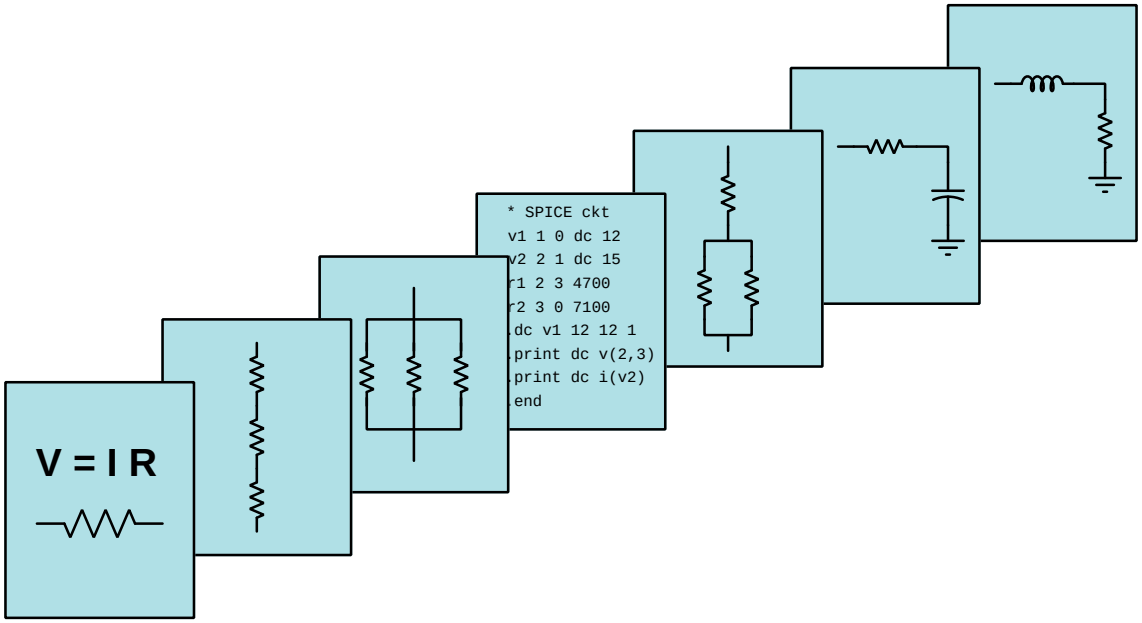
Modular Electronics Learning (ModEL) project



Phasors and AC Circuit Measurements

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Chapter 1

Introduction

1.1 Recommendations for students

Phasor mathematics is a powerful tool for quantifying and analyzing AC circuit parameters, but it must be applied according to how those parameters would actually be measured in a circuit. This module seeks to lay a firm foundation for the use of phasors to quantify AC voltages and currents. The Full Tutorial also previews three-phase AC circuits through one of the examples, showing the paradoxical result of balanced phase currents summing at a node in a Wye-connected system.

Important concepts related to phasor measurements include **voltage**, **current**, **phase shift**, **polarity**, **complex numbers**, **Kirchhoff's Voltage Law**, **Kirchhoff's Current Law**, **voltmeter usage**, **ammeter usage**, properties of **series** networks, properties of **parallel** networks, and **sources** versus **loads**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to mathematically demonstrate the addition of two sinusoids that are phase-shifted from one another, using a computer to perform the arithmetic? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to explore the concept of phasor addition in the "neutral" conductor of a three-phase AC circuit? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What is the significance of the mathematical sign of a digital multimeter's measurement?
- Why are so many possible total voltages possible when two AC voltage sources are connected in series, but only a few possibilities exist for DC sources similarly connected?
- How does the "polar" form of a complex number relate to an AC voltage or current?
- How does the "rectangular" form of a complex number relate to an AC voltage or current?

- What does it mean when an AC source is labeled with polarity symbols or an arrow symbol?
- How is it possible that no current passes through the return conductor of certain three-phase circuits?
- What is the proper way to connect a voltmeter to a circuit component to measure its voltage?
- What is the proper way to connect an ammeter to a circuit component to measure its current?
- Which electrical quantity(ies) add in a series network?
- Which electrical quantity(ies) must be equal throughout a series network, and why?
- Which electrical quantity(ies) add in a parallel network?
- Which electrical quantity(ies) must be equal throughout a parallel network, and why?

1.2 Challenging concepts related to phasor measurements

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Phasors representing AC amplitudes and phase shifts** – a powerful tool used for understanding the operation of AC circuits is the *phasor diagram*, consisting of arrows pointing in different directions: the length of each arrow representing the amplitude of some AC quantity (voltage, current, or impedance), and the angle of each arrow representing the shift in phase relative to the other arrows. By representing each AC quantity thusly, we may more easily calculate their relationships to one another, with the phasors showing us how to apply trigonometry (Pythagorean Theorem, sine, cosine, and tangent functions) to the various calculations. An analytical parallel to the graphic tool of phasor diagrams is *complex numbers*, where we represent each phasor (arrow) by a pair of numbers: either a magnitude and angle (polar notation), or by "real" and "imaginary" magnitudes (rectangular notation). Where phasor diagrams are helpful is in applications where their respective AC quantities *add*: the resultant of two or more phasors stacked tip-to-tail being the mathematical sum of the phasors. Complex numbers, on the other hand, may be added, subtracted, multiplied, and divided; the last two operations being difficult to graphically represent with arrows.

