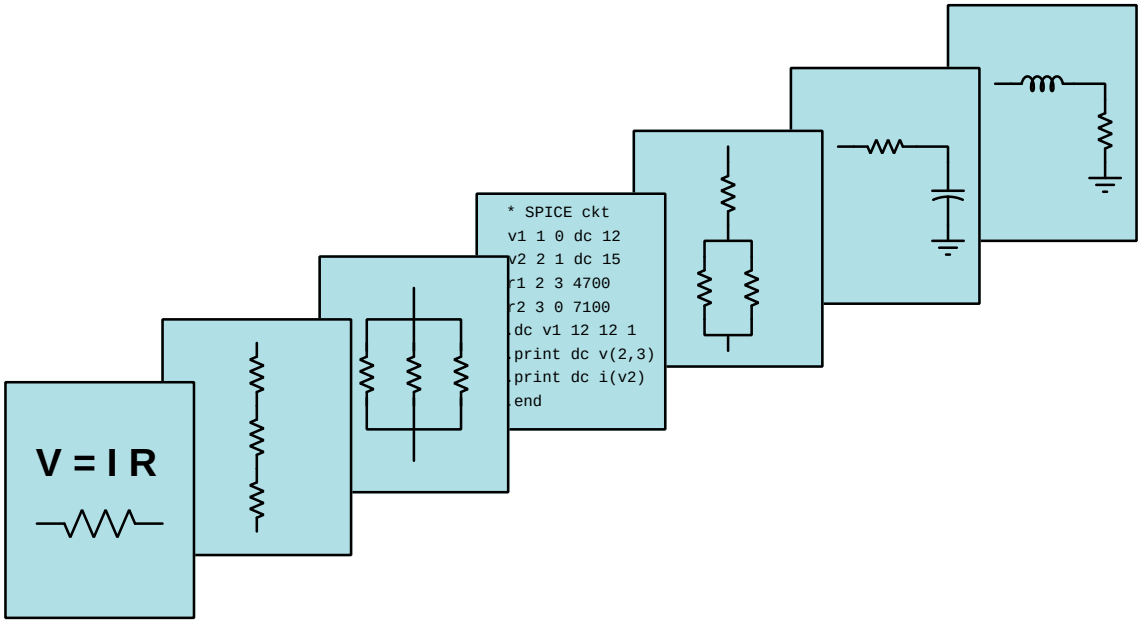
Modular Electronics Learning (ModEL) project



Parallel Circuits and Current Dividers

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Chapter 1

Introduction

1.1 Recommendations for students

This module explores the principles and properties of *parallel* electric circuits. You will note that the Full Tutorial begins with several pages of review from previous modules, on such topics as voltage, current, resistance, and simple circuit behavior. You may also note that this same review appears in other module Full Tutorial sections as well, and that it is no accident. In my experience teaching electric circuit theory to hundreds of students, I have found that a poor grasp of fundamentals leads to major conceptual problems later, and that the "basics" are *always* good to review.

Important concepts related to parallel circuits include **Conservation of Energy**, **Conservation of Electric Charge**, **voltage**, **current**, **resistance**, electric **circuit**, **electrically common** points, **equipotential** points, voltage **polarity**, **sources** versus **loads**, **Ohm's Law**, **Joule's Law**, **voltmeters**, **bonding**, **current divider** networks, **loaded** versus **unloaded** networks, and **potentiometers**.

A very important problem-solving strategy applied throughout the Tutorials is that of *annotating diagrams* with labels showing voltages, polarities, and currents (with direction). Mapping calculated values onto these diagrams is a helpful way to avoid confusion and to maintain proper context for all the quantities. Another important point about problem-solving is the need for *patience*, and the willingness to proceed with calculations even if the final strategy for solving the problem eludes one's immediate grasp.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the properties of parallel electrical networks? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?

- How does a simple circuit respond to a switch being opened?
- How is a *parallel* network defined?
- How do sources differ from loads?
- How do individual component voltages relate to total (source) voltage in a parallel network?
- How do individual component currents relate to total (source) current in a parallel network?
- How do individual component resistances relate to total resistance in a parallel network?
- How do individual component power dissipations relate to total (source) power dissipation in a parallel network?
- How is the total parallel resistance formula derived from foundational principles?
- Which conservation laws apply to parallel networks?
- How do the properties of parallel networks result from more fundamental principles of electricity?
- How do the properties of parallel networks relate to electrical safety?
- What happens when multiple current sources are connected in parallel with each other?
- How do we properly connect a voltmeter to measure the voltage for any particular component in a circuit?
- What is the fundamental definition of an *Ampere* of current?
- How would you describe the meaning of Ohm's Law, in your own words?
- How would you describe the meaning of Joule's Law, in your own words?
- What does it mean to apply Ohm's Law (and other formulae) *in context* when analyzing a circuit?
- What does it mean to "bond" a portion of an electrical circuit, and why might we wish to do this?
- What are some practical applications of current dividers?
- What happens when we "load" a current divider?
- How does a potentiometer function?

1.2 Challenging concepts related to parallel circuits

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Defining a parallel network** – parallel networks are defined by all components spanning the same two sets of electrically-common points. From this definition, and from the Conservation of Energy, follows the conclusion that parallel-connected components must experience the same amount of continuous voltage. One challenge is that sometimes parallel-connected components do not physically or pictorially align in a parallel fashion, even though they are in fact parallel in the *electrical* sense. Another challenge is reasoning logically from this definition of "between the same two sets of electrically-common points" to the necessary conclusion of "same voltage", as many students are so accustomed to memorizing facts without seeing the logical connections between those facts. A helpful way to both identify parallel connections as well as to conceptualize the commonality of voltage is to use color-highlighting to mark all electrically-common points with the same color. This way, all points in one set have one color, all points in the other set have another color, and all components connecting to points with those two colors must be connected in parallel with each other. This coloring approach is used in the Tutorial chapters where points A-B-C-D are all colored blue and points E-F-G-H are all colored green. The *Case Tutorial* section "Example: potential in open and shorted circuits" is a helpful reference for using color-highlighting to denote different amounts of electrical potential in a circuit.

- **Definition versus properties of parallel networks** – again, parallel networks are defined by all components spanning the same two sets of electrically-common points. Everything else we know about parallel networks *derives* from this definition but are not *synonymous* with this definition. For example, the fact that currents divide in a parallel network is a consequence of parallel networks but not a defining quality. The fact that parallel-connected components necessarily share the same voltage is another consequence of parallel networks but not a defining quality. All we need to consider in order to understand this distinction is a counter-example of some electrical network exhibiting either divided currents or equal voltages without being parallel. A good counter-example of the former misconception is a five-resistor bridge network where all five resistor values are unique and non-proportionate, this being a clear example of a network where currents divide despite none of the resistors being in parallel with any other. A good counter-example of the latter misconception is any network that just happens to contain two or more components having the same voltage that are not parallel to each other (e.g. two identical resistors in series with each other). Applying counter-examples to disprove this and similar fallacies is a rhetorical technique called *reductio ad absurdum*, and the habit of looking out for counter-examples when testing claims is a powerful tool for critical thinking.

- **Parallel voltage being the same** – the fact that voltage is the same for all components within a parallel network is a direct consequence of the Conservation of Energy: if a circuit only has two electrically-distinct regions, and energy cannot spontaneously appear or disappear, then the amount of energy absorbed or released by any electric charge traveling between those two regions must be the same regardless of which parallel-connected component those charges happen to pass through. However, some students misconstrue this fact to mean that voltage in a parallel network must be fixed for all time and can never assume different values. For

example, some students assume voltage in a parallel network will remain fixed regardless of component alterations, which is simply not true. This is a (common) case of remembering a phrase ("voltage is the same in parallel") without properly reasoning from first principles.

- **Using Ohm's Law in context.** When applying Ohm's Law ($V = IR$; $I = \frac{V}{R}$; $R = \frac{V}{I}$) to circuits containing multiple resistances, students new to circuit analysis often mix contexts of voltage, current, and resistance. That is, students often mis-apply parameters where they do not belong in a circuit. Whenever applying any equation describing a physical phenomenon, it is important to ensure each variable of that equation relates to the proper real-life value within the problem. For example, when calculating the voltage drop across resistor R_2, one must be sure that the values for current and resistance are appropriate for that resistor and not some other resistor in the circuit. When calculating V_{R_2} using the Ohm's Law equation $V = IR$, one must use the value of *that resistor's* current (I_{R_2}) and *that resistor's* resistance (R_2), not some other current and/or resistance value(s). Some students have an unfortunate tendency to overlook context when seeking values to substitute in place of variables in Ohm's Law problems, and this leads to incorrect results.

- **Voltage being relative between two points** – unlike current, which may be measured at a single point in a circuit, voltage is fundamentally relative: it only exists as a *difference* between two points. In other words, there is no such thing as voltage existing at a single location. Therefore, while we speak of current going *through* a component in a circuit, we speak of voltage being *across* a component, measured between two terminals on that component. An effective strategy for combating this misconception is to continually apply the correct definition for voltage: *the amount of energy difference between an electric charge carrier at one location versus an identical electric charge carrier at another location.* This concept also reinforces (and is reinforced by) the concept of electric circuit components behaving as sources (infusing charge carriers with energy) or loads (depleting charge carriers of energy).

- **The path of least resistance** – a common electrical adage is that "Electricity always follows the path of least resistance." This is only partly true. Given two or more paths of differing resistance, more current will flow through the path of least resistance, but this does not mean there will be zero current through the other path(s)! The saying should probably be revised to state that "Electricity *proportionately* favors the path of least resistance."

- **Parallel resistances diminishing** – series resistances are easy to conceptually manage: it just makes sense that multiple resistors connected end-to-end will result in a total resistance equal to the sum of the individual resistances. However, parallel resistances are not as intuitive. That some total quantity (total resistance) actually *decreases* as individual quantities accumulate (resistors connected in parallel) may seem impossible, usually because the word "total" is generally associated with *addition*.

- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the

given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

A useful resource for practicing circuit analysis is the *Case Tutorial* chapter, containing example circuits with voltage and current values provided to check your work.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how all quantitative results were obtained by the author in the Tutorial chapter's parallel circuit analysis examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Calculate voltage, current, and power values within parallel circuits

  Assessment – Predict all component voltages, currents, and power dissipations in a parallel circuit given resistor values and source value; e.g. pose problems in the form of the "VIRP table for a three-resistor parallel circuit" Quantitative Reasoning question.

- **Outcome** – Apply the concept of current division to parallel resistor network design

  Assessment – Calculate current division ratios for any given parallel resistor network.

  Assessment – Predict necessary resistor values in order to achieve specified current division ratios.

- **Outcome** – Prove the concept of resistive current division by experiment

  Assessment – Design, build, and test a current divider network to achieve a specified current division ratio.

- **Outcome** – Diagnose a faulted parallel circuit

  Assessment – Predict the effect(s) of a single component failing either open or shorted in a parallel circuit; e.g. pose problems in the form of the "Faults in a three-resistor circuit" Diagnostic Reasoning question.

  Assessment – Determine the probability of various component faults in a parallel circuit given symptoms and measured values; e.g. pose problems in the form of the "Problem with a multiple-pump system" Diagnostic Reasoning question.

- **Outcome** – Independent research

  Assessment – Locate potentiometer datasheets and properly interpret some of the information contained in those documents including maximum resistance values, linear versus audio taper, number of turns (for rotary pots), maximum power dissipation ratings, etc.

The study of parallel circuits presents a prime opportunity to discipline students' thinking according to first principles of electricity. A very common tendency among students is to rote-memorize rules for analyzing circuits rather than reason logically from trusted principles. A good way to encourage and reinforce sound reasoning is to have students maintain a list of all the important laws, principles, and problem-solving strategies they learn about electric circuits, and to intentionally reference this list every time they analyze a circuit. At each step in the analysis of a given circuit, ask students to explain *why* they should apply each law or principle in turn!

Chapter 2

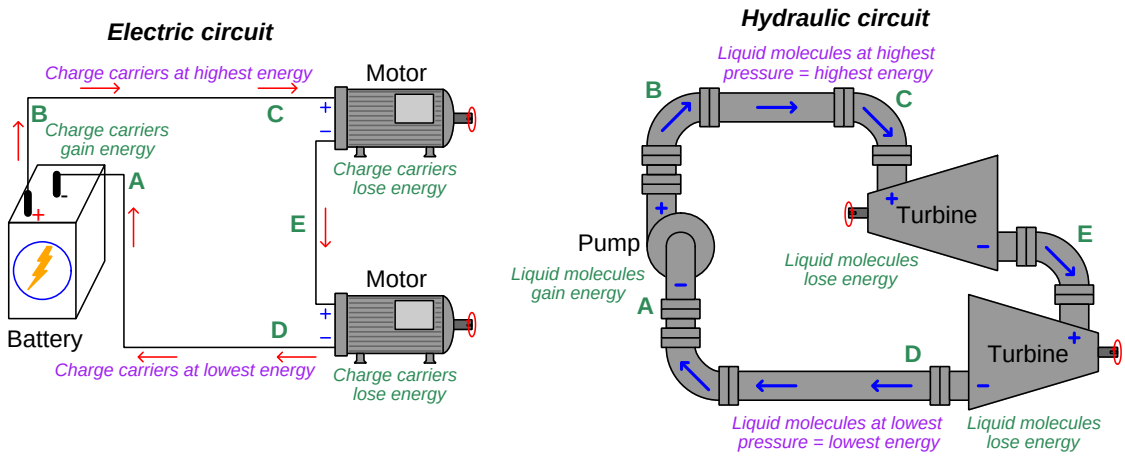
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: potential in open and shorted circuits

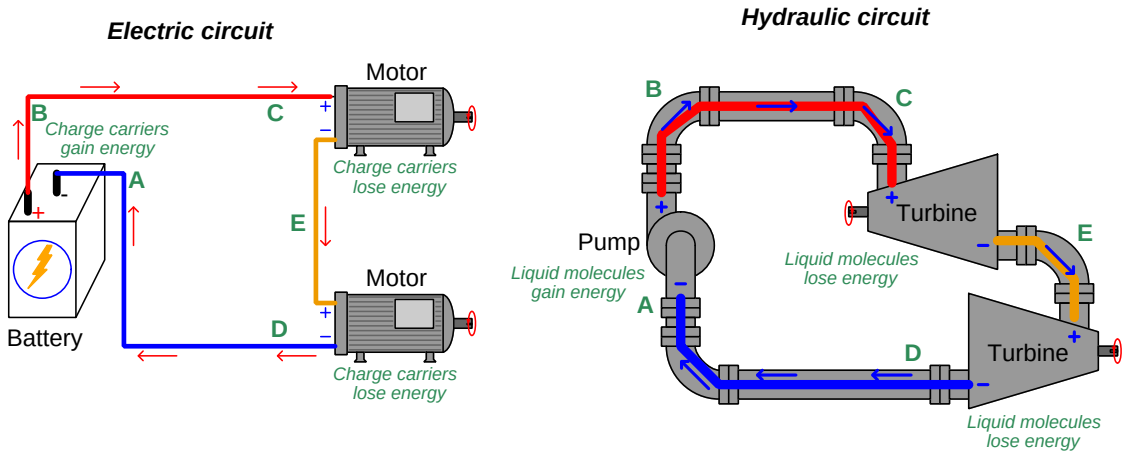
Here we will explore the concept of *potential* in both electric and hydraulic circuits. The following illustrations will be used, each circuit containing one *source* tasked with boosting the potential energy of the flow and two *loads* where that flow releases energy to do useful work:



"Potential" may be thought of as *specific potential energy*, or the amount of potential energy possessed by a certain quantity of mobile fluid particles. In the case of electricity, electric potential is the number of Joules of potential energy carried by one Coulomb (6.2415×10^{18}) of charge carriers. In the case of hydraulic systems, fluid potential is better known as *pressure* and is the number of Joules of potential energy carried by one cubic meter of the fluid.

Differences of potential may be precisely quantified in any circuit, be it *voltage* for electric circuits or *pressure differential* for hydraulic circuits. The "+" and "−" symbol pairs shown in each illustration represent the high-potential and low-potential sides of each difference. Each component's identity as either a source or a load is clear to see from a comparison of these "+" and "−" polarity marks in conjunction with the direction of flow. Note how each source takes in flow on the low-potential (−) side and outputs flow on the high-potential (+) side as it boosts the potential of the flowstream, while each load does just the opposite as energy is released by the flow.

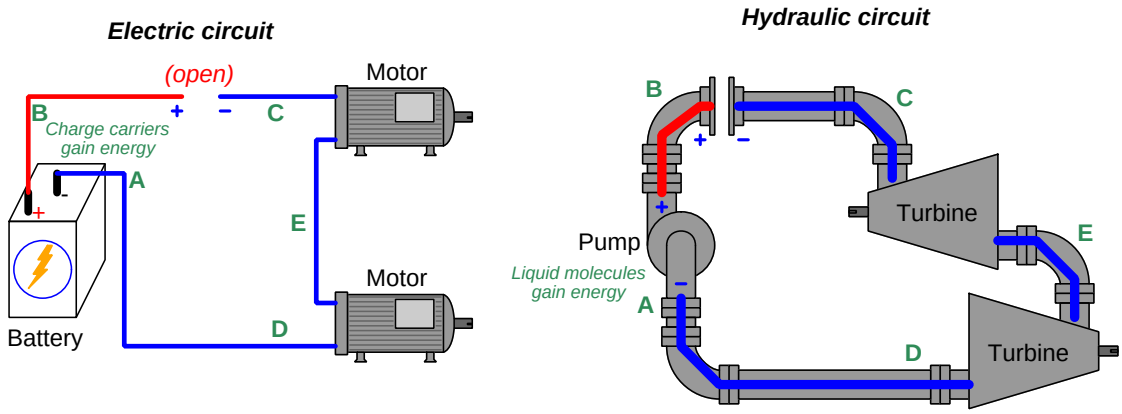
Here we show the circuit illustrations with higher and lower potentials color-coded[1]: red to signify high potential, orange to signify medium potential, and blue to signify low potential. Applying these colors to the original illustrations:



These color-coded potentials are all qualitative in nature. It is really impossible to precisely specify potential at any single location in a circuit, whether electrical or hydraulic, because potential energy depends on *two* conditions: a starting condition and an ending condition. We cannot say with any certainty how much energy is *potentially* released by any given fluid unless we know for certain what its end-state will be. This is why voltage in an electrical circuit is always relative between two points: an expression of how much potential was either gained or lost by charge carriers passing from one location to another. In a similar sense, hydraulic liquid pressure is also relative between two points: even a simple pressure gauge registering liquid pressure inside of a pipe is actually sensing the *difference* in pressure between the fluid within the pipe versus the atmospheric (air) pressure surrounding the pressure gauge. Thus, in both circuits the color-coding merely represents "greater" and "lesser" potential on a relative scale rather than absolute values of potential.

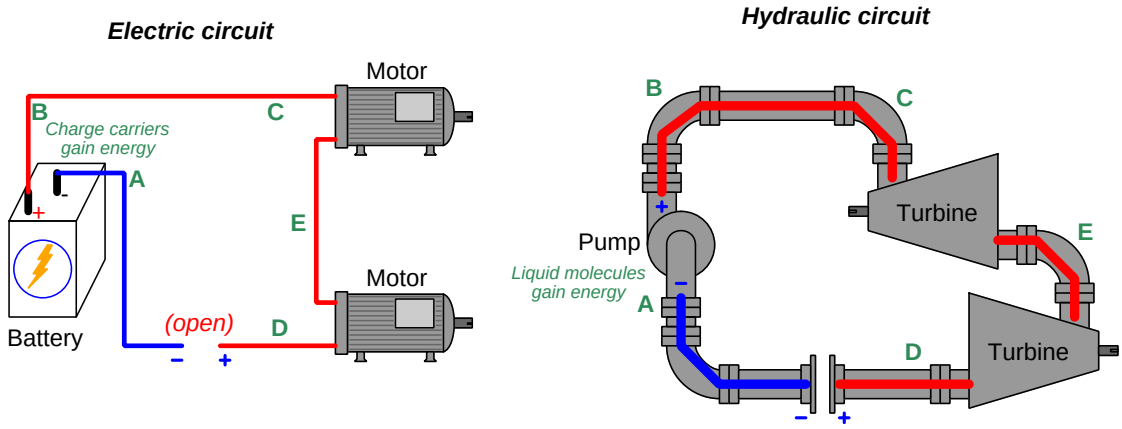
[1] These same colors are often used to signify relative liquid pressures in hydraulic system illustrations.

Now we will place an "open" (i.e. a blockage to flow) on the upper portion of each circuit:



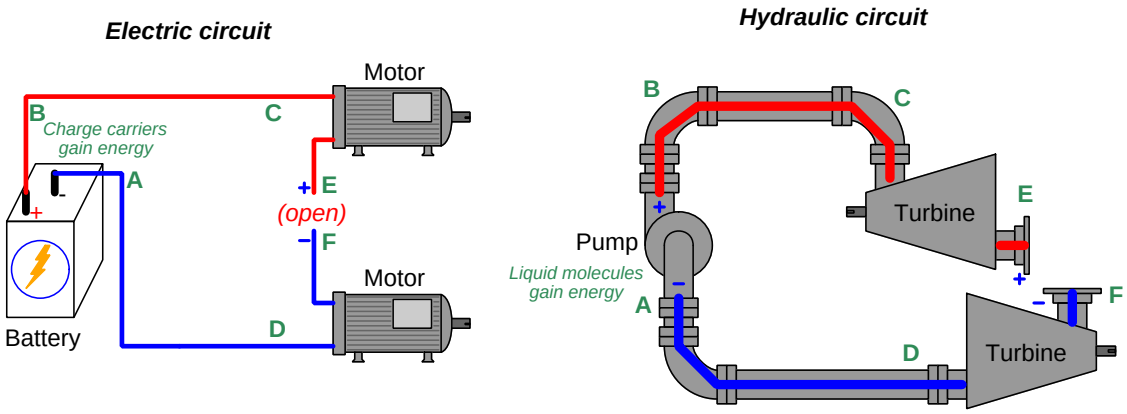
Note how the only portion with high potential lies between the "+" side of the source and the "opened" pathway. All other portions of each circuit immediately equalize in potential to be the same (low). No longer will a difference of potential develop across each load, since the flow ceases everywhere as a result of the "open", but a potential difference does exist across the "open".

Next we will place an "open" on the lower portion of each circuit:



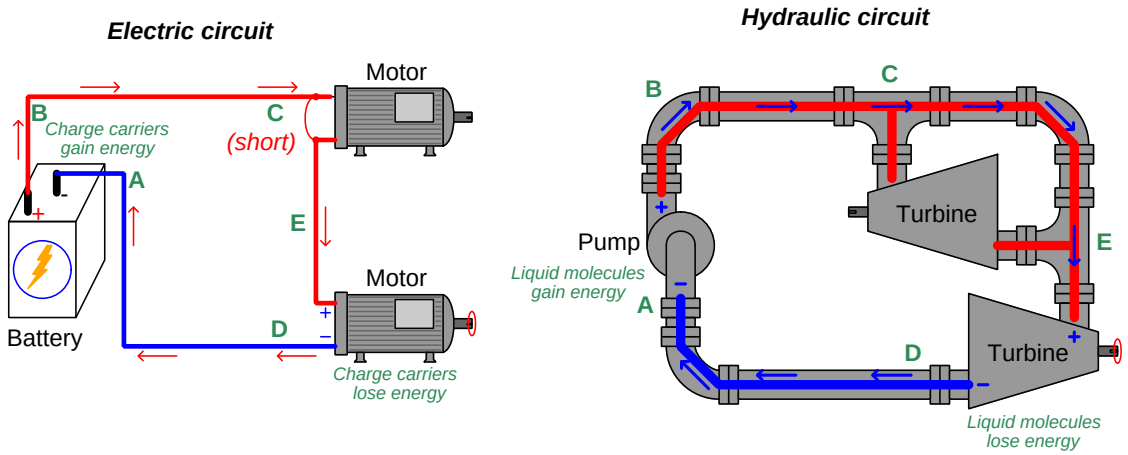
Note how the only portion with low potential lies between the "−" side of the source and the "opened" pathway. All other portions of each circuit immediately equalize in potential to be the same (high). Each load experiences zero difference of potential across it, since the flow ceases everywhere as a result of the "open", but a potential difference does exist across the "open".

