any device generates flicker noise depends on not only the amount of DC current passing through, but also on the type of device.

It is well known, for example, that carbon-composition resistors generate more flicker noise than either metal-film or metal-wire resistors. The tendency for some types of components to generate more flicker noise than others has led to flicker noise sometimes being referred to as *excess noise*. We know that thermal (Johnson) noise is present in all conductive components and cannot be avoided, but that flicker noise varies with component type. Interestingly, the metal alloy Manganin[11] which is often chosen for high-precision metal-wire resistor construction for its zero temperature coefficient (i.e. its resistivity remains constant over wide ranges of ambient temperature) happens to exhibit zero flicker noise as well.

[11] Manganin is a metal alloy consisting of 84% copper, 12% manganese, and 4% nickel.

3.7.4 Burst noise

A type of intrinsic noise unique to semiconductor devices is *burst noise*, also known as *popcorn noise*. As the name suggests, this type of noise takes the form of discrete steps or bursts in signal strength happening at random times, like the sound of popcorn popping in a hot pan. When this occurs in a device conducting an otherwise steady current, the current will be seen to step up and down at random intervals, making the graphed value look similar to a digital signal alternating[12] between high and low states. The frequency range at which burst noise typically occurs is well within the human audio range – usually 100 Hz or less – and indeed may sound like popcorn popping if heard through a loudspeaker.

Burst noise seems to be primarily the result of defects within the crystalline structure of semiconductor devices, and with improvements in semiconductor manufacturing this type of noise has become less and less of a problem for modern circuit designers.

3.7.5 Avalanche noise

When a PN semiconductor junction is operated with a reverse bias of sufficient voltage, minority charge carriers moving "backwards" through the junction may occasionally gain enough kinetic energy to liberate additional electron/hole pairs when they collide with stationary atoms. These liberated charge carriers then drift with the applied voltage and likewise gain kinetic energy. If the voltage is sufficient, these charge carriers may also liberate more charge carriers when they collide with stationary atoms, the result being an "avalanche" of mobile charge carriers which increases conductivity of the PN junction. The avalanche effect does not result in a constant current, though, but rather pulses of current that constitute noise which we call *avalanche noise*.

The "color" of avalanche noise is white, being constant in magnitude across all frequencies.

3.7.6 Intrinsic noise mitigation

Given the random nature of intrinsic noise, a simple and effective means to mitigate its effects is to *average* many samples of the signal over a period of time, the principle being that the random positive and negative variations of the noise will cancel out to yield a more steady signal. Of course, this incurs the price of having to wait longer periods of time to reliably measure the signal, and it also only works in cases where the signal of interest has a low enough frequency that the averaging process will not substantially corrupt it by filtering out (legitimate) fast rates-of-change.

[12] The actual magnitude of these steps will generally be quite small, much less than the steps separating valid digital "high" and "low" states. The point, however, is that burst or popcorn noise lends a randomly "stepped" profile to the graphed quantity.

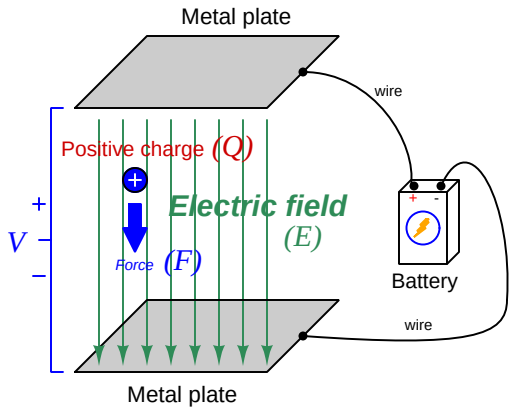
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{E}$ = Electric field (Newtons per Coulomb)

The small "arrow" symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,
$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)
Φ_E = Electric flux (Newton-meter squared per Coulomb)
$\vec{A}$ = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,
Φ_E = Electric flux (Newton-meter squared per Coulomb)
Q = Charge quantity (Coulombs)
ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

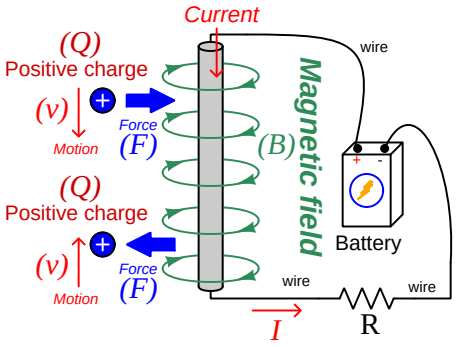
By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field[2]. As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

[2]Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

4.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F, called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

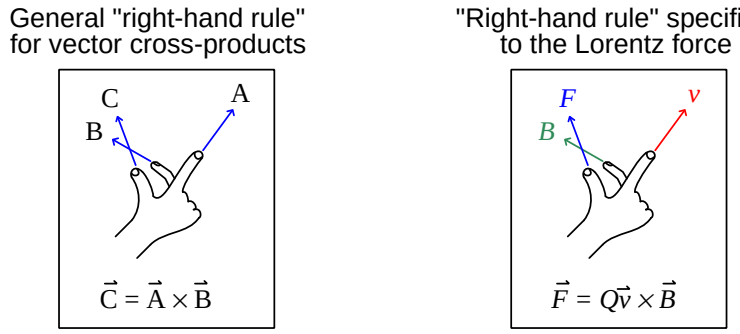
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[3])
$\vec{v}$ = Velocity of moving charge (meters per second)
$\vec{B}$ = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small "arrow" symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The "cross-product" ($\times$) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

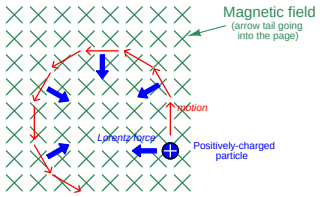
[3] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the "right-hand rule" originates:



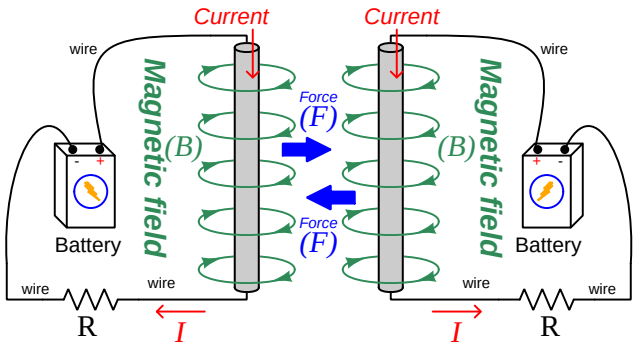
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector ($\vec{v}$), your middle finger in the direction of the magnetic field vector ($\vec{B}$), and your thumb in the direction of the force vector ($\vec{F}$). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current[4] (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **Th**umb points in the direction of the **th**rust (i.e. force) acting upon the moving charge.

The Lorentz force's effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



[4]Some textbooks speak of a "left-hand rule" which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically "electron flow" notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be "positive" and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

$\vec{F}$ = Force exerted on both conductors (Newtons)

I = Current (Amperes)

$\vec{l}$ = Length of wire (meters)

$\vec{B}$ = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\mathrm{C}]\left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\mathrm{C}}{\mathrm{s}}\right][\mathrm{m}] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

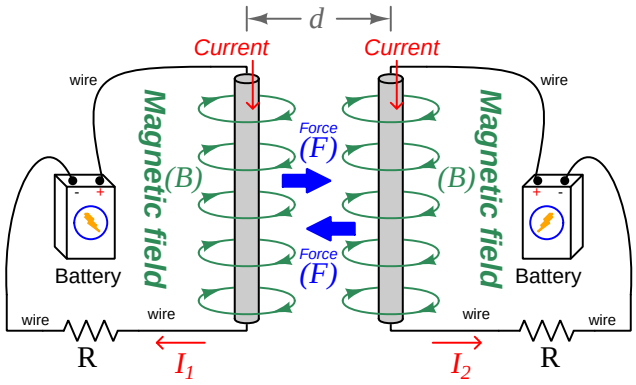
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract[5].

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l\frac{\mu I_1 I_2}{2\pi d}$$



[5]That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

$\vec{B}$ = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

$\vec{A}$ = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss[6].

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

[6] Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at `https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm`, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu A I}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most[7] substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

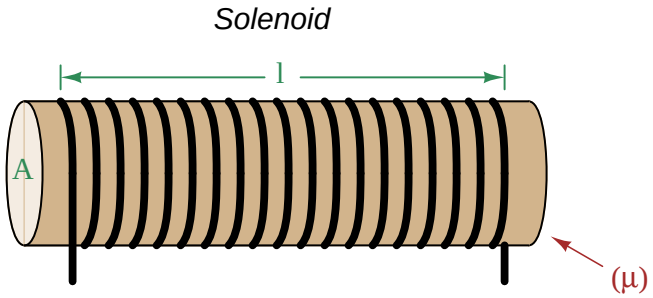
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

[7] Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long[8] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

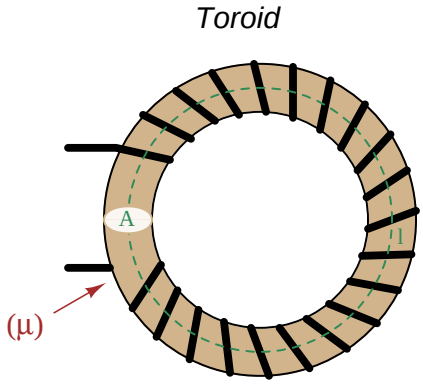
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A.