- **Complex numbers in calculators** – while the ability of certain scientific calculators to perform complex-number arithmetic is an enormously helpful tool for students first learning to analyze AC circuits, some of these calculators prove to be finicky in their handling and entry of these quantities. Advice proven to be sound for all complex-number calculators is to save each and every complex-valued quantity into a memory location and then perform arithmetic operations on those stored variables rather than enter the complex numbers directly into the computation. For example, storing `3 - j4` into memory location `A` and `25` $\angle$ `30`^{o} into memory location `B`, then multiplying `A` $\times$ `B` rather than entering `3 - j4` $\times$ `25` $\angle$ `30`^{o}. Storing values into calculator memory and then retrieving them as needed for calculations is actually sound advice for many reasons, but many students resist taking these "extra" steps and as a result incur all the risks of hand-entering values (e.g. rounding errors due to truncating, keystroke errors when the same value must be used more than once, crowded displays where you cannot see the whole calculation, order-of-operations errors when complex numbers aren't enclosed in parentheses, etc.). Complex-number calculations reward good practices through consistently good results!

- **Complex numbers in measurement** – complex numbers may be expressed in either *rectangular* or *polar* form, either one of these being perfectly valid. However, measurement instruments such as multimeters only provide the magnitude of the polar form of the voltage or current in question. For example, if a component's voltage is 4.8 Volts RMS $\angle$ 35^o with the circuit's source voltage being the phase reference (0^o), an voltmeter reading that voltage will simply register 4.8 Volts RMS. An oscilloscope simultaneously measuring that component voltage on one channel and the source voltage on another will show the 35^o shift on the horizontal axis between the two waveforms.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of phasor addition/subtraction to in-phase AC quantities

  Assessment – Determine whether voltages subtract or add in boost/buck transformer circuits based on winding polarity markings and winding interconnections; e.g. pose problems in the form of the "Boost/buck transformer circuits" Conceptual Reasoning question.

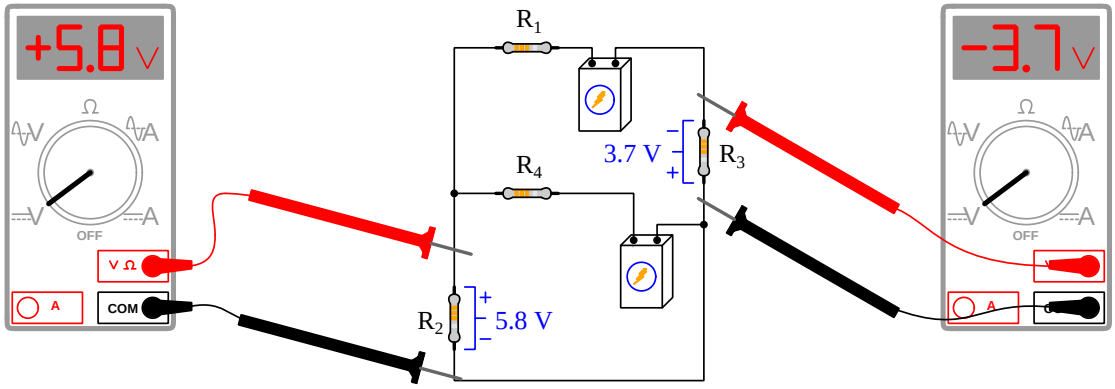
- **Outcome** – Apply the concept of phasor addition/subtraction to out-of-phase AC quantities

  Assessment – Calculate voltages measured between pairs of test points connected to AC voltage sources of varying magnitude and relative phase shift; e.g. pose problems in the form of the "Voltages between specified points" Quantitative Reasoning question.

Chapter 2

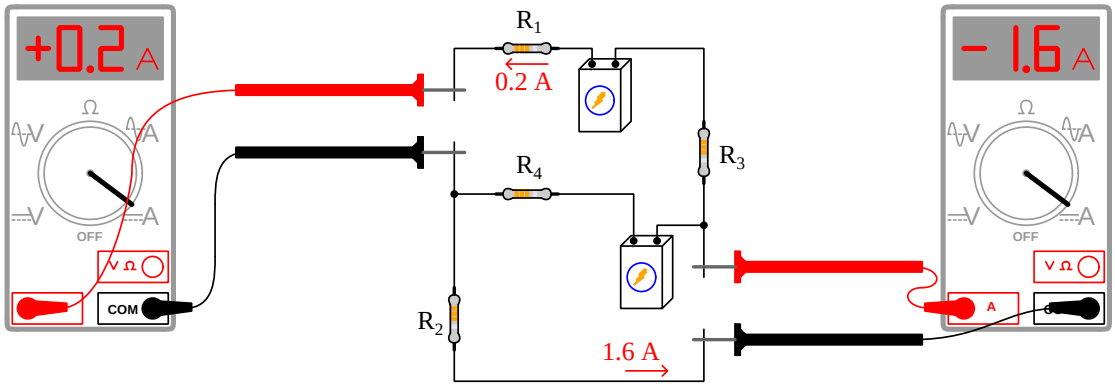
Simplified Tutorial

Suppose we connected a pair of digital multimeters (DMMs) configured as voltmeters to a circuit as shown in this diagram:



The *actual* voltage drops across these two resistors are R_2 dropping 5.8 Volts with positive on top and negative on bottom, and R_3 dropping 3.7 Volts with positive on bottom and negative on top. This is evident from the voltmeter readings and test lead connections. R_2's voltage drop must be positive on top and negative on bottom because its voltmeter registers a positive voltage with its red lead on top and black lead on bottom. R_3's voltage drop must be positive on bottom and negative on top because its voltmeter registers a *negative* voltage with its red test lead on top and black test lead on bottom. In other words, R_2's voltmeter is connected correctly while R_3's voltmeter is connected "backwards".