Our last "open" will be placed between the two loads in each circuit:



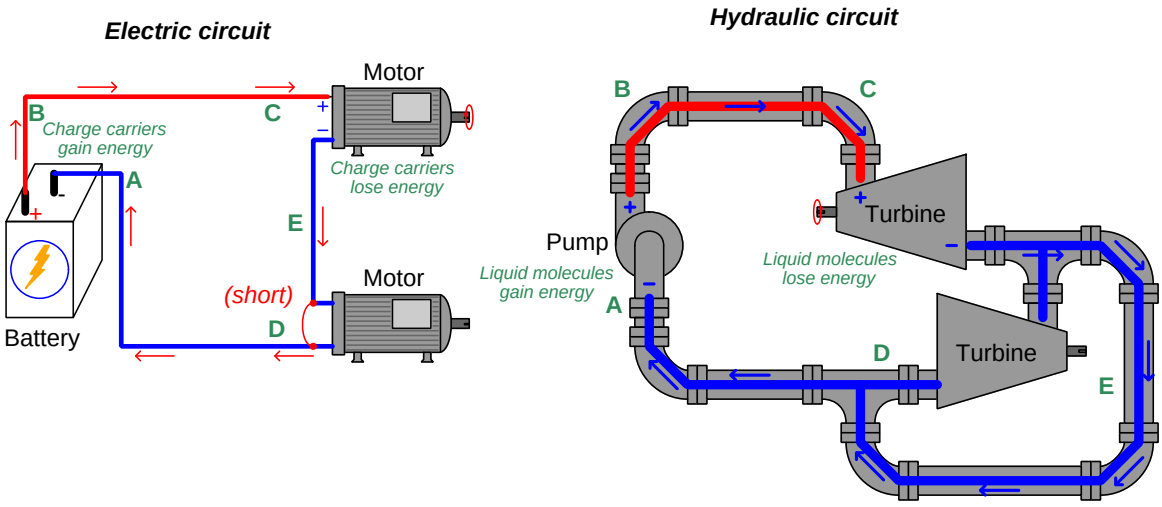
The "open" placed between the two loads creates a new distinct point which we label as **F**. Note how the lower load and its connection to the source maintains a low potential throughout, while the upper load and its connection to the source is at high potential throughout. Neither load has a difference of potential across it, but there is a difference of potential across the "open" in each circuit. The lack of flow, however, means no work is being done.

"Shorting" past loads in a circuit also affects the distribution of potential. Here we see the upper load shorted in each circuit:



Making both ends of the upper load "common" to each other ensures equipotentiality, robbing the upper loads of any potential difference. This means the flow will not lose any energy going past the shorted load, and instead will deliver all its energy to the lower load which still provides resistance to the flow and is therefore able to extract energy from the flow.

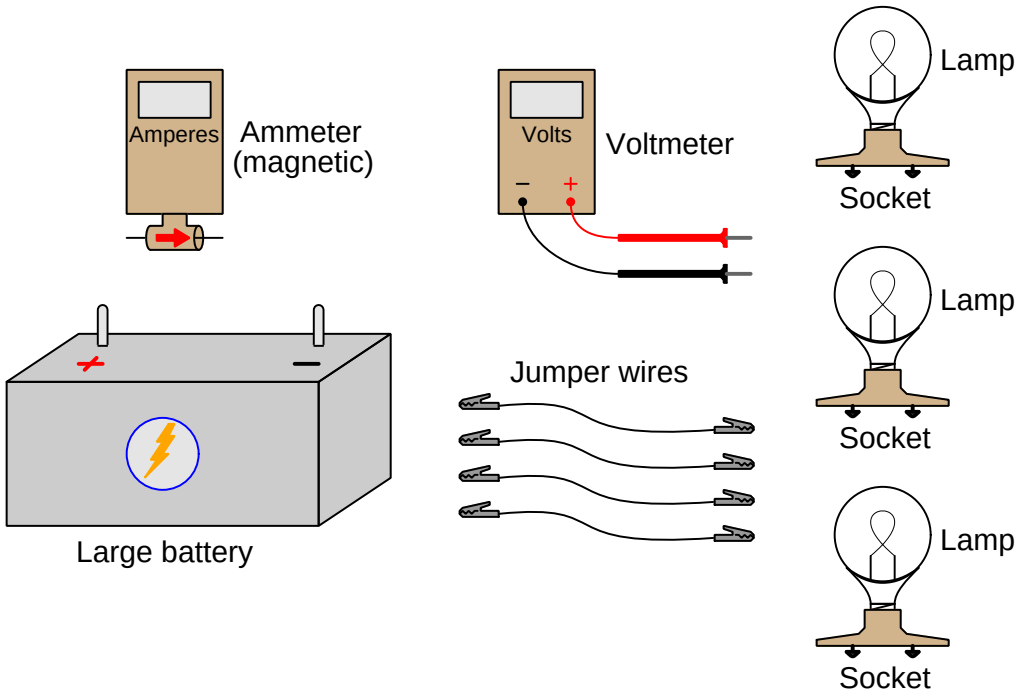
Now we will short past the lower load in each circuit to see the effects:



Making both ends of the lower load "common" to each other ensures equipotentiality, robbing the lower loads of any potential difference. This means the flow will not lose any energy going past the shorted load, and instead will deliver all its energy to the upper load which still provides resistance to the flow and is therefore able to extract energy from the flow.

2.2 Example: Battery, lamps, jumper wires, and meters

Here, a large 12 Volt battery and three 12 Volt-rated lamps are provided for experimentation, along with "jumper" wires consisting of plastic-clad stranded copper conductors terminated with spring-loaded "alligator" clip jaws at either end. A simple voltmeter and magnetic ammeter stand ready to take measurements:



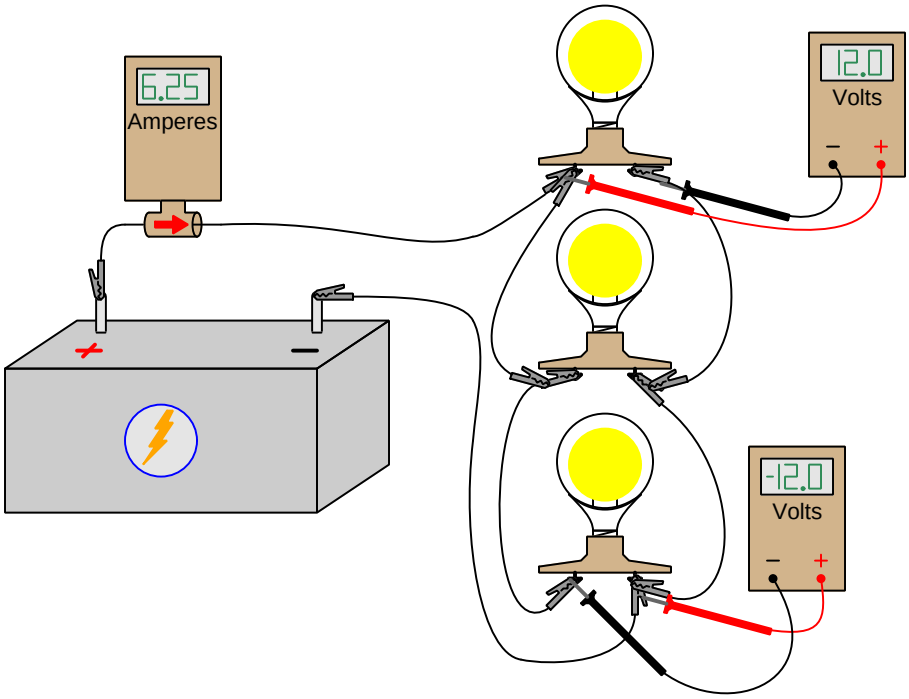
Current is defined as the motion of subatomic "electric charge carriers" existing throughout the bulk of electrically conductive materials such as copper metal. The standard metric unit of measurement for electric current is the *Ampere*, with one Ampere (1 A) being equivalent to 6.2415 $\times$ 10^{18} individual charge carriers drifting past a given location per second of time. The ammeter we will use senses electric current by the magnetic field produced around a current-carrying conductor. A positive indication on the ammeter's display means charge carriers are moving (as interpreted by the "conventional flow" standard) in the direction of the meter's arrow; a negative indication means motion in the opposite direction.

Voltage is defined as the amount of energy either gained or lost by an electric charge carrier between two different locations. The standard metric unit of measurement for voltage is the *Volt*, with one Volt (1 V) being equivalent to one Joule of energy difference per 6.2415 $\times$ 10^{18} individual charge carriers. The voltmeter we will use has two copper-wire *test leads* that are touched to the two locations of interest. The red test lead is the "measurement" lead while the black test lead is the "reference" lead. A positive indication on the voltmeter's display means charge carriers along

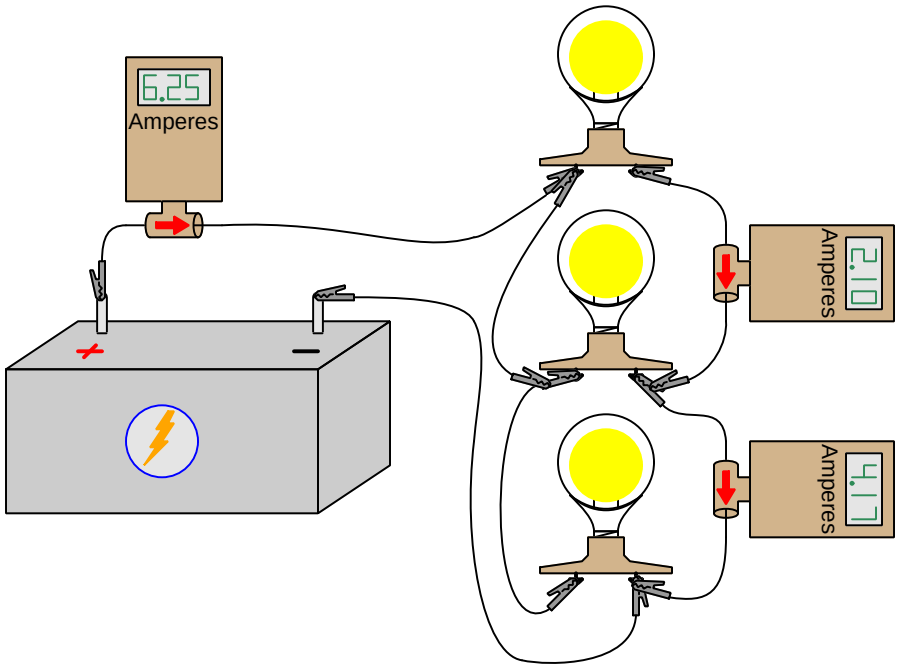
the red lead are at a higher state of energy than charge carriers along the black lead; a negative indication means the opposite.

Resistance converts some of the energy conveyed by moving electric charge carriers into other forms, most commonly heat. The standard metric unit of measurement for resistance is the *Ohm*, with one Ohm (1 Ω) being equivalent to one Volt of "drop" while carrying one Ampere of current.

Connecting the three lamps in *parallel* with each other results in each lamp glowing as brightly as it ordinarily would if connected to the battery by itself. However, the measured current at either battery terminal is three times greater than it would be for a single lamp. Each lamp drops the battery's full voltage of 12 Volts:



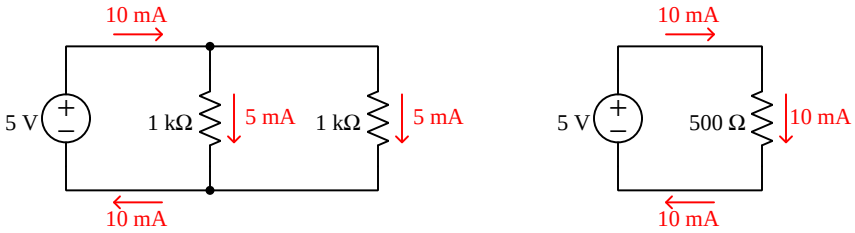
Connecting multiple ammeters into the circuit reveals different amounts of current at different locations:



2.3 Example: equivalent parallel resistances

Let's consider two circuits, each with the same 5-Volt (constant) voltage source and each with one 1 kΩ resistor. Naturally, they will both experience a current of 5 milliAmperes as predicted by Ohm's Law ($I = \frac{V}{R}$):

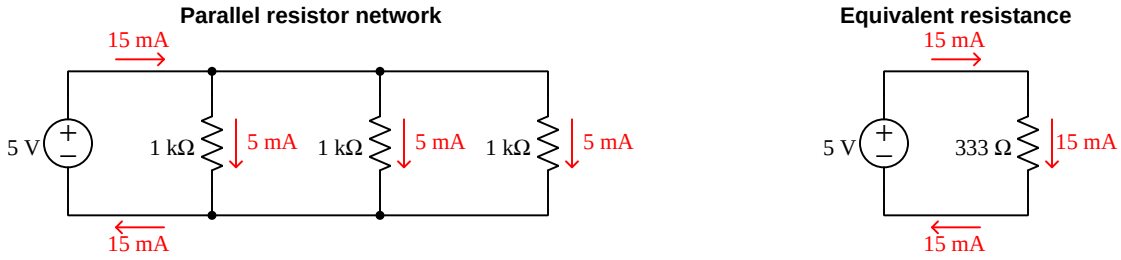
Now, let's add a second 1 kΩ resistor in parallel to the left-hand circuit, and then adjust the value of the resistor in the right-hand circuit to make the two source currents precisely match one another. Since each parallel-connected resistor in the left-hand circuit experiences the exact same voltage of 5 Volts (because all parallel-connected components must share the same voltage), each of those resistors passes 5 milliAmperes of current which makes the left-hand source supply a total of 10 milliAmperes. Adjusting the right-hand circuit's single resistor to match that same 10 milliAmpere source current requires a resistor value of 500 Ω:



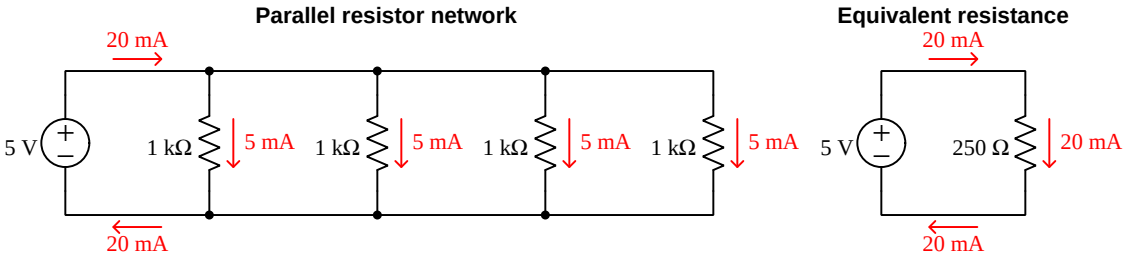
Therefore, we can say that the 500 Ω resistor in the right-hand circuit is *electrically equivalent* to the parallel combination of two 1 kΩ resistors in the left-hand circuit. In other words, the two 5-Volt sources cannot tell the difference between a single 500 Ω resistor and two 1 kΩ resistors connected in parallel with each other.

What we will do next is continue to add more 1 kΩ resistors to the left-hand circuit and adjust the single resistor in the right-hand circuit to be equivalent to that parallel network, making the right-hand resistor's value such that it causes the 5 Volt source to drive the same amount of current. On the next page you will see a sequence of circuits, each successive circuit demonstrating the equivalent resistance of one more 1 kΩ resistor connected in parallel with the others.

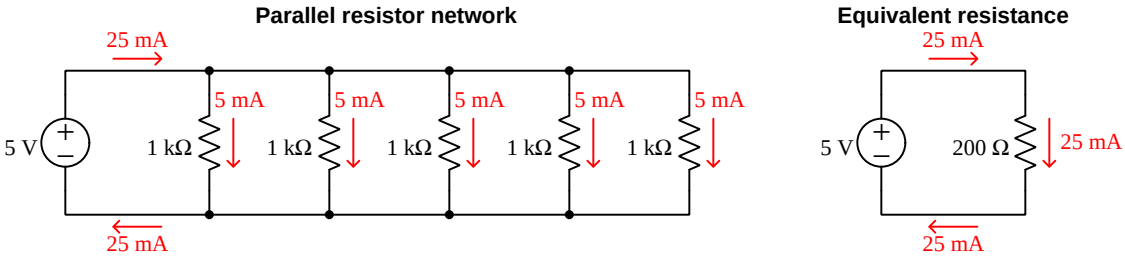
Three 1 kΩ resistors connected in parallel are equivalent to a single 333.3 Ω resistor:



Four 1 kΩ resistors connected in parallel are equivalent to a single 250 Ω resistor:



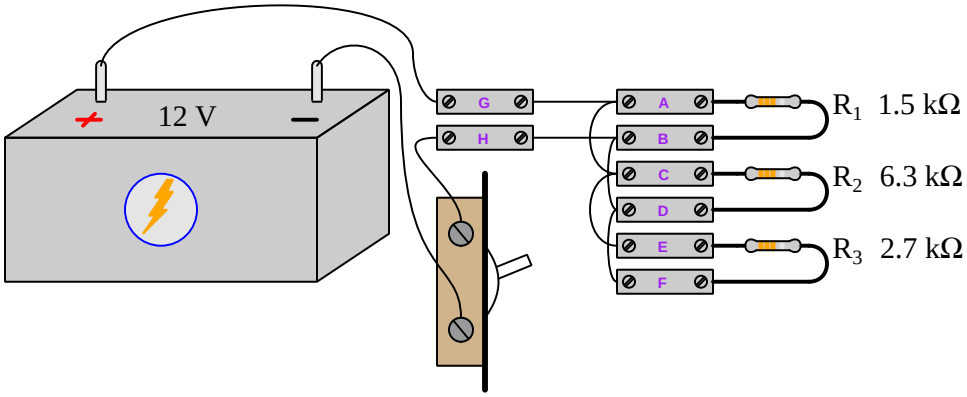
Five 1 kΩ resistors connected in parallel are equivalent to a single 200 Ω resistor:



Each new 1 kΩ resistor stacked on to the parallel resistor network of the left-hand circuit provides a new pathway for current, and an additional 5 milliAmperes of draw from that 5-Volt source. In order for the single resistor of the right-hand circuit to be electrically equivalent to the growing parallel network of the left-hand circuit, that one resistor must *decrease* in value every time.

2.4 Example: Three-resistor circuit

In this scenario we will use *terminal blocks* to neatly organize all wire connections between a battery and three resistors:



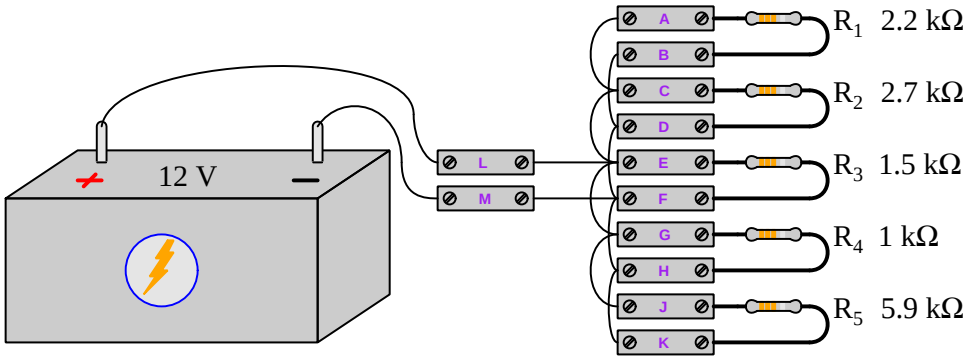
Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as V and I values[2]. With the circuit constructed as shown in the above illustration and the switch in its "on" position, we obtain the following measurements:

- $V_{AB} = 12.00$ Volts
- $V_{CA} = 0.00$ Volts
- $V_{ED} = 12.00$ Volts
- $V_{DB} = 0.00$ Volts
- $V_{BF} = 0.00$ Volts
- $V_{FD} = 0.00$ Volts
- $V_{FE} = -12.00$ Volts
- $I_G = 14.35$ milliAmperes (conventional flow left to right)
- $I_H = 14.35$ milliAmperes (conventional flow right to left)

[2]Double-lettered subscripts for V denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, V_{BC} means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for V represent the voltmeter's red test lead, with the black test lead touching a defined "ground" point in the circuit. Current measurements (I) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for I. Conventional flow notation will be used to specify current direction.

2.5 Example: Five-resistor circuit

In this scenario we will use *terminal blocks* to neatly organize all wire connections between a battery and five resistors:



Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as V and I values[3]. With the circuit constructed as shown in the above illustration, we obtain the following measurements:

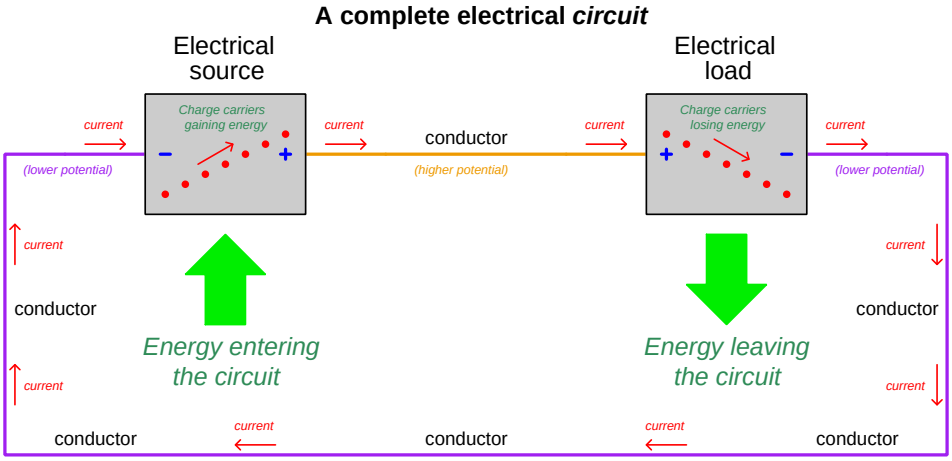
- $V_{FE} = -12.00$ Volts
- $V_{ED} = 12.00$ Volts
- $V_{DC} = -12.00$ Volts
- $V_{CG} = 0.00$ Volts
- $V_{HC} = -12.00$ Volts
- $V_{AF} = 12.00$ Volts
- $V_{KB} = 0.00$ Volts
- $I_M = 31.93$ milliAmperes (conventional flow right to left)
- $I_L = 31.93$ milliAmperes (conventional flow left to right)

[3]Double-lettered subscripts for V denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, V_{BC} means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for V represent the voltmeter's red test lead, with the black test lead touching a defined "ground" point in the circuit. Current measurements (I) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for I. Conventional flow notation will be used to specify current direction.

Chapter 3

Simplified Tutorial

An electric *circuit* is a closed loop formed of electrically-conductive material for electric charge carriers to travel, in which at least one *source* raises the energy level of each charge carrier passing through and at least one *load* extracts energy from each passing charge carrier passing through, the result being that energy is transferred from source to load with electricity as the medium. Note the "+" and "−" symbols used to show relative energy states of charges moving in and out of each component. *Voltage* is the amount of energy gained or lost by charge carriers passing between two points, while *current* is the rate of charge carrier motion:



Fluid power systems are analogous to electric circuits: instead of electric charge carriers flowing through conductors, there is fluid flowing through pipes. A *pump* functions as an energy source by boosting the energy levels of fluid molecules with pressure, and an *actuator* or *fluid motor* functions as a load by allowing the pressurized fluid to do useful mechanical work. *Pressure rise* and *pressure drop* in a fluid power circuit are analogous to voltage in an electric circuit; *fluid flow rate* in a fluid power circuit is analogous to current in an electric circuit. *Resistance* in either type of circuit, is any restriction of flow causing energy to leave the circuit and never return. Power, in either type of circuit, is the rate at which energy transfers.

Switches may be incorporated into a circuit to provide control over the flow of current. An "open" switch prohibits current through it, and so an open switch is able to interrupt current throughout an entire circuit if connected in-line with the other components.

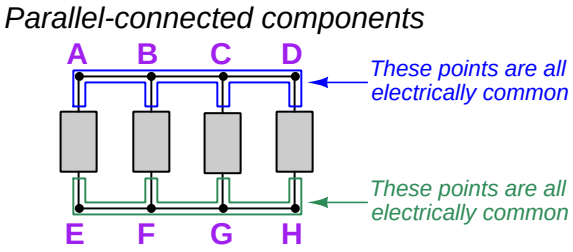
The mathematical relationship between voltage (V) and current (I) for any given resistance (R) is described by *Ohm's Law*:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

Power is mathematically related to voltage, current, and/or resistance by *Joule's Law*:

$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

Components are said to be connected in *parallel* when they all span the same two sets of electrically common points. All parallel networks (i.e. collections of interconnected components) exhibit common properties, including **common voltage**, **additive currents**, and **diminishing resistances**:



First, parallel-connected components experience the same amount of voltage. This is true because the Conservation of Energy makes it impossible for energy to arise from or disappear into nothing, and that is what would have to occur for a set of parallel-connected components to experience different amounts of voltage: two independent charge carriers starting at the same point, passing through different components, and ending up at the same destination, but somehow having different energy levels.

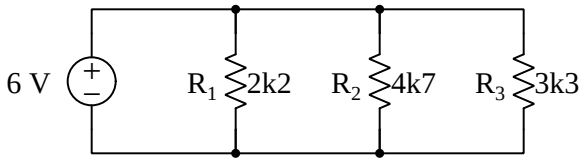
Second, total current for a set of parallel-connected components must be the mathematical sum of the individual component currents. This is true because the Conservation of Electric Charge makes it impossible for charge carriers to simply appear or vanish, and that is what would have to occur for the sum of the individual component currents in a parallel network to be different from the total current of that whole network.

Third, resistance for a collection of parallel-connected components must be less than any of the individual component resistances. Specifically, total resistance is equal to the reciprocal of the sum of the reciprocals of the individual resistances. This is really just a combination of the first and second properties with Ohm's Law: if all component voltages are identical ($V_T = V_1 = V_2 = V_3 \cdots = V_n$) and currents add ($I_T = I_1 + I_2 + I_3 \cdots + I_n$) and $I = \frac{V}{R}$, then $\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \cdots + \frac{V}{R_n}$. Factoring out V from this equation yields $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots + \frac{1}{R_n}$. Solving for total resistance yields $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots + \frac{1}{R_n}}$

This should also make intuitive sense: if charge carriers have multiple alternative routes to make it through a parallel network, then the overall resistance of that network must be less than the resistance of any one route.