[8] These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet[9] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

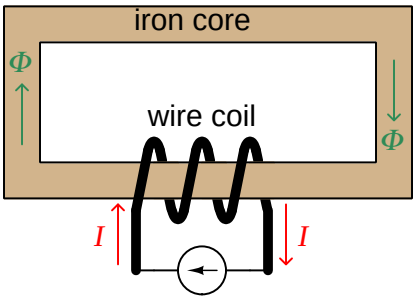
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \qquad\qquad B = \frac{\mu N I}{2\pi r}$$

[9] Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic "circuit" formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to "electromotive force" or "emf" often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current's effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance ($\Re$) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material's permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\Re = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	**Symbol**	**SI unit**	**CGS unit**
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	$\Re$	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the "raw" measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance ($\Re$) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

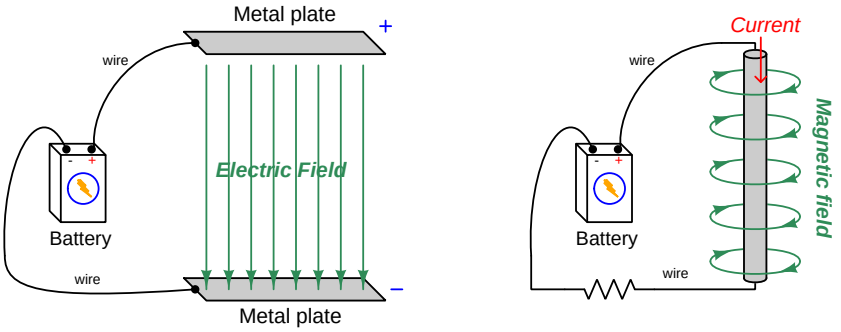
Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the "normalized" measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	**Conversion equivalence**
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance ($\Re$)	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

4.3 Near-field versus far-field regions

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These phenomena exist for DC (direct) as well as AC (alternating) electricity, and are shown in the two following illustrations:



Electric and magnetic fields, however, are not simply manifestations of voltage and current, respectively. Mathematical discoveries made by the Scottish physicist James Clerk Maxwell (and later simplified by Oliver Heaviside) also relate electric fields and magnetic fields *directly to each other*. Consider two[10] of Maxwell's equations shown below:

$$\oint \vec{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

The first equation describes how an electric field ($\vec{E}$) is created by a varying magnetic flux ($\frac{d\Phi_B}{dt}$), which is otherwise known as *Faraday's Law of Electromagnetic Induction*. This is the law exploited in the function of electro-mechanical generators whereby coils of wire are subjected to changing magnetic fields, creating voltage between the coils' end-points. The negative sign in this equation is an embodiment of *Lenz's Law*, which states that any current resulting from the induced voltage will produce its own magnetic field opposing the first magnetic field's direction of change.

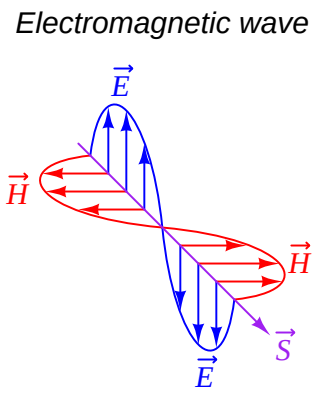
The second equation shown here describes *two* different ways to produce a magnetic field ($\vec{B}$). One way is to use a moving stream of electric charges known as a *current* (I), a fact also known as *Ampère's Law*. This is the law exploited in the function of electromagnets, where we produce a magnetic field by connecting a coil of wire to an electrical source. The second way is to vary an electric flux ($\frac{d\Phi_E}{dt}$) in empty space with surrounding magnetic permeability μ_0 and electric permittivity ϵ_0.

[10] Maxwell's equations of electromagnetism are typically presented as a set of *four*. Here we only list two of these, as the other two are not relevant to electromagnetic waves in particular.

Maxwell's equations suggest that in pulsed DC and AC circuits, where the resulting electric and/or magnetic fields vary over time ($\frac{d}{dt}$) rather than holding steady, those time-varying fields will mutually support one another over space and time. That is, a varying electric field will eventually produce a varying magnetic field, and vice-versa, those fields propagating one another as an *electromagnetic wave* radiating away from the circuit through space at the speed of light[11].

At close range, immediately adjacent to the circuit conductors, electric and magnetic fields are strictly functions of those conductors' voltages and currents, respectively. These fields' shapes follow the basic forms shown in the previous illustrations: magnetic field lines circling current-carrying conductors and electric field lines stretching between conductive surfaces. We generally refer to this region of space around an energized circuit as the *near-field region*, where electric and magnetic fields are distinct from one another in form and in relative magnitude.

At farther distances from the energized circuit, the electric and magnetic fields are found in pairs oscillating at right angles to one another in space, forming a single electromagnetic wave radiating away from the source circuit spherically in all directions. This form of wave is a direct consequence of Maxwell's equations describing how the rate-of-change of one field creates the other field, and vice-versa. Below we see a crude representation of an electromagnetic wave, with an electric $\vec{E}$ field, a magnetic $\vec{H}$ field, and the vector of propagation $\vec{S}$ called the *Poynting vector*:



This region where the electric and magnetic fields exist as coupled pairs comprising a well-formed wave is referred to as the *far-field region*. In this region the electric and magnetic fields are always perpendicular to one another and their relative magnitudes are in a fixed proportion governed by the permeability and permittivity of free space.

The demarcation between near- and far-field regions depends on the physical dimensions of the radiating circuit as well as the *wavelength* of the signal (λ), wavelength being calculated by dividing the speed of light (2.9979×10^8 meters per second in empty space) by the signal frequency in Hertz. There is no simple rule for predicting where the near-field region ends and the far-field region begins, and in fact there is a gradual morphing of one to the other, but one may reliably consider any distance from the circuit *in excess of multiple wavelengths* to be far-field.

[11] In fact, this is precisely what light is: an electromagnetic wave of exceptionally high frequency, well beyond the frequency range of contemporary electric circuits.

The following list compares and contrasts several near-field versus far-field characteristics:

- In the near-field region electric and magnetic field strengths depend greatly on the geometry of the energized circuit's conductors as well as on the specific voltage and current levels existing at each location. In the far-field region, however, the electric and magnetic field strengths are always in fixed proportion to one another.

- Far-field effects may be ignored for low-frequency AC circuits because these circuits' wavelengths are so long. At 60 Hz, for example, the wavelength is nearly 5 *million* meters. If we consider the far-field region of a circuit to begin at least multiple wavelengths from the source, this puts the far-field region of a 60 Hz circuit at least one-quarter of the Earth's circumference away from the circuit in question!

- In the near-field region the proportionality between electric and magnetic field strengths is a function of circuit impedance: high-impedance circuits will have stronger electric fields than magnetic fields, and low-impedance circuits will have stronger magnetic fields than electric fields, all other factors being equal. This is due to Ohm's Law establishing the relationship between voltage and current ($Z = \frac{V}{I}$), and how electric fields originate in a circuit from potential differences while magnetic fields originate in a circuit from currents. In the far-field region, however, where the electric and magnetic fields exist only through mutual support, their strengths are always in the same (fixed) proportion defined as the *characteristic impedance of free space* which is approximately 377 Ohms. This value is equal to the square root of the ratio of magnetic permeability to electric permittivity for free space ($Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$) because these parameters of space dictate just how strongly a varying electric field creates a magnetic field.

- The relationship between field strength and physical distance from the circuit is a very complex one within the near-field region, as this depends greatly on the geometry of the circuit conductors. In the far-field region, however, we find the *Inverse-Square Law* always holds true: as distance from the circuit increases, electromagnetic wave strength simply and reliably diminishes with the *square* of that distance as if the wave were radiating away in a spherical fashion from a point-source[12]. For example, if we double the distance from the radiating circuit (from a point in the far-field region to another point in the far-field region twice as far away in empty space), the power conveyed by the electromagnetic radiation will always be *four times* less. Tripling the far-field distance always weakens the wave's power by a factor of *nine*. This is simply because the area over which the radiated energy spreads increases with the square of the distance from any point-source, viewing that point-source as the center of a sphere.

- In the far-field region every radiating circuit has a definite *radiation pattern* where the strength of the radiated electromagnetic wave as a function of the *angle* from the geometric centerline follows a consistent pattern regardless of distance. This is a very important feature of *antennas*, where different antenna designs feature unique far-field radiation patterns. By contrast, in the near-field region of an antenna the radiation pattern varies significantly with distance.

[12] A "point source" is a hypothetical point in space having no height nor width nor depth (i.e. it has *zero* physical dimensions) that emits radiation. Point-sources are a theoretical concept only. Real sources have spatial dimensions which makes their near-field strength/distance relationships complex, but the farther away you get from them the more their radiative behavior approaches that of a theoretical point-source.