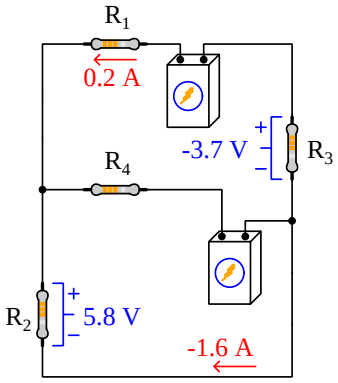
Ammeters may be similarly used to determine actual directions of current in a DC circuit:



It is clear that the left-hand ammeter has 0.2 Amperes of current entering its red lead and exiting its black lead, because that ammeter's measurement is a positive quantity. The right-hand ammeter has 1.6 Amperes of current entering its black lead and exiting its red lead, the direction evident from the "backwards" (negative) sign of its measured value.

When we annotate voltage polarities and current directions in a DC circuit, we typically regard those symbols as having *absolute* meaning. That is to say, when we see "+" and "−" polarity symbols drawn near the terminals of a circuit component, we assume the "+" terminal has a greater potential than the "−" terminal: charge carriers at the positive terminal exist at a higher state of potential energy than charge carriers at the negative terminal. Likewise with current arrows drawn along a conductor in a circuit diagram we interpret those arrows as pointing in the direction of conventional flow (i.e. the actual motion of positive charge carriers). When drawn like this, the voltage and current quantities also shown on the diagram are given as absolute values, never as negatively-signed numbers.

However, it is actually valid to show voltages and currents with polarities and directions "backwards" just as it is perfectly valid to connect a multimeter "backwards" to a circuit: in such cases we know the orientation is backwards because the given value bears a negative sign. The following illustration shows the voltages and currents annotated exactly the same as they were measured by the DMMs:

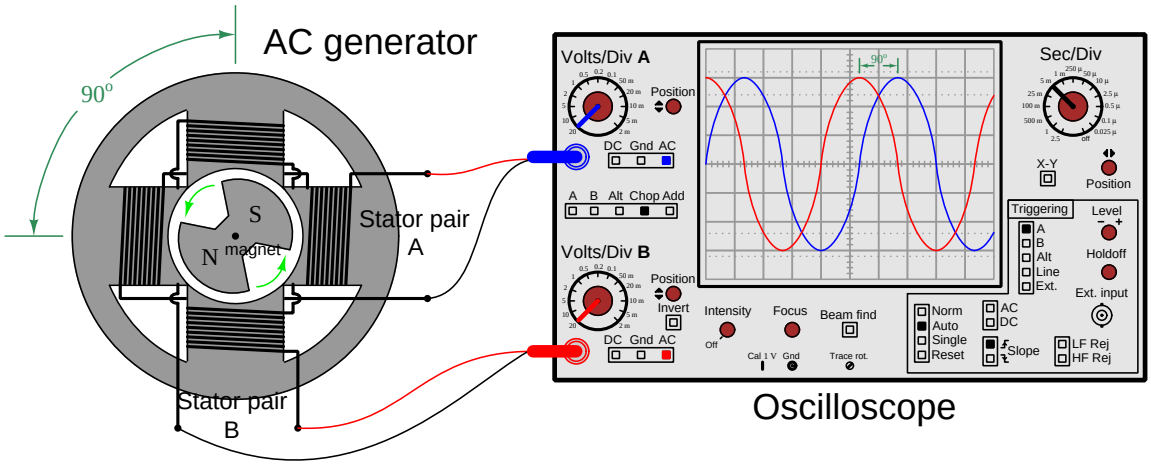


The *actual* polarity of R_3's voltage drop is positive on bottom and negative on top, but the -3.7 Volt value tells us the $+$ and $-$ polarity annotations are representing where a voltmeter's red and black test leads would connect, respectively, and not what the actual potentials are. Likewise, the 1.6 Ampere current is *actually* flowing from left to right, but the direction of that arrow represents how an ammeter's red and black test leads would connect[1] to yield a negative measurement of current. These labels may seem strange at first, but they are mathematically valid because they are paired with the mathematical signs of the values. In other words, the meanings of both the polarity symbols and the arrows are *relative* to the mathematical signs of the values, and have no definite meaning on their own. Likewise, the meaning of a meter measurement's positive or negative sign is incomplete without reference to polarity or arrow notations showing how that meter connects to the circuit.

What has all of this to do with *AC* circuit measurements and *phasors*, you ask? We know voltage polarities and current directions are essential for proper application of Kirchhoff's Laws to DC circuit analysis, and so it's reasonable to expect voltage polarities and current directions will be important in AC circuits too. However, the obvious problem here is that there is no such thing as absolute labeling of voltage polarities or current directions in AC, because AC quantities by definition *oscillate* back and forth. Therefore, in order to apply Kirchhoff's Laws to AC circuits we must apply voltage polarity and/or current direction labels to components in AC circuits, and this will require us to include phasor quantities along with those annotations in order to give them meaning. Just as the $+$ and $-$ voltage polarity symbols and current arrows were relative to the mathematical signs of the quantities shown on the DC circuit diagram, polarity and direction annotations will be relative to the *phase angles* of their respective quantities in an AC circuit diagram.