Additionally, total power is equal to the sum of the individual component powers. This is true because of the Law of Energy Conservation, and applies to *all* types circuit networks, not just parallel.

A tactic many students find helpful for managing calculations during parallel circuit analysis is to draw a *table* listing voltage, current, and resistance as rows, and listing each component as a column, as shown here for the following parallel circuit:



	R1	**R2**	**R3**	**Total**
V				6 V
I				
R	2200 Ω	4700 Ω	3300 Ω	

Properties of parallel circuits apply to values in the same *row* of the table, while Ohm's Law applies to values in the same *column* of the table. Any empty cell within a row becomes an obvious cue to apply the appropriate network property, and any empty cell within a column becomes an obvious cue to apply Ohm's Law.

In this example, a clear first step is to apply the property of parallel voltages being equal in the top row:

	R1	**R2**	**R3**	**Total**
V	6 V	6 V	6 V	6 V
I				
R	2200 Ω	4700 Ω	3300 Ω	

The next clear step is to apply Ohm's Law each resistor's column, filling the empty "current" cells there:

	R1	**R2**	**R3**	**Total**
V	6 V	6 V	6 V	6 V
I	2.7273 mA	1.2766 mA	1.8182 mA	
R	2200 Ω	4700 Ω	3300 Ω	

Now we can apply the property of parallel currents adding up to equal total current to the middle row:

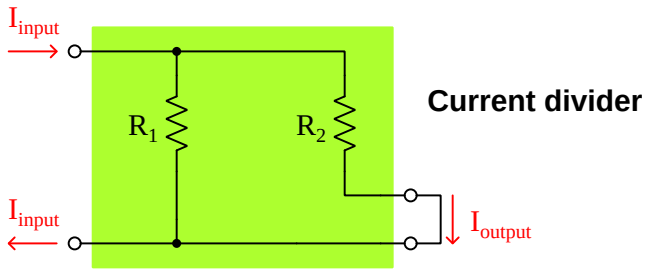
	R1	**R2**	**R3**	**Total**
V	6 V	6 V	6 V	6 V
I	2.7273 mA	1.2766 mA	1.8182 mA	5.8221 mA
R	2200 Ω	4700 Ω	3300 Ω	

Lastly, we may apply Ohm's Law to the "total" column to calculate total circuit resistance (or, alternatively, apply the property of parallel resistances diminishing to equal total resistance):

	R1	**R2**	**R3**	**Total**
V	6 V	6 V	6 V	6 V
I	2.7273 mA	1.2766 mA	1.8182 mA	5.8221 mA
R	2200 Ω	4700 Ω	3300 Ω	1030.56 Ω

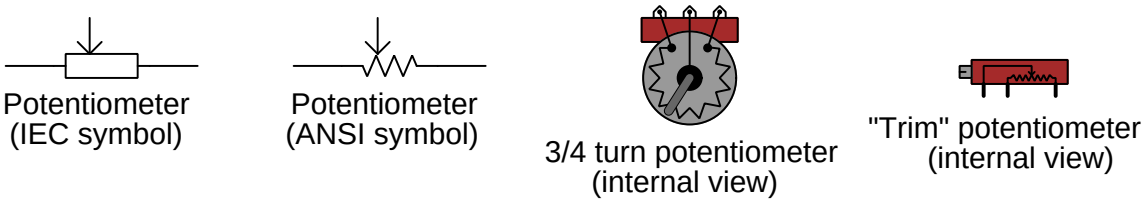
As evidenced by the multiple solutions in the last step, the tabular format helps us check our work by showing us more than one way to calculate (and/or verify) parameters.

Any network of parallel-connected resistors acts to *divide* the total (source) current into fractional portions, each fraction equal to the total (parallel network) resistance divided by that resistance. This is a very practical use for parallel resistor networks, reducing an applied current by a prescribed ratio:

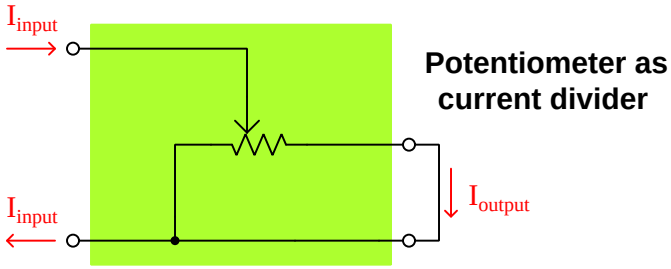


$$I_{output} = I_{input}\left(\frac{R_{total}}{R}\right)$$

A *potentiometer* is an adjustable device intended to provide resistance ratios in one unit. It has three terminals: two connecting to the ends and the third serving as the adjustable "wiper" contacting some point in between. Some potentiometers are coarse, having just 3/4 turn from minimum to maximum, while others are designed to be multi-turn for fine precision adjustment:



This schematic diagram shows how a potentiometer may be used an an adjustable current divider:



Chapter 4

Full Tutorial

4.1 Concept review

First, a review of some foundational principles.

Energy is the ability to set matter into motion. Energy may take many different forms, but is broadly divided into two categories: *kinetic energy* and *potential energy*. "Kinetic" energy is that exhibited through motion: for example, a moving mass has energy because of its ability to set other matter into motion by collision, and its energy is deemed kinetic because it is moving. "Potential" energy is that which exists without motion: for example, a suspended mass has energy because it may potentially cause motion if allowed to fall, but until it does the energy is regarded as "potential" because it is not being put into action (yet).

An interesting property shared by both mass and energy is *conservation*, which means these quantities cannot be created from nothing nor can they be destroyed (annihilated), but are eternal. Both matter and energy can and do, however, change forms.

Just as any mass experience a force when influenced by a gravitational field, some types of matter possess another property called *electric charge*, which is the ability to experience a force when influenced by an *electric field*. Just as mass is a conserved quantity (i.e. eternal), so is electric charge: it cannot be created or destroyed. Electric charges exist in two different types, called *positive* and *negative*. Examples of electrically charged matter include *electrons* (negative) and *protons* (positive), both subatomic particles found in equal numbers within "electrically balanced" atoms.

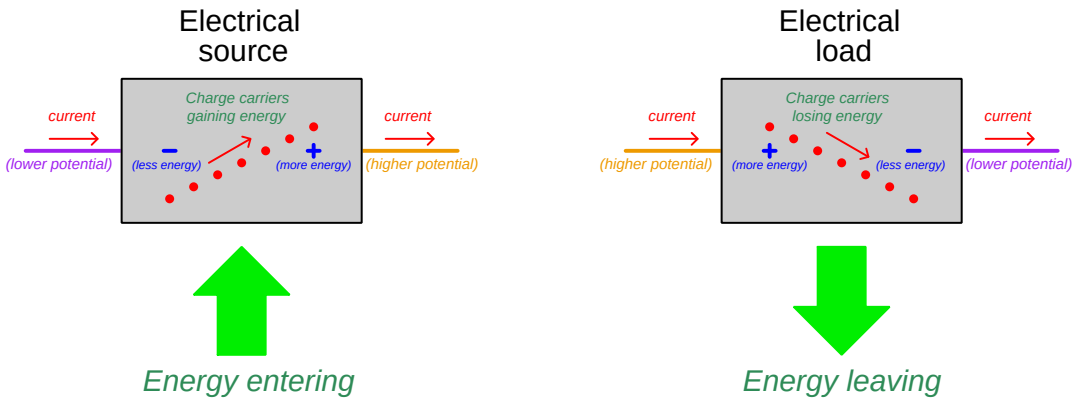
In some forms of matter, electric charges are mobile. Metals, for example, have very loosely-bound electrons which are able to drift through the solid space of the metal object while the remaining portions of the atoms remain bound in place. In some liquids, molecules split into unbalanced atoms called *ions*, some carrying excess electrons (negative charge) and others missing electrons (positive charge), and these ions are free to drift within the liquid. Any mobile electric charges, whether they be single subatomic particles or ions, are generally referred to as *charge carriers*. Substances possessing free charge carriers are called *electrical conductors*, while substances lacking available charge carriers are called *insulators*. The degree of mobility experienced by charge carriers between any two locations is called *electrical resistance*, mathematically symbolized by the variable R and

measured in the unit of the *Ohm* (Ω). A perfect conductor has zero resistance, and a perfect insulator has infinite resistance.

Free charge carriers within conductive substances are constantly moving in random directions, motivated by thermal (heat) energy. If charges are persuaded to move in a particular direction by the presence of an electric field, their net motion is called *current* which is mathematically symbolized by the variable I and measured in the unit of the *Ampere*[1] (A). Since electric charge is a conserved quantity, which means charge carriers cannot simply spring into existence or vanish, the only[2] way to sustain a continuous motion of charge carriers is to provide a closed loop for those charge carriers to circulate, called a *circuit*.

Electric charges may act as carriers of energy, just as masses may possess either kinetic or potential energy. The amount of potential energy exhibited per electric charge carrier is simply called *electric potential*. Any gain or loss of potential between two locations is referred to as *voltage* or *potential difference*, mathematically symbolized by the variable V or E and measured in the unit of the *Volt* (one Volt defined as one Joule of energy transferred per Coulomb[3] of electric charge). A very important feature of voltage, like all forms of potential energy, is that it is relative and not absolute. Just as the potential energy of a suspended weight is quantifiable only between the point of suspension and the point to which it may fall, the amount of potential carried by electric charges is quantifiable only between the location of the charge and another location of different energy, with voltage being the amount of energy per charge either gained or lost between those two points. *There is no such thing as voltage existing at any single point. Voltage only exists between two points, between which charge carriers either gain or lose potential energy.*

If charge carriers gain energy while moving through a device, that device is called a *source*. If charge carriers lose energy while moving through a device, that device is called a *load*:

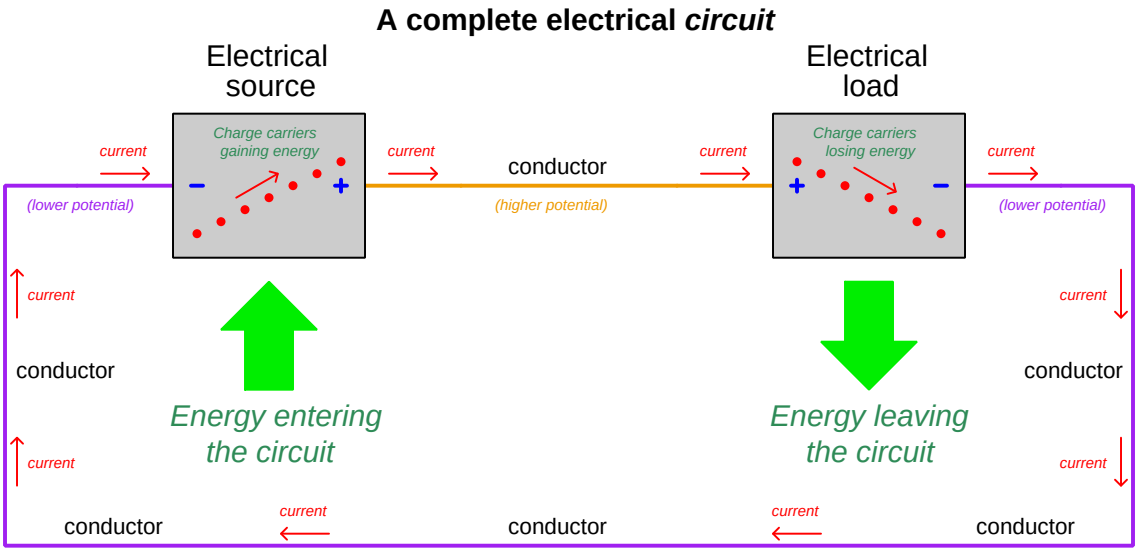


[1] One Ampere of electric current is defined as the passing of one Coulomb of electric charge carriers (equivalent to 6.2415×10^{18} electrons) per second of time.

[2] This concept often causes confusion for students, and so is good to explore in more detail. Try imagining a broken loop with current continuously flowing at all the unbroken points within the loop. What this would inevitably mean is that charge carriers would have to mysteriously vanish where they reach the break, and that other charge carriers would have to mysteriously appear on the other side of the break to enter the circuit. So long as charge carriers are eternal and cannot pass through an insulating (non-conductive) break, it is impossible for a continuous flow to occur in a broken loop.

[3] A Coulomb of charge is equivalent to 6.2415×10^{18} electrons.

Sources and loads usually appear together in circuits, with charge carriers transporting energy from the source to the load. Energy enters the circuit through the source from the outside (e.g. light, heat, sound, chemical reaction, mechanical, etc.) to boost the charge carriers' energy, those charge carriers travel to the load where they release that energy in some form (e.g. light, heat, sound, chemical reaction, mechanical, etc.), and then the charge carriers circle back to the source for another round. An electrical circuit is analogous to a *fluid power circuit* where energy is transferred by means of a circulating fluid, with a *pump* (source) infusing potential energy into the fluid by means of pressure and an *actuator* or *motor* (load) extracting potential energy from the fluid to do useful work such as lifting an object or moving a machine:

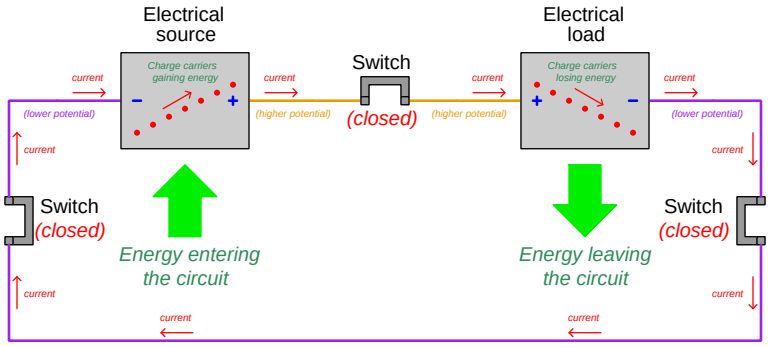


The positive (+) and negative (−) symbols refer to the relative energy levels of charge carriers as they pass through each source and load, and in pairs represents the *polarity* for voltage across each component. These polarity symbols are analogous to "high" and "low" pressure notations for a fluid circuit, revealing the relative pressures of fluid entering and exiting a component. The direction of current shown by the arrows is a convention, based on the assumption that the carriers are all positively charged. This convention for marking the direction of current is used regardless of that actual type(s)[4] of charge carriers involved, and is called *conventional flow notation*.

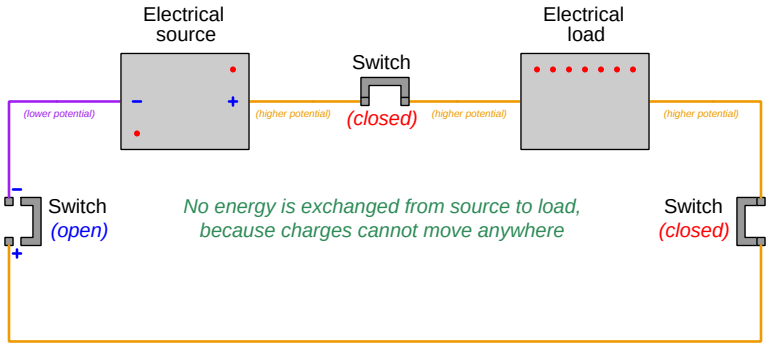
Metal wires, which are very good conductors of electricity (i.e. possess very little resistance R) extract negligible energy from charge carriers flowing through, which explains why the conductors shown in the above diagram have no + or − symbols drawn near. Every point along those metal wires are considered *electrically common* by virtue of their negligible resistance. This forces all charge carriers to be at the same energy level (i.e. *equipotential*) which means there is practically zero voltage between any two points along a good conductor.

[4] In metallic conductors, charge carriers consist of free electrons which are negatively charged and move in the opposite direction from that shown in the illustration. In liquids and ionized gases, charge carriers consist of positive and negative ions with motion in *both* directions (depending on the sign of each carrier's charge). Conventional flow is used as a convenience for representing current through *any* conductor and is not to be taken literally.

Switches are electrical components designed to deliberately establish or sever electrical connections between two or more conductors. Switches are usually designed such that a moving piece of metal either makes or breaks contact with a stationary piece of metal, typically by the actuation of a pushbutton, lever, or some other mechanism. When a switch is positioned to make contact and permit the flow of electric charge carriers through it, it is said to be *closed* or *shorted*. When a switch is positioned to break contact and forbid the flow of current, it is said to be *open*. The following illustration shows a simple circuit with three switches, all closed:



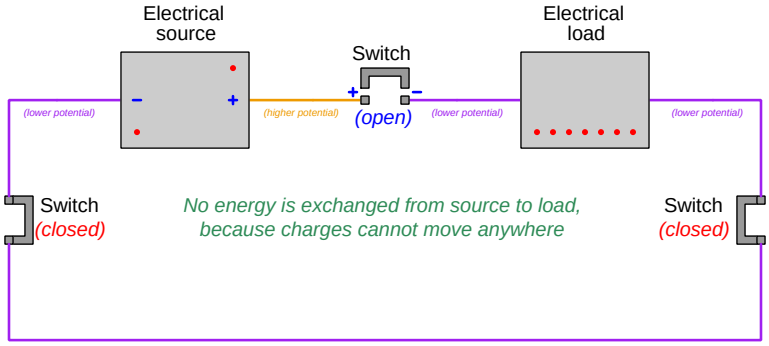
If *any* of these switches is opened, the circuit will become "broken" and current halts throughout. Whichever switch is opened, charge carriers cannot pass through it. This, in turn, immediately ceases the circulation of charge carriers at all points in the circuit due to the Conservation of Charge[5]:



The effect of opening a switch is immediate throughout the circuit: all current ceases, full source voltage appears across the terminals of the open switch, and the load no longer exhibits a voltage across its terminals. All points between the negative pole of the source and the open switch exist at the same low potential, while all points between the positive pole of the source and the open switch exist at the same high potential.

[5]Since charges are eternal and therefore cannot appear from nowhere or disappear into nothingness, the continuous circulation of charge carriers in a circuit demands a complete, unbroken path. If the path becomes broken by an open switch, charge carriers at the "incoming" pole of the switch must stop moving because they cannot vanish at the open air gap of the switch, and charges at the "outgoing" switch pole must also stop moving because no new charge carriers may appear out of the air gap. This situation is analogous to a fluid circuit where the total quantity of fluid is fixed and a valve somewhere shuts off the path of flow: the motion of fluid must stop everywhere in the circuit.

Much the same happens if we open a different switch in this circuit. Current everywhere immediately ceases, and the source's full voltage appears across the terminals of the open switch:



Note how there is a difference between this scenario and the previous one with a different open switch, and that is the absolute energy states of stationary charge carriers within the load. From the perspective of energy transfer, however, this is of no concern. With no charge carriers *moving* through the load, transitioning from high energy to low energy, there is no *potential difference* (i.e. voltage) across the load, and therefore the load receives no energy from the charge carriers.

Voltage (V), current (I), and resistance (R) are proportionately related to one another by a set of formulae called *Ohm's Law*:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

Where,

V = Voltage, in Volts (V)
I = Current, in Amperes (A)
R = Resistance, in Ohms (Ω)

The rate at which energy is exchanged from one form to another is called *power*, mathematically symbolized by the variable P and measured in the unit of the Watt[6] (W). A set of formulae called *Joule's Law* relates power to voltage, current, and resistance:

$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

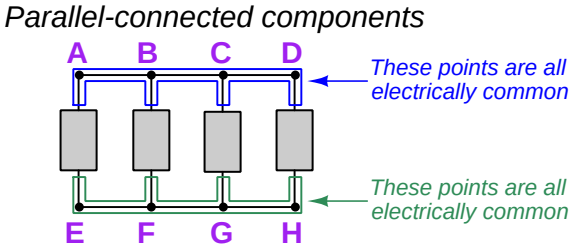
Where,

P = Power, in Watts (W)
V = Voltage, in Volts (V)
I = Current, in Amperes (A)
R = Resistance, in Ohms (Ω)

[6] One Watt is defined as one Joule of energy transferred per second of time.

4.2 Parallel networks

Now that we have reviewed some fundamental terms and principles of electric circuits, let us move on to the topic at hand: *parallel networks*. A "network" is any set of interconnected components, and a "parallel" network is defined as one where all components connect between the same two sets of electrically-common points. The following illustration shows four components (each one represented as a nondescript rectangle) connected in parallel with each other, with lettered points for reference:



If we were to provide a suitable energy source and circuitous pathway to motivate a continuous current, we would form a *parallel circuit*, and find that the amount of voltage across each of the four components – and indeed between *any* two points spanning those components – was identical. No component within a parallel circuit may have less voltage or more voltage than any other component.

The reason for this equality of voltage in any parallel configuration is the equipotential nature of the connecting wires. If the conductors bridging all the upper component terminals together are equipotential (meaning charge carriers neither lose nor gain energy moving through that conductor), then whatever potential difference exists between any two points between those equipotential sets must be the same for *all* points between those equipotential sets. For example, if points A through D are equipotential with each other, and points E through H are also equipotential with each other, then the voltage between any point A-D and any point E-F must be the same (e.g. $V_{AE} = V_{BF} = V_{CH} = V_{DE}$, etc.). Illustrating this mathematically, if one set of variables are equal to each other and another set of variables are also equal to each other, the difference between any member of one set and any member of the other set must be the same:

$$\text{If} \quad A = B = C = D \qquad \text{and if} \qquad E = F = G = H$$

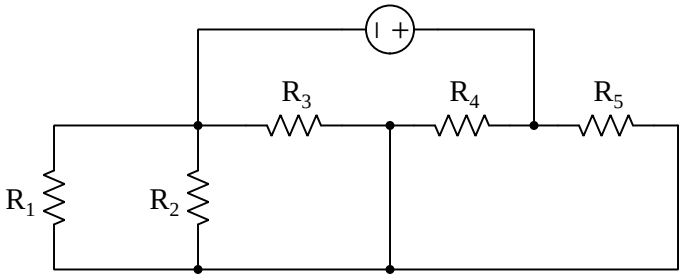
$$\text{Then} \quad (A - E) = (B - F) = (C - H) = (D - E)$$

This is the defining characteristic of a parallel circuit, and the first logical consequence of that characteristic: *all components share the same two sets of electrically common*[7] *points, and as a result all components must experience the same amount of voltage at any given time.*

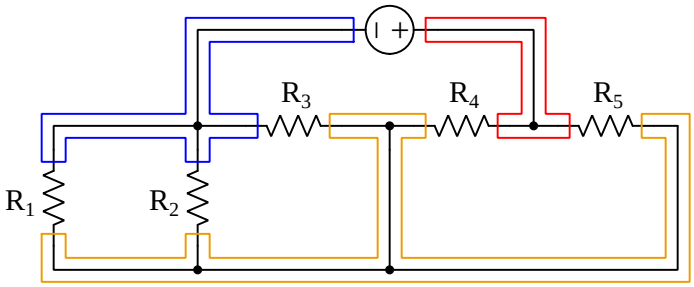
[7] To review, electrically common points are those connected together by negligible resistance, and therefore must be equipotential to each other. Any set of electrically common points and the conductor bridging them together together comprise one electrically common *surface*.

Another way of defining a parallel circuit is to say that only two *electrically distinct*[8] conductors exist among the connected components.

Parallel-connected components can be difficult to identify within some schematic diagrams. Consider the following complex circuit containing five resistors and one voltage source:



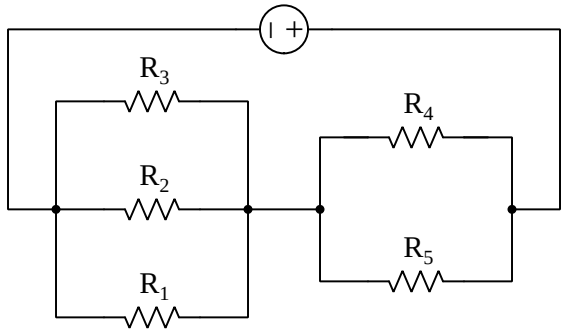
It may not be obvious which of these resistors are in parallel with each other until we apply color-coding to show the relative electric potentials of all the electrically-common point sets. Here we will use red to show the points having the greatest electrical potential, blue to show the points having the least electrical potential, and orange to show the points having some intermediate electrical potential:



Examining this circuit with equipotential points thusly color-coded, we may more easily identify parallel resistors because those resistors will all span the same pair of colors. Thus, R_1, R_2, and R_3 are in parallel with each other because all three span between blue and orange; R_4 and R_5 are also in parallel with each other because they both span between red and orange.

[8] If electrically common points are defined as being connected together by a conductor of negligible resistance and therefore equipotential, then electrically distinct points must have substantial resistance between them and therefore be capable of existing at different potentials.