- The spatial *polarization* of the electromagnetic wave (i.e. the orientation of its perpendicular electric and magnetic fields) is always well-defined in the far-field region but often ill-defined in the near-field region. This means, among other things, that two or more antennas will exchange electromagnetic energy efficiently only if they are appropriately oriented to one another over far-field distances, but may exchange energy fairly well regardless of orientation over near-field distances.

- Any "gain" specifications for an antenna structure apply only to apparent gains in signal power over the far-field range, because they refer to the degree to which an antenna focuses its energy in one direction more than another (i.e. its directionality), implying a distance over which the electromagnetic wave has become well-formed and the antenna's radiation pattern is reliably established. At near-field distances these "gain" figures are meaningless.

4.4 Maxwell's equations of electromagnetism

The Scottish physicist James Clerk Maxwell mathematically proved the interrelationships of electric and magnetic fields, publishing his results in the book *A Treatise on Electricity and Magnetism* in 1873. His equations were later simplified and consolidated by the work of Oliver Heaviside, commonly expressed in the following forms describing electric fields (**E**) and magnetic fields (**B**) within a vacuum:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

We will now explore each of these four equations, relating them to laws codified by other early electrical researchers, as well as describe how they relate to the phenomenon of *electromagnetic waves*.

4.4.1 Gauss's Law of electric fields

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The first of these equations simply describes electric fields as spreading out from a quantity of electric charge (Q). If we imagine a three-dimensional surface completely enclosing an electric charge (e.g. a soap bubble surrounding a charge), the total quantity of electric flux passing through that surface will be the product of the electric field (also known as the electric flux density, **E**) and the enclosing area (**A**). This is also known as *Gauss's Law of electric fields*.

4.4.2 Gauss's Law of magnetic fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

The second of these equations describes magnetic fields passing through a three-dimensional enclosing surface. If we imagine a magnet completely enclosed within a soap bubble, the sum total of magnetic flux passing out through that soap bubble's surface must equal the sum total of magnetic flux re-entering it somewhere else. Literally, this equation says the product of magnetic field (also known as magnetic flux density **B**) and the enclosing area (**A**) must be algebraically equal to zero. This is to say, magnetic fields must always exist as *closed loops*, or alternatively we may say there is no such thing as a magnetic monopole (i.e. a solitary North or South pole without its complement, or a *magnetic charge*). In this way magnetic fields are quite different from electric fields which radiate away from discrete electric charges without ever returning. This is also known as *Gauss's Law of magnetic fields*.

4.4.3 Faraday's Law of electromagnetic induction, extended

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

The third equation states that an electric field ($\mathbf{E}$) will be produced in open space by a varying magnetic flux $\left(\frac{d\Phi_B}{dt}\right)$. Michael Faraday expressed this phenomenon in terms of voltage induced along the length of a conductor by a time-varying magnetic field, but Maxwell demonstrated mathematically that this same thing happens even in empty space!

4.4.4 Ampère's Circuital Law, extended

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

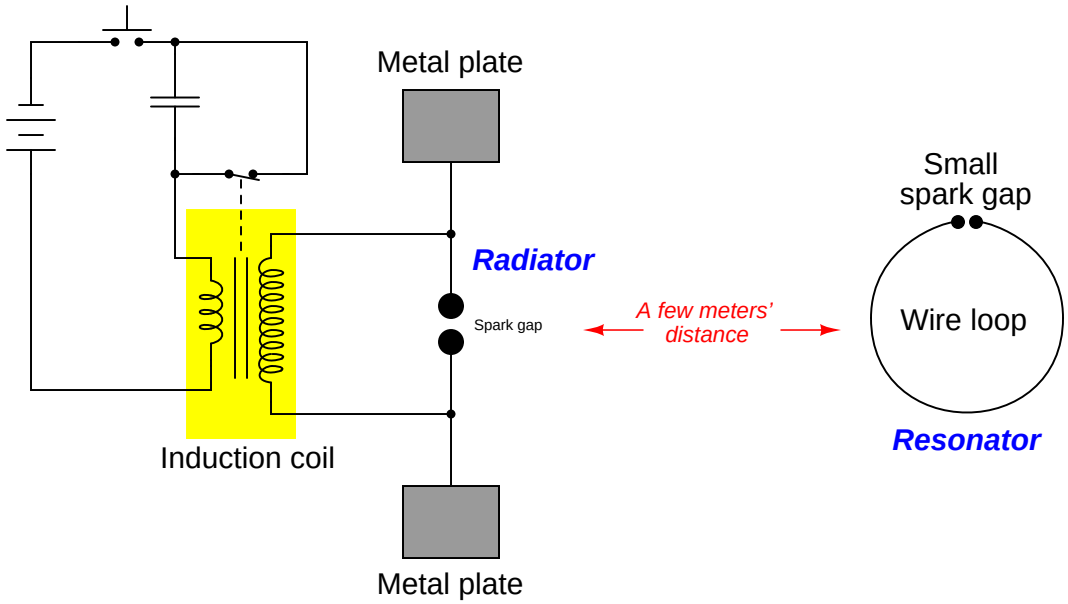
The fourth equation states than a magnetic field ($\mathbf{B}$) will be produced in open space either by an electric current (I) and/or by a varying electric flux $\left(\frac{d\Phi_E}{dt}\right)$. The French scientist André-Marie Ampère described the relationship between current passing through a conductor and the magnetic field created around that conductor by the current, but once again Maxwell extended this concept to include magnetic fields generated in empty space. In empty space, where there are no moving electric charges (i.e. current, I) to generate a magnetic field, only the second term of the equation applies with a time-varying electric field substituting for actual electric current.

4.4.5 Proof of electromagnetic waves

Given the complementary relationship between time-varying electric and magnetic fields, Maxwell reasoned, it was possible for a varying electric field to create a varying magnetic field which would then create another varying electric field, and so on. This cause-and-effect cycle could continue, ad infinitum, with fast-changing electric and magnetic fields radiating off into open space without needing wires to carry or guide them, or a tank circuit to sustain the oscillations. In other words, *these complementary fields self-sustain as they travel*, although they do become weaker due to their spreading out of a finite amount of energy over larger and larger volumes of space.

The Prussian Academy of Science offered a reward to anyone who could experimentally validate Maxwell's prediction, and this challenge was answered by Professor Heinrich Hertz at the Engineering College in Karlsruhe, Germany in 1887, eight years after Maxwell's death. Hertz constructed and tested a pair of devices: a "radiator" to produce the electromagnetic waves, and a "resonator" to receive them.

A simplified diagram showing Hertz's experimental device is shown here:



An "induction coil" (a buzzing device constructed of a self-interrupting relay and step-up transformer winding to generate a continuous pulsing waveform of high voltage) provided an extremely noisy (i.e. harmonically rich)[13] AC signal to the radiator, while a spark gap at the

[13] A fundamental property of waves is that any wave-shape, no matter how complex, is mathematically equivalent to a series of perfectly sinusoidal waves of differing frequencies added together. Waveforms with fast rise/fall times, such as the pulse waveforms produced by switched induction coils, contain many such high-frequency "harmonic" signals, and it is these high-frequency signals that created the radiated electromagnetic waves in Hertz's experiments. The induction coil's secondary winding inductance combined with the parasitic capacitance of the radiator's wires and plates provided the means to momentarily sustain some of these high-frequency oscillations long enough to sustain visible sparks at the resonator's gap.

resonator provided indication that the electromagnetic waves were captured and converted into voltage by the resonator wire.

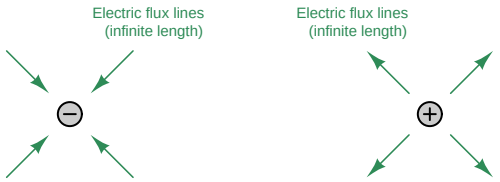
Both the radiator and the resonator are antennas. The purpose of the transmitting antenna (radiator) is to take high-frequency AC power and radiate that power in the form of electromagnetic waves: self-sustaining electric and magnetic fields propagating outward into open space. The purpose of the receiving antenna is to capture those electromagnetic waves and convert them into an electrical signal (i.e. AC voltage and current). All antennas – from historical to the most modern – behave in fundamentally the same way: energize them with high-frequency AC electrical power, and they will radiate electromagnetic waves at that frequency; expose them to electromagnetic waves, and they will produce a very small AC electrical signal at the same frequency as the incident radiation.