[1] Or more simply, the direction in which a clamp-on DC ammeter's arrow would point.

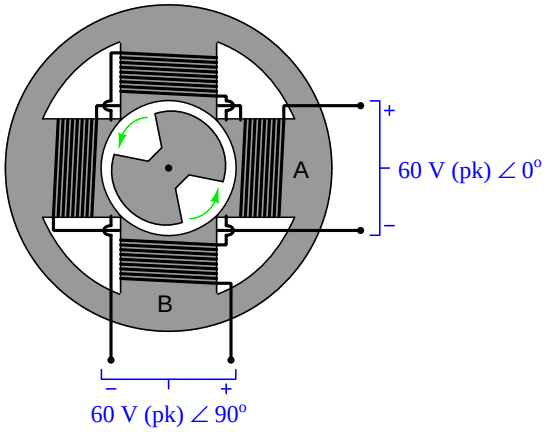
Let's examine an application of this principle on a two-winding AC generator, where each winding pair is offset 90^o from the other, to create two voltages phase-shifted 90^o from each other:



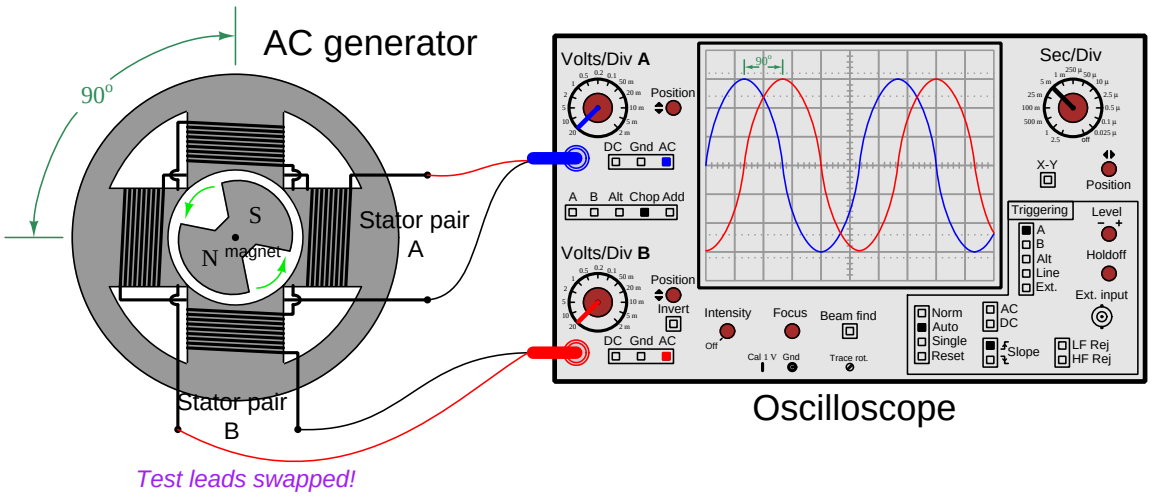
This particular oscilloscope has been configured to trigger its sweep based on Channel A's signal, with trigger level set at 0 Volts with a rising slope. This makes Channel A (the blue-colored trace) appear as a sine wave, with the left-hand border of the screen representing 0 degrees on that sine wave. Since Channel A is our triggering reference signal here, we will assign a value of 0^o as its phase angle which makes Channel B's signal leading by $+90^o$.

$$V_A = 60 \text{ V (peak)} \angle 0^o \qquad V_B = 60 \text{ V (peak)} \angle 90^o$$

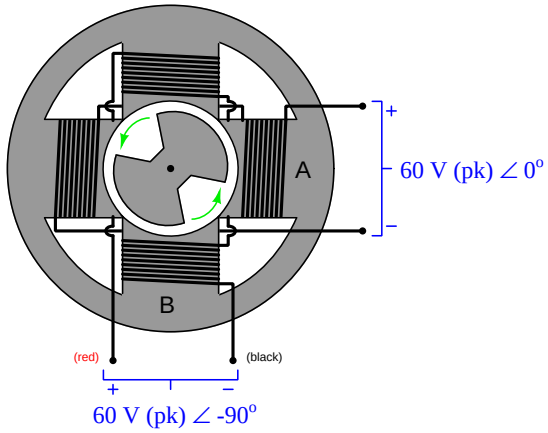
If we denote the red and black test lead connection points on the generator using $+$ and $-$ polarity symbols, we may give these phasor representations of voltage their full meaning in the following diagram:



If we were to swap the test lead connections on Channel B and re-measure these two winding voltages, we would see a different phase shift represented on the oscilloscope's screen. Now Channel B (the red trace) appears to be *lagging* Channel A by -90^o, where before it was *leading*:

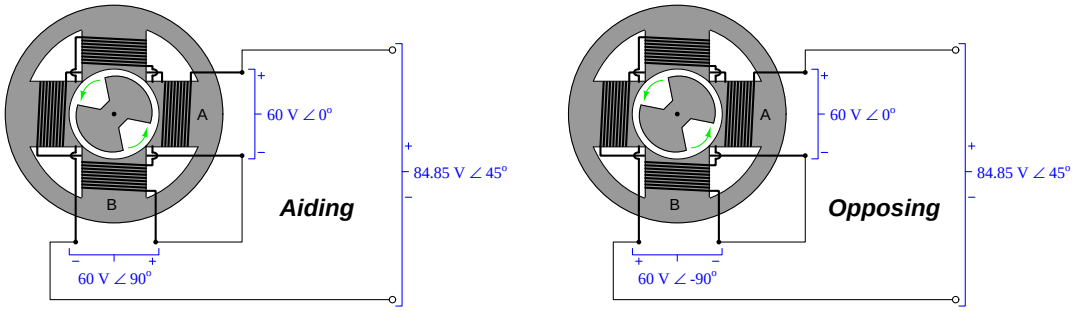


If we denote Channel B's new red and black test lead connection points on the generator using + and − polarity symbols, we may represent the output voltage of stator winding B in a new way:



In either case, the output voltage of stator winding B is the same. Nothing was altered with the generator itself – only our oscilloscope test leads for Channel B were swapped. This means 60 V $\angle$ 90^o with − on left and + on right means exactly the same thing as 60 V $\angle$ -90^o with + on left and − on right.

We may observe how to apply these polarities and phasor values to Kirchhoff's Voltage Law by connecting the generator's two stator windings in series and calculating their total voltage:



For the left-hand circuit where winding B's voltage is represented as 60 V $\angle$ 90^o and the polarities are series-aiding, total voltage is the sum of the two voltages:

$$(60\ \text{V}\angle 0^o) + (60\ \text{V}\angle 90^o) = 84.85\ \text{V}\angle 45^o$$

For the right-hand circuit where winding B's voltage is represented as 60 V $\angle$ -90^o and the polarities are series-opposing, total voltage is the difference between the two voltages:

$$(60\ \text{V}\angle 0^o) - (60\ \text{V}\angle -90^o) = 84.85\ \text{V}\angle 45^o$$

Note how the calculated total voltage is identical for these two scenarios, because the only difference is how we chose to notate winding B's voltage – it's the same exact voltage, just expressed two different ways.

This same principle applies to AC current measurements as well. The arrow notation used to show conventional flow in a diagram merely represents how an ammeter would be connected to the circuit (i.e. current entering the red lead and exiting the black lead), and the actual current is represented by the combination of that arrow symbol and the phasor quantity.

In summary,

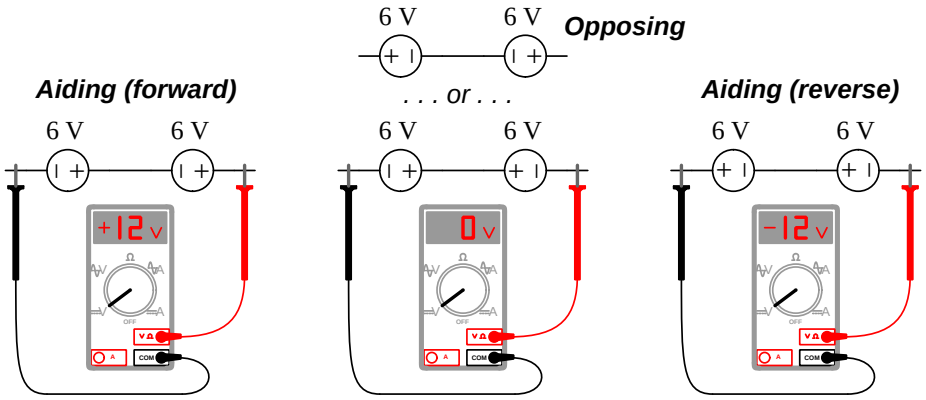
- Polarity symbols drawn on an AC voltage source's terminals declare how we would connect an oscilloscope to measure the phase of that voltage waveform.
- An arrow drawn along a wire declares how we would connect an oscilloscope to measure the phase of that current waveform.
- Polarity symbols and arrows only have meaning in conjunction with a specified phase angle, and vice-versa.
- Refer to the polarity and arrow symbols when applying Kirchhoff's Laws *as though the circuit were DC rather than AC*, using these annotation symbols to determine when to add quantities versus when to subtract quantities. Then, as you set up your equations for calculating total voltage or current, insert the known phasor quantities associated with those symbols and trust the result given to you by the arithmetic.

Chapter 3

Full Tutorial

3.1 Additive voltages

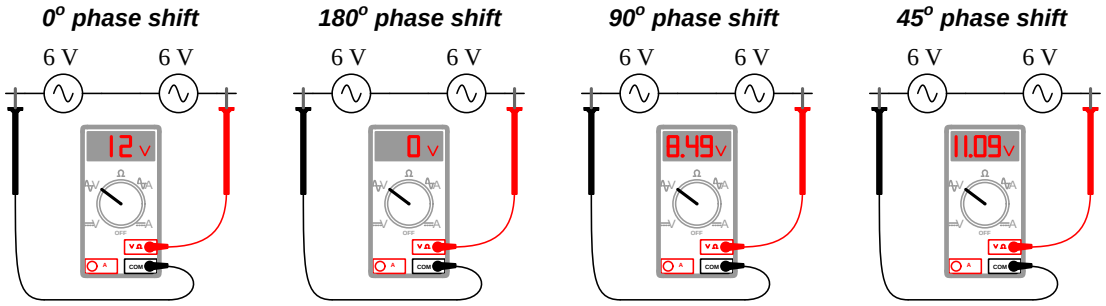
What would be the result of connecting two 6 Volt DC sources in series with each other and measuring their combined voltage with a digital multimeter (DMM)? At first, you might think "12 Volts" to be the obvious answer, but this is not necessarily correct. With two DC voltage sources having two terminals each, there are actually *four* different configurations of series connection. As we see here, only one of the four connections yields a total of +12 Volts:



We must remember that **voltage is always measured between two points**, being by definition the difference of potential energy between electric charges existing at two separate locations. Voltmeters have two test leads, one red and one black, and this guarantees there are always *two different ways to connect a voltmeter* to any voltage source. If the red lead touches a point with more electrical potential than the black lead, the meter will register a positive quantity; if opposite, the meter reads negative. This is why + and − *polarity* symbols are used in schematic diagrams, to give context to the voltage quantities. Without polarity symbols, we don't fully know what "6 Volts" means which is why it's impossible to give a sure answer to the initial question.

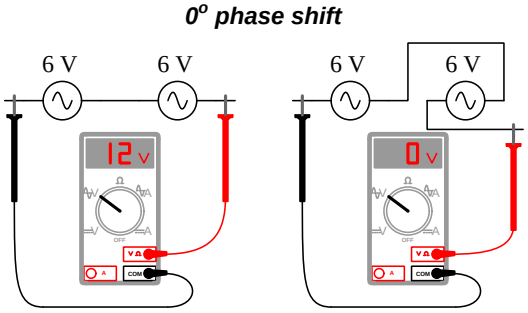
The same is true for DC current: ammeters have both red and black test leads, which means there are always two different ways in which the meter may be connected to a circuit. In schematic diagrams we use arrow symbols to denote direction of the electric current, once again to give context to the current quantities.

What would be the result of connecting two 6 Volt, 60 Hz AC sources in series with each other and measuring their combined voltage with a digital multimeter (DMM)? We face the same fundamental uncertainty here as we did with two 6 Volt DC sources, namely that we have not been informed how the polarities of these two voltage sources relate with each other. At first you might be inclined to think that there is no such thing as polarity with *alternating* quantities because by definition they oscillate between positive and negative, but this is not strictly true. AC quantities sharing a common frequency may differ in *phase*, meaning they do not peak at the same time. Here are just a few examples of series-connected AC voltage sources showing several possible total voltages depending on the amount of phase shift between the two sources:



With two DC voltage sources connected in series we only had three possible results for total voltage: −12 Volts, 0 Volts, or +12 Volts. With two AC voltage sources connected in series, however, we have an *infinite* number of possible results ranging from 0 Volts to 12 Volts inclusive.

The problem becomes even more complicated when we consider the following thought experiment. Imagine two in-phase 6 Volt AC voltage sources connected together in series to make a total of 12 Volts, just like the left-hand example shown above. Each of these two sources reaches their positive and negative peaks at precisely the same time which is why their voltages simply add together to make the total. Now imagine swapping the terminal connections on just one of these two sources without altering the 0^o phase relationship. The sources will still reach their respective peaks at precisely the same time, but now the wire connections have placed those two waveforms at odds against each other instead of aiding each other. The total will now be 0 Volts instead of 12 Volts:

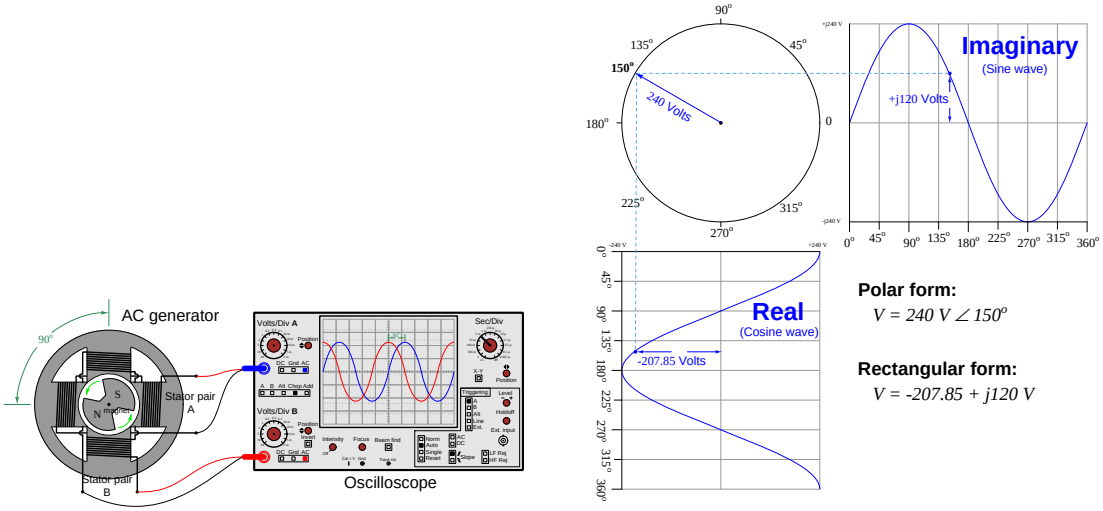


This thought experiment proves there really is such a thing as *polarity* for AC voltages: i.e. it truly matters how the two terminals of an AC voltage source connect to other components.