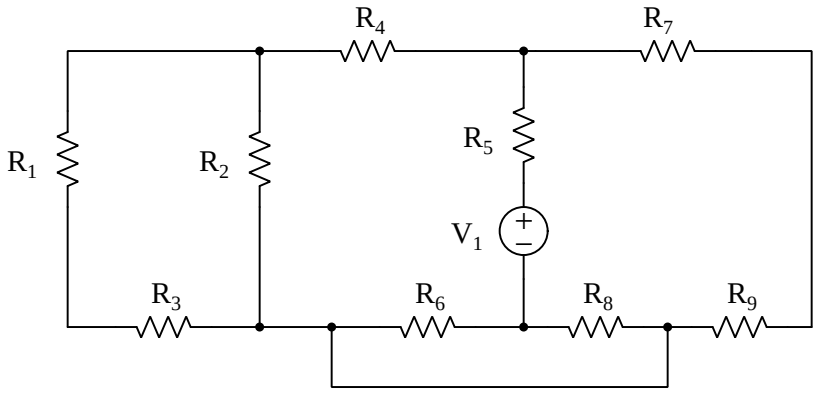
Since electrical schematic diagrams show logical relationships between components rather than spatial relationships, we are at liberty to re-draw the schematic so that all electrically-parallel resistors are *geometrically* parallel to each other, as shown below:



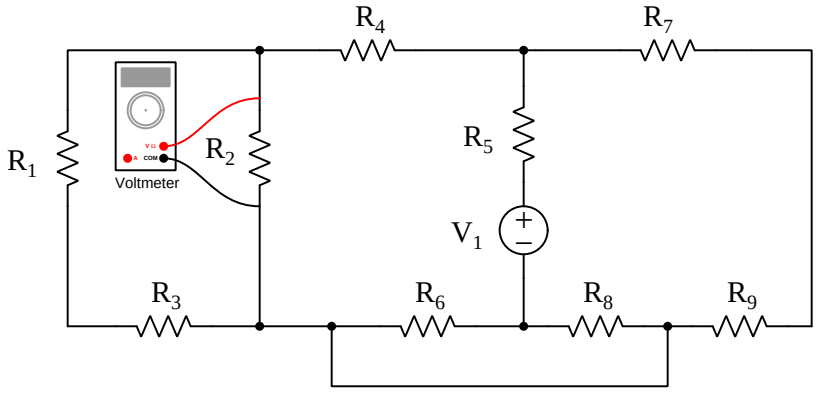
Now it is obvious that R_1, R_2, and R_3 are electrically parallel and that R_4 and R_5 are also electrically parallel. When re-drawing circuits to make series and/or parallel relationships easier to see, a recommendation is to orient the axis of every component in the same direction, in this case horizontal. Wire-lines in a schematic diagram may be stretched and/or bent as necessary to accommodate the new component orientations.

The fact that all parallel-connected components must experience the same voltage at any given time has practical applications. One such application is in the use of *voltmeters* to measure voltage. If we know voltage must be the same for all parallel-connected components, then we may ensure a voltmeter intercepts a particular component's voltage by connecting that voltmeter in parallel with that component.

To see how this principle might be applied, consider the following schematic diagram showing a complex circuit containing nine resistors (R_1 through R_9) and a single voltage source (V_1):



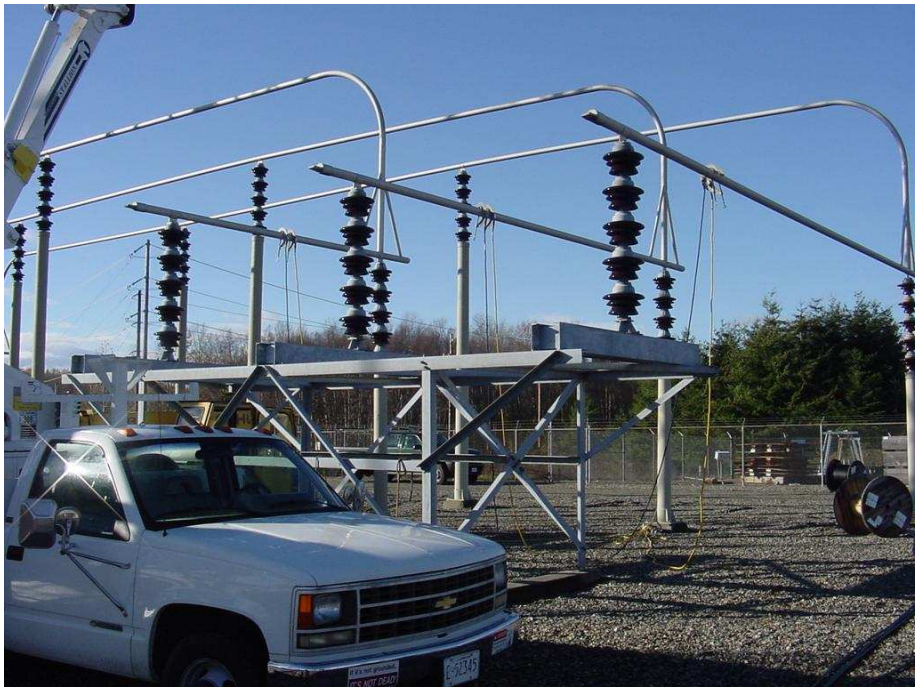
If our task was to measure the amount of voltage dropped by resistor R_2, we must connect the voltmeter's test leads to points common with the upper and lower terminals of R_2, respectively:



The voltmeter is connected in parallel with resistor R_2, and therefore must experience the same voltage dropped across R_2. However complicated and mysterious the rest of the circuit might be, this much we know to be true!

Another practical application of parallel connections is electrical safety, specifically a concept known as *bonding*. To "bond" a set of electrical conductors means to connect them together using another conductor in order to ensure a zero-voltage state between those conductors. In other words, *to ensure a zero-voltage condition between two points, we intentionally connect a wire* (which by definition will have negligible voltage end-to-end) *in parallel with those two points, so that the wire's zero-voltage state will force the connected points to have zero voltage between them as well.*

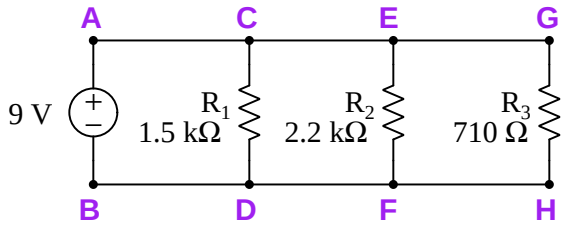
The following photograph shows a work site at a 230 kV electrical substation, where electricians are busy performing maintenance work on a high-voltage component. In addition to opening large switches (called *disconnects*, lying outside the boundaries of this photograph) to isolate this new component from any source of voltage, the electricians have taken the additional step of *bonding* the high-voltage conductors to each other and to Earth ground by means of temporary wire cables. The cables on this work site happen to be yellow in color, and may be seen hanging down from C-shaped clamps attached to three horizontal metal tubes called *busbars* which serve as conductors for electricity in this substation:



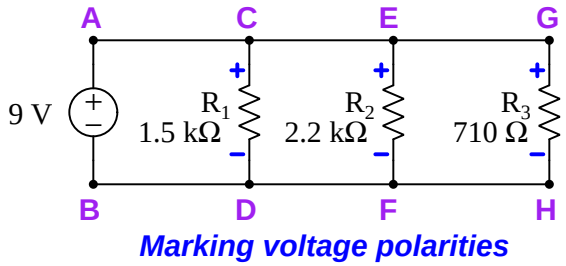
Another way of looking at bonding is to consider it a means to force multiple points in a circuit to become equipotential to each other. The bonding conductor is a *short* (i.e. a low-resistance path) between points, thus forcing those points to have the same potential. Any person whose body comes into contact with those points will not experience any substantial voltage between those points to cause bodily harm. Furthermore, bonding the shorted power conductors to ground forces all of them to be at Earth potential, further protecting human life by ensuring no one standing on the ground will become shocked or burned if they happen to touch one of the conductors.

4.3 Properties of parallel networks

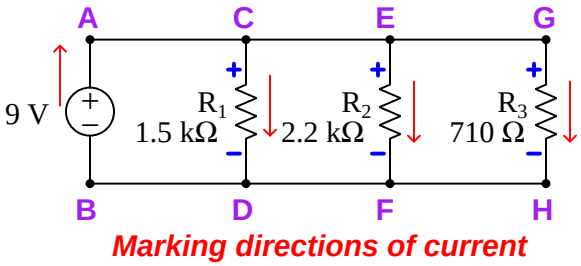
While it is true that parallel-connected components experience the same voltage, this is not the only unique property of parallel circuits. Other properties become apparent when we analyze the relationship between voltage and current for each component in a parallel circuit. Let us consider a three-resistor parallel circuit as a test case, powered by a 9 Volt voltage source. The letters A through H identify different points in the circuit we will later reference:



A good first step in analyzing this circuit is tracing directions of current and marking polarities of voltage. The polarity of the voltage source is already given by the source's + and − symbols, and we also know all three resistors must share this same voltage because they are connected in parallel with each other. Therefore, we may place + and − symbols near each resistor to match the source:



Current directions may now be assigned according to the polarity of voltage and the identity of each component as either a *source* or a *load*, namely that current always exits the positive terminal of a source and enters the positive terminal of a load. Our one voltage source is a source by definition, and resistors are always loads[9], so the current directions must be as follows:



[9]Recall that the function of a load is to extract energy from charge carriers as they pass through. This is what any *resistance* does by its inherent opposition to the passage of current: it depletes some of the energy of those passing charge carriers, converting that energy into heat.

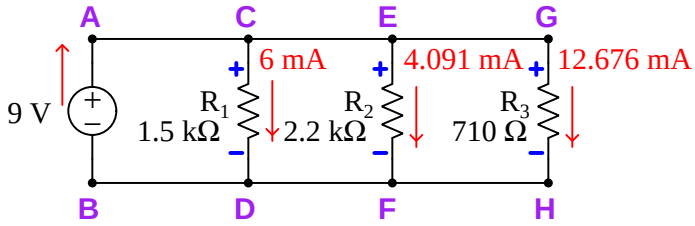
At this point we are ready to apply Ohm's Law to the calculation of each resistor's current, since Ohm's Law (like all physics formulae) requires its variables to be in the same context. That is to say, if we wish to use Ohm's Law ($I = \frac{V}{R}$) to calculate the current through a resistor (I_R), we need to know the voltage across that same resistor (V_R) as well as the resistance of that resistor (R). With only one value of voltage in this circuit (V) shared in common with all three resistors, and three different resistor values (R_1, R_2, and R_3), we will calculate three different current values (I_{R1}, I_{R2}, and I_{R3}, each resistor with its own "branch" current):

$$I = \frac{V}{R} \qquad \text{(General form of Ohm's Law)}$$

$$I_{R1} = \frac{V}{R_1} = \frac{9 \text{ V}}{1.5 \text{ k}\Omega} = 6 \text{ mA}$$

$$I_{R2} = \frac{V}{R_2} = \frac{9 \text{ V}}{2.2 \text{ k}\Omega} = 4.091 \text{ mA}$$

$$I_{R3} = \frac{V}{R_3} = \frac{9 \text{ V}}{710 \, \Omega} = 12.676 \text{ mA}$$



Marking current magnitudes

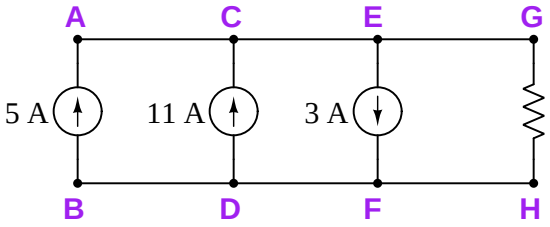
At this juncture it is worth asking, "How much current is there flowing through the voltage source?" A voltage source is only specified for the amount of voltage it outputs, not current. The amount of current output by a voltage source depends on what load(s) the source is powering. In this case, the source is powering three resistors, each of those drawing a known amount of current.

Recalling the definition of current being *rate of electric charge carrier motion*, and knowing in particular that one Ampere is defined as one Coulomb of charge moving by per second of time, we may begin to answer the question of the source's current by examining charge carrier motion through the three resistors. Over one second's worth of time, 6 milliCoulombs of charge pass through resistor R_1 (from C to D). over that same time interval 4.091 milliCoulombs of charge pass through R_2 (from E to F). Lastly, during that same time period 12.676 milliCoulombs of charge move through R_3 (from G to H).

Electric charge, like energy, is a conserved quantity, which means every charge carrier passing through the resistors to deliver energy must also pass through the source to replenish their energy. Totaling the flow of charge carriers through all three resistors, we get 6 mA + 4.091 mA + 12.676 mA = 22.767 mA. This means the rate of charge carrier flow through the source must be equal to 22.767 mA.

Here we see another important property of parallel circuits: *currents add in a parallel circuit*, because the charge carriers passing through each component must be accounted for according to the Law of Electric Charge Conservation.

This is true for parallel-connected sources as well as parallel-connected loads. Consider the following parallel set of current sources:



Imagine the flow of charge carriers through each current source being modeled by the flow of liquid through three pumps. The first pump discharges liquid upward at a rate of 5 units per second, while the second pump does likewise at 11 units per second. The combined flow rates of these two pumps from C to E (and also from F to D) must therefore be 16 units per second of liquid. However, at junctions E and F, the third pump's flow rate *subtracts* from the total of the first two, because that pump is moving liquid downward through its branch of the circuit rather than upward. This leaves a fluid flow rate of only 13 units per second moving from point E to G, through the resistance to H, and then to F. Just as fluid molecules cannot be created or destroyed (the Conservation of Mass), the electric charge carriers moving in this parallel circuit must likewise be accounted for (the Conservation of Electric Charge), leaving 13 A of current flowing downward through the resistor. This solitary resistor would develop a voltage drop as predicted by Ohm's Law ($V = IR$).

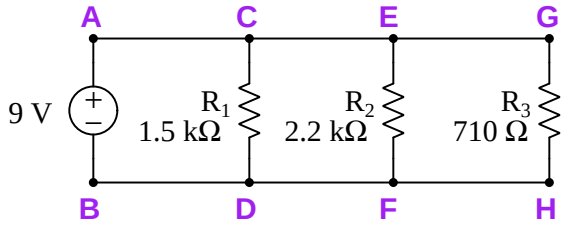
As a general rule, then, we find that the current resulting from a set of parallel-connected components is equal to the algebraic[10] sum of each component's current within that set.

So far we have drawn two important conclusions about parallel-connected electrical components:

1. Parallel-connected components experience the *same voltage* at any given time.
2. The total resultant current for a set of parallel-connected components is equal to the *algebraic sum* of the components' currents.

[10]The term "algebraic sum" simply means a total respecting the mathematical sign of each argument. In the case of the three parallel-connected current sources, their sum total current was 13 milliAmperes because 5 + 11 + (−3) = 13.

One more important conclusion regarding parallel circuits awaits our discovery, regarding *resistance*. If we return to our example circuit powered by the 9 V voltage source, we may ask the question "How great is this circuit's *total* resistance?"



Of course, Ohm's Law is useful in calculating this ($R = \frac{V}{I}$), but as always we must consider all its variables in context. If it is *total* circuit resistance we wish to calculate, then we need to identify the circuit's *total* voltage and its *total* current prior to performing the calculation. Total voltage is the same as the source voltage because this is a parallel circuit, and we have already calculated the total current to be 22.767 milliAmperes. Therefore the calculation for total resistance is:

$$R_{total} = \frac{V}{I_{total}} = \frac{9 \text{ V}}{22.767 \text{ mA}} = 395.31 \, \Omega$$

If you compare this total resistance figure of 395.31 Ω against the individual resistor values given in the schematic diagram, you will notice that total resistance is substantially less than any of the individual resistances. This should make intuitive sense, as we would expect the combined effect of multiple pathways in parallel to offer less resistance (i.e. less opposition to charge carrier motion) than any one of the paths on its own. The precise mathematical relationship between this total value of 395.31 Ω and the three resistors' individual values, however, is not obvious from inspection.

We may derive a mathematical relation by beginning with the statement that total current is equal to the sum of the parallel-connected resistor currents:

$$I_{total} = I_{R1} + I_{R2} + I_{R3}$$

We may substitute Ohm's Law into this statement of summed currents, knowing each resistor's current is equal to the quotient of the common parallel voltage and that resistance ($I_R = \frac{V}{R}$):

$$I_{total} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Factoring out the common voltage value V:

$$I_{total} = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Then, dividing both sides of the equation by V:

$$\frac{I_{total}}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This doesn't quite get us to the definition of total resistance we were seeking, since $R = \frac{V}{I}$ rather[11] than $\frac{I}{V}$. However, observe what happens if we reciprocate both sides of the equation:

$$\frac{V}{I_{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

From our previous calculation of total resistance we know that $\frac{V}{I_{total}} = R_{total}$, so what we have here is really a definition of total resistance for a parallel circuit:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

In summary, we may state one definition of a parallel circuit followed by three important electrical properties unique to parallel circuits:

Definition: *Parallel*-connected electrical components share two sets of electrically common points.

Property #1 Parallel-connected components experience the *same voltage* at any given time. $V_1 = V_2 = V_3 \cdots = V_n$

Property #2 Currents add in parallel: total current for a set of parallel-connected components is equal to the *algebraic sum* of the components' currents. $I_{total} = I_1 + I_2 + I_3 \cdots + I_n$

Property #3 Resistances diminish in parallel: total resistance for a set of parallel-connected resistances is equal to the *reciprocal* of the sum of the reciprocated resistance values. $R_{total} = 1/(1/R_1 + 1/R_2 + 1/R_3 \cdots + 1/R_n)$

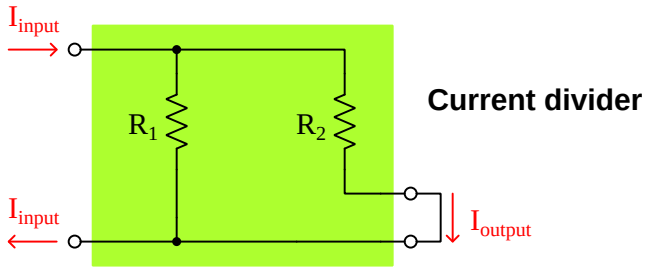
It should be noted that the total power (P) of any parallel circuit is equal to the sum of all load powers. This is simply a consequence of the Law of Energy Conservation, since power is nothing more than the rate of energy transfer over time: the sum total of all energy transfer out of the circuit (at load components) must be equal to the sum total of all energy transfer into the circuit (at source components). However, this is not unique to parallel circuits, but applies to *all* electrical circuits because the Law of Energy Conservation is universal:

$$P_{total} = P_1 + P_2 + P_3 \cdots + P_n$$

[11] $\frac{I}{V}$ actually does have a physical definition, and it is *conductance*. Conductance is a measure of how *easy* it is for electrical charge carriers to pass through a component. Conductance is the reciprocal of resistance and is mathematically symbolized by the variable G, and measured in the unit of *Siemens* (S). The total amount of conductance in a parallel circuit is the sum of all resistors' individual conductances.

4.4 Current dividers

An important application of parallel-connected resistors is a particular configuration known as a *current divider*, the purpose of this network[12] being to divide an applied current into smaller proportions. A simple example of a current divider appears in this schematic diagram, shown by itself with no power source connected, but with terminals provided for input and output currents:



The characteristic formula for a current divider, predicting the amount of current passing through any one of the parallel resistances R, is as follows:

$$I_{output} = I_{input}\left(\frac{R_{total}}{R}\right)$$

In the case of the two-resistor current divider illustrated above, the formula could be written as:

$$I_{output} = I_{input}\left(\frac{\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right)}{R_2}\right)$$

In any case, the ratio $\frac{R_{total}}{R}$ is the *proportion* of I_{input} represented by I_{output}, and so long as the resistor values remain fixed this current division ratio will remain fixed as well. For example, if R_1 and R_2 in the two-resistor divider happen to be equal in resistance, the proportion will be 50% (i.e. I_{output} will be equal to one-half the current of I_{input}). If R_2 happens to be three times the value of R_1, the proportion will be 25% (i.e. I_{output} will be one-quarter[13] the current of I_{input}). The remainder of the input current is diverted through the current divider's other resistor(s).

Any ratio desired (between 0 and 1, inclusive) is possible with a current divider, given the appropriate resistor values. This makes current dividers especially useful in measurement applications (e.g. reducing some high-current quantity to a precisely smaller proportion for easier and safer interpretation), computational applications (e.g. performing fixed-value division on a current representing some other quantity), and electric power applications (e.g. reducing a large source current to a smaller current more suitable for powering certain components and sub-circuits).

[12] A *network* simply refers to a collection of interconnected components, with or without a source.

[13] In case you find this confusing, and were expecting a division ratio of one-third, consider the fact that the division ratio of this two-resistor divider is defined by R_2 compared to R_{total}, not R_2 versus R_1. If R_2 is three times the resistance of R_1, then it will draw one-third as much current as R_1, but only one-quarter of the *total* current. If you still find this confusing, assign some numerical values to it and see for yourself how I_{R1}, I_{R2} and I_{total} (I_{input}) relate.

Tempting[14] though it may be to treat the current divider formula as a new principle of parallel resistor circuits, it is really just an expression of Ohm's Law and familiar parallel-circuit properties. All we need to do is re-associate[15] some of the variables in the formula to make this apparent:

$$I_{output} = I_{input}\left(\frac{R_{total}}{R}\right) \qquad \text{(Original form)}$$

$$I_{output} = I_{input}R_{total}\left(\frac{1}{R}\right) \qquad \text{(Modified form)}$$

The product $I_{input}R_{total}$ is equivalent to voltage (V) in a parallel circuit, following Ohm's Law ($V = IR$): the one and only voltage in this parallel circuit is equal to the total applied current multiplied by the total resistance of the circuit. Substituting V for $I_{input}R_{total}$:

$$I_{output} = V\left(\frac{1}{R}\right)$$

$$I_{output} = \frac{V}{R}$$

This final result, of course, is just another application of Ohm's Law: the current through any one resistor in a parallel network is equal to the voltage applied across that resistor divided by that resistor's value.

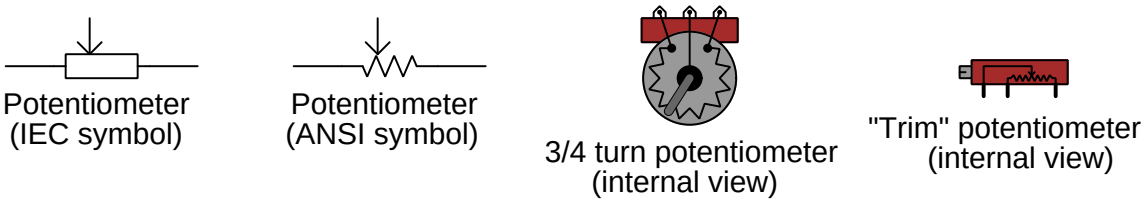
It should be noted that current dividers will output current values faithful to these formulae only when *unloaded*: that is to say, only when negligible voltage is dropped across its output terminals. If any load (for example, another resistor) is connected across the output terminals of a current divider, its output current will "sag" below the value predicted by the current divider formula. Connecting a load to a current divider's output terminals essentially modifies the current divider circuit, since the values of its constituent resistors are no longer the sole factors in determining the division ratio ($\frac{R_{total}}{R}$). A further exploration of this "loading" effect is beyond the scope of parallel circuits, and will be addressed in later modules after the introduction of *series-parallel* circuit analysis.

[14] An extremely important pedagogical principle within the sciences is to *connect* concepts wherever possible, and to preferentially reason from foundational principles as opposed to accumulating new rules. This is just one example in action: showing how the division ratio of a current divider circuit is really just an application of Ohm's Law and the concept of voltage being shared among all parallel-connected components. An unfortunate tendency among many students is to memorize new formulae for various circuit configurations without grasping the common origins of those formulae. I say this is "unfortunate" because the urge to memorize rather than reason represents an aversion to patient observation and logical thinking.

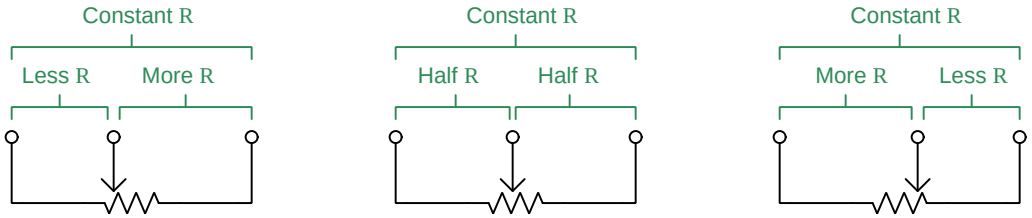
[15] In this case, we are literally employing the *associative property of multiplication* which states that $a(bc) = (ab)c$.

4.5 Potentiometers

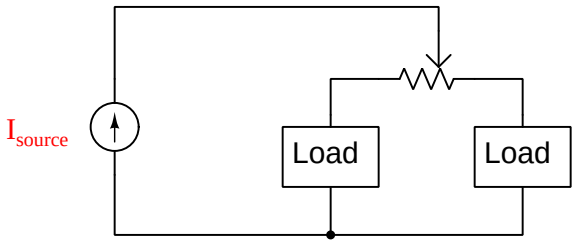
A very important electrical component usable as a variable current divider is the *potentiometer*. This device has three terminals: two connecting to the ends of a resistive strip, and one connecting to a movable "wiper" that contacts the resistive strip at one point.