4.5 Gauss' Law of Electric Fields

One of fundamental principles relating to electric fields is *Gauss' Law of Electric Fields*. A common mathematical expression for this law is as follows:

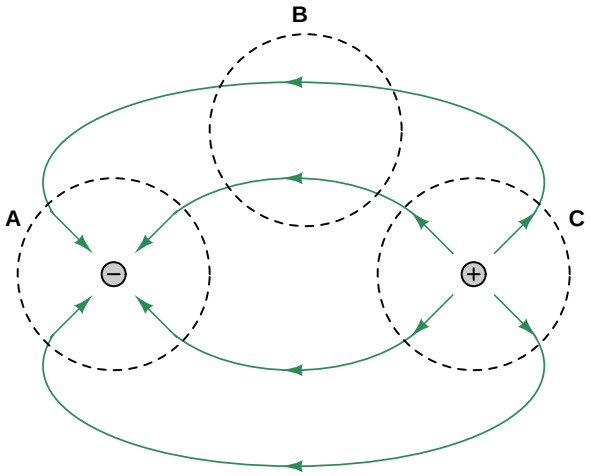
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

This equation simply describes electric fields (E) as lines of flux spreading out from a quantity of electric charge (Q). By convention we draw these electric flux lines as radiating away from positive charges and heading toward negative charges as shown below:



If we imagine a three-dimensional surface completely enclosing any electric charge, like a soap bubble surrounding that charge, the total quantity of electric flux lines passing through that surface will be the product of the electric field (also known as the electric flux density, **E**) and the enclosing area (**A**). This "soap bubble" is formally known as a *Gaussian surface*, and we may arbitrarily place that surface anywhere within space for the sake of solving an electric field problem. The major point to understand here is that *any* Gaussian surface drawn will experience an amount of electric flux penetrating its surface proportional to the amount of electric charge enclosed within that surface regardless of the size or shape.

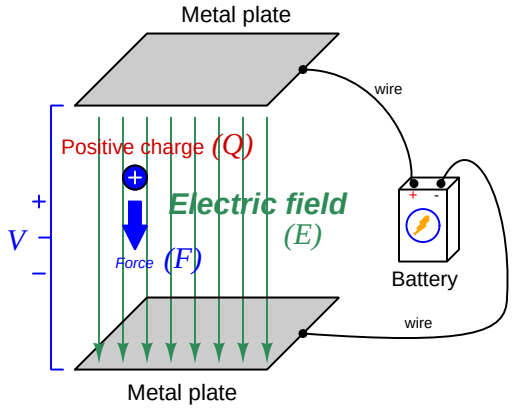
Observe what happens if we connect the electric flux lines together from two opposite charges, and then draw Gaussian surfaces in different regions of space labeled **A**, **B**, and **C**:



Gaussian surface **A** encloses a single negative charge and we see inward-pointing electric flux lines penetrating that surface. Gaussian surface **C** encloses a single positive charge and we see outward-

pointing electric flux lines penetrating that surface. Gaussian surface **B** contains no electric charges at all because it encloses empty space, and we see the number of electric flux lines entering the enclosed volume exactly equaling the number of flux lines leaving it. In other words, for Gaussian surface **B** there is zero net electric flux enclosed within it and also zero *net* penetrations since the number going in balances the number going out.

If we apply an electric field across open space containing a single positive charge, that charge will experience a force proportional to its charge-quantity and the strength of the electric field. The direction of this force will be downward to the negative plate and away from the positive as shown in the following illustration:



$$\vec{F} = Q\vec{E}$$

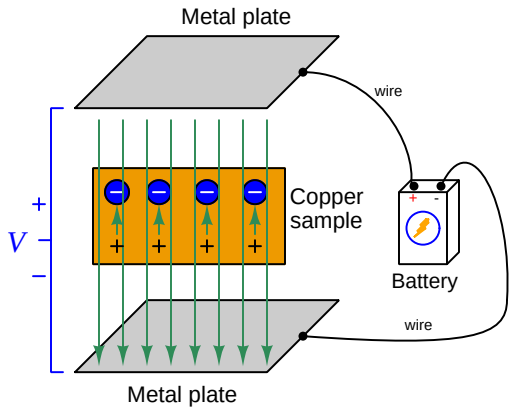
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[14])
$\vec{E}$ = Electric field (Newtons per Coulomb)

Unsurprisingly, a negative charge in that same space will experience a force in the opposite direction, upward to the positive plate and away from the negative. With nothing to impede the motion of these charges, they will accelerate as a result of the applied force from the electric field, their acceleration predicted by the familiar equation $F = ma$.

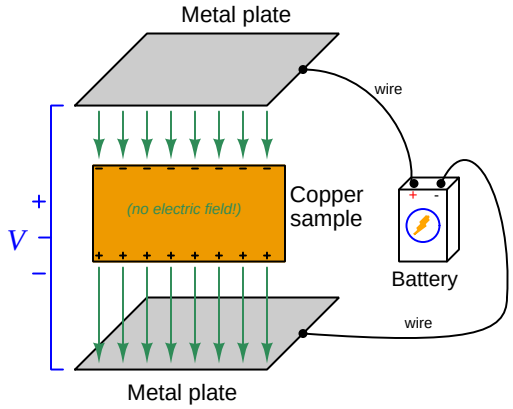
[14] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Now imagine a solid piece of electrically conductive material such as copper metal suspended between the same two metal plates. The outer-most electrons of the copper atoms are free to drift throughout the metal, which is the fact that makes copper metal a conductive material. The effect of the two plates' electric field on the free electrons within the copper sample will be much the same as upon free electrons in a vacuum space between two metal plates: those free electrons will accelerate upward in the direction of the positive plate and away from the negative. This migration of free electrons toward the upper surface of the copper sample leaves positively-charged regions near the lower surface of the copper:



The electrons will migrate as far as they are able, which in this case is the upper surface of the conductive copper sample. The separation of negative and positive charges within the copper sample – the negative charges being mobile electrons and the positive charges being immobile protons in the copper atom nuclei – itself forms an electric field within the copper, pointed in the direction opposite that of the battery's external field. These short vector-arrows pointed upward in the above illustration represent the flux lines from the positive stationary nuclei to the displaced negative electrons.

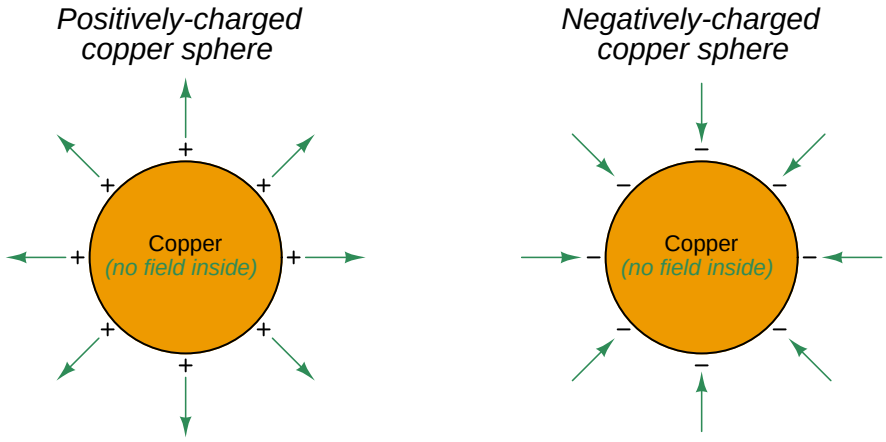
Since these two sets of electric flux lines point in opposite directions, their net effect is to cancel each other out. As more electrons drift upward this cancellation continues until there is zero net electric field within the copper sample. When the net electric field reaches zero, the electrons no longer experience any driving force upward. In this *equilibrium* condition of zero net force on all electric charges, the interior of the copper sample contains no net electric field:



It should not come as a surprise to anyone familiar with basic concepts of electric circuits that this sample of copper naturally excludes any electric field imposed externally upon it, since the mass of copper may be thought of as a large collection of electrically common points, and we know one of the characteristics of electrically common points is equipotentiality. *And, where there is no difference of electrical potential between two points (i.e. no voltage), there can be no electric field between those points.* A conductive object in a state of electrical equilibrium, therefore, contains no internal electric field(s).

This phenomenon of electrical conductors naturally cancelling out internal electric fields finds wide application in electrical and electronic systems. One such application is the *shielding* of sensitive components by surrounding them with a continuous conductive surface. Another application is in insulated-gate field-effect transistors (also known as MOSFETs) where an externally-applied electric field causes electron and hole charge carriers to reposition themselves and in so doing cause a conductive channel transverse to that field to either grow wider or narrower.

An interesting consequence of Gauss' Law of Electric Fields is that any net gain or loss of electric charge by a conductive object results in the unbalanced charges moving to the very exterior surface of the object. For example, if we inject extra electrons into a copper block, those electrons quickly settle into positions on the very outer skin of that block where they remain. Likewise, if electrons are extracted from a copper block, the remaining electrons will migrate within the block so as to leave the resulting positive charges (i.e. electron deficiencies) at the outer surface of that block:



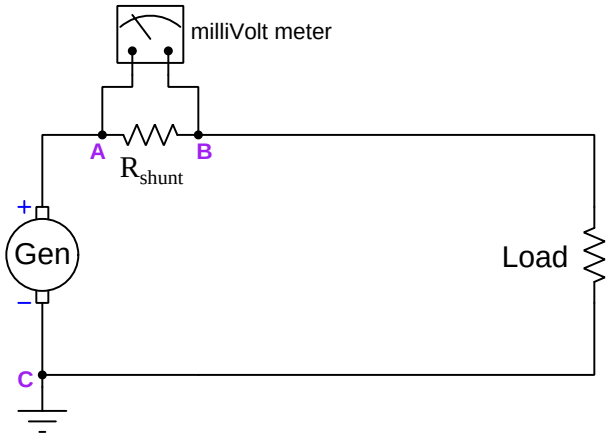
All unbalanced charges migrate to the outside of the object

We may understand this by appealing to the proven fact that a conductive object in a state of electrical equilibrium must have no electric field within its interior. If we imagine drawing a Gaussian surface just beneath the skin of that conductive object, we may conclude from Gauss' Law that the interior of that Gaussian surface must contain no net electric charge (i.e. all positive and negative charges within it are perfectly balanced) because zero net electric field penetrating that Gaussian surface implies zero net electric charge within. This must mean that any and all unbalanced electric charges in that conductive object must reside *outside* the area of no electric field; i.e. outside the Gaussian surface on the conductive object's outer surface. Thus, a Gaussian surface drawn anywhere inside a charged conductive object experiences no electric flux lines but a Gaussian surface drawn outside that same object does.