3.2 Phasor polarity

It would make little sense to permanently label the two terminals of an AC voltage source such as an electromechanical generator with + and − symbols as we do for DC voltage sources such as batteries because we know AC voltage by definition *alternates* polarity over time. However, what we can do is label an AC source with + and − polarity markings showing its actual voltage polarity when that source is at its 0^o point. For an AC generator, that would be at a particular shaft position while the shaft is rotating in a particular direction.

Consider the following illustration of a two-phase AC generator with two pairs of stator windings oriented perpendicular to each other, such that these windings' AC voltages will always be phase-shifted by 90 degrees:

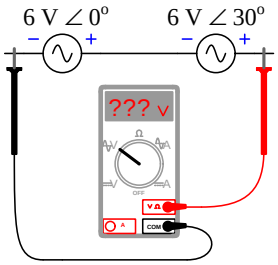


If we graphically represent the generator's rotor position as a phasor-arrow, we see that each stator winding's AC voltage is simply a projection of that phasor arrow's tip as rotates counter-clockwise: stator winding pair A outputting a sine wave and stator winding B outputting a cosine wave. It is as though the generator were creating a constant voltage of 240 Volts, but each stator winding only outputs a fraction of that 240 Volts based on the sine or cosine function of the shaft angle. This phasor-arrow's label of 240 Volts ∠ 150 degrees means that at a shaft position of 150 degrees (moving counter-clockwise) the instantaneous voltage output by winding A will be +120 Volts while the instantaneous voltage output by winding B will be −207.85 Volts.

Since the actual +/− polarity of each stator winding's voltage is a known function of shaft position, it actually makes sense that we could label those winding terminals with "+" and "−" symbols at a shaft position where the winding's voltage peaks. In the example shown above, stator winding pair B ("Real") reaches peak voltage at a shaft position of 0 degrees and again at 180 degrees, while winding A ("Imaginary") peaks 90 degrees later in both cases. Knowing this, we could mark winding B's terminals with "+" and "−" for its peak voltage polarity when the shaft is at 0 degrees, and mark winding A's terminals with "+" and "−" for its own peak voltage polarity 90 degrees later. Then, to make these notations complete we would state each winding's voltage in

terms of peak value (240 Volts) at angles reflecting their relative phase shifts: either designate B as 240 V $\angle$ 0^o and A as 240 V $\angle$ -90^o, or B as 240 V $\angle$ 90^o and A as 240 V $\angle$ 0^o.

Let's explore this principle by working through an example problem. Consider a pair of series-connected AC voltage sources at the same frequency, but phase-shifted 30^o from each other. We will arbitrarily assign one of these sources a phase angle of 0^o to make it the phase reference, and show the phase shift by giving the other source a phase angle of 30^o. This means the 0^o source *lags* and the 30^o source *leads* (i.e. when the reference source's waveform is at its 0^o point, the other source's waveform will already be at its 30^o point). Finally, we will annotate the terminals of each source with + and − symbols, denoting the actual voltage polarity when each of the sources peaks at its 0^o point[1] in time:



With these annotations in place, we may proceed to calculate the total voltage. First, we note the polarities of the sources with respect to their interconnection, and it appears they are series-aiding (i.e. the + of one connected to the − of the other). This means we will calculate total voltage by Kirchhoff's Voltage Law, using the same arithmetic as if these were DC voltage sources. The only difference in this case is that we will be adding two *complex* numbers together[2]:

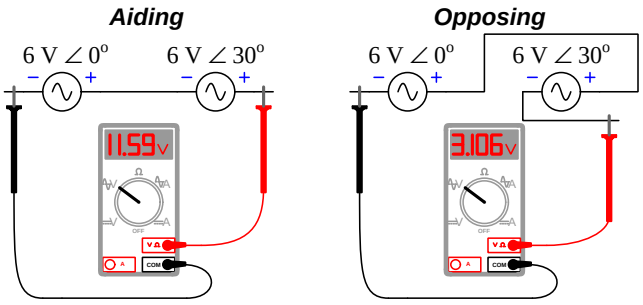
$$(6\ \text{V}\angle 0^o) + (6\ \text{V}\angle 30^o) = 11.59\ \text{V}\angle 15^o$$

[1] In other words, each of these two sources will have a lesser potential on the left terminal (−) and a greater potential on the right (+) when they individually reach their positive peak values (0^o on a cosine wave). The 30^o phase shift means the right-hand source reaches its peak value 30^o ahead of the left-hand source.

[2] Some electronic calculator models directly support complex-number arithmetic, and are valuable tools for AC circuit analysis!