As the wiper forms a single point of contact with the resistive strip, it essentially makes the potentiometer appear as a pair of series-connected resistors having complementary values: when the wiper is in the exact center, each wiper-to-end resistance will be equal (each one half the total end-to-end resistance); when the wiper position is offset from center, one of those resistances will be greater than half and the other resistance will be less than half. End-to-end resistance is always constant regardless of wiper position:

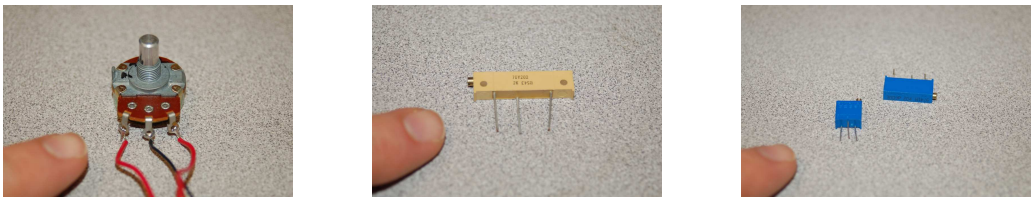


When the outer two terminals of a potentiometer are connected to a current source, the wiper provides a connection point to a node where current divides between two paths. Moving the wiper back and forth adjusts the current division ratio. In the example shown below, moving the wiper toward the right causes the right load to receive more of the source's current, and the left load to receive less:



Potentiometers are extremely useful devices: as volume controls in audio circuits, calibration adjustments on electronic measuring instruments, sensors to detect angle or position, and many other applications. One common application for potentiometers is to form a single, variable resistance (called a *rheostat*) as shown below:

Photographs of various potentiometers with *rotary* adjustments are shown here:



The left-hand image shows a 3/4 turn potentiometer with wires soldered to its three terminals. The black-colored wire connects to the potentiometer's "wiper" terminal while the two red-colored wires each connect to opposite ends of the potentiometer's resistive strip. The middle and right-hand images show "trimmer" potentiometers with multi-turn adjusting screws and leads designed for mounting to a printed circuit board (PCB).

Linear potentiometers are also manufactured, where the resistive strip and wiper motion range fall along a straight line. Linear potentiometers are commonly used for audio mixer controls (where each audio channel has a "slide" control for volume) and as machine position measurement sensors.

As with fixed-value resistors, the physical size of a potentiometer is no indication of resistance, but rather a reflection of its *power* rating. It is possible to design potentiometers for a wide range of resistance values in the same size package, but power dissipation requires surface area to shed heat which is why physical size is a reliable indication of power rating but not resistance.

Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Derivation of parallel resistance formulae

The parallel resistance formula $R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$ seems rather strange and unintuitive at first glance. While it may be derived from Ohm's Law and the properties of parallel networks (specifically, branch currents adding to equal total current), there is another approach as well involving *conductance*.

Conductance is the reciprocal of resistance. It is symbolized by the variable G and measured in the unit of Siemens[1] (S). If we were comparing two resistors, we could say that resistor A had more resistance than resistor B, or we could communicate the exact same idea by saying resistor B *was more conductive* than resistor A. For example, a resistance of 5 Ohms is equal to a conductance of $\frac{1}{5}$ Siemens.

Just as series-connected resistances add to create a greater total resistance, parallel-connected conductances add to create a greater total conductance. This should make intuitive sense, as the more branches we provide in a parallel network for current to split and flow through, the easier it must be overall for current to flow. If we had a three-branch parallel network, we could express this idea of added conductances as follows:

$$G_{total} = G_1 + G_2 + G_3$$

Now, since we happen to know that conductance is merely the reciprocal of resistance (i.e. $G = \frac{1}{R}$), we may make this substitution in each term of the above equation:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Reciprocating both sides of this equation to solve for R_{total} in a parallel network:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

It should be easy to see that this general form of the equation holds true regardless of the number of branches in the parallel network, and so we may express this truth more generally:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$$

[1] An obsolete unit of measurement for conductance is the *mho*, which is just "ohm" spelled backwards. The symbol for the mho was an upside-down Greek Omega letter. And you didn't think electrical engineers had a sense of humor?

For *two-resistance* parallel networks there exists an alternative formula for calculating total resistance, often called the *product over sum* formula. Like the first, this formula seems counter-intuitive at first sight:

$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

Let us begin our derivation of this formula by starting with the other form which we have already derived:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

We may evaluate the sum within the denominator of this fraction by finding a common denominator for $\frac{1}{R_1}$ and $\frac{1}{R_2}$. One way to do this is to multiply $\frac{1}{R_1}$ by the "unity fraction" $\frac{R_2}{R_2}$, and to multiply $\frac{1}{R_2}$ by $\frac{R_1}{R_1}$, since both "unity fractions" have numerical values of 1 and as such do nothing to change the values of the terms they're multiplied with:

$$R_{total} = \frac{1}{\frac{1}{R_1}\frac{R_2}{R_2} + \frac{1}{R_2}\frac{R_1}{R_1}}$$

$$R_{total} = \frac{1}{\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}}$$

$$R_{total} = \frac{1}{\frac{R_1 + R_2}{R_1 R_2}}$$

Now we just re-write the right-hand side of this equation so that it is no longer a compound fraction, and we will have derived our two-resistance parallel formula:

$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

5.2 Metric prefixes

The *metric system* of measurement specifies certain units of measurement for various physical parameters, some of them listed here:

Parameter	**Unit**	**Symbol**
Length	Meter	m
Temperature	Celsius or Kelvin	C or K
Mass	Gram	g
Force	Newton	N

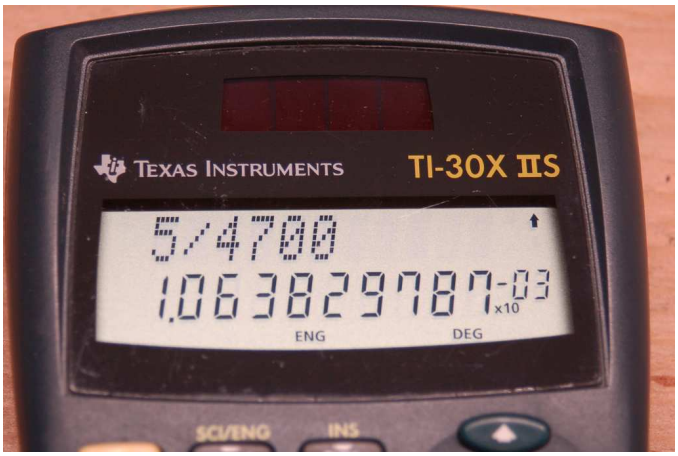
In order to make the expressions of large and small measurements more convenient, the metric system also includes *prefixes* representing power-of-ten multipliers, some of which are listed here:

Prefix	**Symbol**	**Power-of-ten**
Peta	P	$10^{15} = 1,000,000,000,000,000$
Tera	T	$10^{12} = 1,000,000,000,000$
Giga	G	$10^{9} = 1,000,000,000$
Mega	M	$10^{6} = 1,000,000$
Kilo	k	$10^{3} = 1,000$
Milli	m	$10^{-3} = \frac{1}{1,000}$
Micro	μ	$10^{-6} = \frac{1}{1,000,000}$
Nano	n	$10^{-9} = \frac{1}{1,000,000,000}$
Pico	p	$10^{-12} = \frac{1}{1,000,000,000,000}$
Femto	f	$10^{-15} = \frac{1}{1,000,000,000,000,000}$

In order to represent very large or very small quantities all we need to do is combine the right prefix with the unit. For example, a distance of 12,509 meters could be written as 12.509 *kilo*meters (12.509 km) because "kilo" simply means *thousand*. Similarly, a mass of 0.000000421 grams could be written as 0.421 *micro*grams (0.421 μg) or 421 *nano*grams (421 ng) because "micro" means *one-millionth* and "nano" means *one-billionth*.

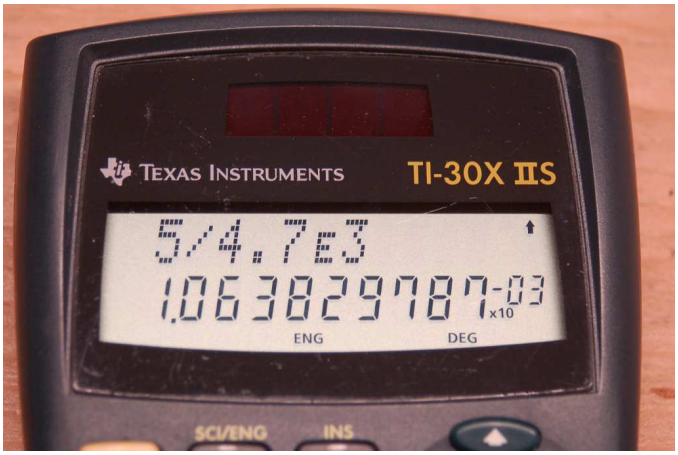
A useful way to think about metric prefixes is to consider the word *dozen*. Saying that we've collected three dozen eggs from a flock of chickens means the same thing as saying we've collected thirty-six eggs, because one "dozen" equals a quantity of twelve, and three times twelve is thirty-six. Here the word "dozen" is nothing more than a convenient multiplier for expressing some whole-numbered quantity of things. Metric prefixes constitute the same basic concept: they are simply multipliers used to help express a wide range of quantities while using numbers that are easier for us humans to handle.

Scientific hand calculators offer display modes and entry functions tailored for the purpose of expressing results and accepting input using standard metric prefix power-of-ten notation. Consider the following photograph of a Texas Instruments model TI-30X IIS calculator computing the current through a 4700 Ohm resistor energized by a 5 Volt source (using Ohm's Law, $I = \frac{V}{R}$):

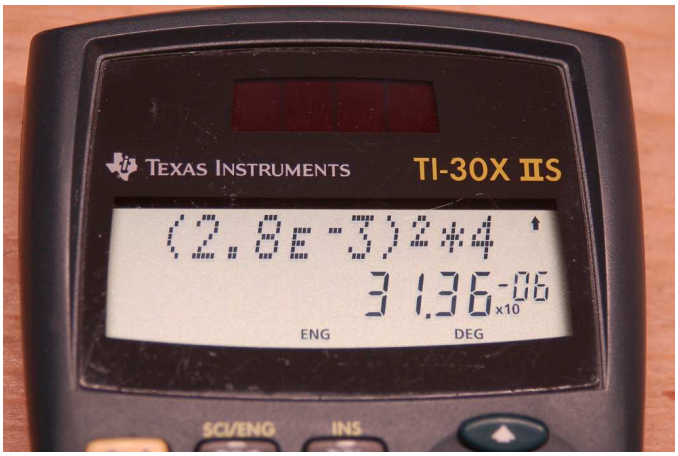


When the calculator has been set to the *Engineering* display mode, all results appear with a $\times 10^x$ label where x is an integer multiple of three to match with the metric prefixes listed previously. Here, the quotient $\frac{5}{4700}$ results in 1.0638×10^{-3}, or 1.0638 milliAmperes (1.0638 mA). It would be equally correct for the calculator to display the result as `0.001638` Amperes, but the 10^{-3} multiplier forced by Engineering mode results in a displayed number of 1.0638 that is easier for us to accurately read, write, and remember than 0.001638 with all the leading zeroes.

A button marked `EE` (sometimes $\times 10^x$ or simply `E`) allows entry of numerical values in either scientific notation or "engineering" form. Here, the same current calculation is performed entering the 4700 Ohm resistor's value as 4.7 kiloOhms (4.7 kΩ), entered as `4.7E3` in the calculator:



Another example shows the calculation of power dissipated by a 4 Ω resistor passing a current of 2.8 mA, using Joule's Law ($P = I^2R$) and once again making use of the `EE` button when entering the current value as `2.8E-3`:



Here, the resulting power dissipation is 31.36 microWatts (31.36 μW).

Some hand calculators provide the Normal/Scientific/Engineering display setting as a menu option, while others such as the TI-30X IIS shown here designate key functions to do the same. Note the `SCI/ENG` function visible in yellow-colored text above one of the buttons in these photographs.

Engineering mode and the `EE` button both require a knowledge of the powers-of-ten associated with common metric prefixes, but once those prefix multipliers are memorized they become very valuable in saving you the trouble of reading and entering numerical values filled with leading or trailing zeroes. As previously mentioned, without Engineering mode set our calculated current of 1.0638 mA would have been displayed by the calculator as 0.001638. Those leading zeroes, while mathematically accurate, are easy for humans to mis-count. The same is true for very large numbers with many trailing zeroes. *This is the chief purpose of metric prefixes: to reduce human error when reading, writing, and remembering very large and very small quantities alike.*

Mathematical formulae in science and engineering generally assume the use of *base units* in their variables. For example, *Ohm's Law* assumes voltage expressed in plain Volts (V), current expressed in plain Amperes (A), and resistance expressed in plain Ohms (Ω). However, for very large and very small quantities, it is more convenient for humans to read and write values using either metric prefixes or scientific notation rather than base units. For example, a voltage of 72 Volts applied across a resistance of 3,500,000 Ohms will result in a current of 0.00002057143 Amperes; but these quantities are much more easily written as 72 Volts, 3.5 megaOhms, and 20.57143 microAmperes, respectively.

Again, metric prefixes exist to make very large and very small numbers easier for human beings to manage. At root a metric prefix is nothing more than a particular power-of-ten multiplier. Thus, 3.5 MΩ is equal to 3.5×10^6 Ohms, or 3.5 *million* Ohms; likewise, 20.57143 μA is equivalent to 20.57143×10^{-6} Amperes, or 20.57143 *millionths* of an Ampere.

A very common error made by students new to the use of metric prefixes is to ignore them when inserting values into scientific and engineering formulae. For example, if someone tries to apply Ohm's Law to the example quantities of 72 V and 3.5 MΩ to find current ($I = \frac{V}{I}$) by simply dividing 72 by 3.5, their result will be grossly erroneous, as $72 \div 3.5 = 20.57143$. What this incorrect calculation *actually* finds is that 72 Volts applied across 3.5 Ohms yields a current of 20.57143 Amperes. When you enter "3.5" into a calculator, that calculator has no idea you really mean 3.5 *million* (3.5 *mega*) unless you specifically tell it so. This either requires you manually convert 3.5 MΩ into base units of Ohms (3,500,000 Ohms) by entering `3500000` into the calculator, or by using the calculator's power-of-ten entry key by entering `3.5E6` which means 3.5×10^6. In all cases, it is your responsibility to convert all metric-prefixed values into base-unit forms suitable for the formula(e) being calculated.

An interesting twist on the theme of base units is any formula involving *mass*, in which case the value for m must be computed in *kilograms* rather than grams. This is in accordance with the meter-kilogram-second (MKS) convention of modern[2] metric notation where the kilogram rather than the gram is considered to be the "base" scientific unit for mass.

For example, suppose we wished to calculate the kinetic energy of an 80 gram mass traveling at a velocity of 0.79 kilometers per second using the formula $E = \frac{1}{2}mv^2$. To properly calculate energy (E) in Joules, we would need to enter 0.080 for mass (m) in kilograms because 0.080 kg = 80 g and kilograms are the base unit for mass; we would also need to enter 790 or 0.79×10^3 or `0.79E3` for velocity (v) in meters per second because 790 m/s = 0.79 km/s with meters and seconds being base units. The proper result would be 24,964 Joules, or 24.964 kJ. If we were to ignore these metric prefixes by entering 80 for m and 0.79 for v, we would erroneously obtain a result of 24.964: an answer to a very different scenario where the mass is 80 kilograms rather than 80 grams and has a velocity of 0.79 meters per second rather than 0.79 kilometers per second.

So, just like the Ohm's Law example shown previously, we must first properly identify the base units required by the formula we're using, and then ensure our calculator sees all values expressed either directly in those base units or using powers-of-ten meaning the same thing. With the exception of mass where the *kilo*gram is considered the standard scientific base unit for all formulae, metric prefixes are nothing more than convenient ways to express large and small numerical values in order to avoid reading, writing, and remembering numbers containing a multitude of place-holding zeroes.

[2] The older metric convention was centimeter-gram-second (CGS) whereby all formulae using distance or displacement required casting that variable in *centi*meters rather than meters.

5.3 Resistor labeling

Resistors dissipate electrical energy in the form of heat, dropping voltage proportional to the amount of current passing through. This ratio of voltage to current is called *resistance* (R), measured in the unit of the *Ohm* (Ω), and it is the primary characterstic of any resistor. A popular method of labeling resistance values utilizes colored bands[3] to represent resistance values and tolerances. An example of this resistor style appears in the following photograph:

Four-band resistors are the most popular of the banded style, with the first and second bands representing significant digits, the third band representing a power-of-ten multiplier, and the fourth band representing tolerance.

Band color	**First digit value**	**Second digit value**	**Multiplier**	**Tolerance**
Black	0	0	10^0	
Brown	1	1	10^1	$\pm$ 1%
Red	2	2	10^2	$\pm$ 2%
Orange	3	3	10^3	$\pm$ 0.05%
Yellow	4	4	10^4	$\pm$ 0.02%
Green	5	5	10^5	$\pm$ 0.5%
Blue	6	6	10^6	$\pm$ 0.25%
Violet	7	7	10^7	$\pm$ 0.1%
Grey	8	8	10^8	$\pm$ 0.01%
White	9	9	10^9	
Gold			10^{-1}	$\pm$ 5%
Silver			10^{-2}	$\pm$ 10%
None				$\pm$ 20%

Applying this color code to the resistor shown above, we see that yellow = 4, violet = 7, red = 2, and silver = $\pm$ 10%. Therefore, this resistor's value is 47 $\times$ 10^2 Ohms (i.e. 4700 Ω) plus or minus 10%, which means it could be as low as 4230 Ω or as high as 5170 Ω.

[3] A useful mnemonic for associating these colors with decimal digits 0 through 9 and the percentages 5-10-20% is as follows: *"Better Be Right Or Your Great Big Venture Goes Wrong. Get Started Now."*.

Precision resistors require more significant digits to precisely quantify their values, and so use a *five-band* code where the first three bands represent digits, the fourth band represents the power-of-ten multiplier, and the fifth band represents tolerance. Both the four-band and five-band color codes are standardized as part of the international standard IEC 60062.

Many modern resistors are labeled with text rather than color-coded bands. For these resistors, a letter is used in place of a decimal point, the letter either being `R` (unit), `K` (kilo), `M` (mega), `G` (giga), `T` (tera), or `L` (milli). This is commonly referred to as the *RKM* code. Some labeling examples are shown in the following list:

- R82 = 0.82 Ω
- 3R9 = 3.9 Ω
- 47R = 47 Ω
- 560R = 560 Ω
- 3K3 = 3.3 kΩ
- 27K = 27 kΩ
- 100K = 100 kΩ
- 2M2 = 2.2 MΩ
- 15M = 15 MΩ
- 2L7 = 2.7 mΩ

Tolerances for text-labeled resistors use the following letter codes, the tolerance letter always being the last character of the resistor's text label:

Letter code	**Tolerance**
B	± 0.1%
C	± 0.25%
D	± 0.5%
F	± 1%
G	± 2%
H	± 3%
J	± 5%
K	± 10%
M	± 20%
N	± 30%

For example, a resistor labeled `6K8J` would be 6.8 kΩ ± 5%.

Surface-mount device (SMD) resistors are often labeled with numbers only, either a three-digit numerical code or a four-digit numerical code. These codes follow the same pattern as four- and five-band color codes (except with no specified tolerance, just **digit-digit-multiplier** or **digit-digit-digit-multiplier**). For example, `473` would be 47 kΩ and `2711` would be 2.71 kΩ.

An exception to this general rule appears in the following photograph of a 5 milli-Ohm surface-mount resistor with a tolerance of ± 1%. The `5L0` portion of the code indicates 5.0 mΩ, while the `F` portion indicates the ±1% tolerance:

This particular SMD resistor happens to be useful as a high-current *shunt resistor* developing a precise voltage drop proportional to current. Its low resistance value does little to impede the flow of charge carriers to the load, while its tight tolerance ensures its voltage drop will be a fairly good representation of current through it.

5.4 Ground in electric and electronic circuits

Ground and *grounding* within both electric power and electronic circuits are commonly misunderstood topics, largely because these terms can often have very different meaning depending on context. Here we will explore some of these different meanings and hopefully eliminate some of the confusion surrounding them.

The very first concept we must understand in order to comprehend what "ground" is in any circuit is that *voltage* is fundamentally a *relative* measurement of potential energy different *between two points*. This is really true of all forms of potential energy, because "potential" means energy that has not yet been put to work setting matter into motion, implying a final state yet to be realized. When we say an electrical charge carrier possesses potential energy (i.e. electrical potential), this means it *could* (potentially) release energy to do work if it were to fall into a lower state of energy. The difference in energy held by that electric charge carrier from its original (high-potential) state to some other lower-potential state will be the amount of energy released by that charge carrier to do work, with that amount of work done dependent on both the charge carrier's initial energy level and its final energy level. *Voltage* is the term we use to describe such a difference in electrical potential.

We say that voltage always exists "between two points" or "between two locations" in a circuit because voltage is the amount of potential energy either lost or gained by electric charge carriers from one position in a circuit to another. It is impossible to quantify how much energy will be gained or lost by charge carriers unless we know both the starting point and the ending point for those charge carriers. In order to quantify the absolute amount of potential energy possessed by anything, we would have to know where or how it could lose every bit of its stored energy to be left with zero capacity to do work[4]. This is really unknowable from a practical perspective, and so the best we can do is quantify gains or losses in potential energy between different states.

This fact about voltage is also why we label it with *pairs* of "+" and "−" symbols, the "+" symbol representing the point of *higher potential* and the "−" symbol representing the point of *lower potential*. We cannot place an absolute value on electrical potential at any single location because no one can say where the location of zero energy is in the universe for electric charges. The best we can do is to rate one location as having higher or lower electrical potential than another, quantifying only the difference in potential between those two locations.

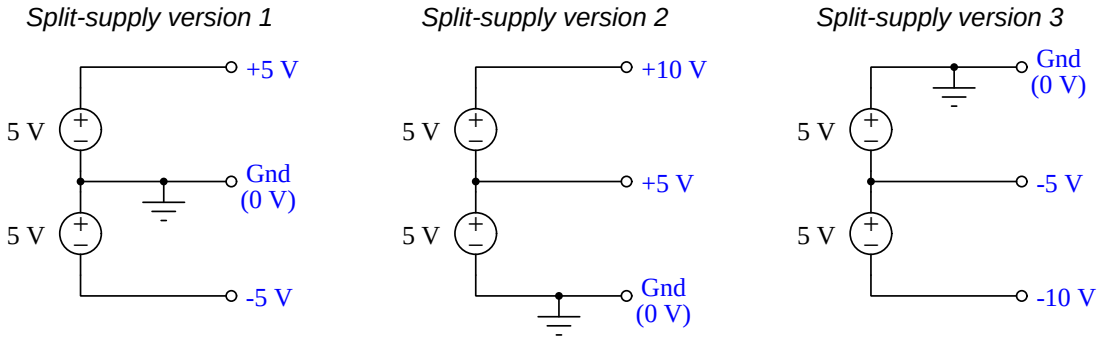
Now that we have reviewed this property of voltage and of potential energy in general, we are ready to explore what is meant by *ground* and *grounding*.

[4] Using gravitational potential energy as an example, a weight hoisted above the Earth's surface certainly possesses potential energy, and we might be tempted to quantify this potential energy by simply measuring its weight and its height above ground level and then multiplying those two quantities (i.e. potential energy equals force times distance). However, there are more places this weight could *potentially* fall than just ground level. That same weight, hoisted above the Earth's surface to the same height, will be assessed to have a far greater amount of potential energy if we consider it potentially falling down a deep hole rather than just falling to ground level. For that matter, one could even argue the weight has a *negative* quantity of potential energy with respect to what it would possess at the summit of the highest mountain! Potential energy, defined as the amount of work that *could* be done, necessarily depends on a system's initial condition *and* any number of different final conditions of which none are known to be truly zero-energy.

How do we rate the height of mountain peaks? Certainly, we may rate one mountain peak as being some number of meters higher or lower than another, but generally what people do instead is measure a mountain's peak height compared to *Mean Sea Level*. Since water is a liquid, and liquid tend to settle at equal heights under the influence of gravity when they are free to move about, the level of oceans throughout the world provide a relatively stable reference point for measurements of altitude. If we imagine a world without oceans, perhaps one with nothing but dry land and no open bodies of water anywhere, people living at different locations in this world would likely choose some flat-land reference points local (to them) for their measurements of nearby mountains, but these localized altitude measurements would be confusing if not meaningless for world-wide comparisons of mountain peaks.

Much the same is true of electrical and electronic circuits, where voltage is an important measure of electrical potential energy as well as a measure of signal strength where electricity is used as an analogue for representing other things[5]. One way to conveniently rate electrical potentials at different points in a circuit is by arbitrarily choosing one of those points to be the "zero-energy" reference point, and calling it *ground*. Ground in an electrical or electronic circuit is analogous to sea level for rating mountain peak heights or a *datum* or *benchmark* used for surveying land: simply a point of reference agreed-upon to be "zero" for the sake of ranking all other potentials to it.