4.6 Common-mode voltages and currents

A topic of much misunderstanding in electronic circuits, often relevant to precise measurements of voltage and current, is that of *common-mode signals*. The topic is especially confusing in regard to *voltage*, as voltage is fundamentally a differential quantity always existing *between two points* and never at a single location. We will explore this topic in some detail here, through practical examples.

Suppose we install a shunt resistor and a milliVolt meter on a power circuit to infer how much current a DC generator sends to a load. Given the existence of three electrically-distinct points in this circuit (**A**, **B**, and **C**) we have three unique voltages – the voltage registered by the milliVolt meter (V_{AB}), the voltage across the load (V_{BC}), and the voltage output by the generator (V_{AC}):

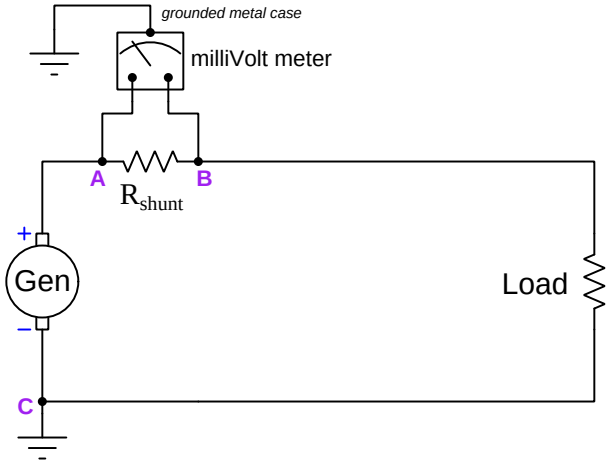


Supposing the generator output 300 Volts DC to a load that happens to be 40 Ohms at some given point in time, with the shunt resistor being a fixed value of 0.1 Ohms, these three voltages would be as follows:

- V_{AB} = 74.81 milliVolts DC
- V_{BC} = 299.25 Volts DC
- V_{AC} = 300 Volts DC

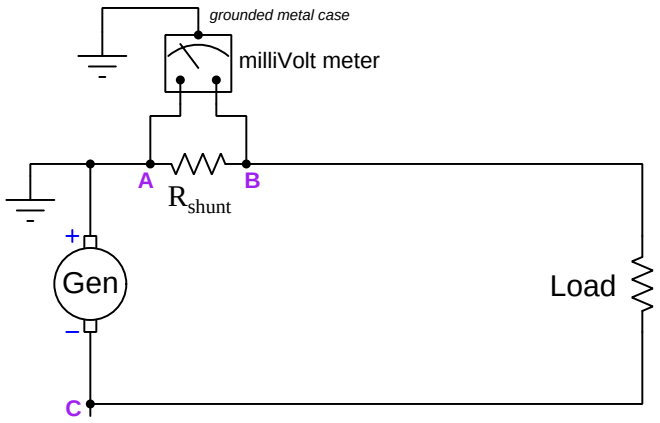
The milliVolt meter's purpose in this circuit, of course, is to infer load current by reporting the amount of potential difference dropped across R_{shunt}. In this case, the registered differential voltage between test points **A** and **B** is 74.81 milliVolts which equates to 7.481 Amperes of load current. The milliVolt meter should be completely immune to any changes in generator voltage, and only respond to current in this circuit.

Now suppose we add one more consideration to this seemingly simple circuit – we consider the milliVolt meter's metal casing attachment to a metal pole planted in the soil. This places the meter's case at ground potential (i.e. the potential of point **C**):



One would expect this to have no effect whatsoever on the meter's proper measurement of differential voltage between its test leads (points **A** and **B**), and in many cases this would be a correct assumption. However, there may be a *safety* concern if the milliVolt meter is not rated to handle such a high amount of voltage between either of its test leads and its metal case, in this instance 300 Volts between **A** and the case, and 299.25 Volts between **B** and the case. If the electrical insulation inside the milliVolt meter were insufficient to handle this much potential difference, it could break down over time and result in a fault where substantial current passes from either test lead to the case, through Earth ground, and back to the generator!

An interesting solution to this dilemma is to relocate the generator's ground connection from point **C** to point **A** as such:

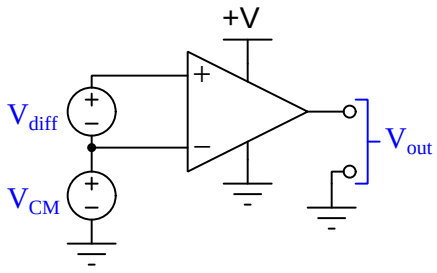


Now the voltage between point **A** and the meter's metal case is 0 Volts, and the potential between point **B** and the meter's metal case is −74.81 milliVolts. Clearly there no longer exists any hazard of electrical insulation breakdown inside the meter! This relocation of Earth-grounding point in the main power circuit has absolutely no effect whatsoever on the generator or on the load or even on the accurate reporting of current as inferred by the shunt resistor, but it certainly has an impact on how much potential difference the meter's internal insulation must withstand, and therefore on the meter's long-term reliability and safety.

By moving the circuit's Earth ground reference point we managed to vastly reduce the amount of electrical insulation stress inside the meter without affecting the generator or the load in the slightest. The meter still registers the shunt resistor voltage drop it's designed to measure, but without the hazard of insulation breakdown we faced before. It would be convenient to have a general term to refer to this change of stress on the meter, which is clearly related to voltage in a general sense but not to the differential voltage across the shunt resistor the meter was set up to read.

Such a term does exist, and it is called *common-mode voltage*. Since in this case we're concerned about the voltage impressed across the insulation protecting wires inside of the meter, the "common" element here would be the meter's metal case, since that is the other pole of any high voltage that any wire inside the meter would experience. We might interpret "common-mode voltage" to mean *any voltage shared in common between the two meter test leads and the metal case*, but this is a somewhat ambiguous definition. How much voltage, exactly is "common" between the 300 Volts (between **A** and case) and 299.29 Volts (between **B** and ground) when this circuit's Earth ground point is located on the generator's negative pole? We might define "common-mode" to be the lower of the two voltages (299.25 Volts), or perhaps the higher of the two (300 Volts), or maybe even the average of the two (299.63 Volts). A compelling argument could be made that the best way to define "common" in this context would be the greater of the two ground-referenced voltages (300 Volts) because what we're ultimately concerned about is over-stressing electrical insulation.

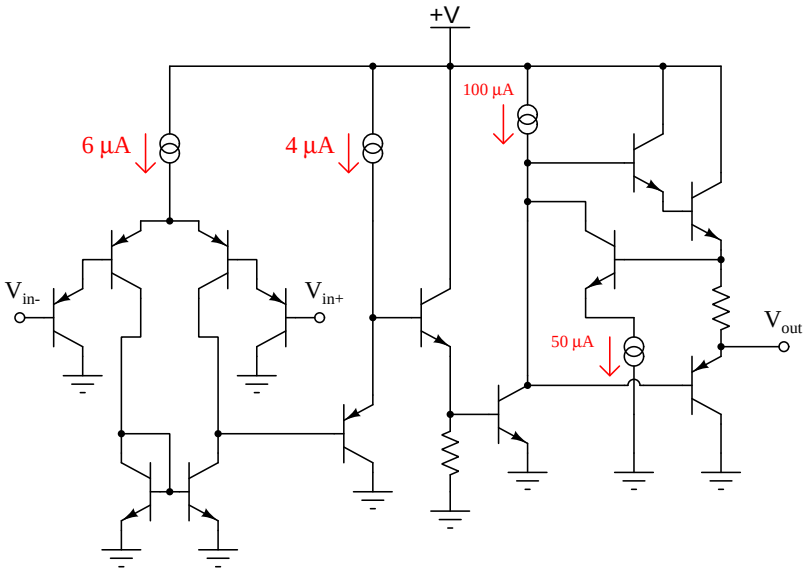
Another practical application of "common-mode voltage" is in differential amplifier circuits, which like the milliVolt meter in the prior example are designed to sense a difference of electrical potential between two input terminals but ignore any voltage with reference to ground or other common points in the circuit. Shown below is a typical illustration of common-mode voltage (V_{CM}) as applied to the input terminals of a differential amplifier (the triangular symbol):



Again, the purpose of any differential voltage amplifier is to respond to V_{diff} but ignore V_{CM}, the ground-referenced output signal V_{out} ideally being a function only of the differential input voltage V_{diff}. This is the ideal situation, but real differential voltage amplifiers do indeed respond (albeit slightly) to common-mode voltage, and if the common-mode voltage becomes too great the amplifier may cease to function entirely. For this reason we must be aware of how common-mode voltage is defined and why it matters to the internal operation of such amplifiers.

If we take this illustration literally, it would mean the lower input terminal of the amplifier experiences V_{CM} with reference to ground, while the upper input terminal experiences V_{diff} more than that, making "common-mode" voltage be the lesser of the two potentials with reference to ground. But, just as in the case of the milliVolt meter in the prior example what might actually *matter* in terms of real consequences might not be voltage measured with reference to ground. In other words, we might express V_{CM} in reference to ground, but there might be some other commonly-shared reference point that actually matters more to the proper operation of the amplifier.

Take for example the internal schematic diagram (simplified) for a model 324 operational amplifier:



Note how the transistors connected to the V_{in+} and V_{in-} input terminals of the amplifier are part of a Darlington pair array, with emitter arrows pointed in such a direction to indicate base currents must flow *out* from the amplifier to whatever device(s) provide the input terminal potentials. In order to ensure base current flows through each of these input transistors, the voltage as measured between either of these terminals and the amplifier's negative DC power terminal (i.e. in this case, ground) must never rise so high as to leave too little voltage across the base-emitter PN junctions of all four input transistors and the 6 μA current regulator for them to operate. This means the electrical potential of either input terminal must always be more than 1.4 Volts *lower* than the +V DC power supply "rail" potential in order to forward-bias two PN junctions (0.7 Volts each) plus provide enough voltage drop across the 6 μA current regulator for it to do its job properly.

One datasheet I consulted on the model LM324 operational amplifier states the common-mode input voltage range as such:

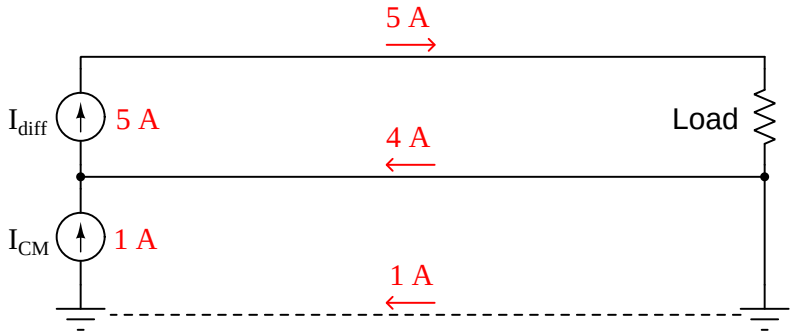
> Maximum V_{CM} common-mode voltage = V_{CC} − 2 Volts *(Where V_{CC} is the positive DC power rail)*

According to this datasheet, this model 324 amplifier circuit requires at least 2 Volts of drop spread between the two base-emitter PN junctions and the 6 μA current regulator in order to maintain a healthy amount of current through them all. So, even though we may define V_{CM} in terms of voltage measured between any amplifier input terminal and ground, what actually matters is maintaining a minimum common-mode voltage drop between any input terminal and the +V DC power supply terminal.

The moral of this story is that common-mode voltage may be defined in more than one way, and also that the way in which we define how it's measured may not necessarily directly relate

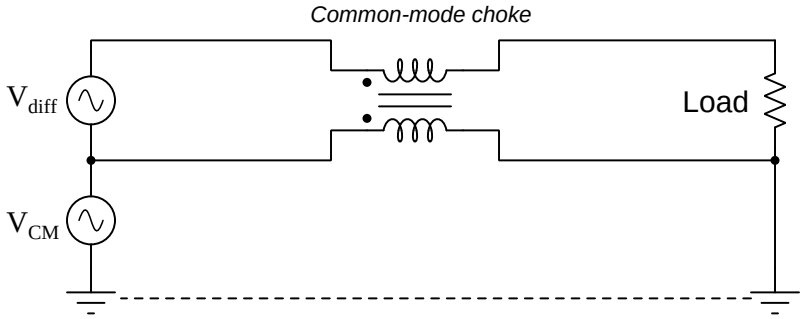
to the threat posed by having too much (or not enough) common-mode voltage! As usual, first principles matter: in the case of the milliVolt meter connected across a high-side shunt resistor we were concerned with possibly breaking down electrical insulation with too much potential difference between a test lead and the meter's metal case; with the model 324 operational amplifier we were concerned with keeping the input transistors in their active operational modes (conducting current).

In case you have been wondering, yes there is such a thing as common-mode *current* as well as common-mode voltage. In the case of common-mode current, it is defined as the net amount of electrical current among a multiple parallel conductors. An illustration is shown below:

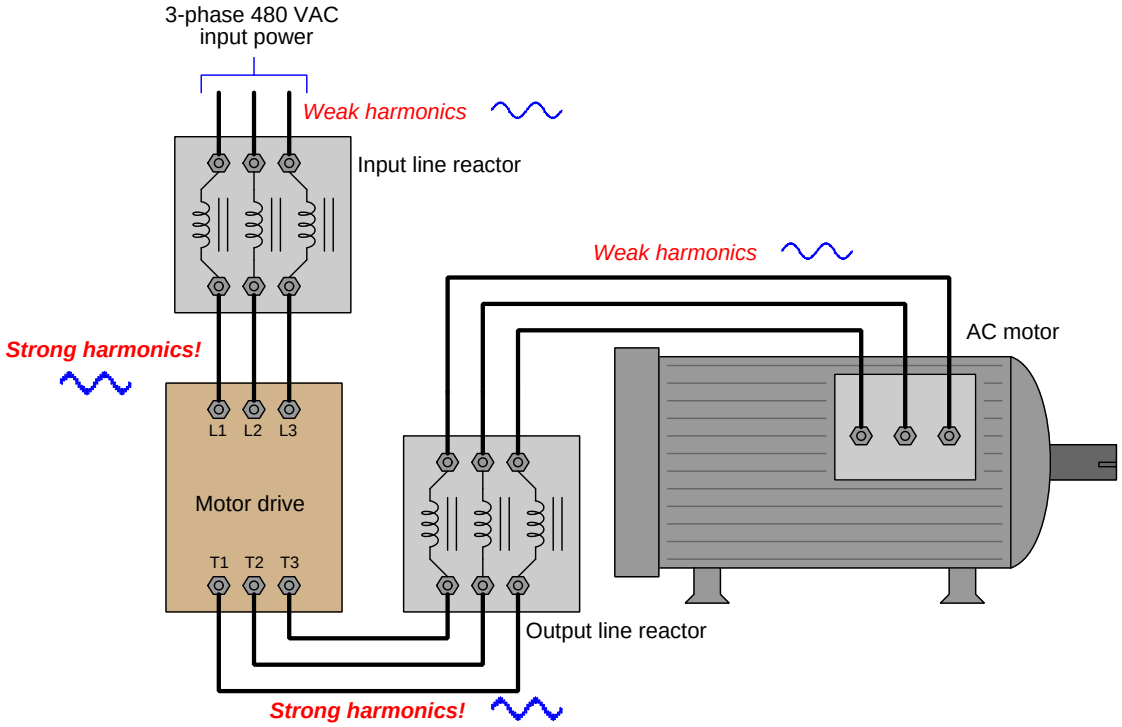


Here the differential current seen by the load is 5 Amperes, but the common-mode current through the two wires connected to that load is 1 Ampere.

In AC circuit applications where common-mode currents are undesirable, a popular way to attenuate them is to use a pair of magnetically-coupled inductors as a *common-mode choke*. The operating principle here is that the inductors' magnetic fields will be equal and opposite – and therefore completely cancel out – for differential current, but will aid each other to create a high impedance standing in the way of any common-mode current:



Common-mode chokes are often found in three-phase AC power systems where *triplen harmonics*[15] are found, as these harmonic currents exhibit zero phase shift between each other and so always flow in the same direction at any given time through the three power conductors. An example circuit shown below places two such three-phase common-mode chokes on either side of a variable-frequency motor drive (VFD):



[15] A "triplen" harmonic is any harmonic that is a multiple of three; e.g. 3rd, 6th, 9th, 12th, etc.

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Electric field

Magnetic field

Electromagnetism

Faraday's Law of Electromagnetic Induction

Lenz's Law

Electrical source

Electrical load

Capacitance

Inductance

AC

DC

Frequency

Parasitic effect

Mutual induction

Sinusoidal decomposition (i.e. Fourier's Theorem)

Fundamental frequency

Harmonic frequency

Common-mode voltage signal

Differential voltage signal

Noise

Bus

Permeability

Ground loop

Noise immunity

Filter

Digital signal

Analog signal

5.1.3 Compass in a lightning storm

When lightning strikes, nearby magnetic compass needles may be seen to jerk in response to the electrical discharge. No compass needle deflection results during the accumulation of electrostatic charge preceding the lightning bolt, but only when the bolt actually strikes. What does this phenomenon indicate about voltage, current, and magnetism?