A good example of this is an electrical network popularly known as a *split DC power supply*. Three different versions of this power supply network are shown with the same pair of 5-Volt DC sources, the only difference being the arbitrary location of the "Ground" (Gnd) point:

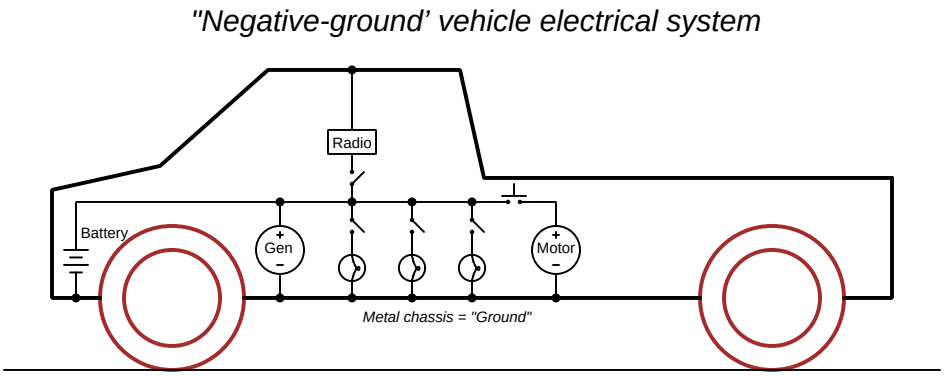


Each terminal's voltage label represents the amount of electrical potential there *relative to the "Ground" terminal*; i.e. "Ground" is simply a "sea-level" reference point against which all other potentials may be measured. The three different versions shown here illustrate just how arbitrary the location of "Ground" really is. Any load connected between the same pairs of terminals on each power supply would experience the same amount of voltage regardless of the "Ground" terminal assignment.

This is one meaning of the word "ground" in electric and electronic circuits: an arbitrary point of reference for the measurement and comparison of electrical power and signal potentials at different locations in the circuit.

[5] Many, many examples exist of voltage being used to represent non-electrical quantities. In digital computer systems, DC voltage levels represent binary "1" and "0" number states. In analog electronic sensor systems, voltage may represent temperature, speed, force, weight, sound pressure, or any number of other physical quantities.

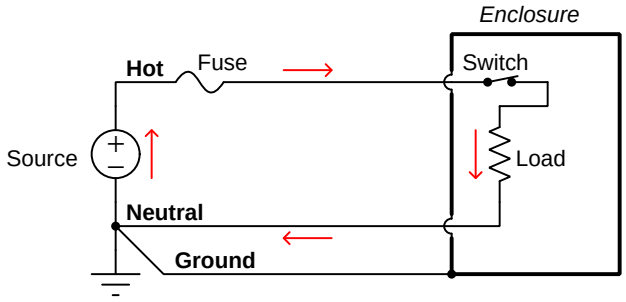
Another meaning of the word "ground" in circuits is a metallic surface used as a common conductor throughout some device or system for reasons of wiring economy. Automobiles with metal frames and chassis typically use their metallic bodies as an electrical conductor for nearly all circuits contained within. Doing so eliminates the need to install separate copper-wire conductors for every source and every load, thereby saving expense and weight:



Modern automotive electrical systems are "negative-ground" which means the vehicle chassis is always the lowest-potential point in any portion of the circuit. Interestingly, some early automobiles used "positive-ground" wiring which simply had the reverse polarity on all sources and loads: the vehicle chassis being the highest-potential point and all other points being either equal to it or at lower (−) potentials. This choice of polarity is really arbitrary, as electric circuits may be designed and built to function just as well either way. The "negative-ground" standardization we see today is really nothing more than automobile manufacturers agreeing to wiring their different vehicles the same way in order to enjoy interoperability of components.

This usage of the word *ground* – describing the metal chassis of some complex system exploited as a common electrical conductor for multiple circuits – is not incompatible with the previous usage of *ground*: a point of reference in a circuit arbitrarily defined to be "zero" potential. An automotive technician troubleshooting circuit faults in a vehicle often connects the black test lead of their voltmeter to the metal chassis, expecting to measure positive or zero voltage values on that meter when probing circuit test points with the red test lead. In a negative-ground electrical system, the chassis' electrical potential is *defined* as zero for the purpose of specifying voltages at other points in the circuit(s).

Yet another electrical meaning of the word "ground" is associated with *safety*, in which one metallic conductor of a power circuit is made electrically common to the Earth through a *grounding rod* buried in the soil. In the following circuit, the ground symbol in the lower-left corner is not merely a reference marker intended for anyone viewing the schematic diagram, but actually represents a metal rod driven deep into the soil and connected to one terminal of the voltage source with intent to achieve equipotentiality with the Earth:



Here the purpose of "grounding" is more important than merely establishing a reference point for voltage measurements. Instead, designating the conductor attached to this metal rod as "Ground" and attaching it to any conductive surface on or around an otherwise energized electrical appliance forces that surface to be equipotential to the same Earth that any humans and animals are standing on. This greatly mitigates the risk of electric shock should there be an internal fault within that appliance by making it impossible to have a difference of potential (i.e. voltage) between the grounded surface and the soil.

The following photograph shows the grounding rod for a commercial building, a large-gauge bare copper conductor clamped to the top of the rod which leads into the building's electrical room:

Ground wires in electric power systems should never carry current during normal operation. In fact, the presence of current through a ground conductor indicates a problem in the system! Note the two current-bearing power conductors in the previous schematic, specifically how the current-carrying conductor made common to Earth ground is called the *neutral*, while the other (non-grounded) power conductors is called *hot* by comparison. The Hot/Neutral designations always refer to which is grounded and which is not, rather than polarity. For example, if we were to reverse

the polarity of the DC voltage source shown in that schematic, the upper conductor (now −) would still be considered "hot" and the lower conductor (now +) would still be considered "neutral".

Chapter 6

Programming References

A powerful tool for mathematical modeling is text-based *computer programming*. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, *C++* and *Python*.

6.1 Programming in C++

One of the more popular text-based computer programming languages is called *C++*. This is a *compiled* language, which means you must create a plain-text file containing C++ code using a program called a *text editor*, then execute a software application called a *compiler* to translate your "source code" into instructions directly understandable to the computer. Here is an example of "source code" for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer's console:

```
#include <iostream>
using namespace std;

int main (void)
{
  float x, y;

  x = 200;
  y = -560.5;

  cout << "This simple program performs basic arithmetic on" << endl;
  cout << "the two numbers " << x << " and " << y << " and then" << endl;
  cout << "displays the results on the computer's console." << endl;

  cout << endl;

  cout << "Sum = " << x + y << endl;
  cout << "Difference = " << x - y << endl;
  cout << "Product = " << x * y << endl;
  cout << "Quotient of " << x / y << endl;

  return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other "whitespace" is largely irrelevant in C++ code, and is included only to make the code more pleasing[1] to view.

[1] Although not included in this example, *comments* preceded by double-forward slash characters (`//`) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.

Let's examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }, often referred to as "curly-braces").

- `int main (void)` labels the "Main" function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an *integer* number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program's completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called *arguments* to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer's memory for two *floating-point* variables, in this case the variables' names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line's end is necessary because C++ ignores whitespace on the page and doesn't "know" otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the "standard output" stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent *toward* the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer's console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.

After saving this *source code* text to a file with its own name (e.g. `myprogram.cpp`), you would then *compile* the source code into an *executable* file which the computer may then run. If you are using a console-based compiler such as *GCC* (very popular within variants of the Unix operating system[2], such as Linux and Apple's OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the `GCC` compiler to take your source code (`myprogram.cpp`) and create with it an executable file named `myprogram.exe`. Simply typing `./myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio[3], you may simply create a new console application "project" using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on
the two numbers 200 and -560.5 and then
displays the results on the computer's console.

Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

[2] A very functional option for users of Microsoft Windows is called *Cygwin*, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

[3] Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the "Hello World" `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile ("Build") and run your new program. Upon execution a console window will appear showing the output of your program.

learn computer programming by closely examining others' example programs and modifying them than it is to write your own programs starting from a blank screen.

6.2 Programming in Python

Another text-based computer programming language called *Python* allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an *interpreted* language: a software application called an *interpreter* reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3`[4] and pressing the Enter key:

```
Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>>
```

The `>>>` symbols represent the prompt within the Python interpreter "shell", signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

[4] Using version 3 of Python, which is the latest at the time of this writing.

More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which "imports" these functions from Python's math *library* (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in *radians* rather than *degrees*, and how Python provides convenient functions for translating between the two. Logarithms assume a base of e unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person's first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.

Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using *phasors*[5] as shown in the following example. Here we see Python's interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor's impedance (`zc`) as $X_C \angle -90^o$ with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values ($0 - jX_c$ Ω and $0 + jX_l$ Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω $\angle$ -25.102^o). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in *radians*; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python's interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

[5] A "phasor" is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.

assignment of variables as well as a convenient text record[6] of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter's shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

```
x = 200
y = -560.5

print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
```

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace *is* significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console's prompt, at which point it would display the following result:

```
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
```

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

[6] Like many command-line computing environments, Python's interpreter supports "up-arrow" recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.

If you are interested in learning more about computer programming in *any* language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.

6.3 Modeling a parallel circuit using C++

Here is an example C++ program intended to display a crude representation of a three-resistor parallel circuit and then calculate all relevant voltages, currents, and powers:

```
#include <iostream>
#include <cmath>
using namespace std;

int main (void)
{
  float i1, r1, r2, r3, rtotal, v, ir1, ir2, ir3, pr1, pr2, pr3, ptotal;

  i1 = 90e-3;
  r1 = 2.2e3;
  r2 = 710;
  r3 = 1e3;

  cout << "+----+---+---+ " << endl;
  cout << "|    |   |   | " << endl;
  cout << "I1  R1  R2  R3 " << endl;
  cout << "|    |   |   | " << endl;
  cout << "+----+---+---+ " << endl;

  cout << "I1 = " << i1 << " Amperes" << endl;
  cout << "R1 = " << r1 << " Ohms" << endl;
  cout << "R2 = " << r2 << " Ohms" << endl;
  cout << "R3 = " << r3 << " Ohms" << endl;

  rtotal = 1 / ((1 / r1) + (1 / r2) + (1 / r3));

  v = i1 * rtotal;

  cout << "V = " << v << " Volts" << endl;

  ir1 = v / r1;
  ir2 = v / r2;
  ir3 = v / r3;

  cout << "IR1 = " << ir1 << " Amperes" << endl;
  cout << "IR2 = " << ir2 << " Amperes" << endl;
  cout << "IR3 = " << ir3 << " Amperes" << endl;

  pr1 = pow(ir1,2) * r1;
```

```
   pr2 = pow(ir2,2) * r2;
   pr3 = pow(ir3,2) * r3;

   ptotal = pr1 + pr2 + pr3;

   cout << "PR1 = " << pr1 << " Watts" << endl;
   cout << "PR2 = " << pr2 << " Watts" << endl;
   cout << "PR3 = " << pr3 << " Watts" << endl;
   cout << "PTOTAL = " << ptotal << " Watts" << endl;

   return 0;
}
```

When compiled and executed, this program generates the following output:

```
+----+---+---+
|    |   |   |
I1  R1  R2  R3
|    |   |   |
+----+---+---+
I1 = 0.09 Amperes
R1 = 2200 Ohms
R2 = 710 Ohms
R3 = 1000 Ohms
V = 31.4356 Volts
IR1 = 0.0142889 Amperes
IR2 = 0.0442755 Amperes
IR3 = 0.0314356 Amperes
PR1 = 0.44918 Watts
PR2 = 1.39183 Watts
PR3 = 0.988197 Watts
PTOTAL = 2.8292 Watts
```

Chapter 7

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

7.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Conservation of Electric Charge

Reductio ad absurdum

Voltage

Conductors versus Insulators

Resistance

Current

Electric circuit

Electrical source

Electrical load

Equipotential points

Electrically common points

Open

Short

Switch

Ohm's Law

Joule's Law

Parallel

Bonding

Voltage source

Current source

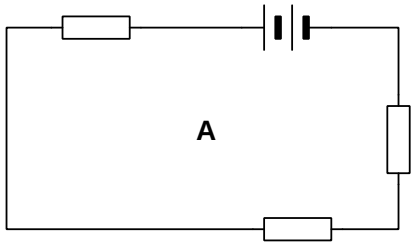
Properties of parallel circuits

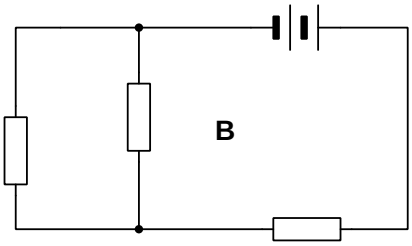
Current divider

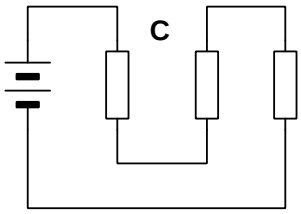
Potentiometer

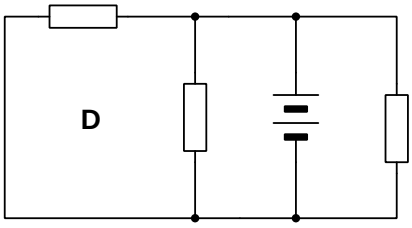
7.1.3 Identifying parallel circuits

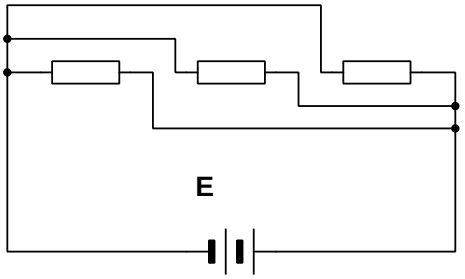
Identify which of these circuits is a *parallel* circuit (there may be more than one shown!)

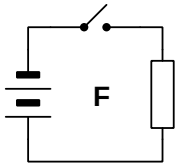












For every circuit that is *not* parallel, explain why.

How do component voltages relate to total (source) voltage in a parallel circuit, and why?

How do component currents relate to total (source) current in a parallel circuit, and why?

How do component resistances relate to total resistance in a parallel circuit, and why?

Challenges

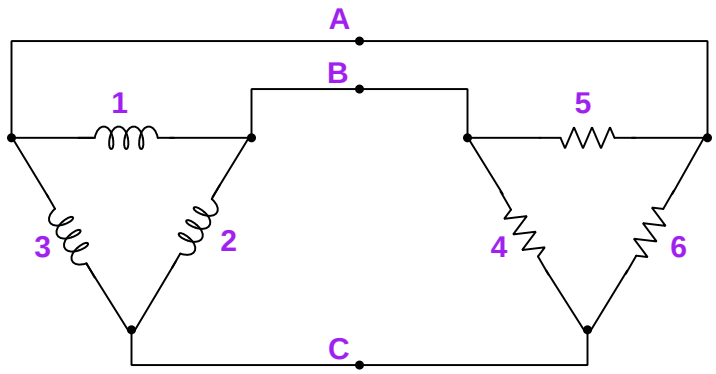
- A common practice in industrial control system wiring is to label every wire with a number,

and to use common numbers for wires that are equipotential with each other. Apply this principle to any of the circuits shown.

- Identify a place where a single fuse might be installed, which would interrupt power to all loads when it blows.

7.1.4 Identifying parallel points and elements

Identify all labeled points as well as elements (i.e. components) in this circuit that are connected in *parallel* with each other):



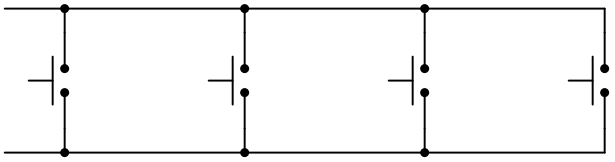
Challenges

- If component 5 happens to fail open, will any of the parallel-connected relationships change?

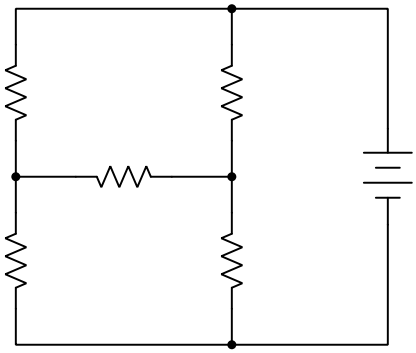
7.1.5 Parallel misconceptions

A very common misconception among new students learning about parallel circuit networks is to try to identify parallel connections by the presence of multiple pathways for current. For example, you may hear a new student say something like, "parallel connections are where current splits and goes different directions."

Examine the following parallel network, comprised of three switches (all in their *open* positions), and explain why that false definition of parallel fails:



Examine the following network comprised of five resistors, *none* of them being in parallel with one another, and explain why that false definition of parallel fails:



Finally, provide an accurate definition for *parallel* that cannot be mis-applied in either of these networks.

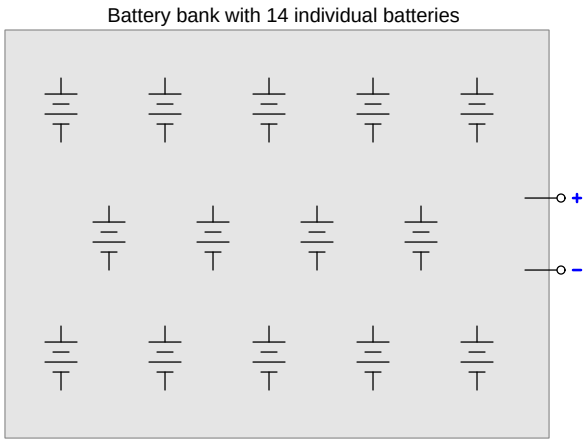
Challenges

- Sketch another circuit or network that serves to reveal the incorrect nature of the false parallel definition ("current splits").

7.1.6 Battery bank connections

Secondary-cell batteries are electro-chemical devices capable of absorbing and releasing electrical energy, i.e. alternately acting as loads and as sources. They are often used as energy-storage media for off-grid electrical power systems where the dominant energy sources are solar (photovoltaic) and wind power, which of course do not reliably deliver energy at all times of the day.

One battery by itself does not store much energy, and so multiple batteries are typically connected to form a larger "bank" for practical energy storage purposes. Suppose we were designing such a battery bank where each battery was rated at 12.6 Volts and a maximum output current of 75 Amperes. In order to store enough energy to serve our needs, it has been decided that fourteen of these batteries are necessary. Determine how these batteries could be connected together to achieve maximum current for the bank as a whole, and then calculate how much that maximum current would be. A partial schematic diagram has been provided for you below, showing all fourteen secondary-cell batteries and the bank's output terminals:

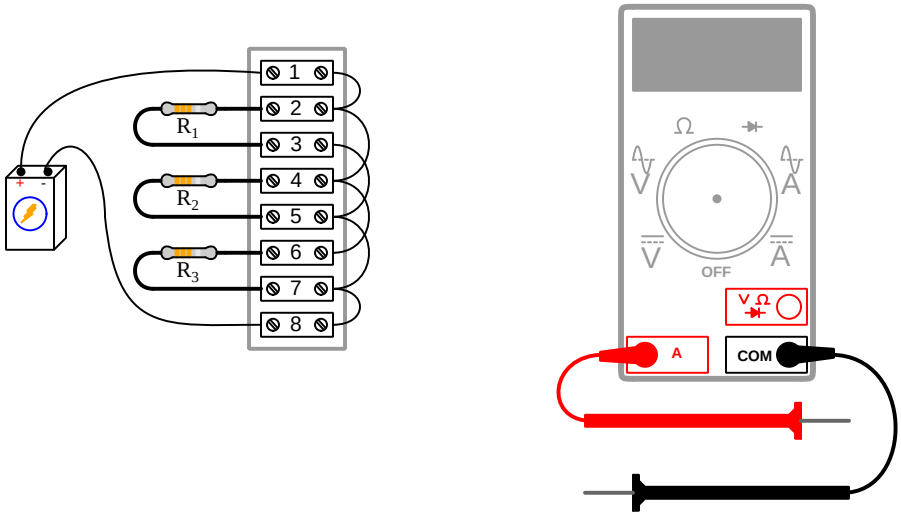


Challenges

- With the bank's batteries wired together for maximum output current, calculate the bank's nominal output voltage value.
- With the bank's batteries wired together for maximum output current, what will happen if any one of the batteries fails "open"?
- With the bank's batteries wired together for maximum output current, what will happen if any one of the batteries fails "shorted"?

7.1.7 Measuring current in a parallel circuit

Identify appropriate ways to connect an ammeter to measure current through each of the three resistors in this parallel circuit, one resistor at a time. Be sure to specify which terminal the meter's red lead should contact, versus which terminal the meter's black lead should contact, in order to ensure a *positive* reading:



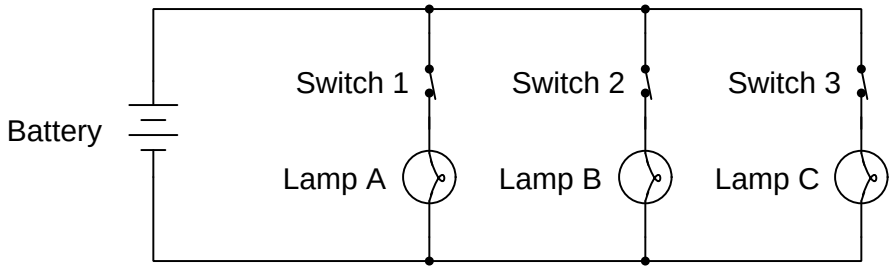
Note: multiple correct answers exist for every current measurement in this circuit!

Challenges

- It is surprisingly easy to blow the fuse inside of an ammeter by connecting it incorrectly to the circuit under test. Identify at least two different ways the ammeter might be connected to this circuit which would blow the fuse.
- Identify which terminals to touch with the ammeter probes to measure the combined current through R_2 and R_3.
- Predict the effects of resistor R_1 failing open.
- Predict the effects of resistor R_2 failing open.
- Predict the effects of resistor R_3 failing open.
- Predict the effects of resistor R_1 failing shorted.
- Predict the effects of resistor R_2 failing shorted.
- Predict the effects of resistor R_3 failing shorted.

7.1.8 Parallel lamp circuit with switches

Three incandescent lamps are wired in parallel with each other, each lamp having its own control switch wired in-line with it. With all three control switches in the "closed" state (as shown), the three lamps glow at normal brightness:



Describe the effects of opening switch 2, while leaving switches 1 and 3 closed.

Describe the effects of opening switches 1 and 2, while leaving switch 3 closed.

Describe the effects of opening all three switches.

Challenges

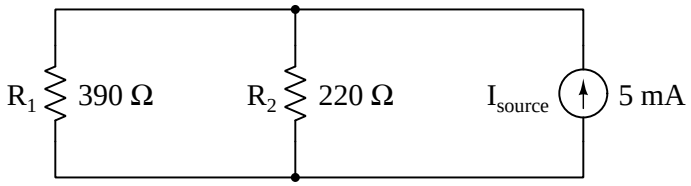
- Choose a good location for a *fuse* to protect all conductors within this circuit from overcurrent.
- Choose a good location for a *fuse* to protect just those conductors serving lamps B and C from overcurrent.
- Which combination of switch states results in the circuit exhibiting minimum resistance?
- Which combination of switch states results in the circuit exhibiting maximum resistance?
- Identify how a second battery could be installed in the circuit to make the lamps glow brighter than they do now.

7.1.9 Explaining the meaning of calculations

An unfortunate tendency among beginning students in any quantitative discipline is to perform calculations without regard for the real-world meanings of the values, and also to follow mathematical formulae without considering the general principles embodied in each. To ignore concepts while performing calculations is a serious error for a variety of reasons, not the least of which being an increased likelihood of computing results that turn out to be nonsense.

In the spirit of honoring concepts, I present to you a quantitative problem where all the calculations have been done for you, but all variable labels, units, and other identifying data have been stripped away. Your task is to *assign proper meaning* to each of the numbers, identify the correct unit of measurement in each case, apply any appropriate metric prefixes to those values, explain the significance of each value by describing where it "fits" into the circuit being analyzed, and identify the general principle employed at each step.

Here is the schematic diagram of the parallel circuit:



Here are all the calculations performed in order from first to last:

1. $\frac{1}{\frac{1}{390}+\frac{1}{220}} = 140.66$
2. $(5 \times 10^{-3}) \times (140.66) = 0.7033$
3. $\frac{0.7033}{390} = 1.803 \times 10^{-3}$
4. $\frac{0.7033}{220} = 3.197 \times 10^{-3}$
5. $(1.803 \times 10^{-3}) + (3.197 \times 10^{-3}) = 5 \times 10^{-3}$
6. $(1.803 \times 10^{-3}) \times (0.7033) = 1.268 \times 10^{-3}$
7. $(3.197 \times 10^{-3}) \times (0.7033) = 2.248 \times 10^{-3}$
8. $(5 \times 10^{-3})^2 \times (140.66) = 3.516 \times 10^{-3}$
9. $(1.268 \times 10^{-3}) + (2.248 \times 10^{-3}) = 3.516 \times 10^{-3}$

Explain what each value means in the circuit, identify its unit of measurement and appropriate metric prefix, and identify the general principle used to compute it!

Challenges

- Explain how you can check your own thinking as you solve quantitative problems, to avoid the dilemma of just "crunching numbers" to get an answer.
- For each calculated step shown, identify the physical Law or electric circuit principle being applied.
- Identify any of the calculations shown above whose order could be changed. In other words, can this problem be solved in a different order?
- Identify any of the calculations shown above that are not strictly necessary to the complete analysis of the circuit, explaining why it has any merit at all.
- Do you see any alternative paths to a solution, involving specific calculations not shown above?
- When performing addition or subtraction on physical quantities, all units of measurement must be identical. Identify where you see this principle being applied in the steps above.

7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

7.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

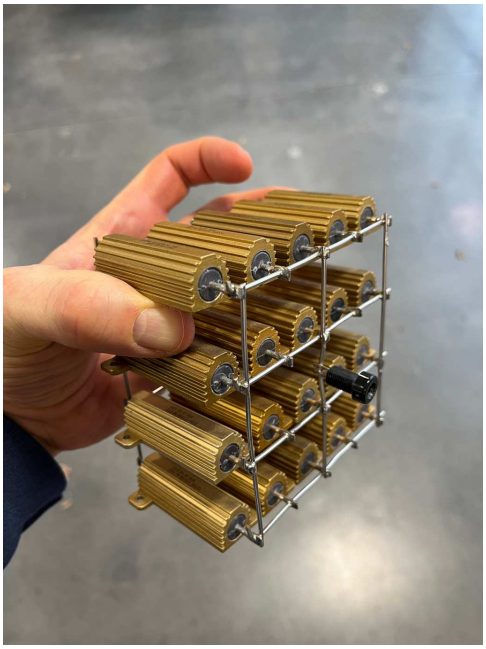
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

7.2.3 Resistor network

Determine the total electrical resistance of this soldered resistor network, based on the fact that each of these resistors is identical and has a 20 Ohm rating:

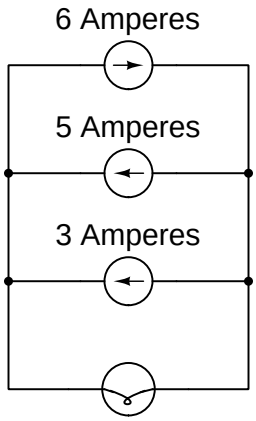


Challenges

- If each resistor dissipated 1.5 Watts of power, how much would the total power dissipation of the network be?

7.2.4 Three-source, one-lamp circuit

How much current does the lamp experience in this circuit? Explain your answer.



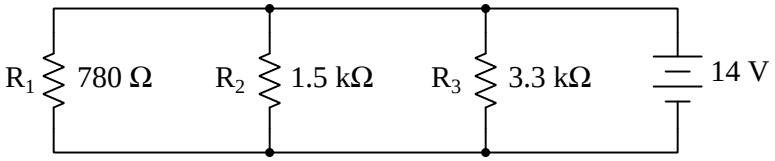
Also, identify the polarity of the voltage across the lamp (mark with "+" and "−" signs), and sketch arrows showing the direction of current through the lamp.

Challenges

- Describe how to re-connect these three sources to achieve the brightest possible operation of the lamp.
- Describe how to re-connect these three sources to *double* the amount of current through the lamp.
- Describe how to re-connect these three sources to achieve the dimmest possible operation of the lamp.

7.2.5 VIRP table for a three-resistor parallel circuit

Complete the table of voltage, current, resistance, and power values (V, I, R, and P) for the circuit shown:



	R1	R2	R3	Total
V				14 V
I				
R	780 Ω	1.5 kΩ	3.3 kΩ	
P				

Also, show how you may double-check your work without necessarily using a calculator (i.e. use "mental math" and estimation techniques).

Challenges

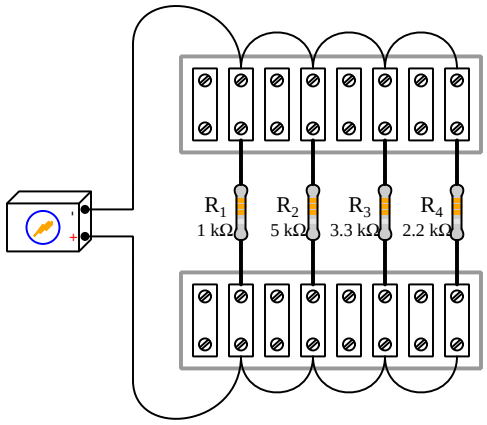
- Identify the effects of any single resistor failing open.
- Identify the effects of any single resistor failing shorted.
- Identify an alteration one could make to this circuit to increase the total current, without changing anything about the source.
- Identify an alteration one could make to this circuit to decrease resistor R_2's voltage.
- Identify an alteration one could make to this circuit to increase resistor R_1's voltage.
- Identify which data represented in this table illustrate a general principle of parallel circuits.
- Identify which data represented in this table illustrate the *Law of Energy Conservation* in action.
- Identify which data represented in this table illustrate the *Law of Electric Charge Conservation* in action.

7.2.6 Solving for a parallel branch resistance

A four-resistor parallel network contains a 10 kΩ resistor, a 5.1 kΩ resistor, a 3.9 kΩ resistor, and a fourth resistor with an undecided value. Calculate the necessary resistance value for this fourth resistor to give the entire parallel network a resistance value of 1.3 kΩ.

7.2.7 Current divider circuit

The circuit shown below is commonly called a *current divider* because it splits the source's current into multiple, smaller currents of fixed proportion:



Calculate the current division percentage for each resistor (i.e. the proportion of source current that each branch current represents):

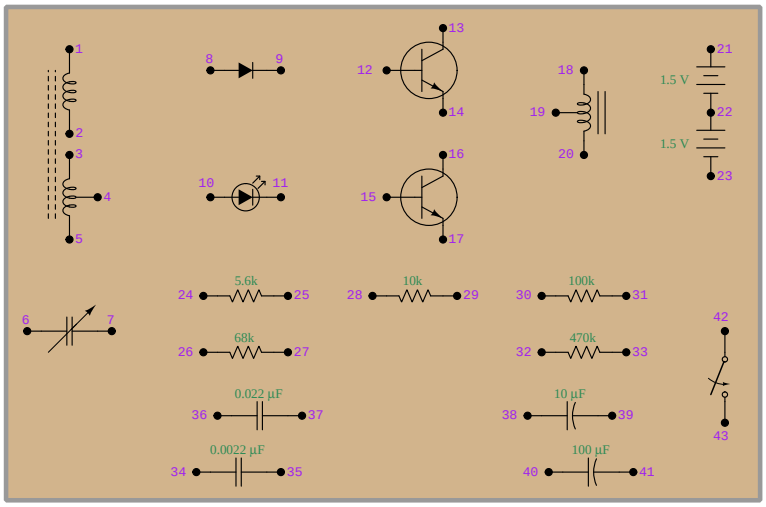
- I_{R1} percentage =
- I_{R2} percentage =
- I_{R3} percentage =
- I_{R4} percentage =

Challenges

- An appropriate problem-solving technique for portions of problems that are qualitative (i.e. no numerical figures) is to incorporate your own numerical values and use quantitative techniques to analyze. Identify a way you could incorporate a definite quantity into this problem to make it easier to solve.
- Predict the effects of resistor R_1 decreasing in value, on all the current division proportion (percentage) values.
- Predict the effects of resistor R_3 increasing in value, on all the current division proportion (percentage) values.
- Predict the effects of any one resistor failing open, on all the current division proportion (percentage) values.

7.2.8 Current divider built using a Radio Shack 30-in-one kit

Decades ago a series of experimenter kits were sold by Radio Shack where components were fixed to a cardboard panel and connections between them made using lengths of hook-up wire inserted into spring clips at each component terminal. Each of these spring clips bore a unique identifying number, which meant circuit configurations could be documented via a "Wiring sequence" calling out which of the spring clips needed to be made electrically common to each other.



Sketch wires connecting the terminals specified in the following wiring sequence, and then calculate each of the listed currents. Be sure to describe which direction current flows through the component, assuming conventional flow notation (i.e. positive charge carriers):

Wiring sequence: `22-31-29-26, 21-28-27-30`

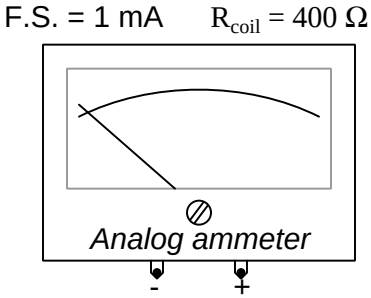
- I_{10k} =
- I_{100k} =
- I_{68k} =

Challenges

- What are the properties of parallel resistor networks?

7.2.9 Shunt resistor for an ammeter

Ammeters are designed to measure the amount of electric current passing through them, and in order to present minimal burden to the circuit being tested, ammeters are also designed to have as little internal resistance as possible. The following illustration is of an *analog*[11] ammeter based on a moving coil of wire and a stationary magnet, with its full-scale ("F.S.") current rating and internal coil resistance values given:



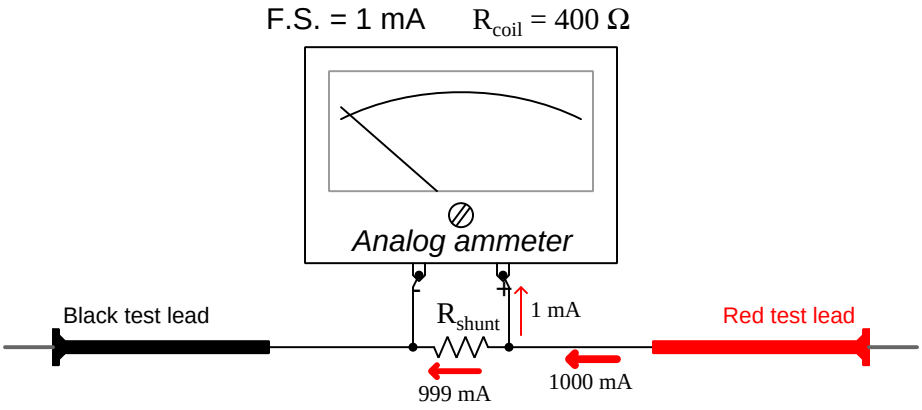
This means the ammeter's needle will be deflected precisely to the full-scale (fully-right) position with exactly 1 milliAmpere of current passing through the 400 Ohm internal wire coil. In other words, this ammeter has a range of 0 to 1 milliAmpere (0-1 mA), and no more.

One milliAmpere is not a lot of electrical current, and so this ammeter will be of limited use to us in its present form. It would be greatly beneficial to our needs if we could figure out a way to extend the range of this ammeter to be able to measure higher currents. Fortunately, a simple technique exists for this, which requires the connection of a *shunt resistor* in parallel with the analog ammeter. The purpose of this "shunt"[12] resistor is to bypass a precise fraction of current past the meter unit so that a larger (total) current is necessary to cause full-scale deflection of the needle.

[11] An "analog" system is one where all quantities are *continuous* in nature, as opposed to a "digital" system where quantities exist in discrete units. An analog ammeter uses a needle (pointer) over a printed scale, representing current magnitude by how far the needle gets deflected over the range of the scale. Since the needle is free to move in a continuous manner up and down the scale (i.e. there is no minimum amount of motion it is restricted to), the analog ammeter can – at least in theory – represent vanishingly small increments in current. A digital ammeter, by contrast, uses decimal digits to display the measured current, and as such has limited resolution (i.e. a meter with four digits cannot show increments of current smaller than that least-significant digit).

[12] The term *shunt* is a synonym for *parallel*.

For example, if we wanted to extend the range of this analog ammeter to 0-1 Ampere, we would have to connect a shunt resistor that bypassed 999 mA of current while letting 1 mA go through the analog meter unit at a measured current of 1 A (1000 mA):



Calculate the value of shunt resistor for the range shown (0-1 A), and then calculate the necessary shunt resistor values for the following ranges as well, all assuming the use of the same 0-1 mA, 400 Ω analog ammeter unit:

- 0-5 A range ; R_{shunt} =
- 0-250 mA range ; R_{shunt} =
- 0-100 mA range ; R_{shunt} =

Challenges

- What will happen if someone happens to connect this ammeter incorrectly to a circuit and cause current to go the opposite direction through it (i.e. conventional flow in the black test lead and out the red)?
- Predict the effects of the shunt resistor failing open.
- Predict the effects of the shunt resistor failing shorted.
- Explain why no R_{shunt} value is capable of giving this ammeter a full-scale measurement range of *less than* 1 mA.

7.2.10 Designing an electric heater

Suppose you need to design an electric heater for an industrial enclosure which will contain moisture-sensitive equipment. The purpose of this heater is to maintain an elevated temperature inside the enclosure so that moisture (i.e. dew) never condenses on this equipment.

A mechanical engineer determines that a heat output of 130 Watts should be sufficient for the task. The enclosure already contains a 25 Watt incandescent lamp that is always energized. Your task is to choose the necessary heating element resistance value for the electric heater such that the total heat dissipated inside this enclosure will be 130 Watts.

Assume an electrical power source of 115 Volts.

Challenges

- Does your calculated heater resistance value represent a *minimum* or a *maximum* resistance value in order to maintain a dew-free environment inside the enclosure?
- If a new series of enclosures destined for the European market were being designed, where the line voltage is typically 220 Volts instead of 115 Volts as it is in North America, would the heater need a greater or a lesser resistance value to dissipate the same amount of power?
- Identify any assumptions implicit in this design problem, that may not necessarily be true for a real enclosure housing real equipment in a real environment.

7.2.11 Interpreting a SPICE analysis

A computer program called SPICE was developed in the early 1970's, whereby a text-based description of an electric circuit (called a *netlist*) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Here is a netlist text for a three-resistor parallel circuit powered by a DC voltage source, each of the three resistors equipped with its own ammeter in the form of a zero-voltage source:

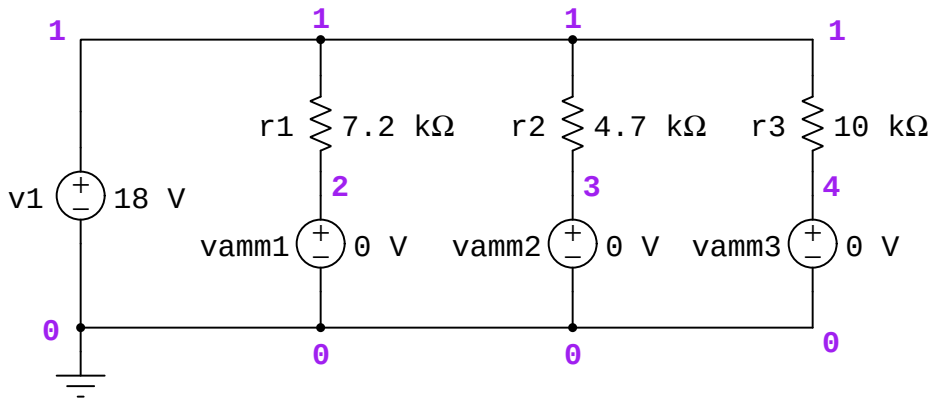
```
* Three-resistor parallel DC circuit
v1 1 0 dc 18
r1 1 2 7200
r2 1 3 4700
r3 1 4 10k
vamm1 2 0 dc 0
vamm2 3 0 dc 0
vamm3 4 0 dc 0
.dc v1 18 18 1
.print dc v(1,2) v(1,3) v(1,4)
.print dc i(vamm1) i(vamm2) i(vamm3)
.print dc i(v1)
.end
```

One of the fundamental concepts in SPICE programming is that connections between components in the virtual circuit to be analyzed are specified by identifying *node* numbers. All points in the circuit known to be equipotential must bear the same number. In this particular example, the only nodes in existence are `0` through `4`. SPICE always assumes that node `0` is "ground" (i.e. a common point of reference for default voltage measurements).

Early versions[13] of SPICE provided no means for displaying current through components other than voltage sources, and so three "dummy" voltage sources (each one having a DC voltage of zero Volts) have been included in this circuit as "ammeters" so we will be able to see the calculated value for current through each of the three resistors.

[13] In the interest of my readers, my goal is to provide netlists compatible with all versions of SPICE, new and legacy, wherever possible.

The following schematic diagram[14] shows one possible interpretation of this SPICE netlist, with each of the nodes numbered:



Identify how each component's connecting nodes and essential parameters (i.e. values) are specified in the netlist. Based on the what you can identify in this netlist, perform your own calculations using Ohm's Law to predict all other values in this circuit.

When SPICE (software version 2G6) processes the contents of this "netlist" file, it outputs a text description of the analysis. The following text has been edited for clarity (e.g. blank lines, extraneous characters, and statistical data removed):

```
 v1           v(1,2)       v(1,3)       v(1,4)
1.800E+01     1.800E+01    1.800E+01    1.800E+01

 v1           i(vamm1)     i(vamm2)     i(vamm3)
1.800E+01     2.500E-03    3.830E-03    1.800E-03

 v1           i(v1)
1.800E+01    -8.130E-03
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer's analysis agree with your own calculations.

Once you become familiar with the analysis data format of SPICE, you will be able to use the "Gallery" of SPICE simulations found in the *"SPICE Modeling of Resistor Circuits"* module as practice problems for developing your own circuit analysis skills.

[14] A recommended procedure to follow when generating your own SPICE netlist is to start with a schematic diagram, numbered each and every connecting wire in the circuit, with every equipotential point bearing the same node number. Once all nodes have been identified and numbered, it becomes easy to write each component line of the netlist.

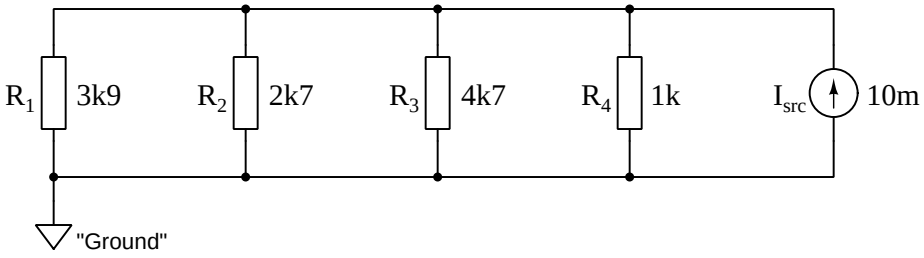
Challenges

- SPICE utilizes *power-of-ten* notation for small and large values. The standard representation of this in a plain-text format is a capital letter `E` followed by the power value. Identify any such figures in the SPICE output listing, and express them in regular decimal notation.
- Explain the purpose of having a "shunt" resistor in this circuit, and also why its resistance value is so low compared to the other resistor values.
- Modify this SPICE netlist to utilize a voltage source with a value of 6 Volts instead of 24 Volts.
- Modify this SPICE netlist to specify resistor R_1's resistance using scientific (power-of-ten) notation.
- Modify this SPICE netlist to specify resistor R_1's resistance using metric prefix ("kilo") notation.

7.2.12 Using SPICE to analyze a parallel circuit

A computer program called SPICE was developed in the early 1970's, whereby a text-based description of an electric circuit (called a *netlist*) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Write a netlist and perform a SPICE analysis on the following four-resistor circuit (this particular schematic utilizing European-style component symbols), instructing the computer to calculate current through each of the four resistors, as well as the voltage across the source:



If you are using a legacy version of SPICE such as version 2G6, the only way to instruct the computer to print a current value is through a voltage source. This means you will have to include "dummy" voltage sources (each one set to a voltage value of zero) in your circuit to function as ammeters.

Note: one of the most compelling reasons for beginning students of electronics to learn how to perform computer simulations of simple DC circuits is so they will be able to make their own practice problems for circuit analysis. If you learn to use SPICE as a tool, you will never lack for having practice problems!

7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

7.3.1 Interpreting an ammeter measurement

Suppose a technician uses an ammeter to measure 0.00 milliAmperes between two test points in a circuit. Identify which (if any) of the following statements are true, based on this one measurement:

1. Those test points are directly connected together by low resistance (i.e. they are "electrically common" or "shorted" to each other).
2. Those test points are connected together by high resistance.
3. Those test points are connected directly to a voltage source.
4. The circuit's power source is dead.

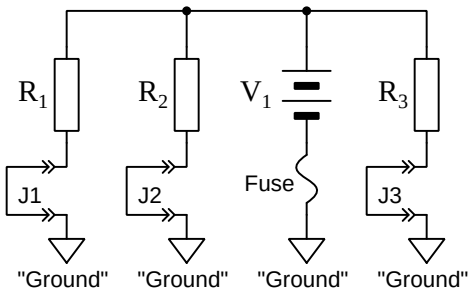
Now suppose you happened to know that those two test points in the circuit are supposed to have 33 milliAmperes of current passing between them through the ammeter, which is the normal operating current of the circuit's only source. Does this information alter any of your conclusions about the 0.00 milliAmpere meter reading? Why or why not?

Challenges

- Sketch a complete diagram of a simple circuit, identifying two points within it that would normally have 33 milliAmperes flowing from one, through the ammeter, to the other but due to a fault that you propose now have no current flowing at all.

7.3.2 Faults in a three-resistor circuit

Examine this schematic diagram of a three-resistor circuit, drawn using European component symbols, and also using a four "Ground"[15] symbols to represent one of the conductive connections to the battery:



A set of removable "jumpers" (J1 through J3) are provided to facilitate connecting an ammeter to measure branch currents in this parallel circuit. A single fuse protects all conductors and components from overcurrent.

Predict how all jumper currents in this circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):

- Resistor R_1 fails open:
- Resistor R_2 fails open:
- Resistor R_3 fails open:
- Solder bridge (short) past resistor R_1:
- Solder bridge (short) past resistor R_2:
- Solder bridge (short) past resistor R_3:

Challenges

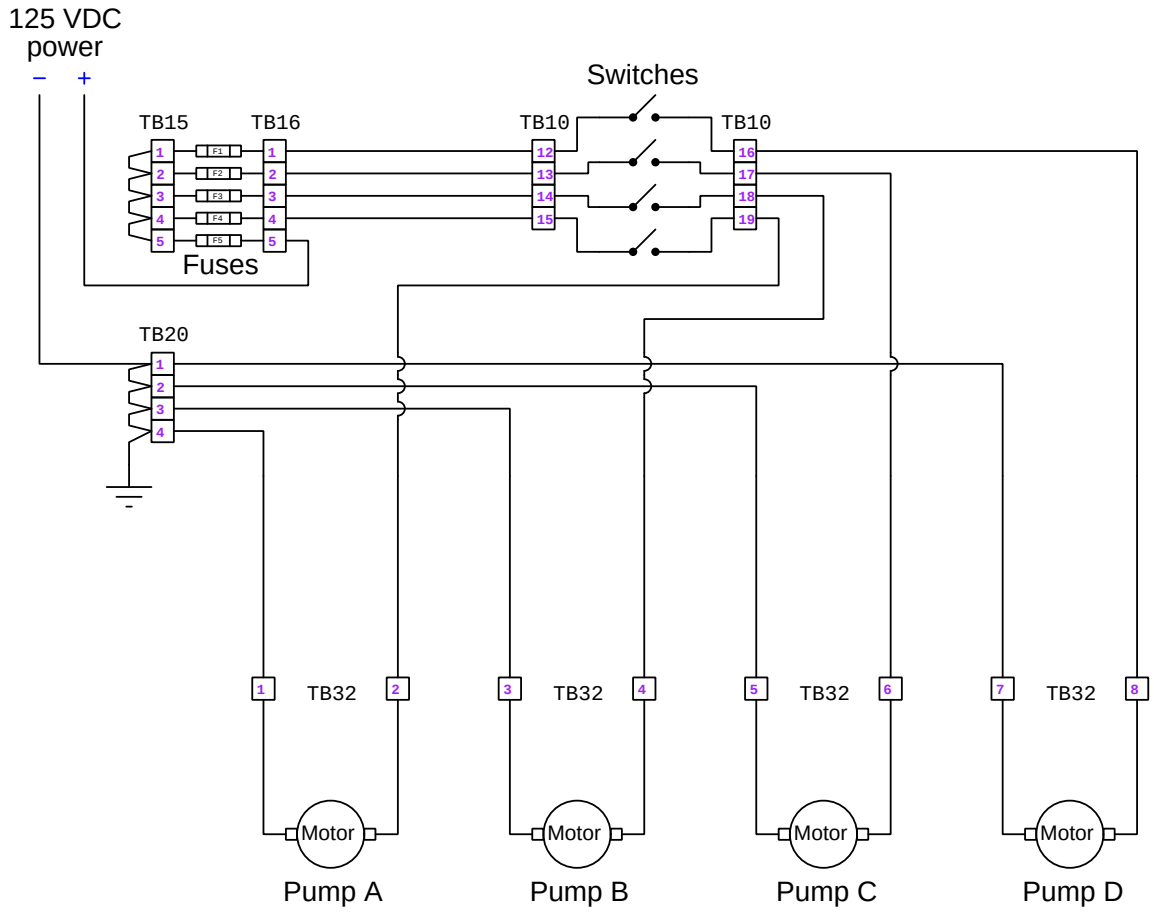
- An appropriate problem-solving technique for qualitative (i.e. no numerical figures) problems such as this is to incorporate your own component values and use quantitative techniques to analyze. Identify some "easy" values you could apply to this circuit to convert it from a qualitative problem into a quantitative problem.
- Suppose this circuit were built without a fuse. Would this alteration change any of your predictions for the effects of resistor faults?

[15] Here, "ground" does not literally refer to a connection to the Earth, but rather to a conductive surface or wire that is arbitrarily deemed the "reference" point or "ground-level" for electrical potential in this circuit. Any schematic diagram containing multiple "ground" symbols should interpreted as those symbols being connected directly together by a wire.

- Re-draw this circuit without using “ground” symbols.