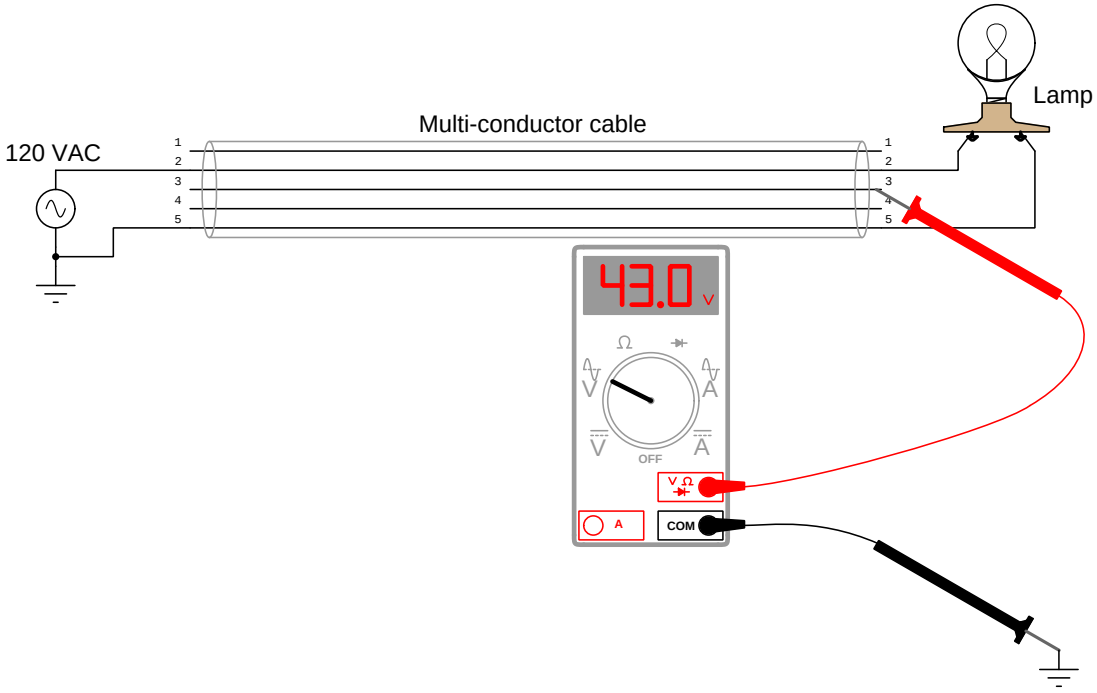
Do you suspect a sensitive electronic circuit might be affected more by the gradual accumulation of electric charge, or by the instantaneous strike of the lightning bolt? Will the circuit be affected at the same time and in the same way as the magnetic compass, or will it be susceptible in different ways?

Challenges

- What would a sensitive *electroscope* (an instrument designed to detect electric fields) register alongside the compass during a lightning storm.

5.1.4 Phantom voltage measurements

A technician uses a digital multimeter to check for the presence of dangerous voltage between conductor #3 and Earth ground prior to touching that conductor. This safety check is done as a matter of precaution, even though the technician has reason to believe that conductor is electrically distinct (i.e. not electrically common to any other conductors) and should not be "live". To this technician's surprise, he measures 43 Volts AC between conductor #3 and ground!



Puzzled, this technician consults another technician to ask how it is possible to measure a possibly dangerous voltage level between a conductor and ground when the conductor in question is not connected to anything else. "Oh," says the other technician, "That's probably just a *phantom voltage*. Don't worry about it!"

Explain this phenomenon. Exactly how is the AC voltmeter registering a voltage on what should be a "dead" wire?

Should you be concerned about the safety hazard of so-called "phantom voltages"? Why or why not?

Some voltmeters made for general electrical use (rather than precision electronic use) are specially built to have less input impedance than typical: tens of kiloOhms instead of megaOhms. This design helps minimize "phantom" voltage measurements. Explain how a voltmeter with less input impedance (also known as *insertion resistance*) than usual is less liable to be "fooled" when taking measurements on an unattached conductor such as this.

Challenges

- Are "phantom" voltages strictly an AC phenomenon, or may they manifest in DC circuits as well?
- Identify some parameters of the multi-conductor cable which could be modified (e.g. length, wire gauge, insulation thickness, etc.) to exacerbate the "phantom" voltages effect.
- Experiment with phantom voltages by inserting one test lead of an AC voltmeter into the "hot" socket of a 120 Volt AC power receptacle, and leaving the other test lead "floating" in air. Also, try connecting one test lead of a voltmeter to Earth ground while placing the other lead *close to* a "hot" AC conductor. What, by definition, makes a conductor "hot"?
- Where is the rest of the 120 Volts dropped in this "circuit" between the source and the multimeter?

5.1.5 Digital versus analog signal cables

Suppose you must run two signal cables from field-mounted instruments to a central room where the control system is located. Two different electrical conduits stretch from the field location to the control system room: one with 480 VAC power wiring in it, and another with low-level control signal wiring in it. The two cables you must run through these conduits are as follows:

- One twisted-pair cable carrying a 4-20 mA analog DC signal
- One twisted-pair cable carrying a Modbus digital signal (RS-485 physical layer)

At first, you plan to run both these cables through the signal wire conduit. However, you soon discover this signal conduit only has room to accommodate one cable but not both. The power wire conduit, however, has plenty of available room.

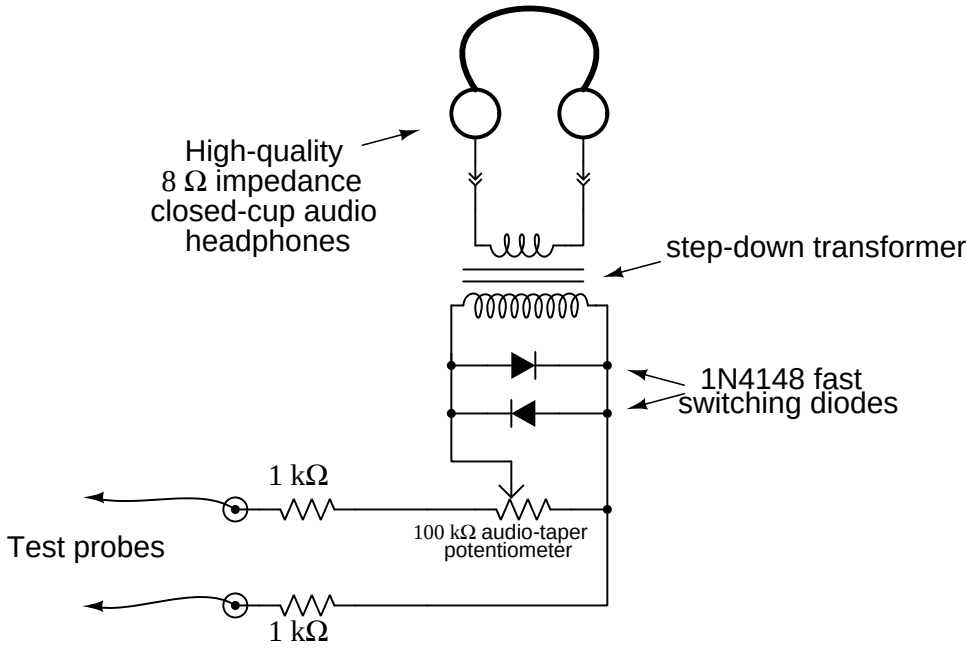
Which cable would you run through which conduit, and why?

Challenges

- Explain why it is best to run all *signal* cables in conduit completely separate from *power* cables.

5.1.6 Sensitive audio detector

A very educational project to construct is this *sensitive audio detector*, designed to let you listen to very small AC voltages and currents with frequencies in the audio range (approximately 20 Hz to 20 kHz, depending on how acute your hearing is):



Explain how you might detect AC *electric fields* with this instrument.

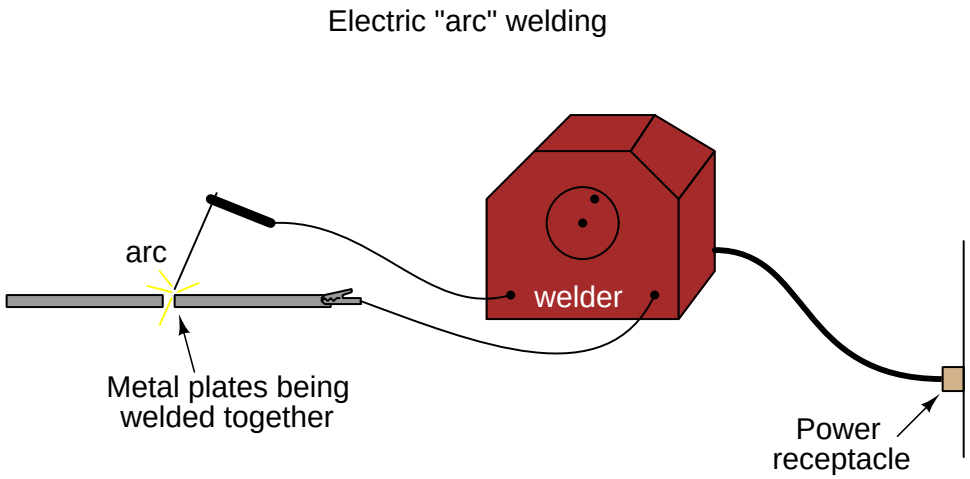
Explain how you might detect AC *magnetic fields* with this instrument.