7.3.3 Problem with a multiple-pump system

A set of electrically-powered water pumps is used to pump water out of the bilge of a large ship. Each of the four pumps is rated at 5 gallons per minute water flow when running, discharging that water overboard through a common pipe equipped with a water flowmeter. A schematic diagram shows how the four pumps (all located in the bilge, at the very bottom of the ship) receive power through four switches located in the engine control room above the bilge. Each of the "box" symbols in this schematic diagram represents a terminal block where wires connect with other wires:



One day there happens to be a major water spill into the ship's bilge, and the engine room operator on shift turns on all four pumps to pump that water overboard. However, he happens to notice the flowmeter register only 14 gallons per minute with all switches turned "on", when it normally registers a total flow rate of 20 gallons per minute. The electrician on shift uses a voltmeter to measure voltage across each of the fuses, and obtains 0 Volts for each one while all four switches remain in the "on" state.

Identify the likelihood of each specified fault for this bilge pump system. Consider each fault one at a time (i.e. no coincidental faults), determining whether or not each fault is compatible with *all* measurements and symptoms in this circuit.

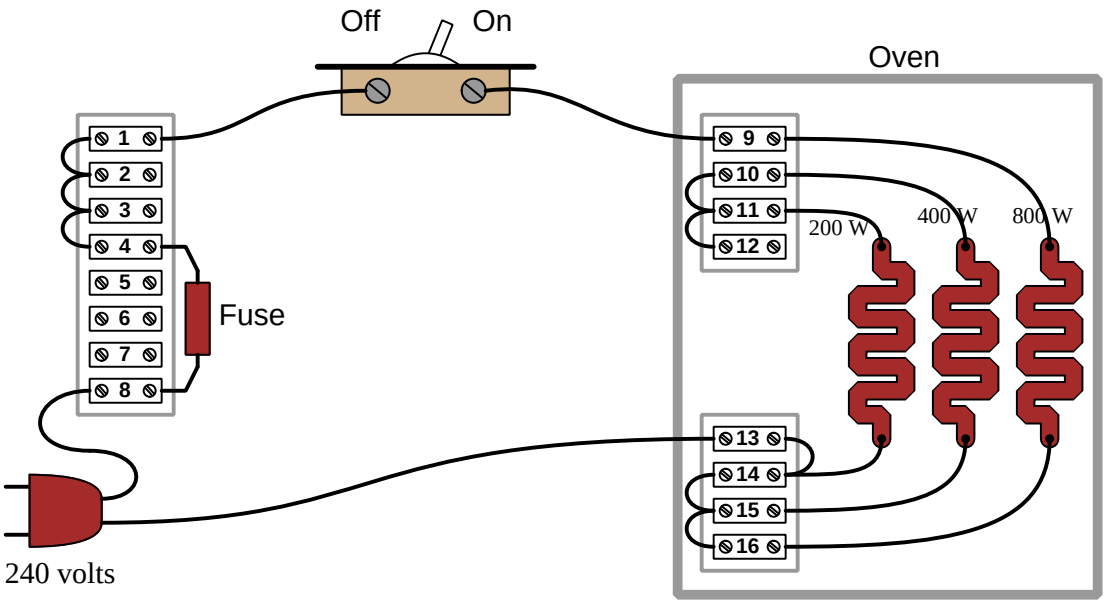
Fault	**Possible**	**Impossible**
Wire between TB15-5 and DC source failed open		
Wire between TB20-1 and TB32-7 failed open		
Wire between TB10-18 and TB32-4 failed open		
Wire between TB15-1 and TB15-2 failed open		
Wire between TB15-2 and TB15-3 failed open		
Wire between TB15-3 and TB15-4 failed open		
Switch between TB10-12 and TB10-16 failed open		
Wire between TB10-19 and TB32-2 shorted to ground		
Fuse F1 blown		
Fuse F2 blown		
Fuse F3 blown		
Fuse F4 blown		
Fuse F5 blown		
125 Volt DC source dead		

Challenges

- For each of the proposed fault, explain *why* you believe it is either possible or impossible.
- Suppose all the switches had been turned "off" when the electrician performed the voltage measurements across each fuse. Would this affect your assessment of possible faults? Why or why not?
- A helpful problem-solving technique when given a circuit in pictorial form is to re-draw that circuit in *schematic* form so that it is easier to analyze. Do so for this circuit, showing the proper placement of all the terminal numbers in the circuit.
- Explain the purpose of fuse F5, given the existence of fuses F1 through F4.

7.3.4 Malfunctioning oven

This electric oven has a problem somewhere – with the switch "On" and the plug inserted into a 240 Volt power receptacle, it heats up at a slower rate than usual. The fuse, which is supposed to automatically "open" to stop current if it becomes excessive, is rated at 8 Amperes. Inside the oven's heating chamber are three separate heater elements rated at 200 Watts, 400 Watts, and 800 Watts individually:



Suppose a technician uses a digital multimeter (DMM) to measure current, by connecting the ammeter across the switch terminals and then turning the switch to the "Off" position. The ammeter reads a current value of 4.2 Amperes.

First, explain why the technician connected the ammeter to the circuit as described. How, exactly, does this procedure ensure the meter will faithfully register the oven's total current? Is this the only way to measure current in the circuit? If not, how else could the ammeter have been connected into this circuit?

Next, identify the most likely fault in the circuit which could explain the oven's abnormally slow heating rate.

Finally, describe any subsequent diagnostic steps the technician could take to further isolate the identify and/or location of the fault.

Challenges

- What circumstances might lead to the fuse "blowing" open?.

- Suppose someone told you the fuse's terminals would be safe to touch with your finger because there should be negligible voltage dropped across an intact fuse. Explain why this is a misguided conclusion.

Chapter 8

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

8.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

8.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures[1] will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. "Live" work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact*[2] *with an energized conductor, no matter what the circuit's voltage*[3] *level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

[1] Professor Charles Dalziel published a research paper in 1961 called "The Deleterious Effects of Electric Shock" detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel's subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn't require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

[2] By "electrical contact" I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

[3] Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it's generally wise (and *always* a good habit to build) to "power down" *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. "cord grips" used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use "touch-safe" terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always.* This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always.* A fuse does no good if the wire or printed circuit board trace will "blow" before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always.* Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one's body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a "hot" line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert.* If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

8.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary[4]. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards[5], and wires that are simply twisted together.

- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work**. Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

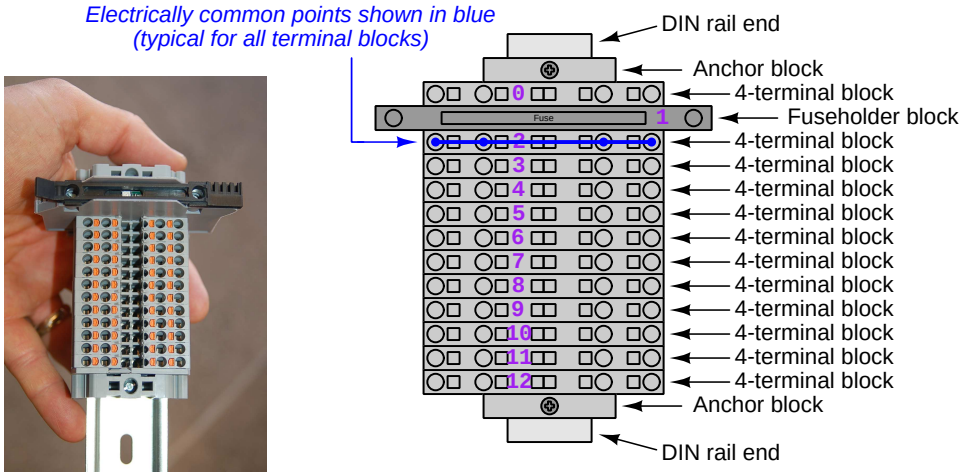
[4] An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

[5] Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

8.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems, and are also quite useful for student-built experiments, providing a robust alternative to solderless breadboards[6]. Terminal blocks provide highly reliable connections rated for significant voltage and current magnitudes, and they force the builder to think carefully about component placement which is an important mental practice. Terminal blocks designed to mount on standard 35 mm DIN rail[7] come in many types and capabilities.

Every student of electricity should build their own terminal block array consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other[8] and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a unique number starting from 0. An example is shown in the following photograph and illustration:



Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others such as the Phoenix Contact model 3211019 blocks shown in the photograph provide buttons[9] for this task which may be pressed using the tip of any suitable tool such as a small screwdriver or multimeter test probe tip.

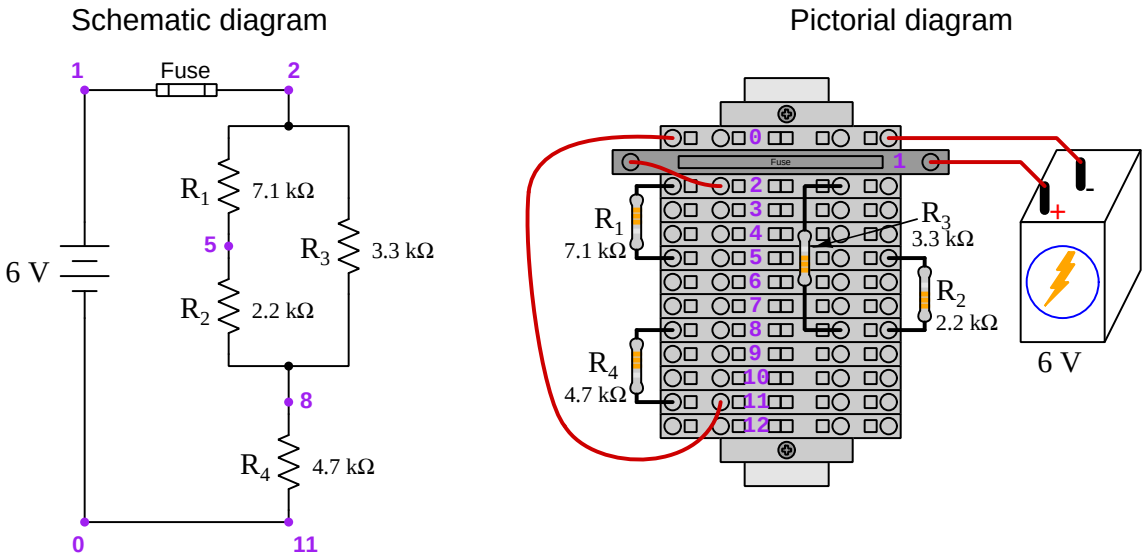
[6] Solderless breadboard are preferable for complicated electronic circuits with multiple integrated "chip" components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for "chip" circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

[7] DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

[8] Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

[9] The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:



Numbering on the terminal blocks provides a very natural translation to SPICE[10] netlists, where component connections are identified by terminal number:

```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rfuse 1 2 0.01
rwire 0 11 0.01
.op
.end
```

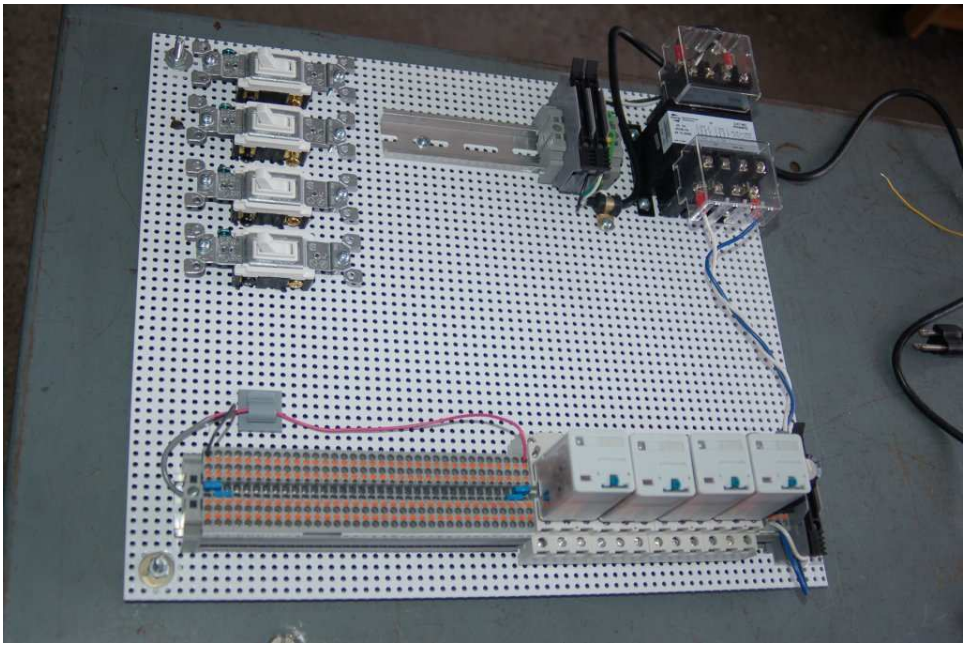
Note the use of two resistances `rfuse` and `rwire` to describe fuse resistance between terminals 1 and 2 and "jumper" wire resistance between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm appropriate to their real-life resistance values.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a

[10] SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

"wiring sequence" may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and "ice-cube" style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel[11]. This "terminal block board" hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT "ice-cube" relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed "feet" support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord's ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer's screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer's 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a "terminal block board" is an inexpensive[12] yet highly flexible means to construct physically robust circuits using industrial wiring practices.

[11] An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

[12] At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.

8.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

> An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*[13]. In order for an hypothesis to be valid, it must be testable[14], which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the "baseline" variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available[15].

[13] Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scient*ists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but scientific *method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one's hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

[14] This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

[15] A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting "data" from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even "bad" data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the "tolerance" of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of "successful" experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

```
Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional
to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage
DC power supply.  Use an ammeter in series to measure resistor current and
a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/
or pose a burn hazard, while excessive voltage poses an electric shock hazard.
30 Volts is a safe maximum voltage for laboratory practices, and according to
Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts
(P = V^2 / R), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:
  (Voltage)    (Current)         (Voltage)   (Current)
   0.000 V   =  0.000 mA          8.100    =  7.812 mA
   2.700 V   =  2.603 mA          10.00 V  =  9.643 mA
   5.400 V   =  5.206 mA          14.00 V  =  13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected
exactly one milliAmpere per Volt the actual current was usually less than
that.  The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts)
to a high of 1037.81 (at 14 Volts), but this represents a variance of only
-0.0365% to +0.0541% from the average, indicating a very consistent
proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself.  I
did not measure it, but simply assumed color bands of brown-black-red meant
exactly 1000 Ohms.  Based on the data I think the true resistance is closer
to 1037 Ohms.  Another possible explanation is multimeter calibration error.
However, neither explains the small positive and negative variances from the
average.  This might be due to electrical noise, a good test being to repeat
the same experiment to see if the variances are the same or different.  Noise
should generate slightly different results every time.
```

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

```
Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be
exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single
1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis
from 0 Volts to 25 Volts in 5 Volt increments.

    * SPICE circuit
    v1 1 0 dc
    r1 1 0 1000
    .dc v1 0 25 5
    .print dc v(1) i(v1)
    .end

RISKS AND MITIGATION: none.

DATA COLLECTED:
       DC transfer characteristic  Thu Feb 14 13:05:08  2019
    ------------------------------------------------------------
    Index   v-sweep         v(1)            v1#branch
    ------------------------------------------------------------
    0       0.000000e+00    0.000000e+00    0.000000e+00
    1       5.000000e+00    5.000000e+00    -5.00000e-03
    2       1.000000e+01    1.000000e+01    -1.00000e-02
    3       1.500000e+01    1.500000e+01    -1.50000e-02
    4       2.000000e+01    2.000000e+01    -2.00000e-02
    5       2.500000e+01    2.500000e+01    -2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely
1/1000 of the applied voltage for all values.  Anything other than perfect
agreement would have probably meant my netlist was incorrect.  The negative
current values surprised me, but it seems this is just how SPICE interprets
normal current through a DC voltage source.

ERROR SOURCES: none.
```

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

8.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead.*
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

8.2 Experiment: Ohm's Law in a three-resistor parallel circuit

In this experiment you will confirm the validity of parallel circuit principles as it applies to carbon composition resistors powered by a battery. For this experiment you will need:

- A primary-cell battery (6 or 9 Volts is sufficient)
- An assortment of carbon composition resistors between 1 kΩ and 100 kΩ
- A digital multimeter (DMM) capable of measuring DC voltage and DC current (milliAmperes) and resistance
- Alligator-clip style jumper wires to connect the battery to a resistor
- (Optional) A solderless breadboard or set of terminal blocks suitable for holding the resistors

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Choose at least three resistors with different resistor values, all between 1 kΩ and 100 kΩ[16]. Decode each resistor's color-code bands to determine its nominal resistance value as well as its tolerance. Then, use your DMM (set to measure resistance) to verify each resistor's actual resistance value. Document these nominal and actual values for all three resistors, labeling them R_1 through R_3.

Next, use your DMM to measure the battery's voltage. Document this value as well. Based on this source voltage measurement, predict how much voltage should exist across each of the three resistors when connected in parallel with the source.

Sketch a complete schematic diagram of the circuit as you intend it to be built. Double-check the schematic to ensure there are no "opens" that will prohibit current where needed, and no "shorts" prohibiting necessary voltage drops and/or causing excessive current. After you are satisfied with your plan, construct the circuit.

Connect your DMM (set to measure voltage) and measure the voltage across each of the three resistors. Compare these measured voltage values with the predicted voltage values from the previous step.

Use Ohm's Law to calculate the expected amount of current for a circuit where all three resistors are connected in parallel with each other, and with the battery. Document these predicted current values, as you will be comparing them with measured (empirical) values of current in the next step.

[16] Avoiding resistor values below 1 kΩ and 100 kΩ helps avoid errors caused by meter burden and test lead connection resistance.

Connect your DMM (set to measure current) and measure the current at each resistor[17]. Compare these measured current values with the predicted current values from the previous step. Notate on the schematic diagram where you took the three current measurements.

How well are the theoretical predictions for series components supported by the empirical (measured) data from this experiment? Are there any noteworthy sources of error in this experiment?

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

Challenges

- Identify unique properties of *parallel*-connected components, and then use your experiment to prove this property is true either by appealing to empirical data you have recorded or by demonstrating those measurements live.
- Describe the mathematical relationship of individual versus total resistances in a *parallel* circuit, and then demonstrate this relationship empirically using your DMM.
- Predict the effects of a particular resistor failing open.
- Predict the effects of a particular resistor failing shorted.
- A good self-test of an ohmmeter is to connect its test leads directly together (otherwise known as "shorting" the leads because a direct connection is the shortest, lowest resistance electrical path possible). What should an ohmmeter register when its leads are shorted together? Why?
- Explain why you cannot obtain a reliable resistance measurement using an ohmmeter when the component in question is energized by a source.
- Sometimes you will find your multimeter yields a non-zero resistance measurement when you would expect it to measure zero or very nearly zero. Explain why this is.
- Identify any data points (empirical) that disagree with theory (prediction), and suggest sources of error.
- Explain why connecting an ammeter directly to the terminals of a voltage source such as a primary-cell battery is a bad idea.

[17] Exercise caution when connecting your ammeter to any circuit! If you connect it in parallel with a voltage source, you will create a *short-circuit* whose low resistance value will likely permit a large enough amount of current to blow the fuse inside your meter (and possibly damage the meter as well). One way to ensure your ammeter will sense the current going through a particular component is to (1) Disconnect the source so that you will not be touching an energized circuit, (2) Disconnect one of the component's two leads from the rest of the circuit, (3) Connect your ammeter between that "lifted" component lead and the point where it used to attach to the circuit, (4) Re-connect the source and note the ammeter's measurement.

- The manner in which you connect the DMM to a resistor to measure its resistance may introduce a measurement error. In particular, contacting each of the meter's test probes with your skin while simultaneously contacting the resistor's terminals places your body in "parallel" with the resistor, providing an alternate path for the meter's test current to flow. Will this measurement error be positive (i.e. registering more than the resistor's true value) or negative (registering less than the resistor's true value)? Explain your reasoning.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

8-9 September 2025 – added a new bullet-point to the list in the "Challenging concepts related to parallel circuits" section of the Introduction chapter regarding the misconception of voltage being the same throughout a parallel circuit, how some students imagine this to mean a parallel circuit can never have but one value for voltage throughout all time. Also made a minor edit to image_0122 so that some numerals were not too close to a wire as to make them challenging to read. Also clarified what a "network" is in relation to a "circuit". Also corrected multiple typographical errors.

24 August 2025 – added a new bullet-point to the list in the "Challenging concepts related to parallel circuits" section of the Introduction chapter regarding the confusion of definitions versus properties of parallel networks. Also made some very minor edits to the Full Tutorial.

30 May 2025 – added a switch to the schematic diagram of image_2093 used in the Case Tutorial section "Three-resistor circuit".

5 February 2025 – corrected a typographical error in the Full Tutorial where it said 13 mA rather 13 A.

31 January 2025 – added another Quantitative Reasoning question, this one based on the legacy Radio Shack 30-in-one project kit where connections between components are made using tension-spring connectors and wires.

21 December 2024 – added another Quantitative Reasoning question, this one challenging students to identify the resistance of a soldered resistor network based on its photograph.

2 October 2024 – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

11 September 2024 – added another page to the Tutorial chapter elaborating on hard-to-spot parallel components.

29 August 2024 – added another section to the Introduction chapter specifically for challenges related to this module's topic.

26 June 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures.

30 September 2023 – added content to the Potentiometers section of the Full Tutorial discussing their use as rheostats.

12 September 2023 – edited illustrations contained within the Concept review section of the Full Tutorial chapter (review_01 file) to show areas of higher and lower electrical potential by means of color-coding conductors.

25 June 2023 – added Technical Reference section on grounding.

22-23 June 2023 – added a requirement in the "Explaining the meaning of calculations" Conceptual Question to also express each quantity using appropriate metric prefixes. Also fixed a typographical error courtesy of Annika Maki.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

15 September 2022 – added a new Case Tutorial section showing how adding more resistors in parallel decreases total (equivalent) resistance.

13 September 2022 – replaced incorrect quotation marks in a few places.

7 June 2022 – added a conceptual definition of resistance to the first page of the Simplified Tutorial chapter.

10 February 2022 – divided the Full Tutorial chapter into sections.

7 February 2022 – added a new sub-question to "VIRP table..." Quantitative Reasoning problem challenging students to check their work using estimation.

15 September 2021 – added more challenge questions.

18 August 2021 – minor edits to a schematic diagram in the Quantitative Reasoning section.

23 July 2021 – added foundational concept review to the "Identifying series circuits" Conceptual Reasoning question.

25 June 2021 – added V/I/R table tactic to the Simplified Tutorial.

22 June 2021 – added technical reference on resistor labeling.

28 May 2021 – minor edits to the Simplified Tutorial.

18 March 2021 – corrected one instance of "volts" that should have been capitalized "Volts".

9-10 February 2021 – minor additions to the Introduction chapter, and also edited the schematic diagram for the potentiometer-based current divider (image_0445). Minor edits to the Full Tutorial as well.

7 January 2021 – typographical error correction ("carries" should have been "carriers" in the first sentence of the Simplified Tutorial), thanks to Brett Deaton.

16-17 September 2020 – fixed typographical error in the Tutorial: a portion of the text said R_2 when it should have said R_1. Also, fixed minor wiring error in the "Problem with a multiple-pump system" question, where the main fuse (F5) was directly feeding the Pump A circuit in addition to indirectly feeding the other pumps' individual fuses.

15 September 2020 – minor additions to the Introduction chapter.

3 September 2020 – added Technical Reference section on metric prefixes.

29 August 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

11 March 2020 – minor addition to the Derivation section on parallel resistance formulae.

2 January 2020 – removed from from C++ code execution output, to clearly distinguish it from the source code listing which is still framed.

1 January 2020 – changed `main ()` to `main (void)` in C++ programming example.

19 September 2019 – minor edit to footnote in the Case Tutorial chapter.

14 September 2019 – added a Case Tutorial chapter.

26 August 2019 – added mention of difficult concepts to the Introduction.

16 June 2019 – minor edits to diagnostic questions, replacing "no multiple faults" with "no coincidental faults".

27 February 2019 – minor edit, replacing double-quote characters with single-quote for cleaner LaTeX formatting.

10 February 2019 – added requirement in "Explaining the meaning of calculations" Conceptual Question to identify the principle applied in each step.

17 December 2018 – added an example C++ program showing simple parallel circuit calculations.

October 2018 – minor edit to parallel properties frame, noting that parallel networks are defined by the sharing of two electrically common point sets, not just two equipotential point sets.

September 2018 – added mathematical derivations of parallel resistance formulae.

August 2018 – added content to the Introduction chapter.

May 2018 – minor edit to "parallel" illustration, annotating connected points as electrically common to each other, not just equipotential to each other.

April 2018 – added a Simplified Tutorial chapter. Also added photographs of rotary potentiometers, and a paragraph about linear potentiometers.

November 2017 – minor edits.

August 2017 – added a conceptual question.

July 2017 – substituted "property" for "characteristic" when describing parallel circuits, because this is a more mathematically conventional term. Also enhanced the definition of "parallel" in terms of electrically distinct points. Finally, capitalized all instances of the word "Earth".

May 2017 – added discussion of potentiometers to the tutorial.

January 2017 – minor clarifications made to the definition of a parallel network.

October 2016 – created changelog for future use.

September 2016 – document first published.

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