Challenges

- What purpose do the two diodes serve in this circuit? Hint: if you remove the diodes from the circuit, you will not be able to hear the difference in most cases!
- The purpose of the transformer in the sensitive audio detector circuit is to increase the effective impedance of the headphones, from 8 Ω to a much larger value. Calculate this larger value, given a transformer turns ratio of 22:1.
- A transformer salvaged from a microwave oven works extremely well for this project. Explain why.

5.1.7 Arc welder electric/magnetic field mitigation

An *electric arc welder* is a low-voltage, high-current power source designed to supply enough electric current to sustain an arc capable of welding metal with its high temperature:



Typical voltage between the two output cables will be anywhere between 20 Volts and 90 Volts AC depending on the welder's design as well as welding conditions, with active current values easily exceeding 100 Amperes AC while welding.

One arc welder owner's manual[4] provides recommendations for minimizing personal exposure to the electric and magnetic fields produced by an arc welder when in operation. These recommendations include:

- Keep cables close together by twisting or taping them, or using a cable cover.
- Do not place your body between welding cables. Arrange cables to one side and away from the operator.
- Do not coil or drape cables around your body.
- Connect [ground] clamp to workpiece as close to the weld as possible.

Explain *why* each of these recommendations works to mitigate electric fields, magnetic fields, or both.

Challenges

- Will the field emissions from an arc welder be mostly electric, mostly magnetic, or evenly balanced?

[4] Miller Electric arc welder manuals circa 2023 use a common section of text describing "EMF" hazards and their mitigation.

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[5]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[6] on an answer key!

[5] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[6] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[7] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[7] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[8] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[9] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[10] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[8] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[9] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[10] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

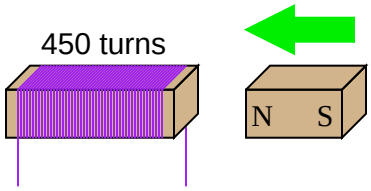
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[11] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

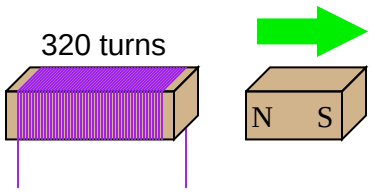
[11] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 Induced voltage

If a wire coil with 450 turns is exposed to a magnetic flux increasing at a rate of 0.008 Webers per second, how much voltage will be induced across the coil?



If a wire coil with 320 turns is exposed to a magnetic flux decreasing at a rate of 0.03 Webers per second (as shown in the illustration), how much voltage will be induced across the coil, and what will its polarity be?



Challenges

- Are the motions shown in the illustrations the only valid way to physically expose these coils to changing magnetic flux?

5.2.4 Magnetic field detector coil

Suppose you were designing a wire coil to be used as part of a magnetic field detection apparatus. It will detect the presence of oscillating (AC) magnetic fields by producing a voltage that will be sensed by a voltmeter.

If this coil is to be placed at a distance from an oscillating magnetic field where the coil covers an area large enough to experience a peak rate-of-change of flux ($\frac{d\Phi}{dt}$) equal to 0.2 Webers per second. How many "turns" of wire would this coil have to possess in order to generate a peak voltage of 5 Volts?

Challenges

- Identify one way for the coil to generate this much voltage, given the same magnetic field source, using fewer turns of wire.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

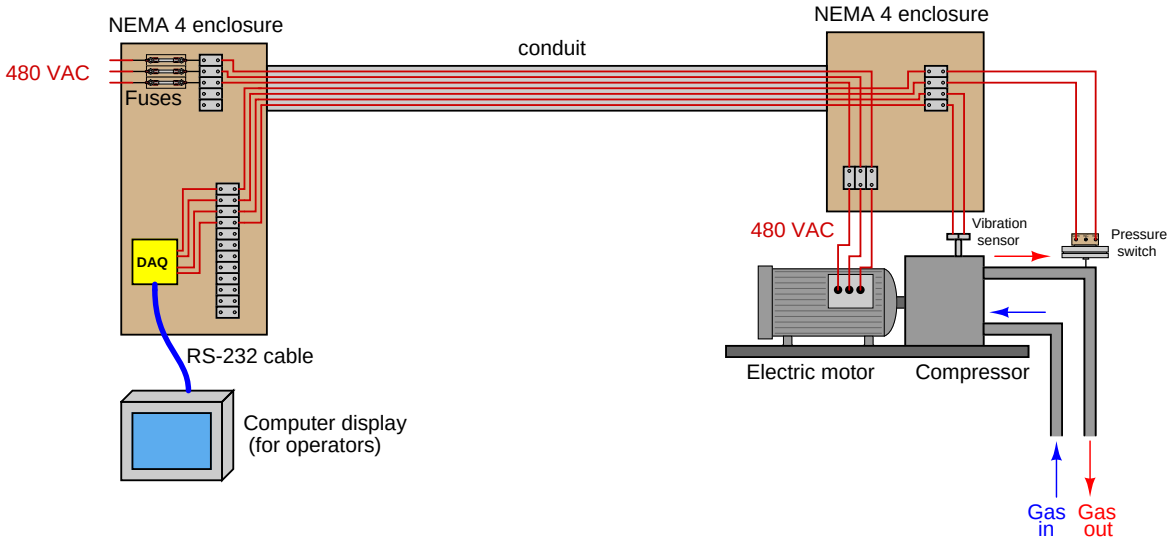
You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Compressor system wiring

A team of technicians recently built this gas compressor system, and are nearly ready to start it up. The system provides over-current protection for the compressor's drive motor using fuses, while a data acquisition unit (DAQ) monitors voltage signals from pressure and vibration sensors on the compressor.

Just days before the planned start-up, you happen to perform an inspection of their work. You see 480 VAC (three-phase) power routed to the compressor's drive motor through fuses in the first enclosure. You also see two sensors at the compressor – a high-pressure alarm switch (24 VDC discrete on/off signal) and a vibration probe (outputting milliVolt signals proportional to compressor vibration) – connected to a data acquisition unit located at the first enclosure. All wiring is in the form of individual conductors, with the exception of the multi-conductor RS-232 cable connecting the DAQ to the operator's computer display:



Your answer should consist of two parts: (1) identify a definite problem you see in the measurement wiring, and (2) explain how you would remedy that problem.

Challenges

- Suppose an engineer recommends installing a *VFD* in this compressor system to be able to adjust the speed of the electric motor. How will this alteration affect the signal-measurement system, if at all?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

24 April 2025 – added another Technical References section, this one on common-mode signals.

2 April 2025 – added another Technical References section, this one on Gauss' Law of Electric Fields.

24 September 2024 – added a new Technical References section on Maxwell's equations.

16 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

19-20 February 2024 – added a new section to the Technical References chapter on near-field versus far-field effects. Also, added a new Challenge question to the "Phantom voltage measurements" Conceptual Reasoning question.

14 November 2023 – added a new Case Tutorial section on empirically determining signal rates of change.

25 October 2023 – minor edits to image_6764 and image_6766.

26 September 2023 – corrected a grammatical error in the Tutorial.

20 July 2023 – added a new Conceptual Reasoning question regarding electric and magnetic field mitigation techniques when using an electric arc welder.

28 June 2023 – added new sections to the Tutorial on reducing generated electric and magnetic fields, as well as a new Case Tutorial section on high-voltage corona. Also corrected some typographical errors.

21 February 2023 – minor edits to the Tutorial, and new questions to the Intro chapter.

8 January 2023 – modified image_4025 (sensitive audio detector schematic) showing 1N4148 fast-switching diodes which are more appropriate for high-audio frequencies than the 1N4001 rectifying diodes shown previously.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

26 April 2022 – added more content to the Tutorial about rates of change, including an oscilloscope screenshot of a $\frac{dV}{dt}$ measurement.

4 November 2021 – edited Case Tutorial examples showing electric and magnetic signal coupling for better clarity.

8 May 2021 – commented out or deleted empty chapters.

29 March 2021 – minor additions to the Tutorial on flicker noise.

22 March 2021 – added content to the Tutorial on noise definitions and also intrinsic sources of noise.

22 February 2021 – edited image_3103 to show AC rather than DC sources.

8 October 2020 – added a Case Tutorial chapter, with examples showing both capacitive coupling and inductive coupling between conductors within a two-conductor cable.

7 October 2020 – minor additions to the Introduction, Tutorial, and Questions chapters.

29 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions. Also, fixed an omission in the Tutorial where I referred to a section that did not exist.

12 August 2020 – added mention of crosstalk to Tutorial.

1 July 2020 – document first created.

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