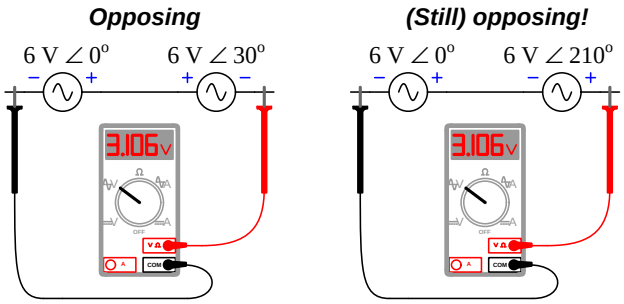
Now let us suppose we reverse the terminal connections on the 30^o leading source and re-measure total voltage. Now the two sources appear to be series-*opposing* (i.e. + connected to +) which means we must use *subtraction* to calculate total voltage:

$$(6\ \text{V}\angle 0^o) - (6\ \text{V}\angle 30^o) = 3.106\ \text{V}\angle -75^o$$



Reversing the terminal connections on the 30^o leading source so that its + terminal connects to the + terminal of the reference (0^o) results in less total voltage than before because the polarities are more opposed than they were before. Mathematically, we noted the opposing polarities and concluded we must *subtract* to calculate total voltage. It is important to note, however, that we cannot simply assume the total magnitude will be less because the polarity marks oppose. Like DC sources measured with digital multimeters, the polarity of the source *and* the mathematical sign registered by the DMM *together* define the voltage.

To illustrate this point, we will re-draw the subtractive scenario in a different manner and contrast it with the former series-opposing diagram, with polarity symbols (seemingly) additive but 180^o added to the phase angle of non-reference source so that it is now 210^o instead of 30^o:



In either network total voltage is the same:

$$(6\ \text{V}\angle 0^o) - (6\ \text{V}\angle 30^o) = 3.106\ \text{V}\angle -75^o$$

$$(6\ \text{V}\angle 0^o) + (6\ \text{V}\angle 210^o) = 3.106\ \text{V}\angle -75^o$$

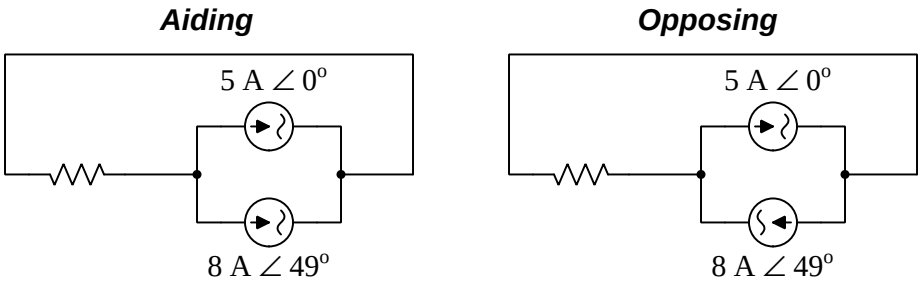
Denoting an AC source as 6 V $\angle$ 210^o instead of 6 V $\angle$ 30^o with reversed polarity markings is analogous to internally re-wiring a DC battery for backwards polarity and labeling it as outputting -6 Volts instead of $+6$ Volts. With DC we never had to worry about this sort of ambiguity because polarity markings were always absolute in their meaning. Here with AC, polarity marks represent true polarity only at that point in time where the voltage waveform reaches its maximum instantaneous value (at 0^o for a cosine wave). Whether the final result will be aiding or opposing depends not just upon the polarity as shown but also upon the amount of phase shift between the connected quantities.

This complexity tends to diminish one's own confidence in *qualitative* AC circuit analysis, because the relative magnitude of the final result is much more difficult to predict than in a DC circuit. Graphical techniques such as *phasor diagrams* exist to aid in understanding these results, but when calculations are required it is advisable to *pay attention to the polarity symbols only for setting up calculations, and then trust the results your calculator returns to you!*

3.3 Additive currents

AC currents summing together at nodes in accordance with Kirchhoff's Current Law are handled in much the same way as AC voltages summing in series in accordance with Kirchhoff's Voltage Law. We use both arrows showing direction of current when each source is at its 0^o point in time, using those arrows as guides for setting up our Kirchhoff's Current Law equation, then add or subtract the phasor quantities specified to us for those sources.

To illustrate, we will consider a pair of AC current sources connected in parallel, driving current through a resistive load. In the left-hand network the two sources appear to be aiding each other due to the directions of their arrows, while in the right-hand network they appear to be opposing:

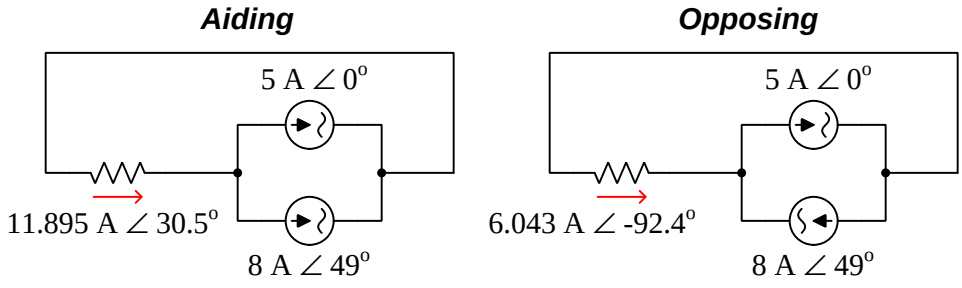


Calculating total current in each network:

$$(5\ \text{A}\angle 0^o) + (8\ \text{A}\angle 49^o) = 11.895\ \text{A}\angle 30.5^o$$

$$(5\ \text{A}\angle 0^o) - (8\ \text{A}\angle 49^o) = 6.043\ \text{A}\angle -92.4^o$$

Showing how these total current values would be represented through the resistor using arrow notation:

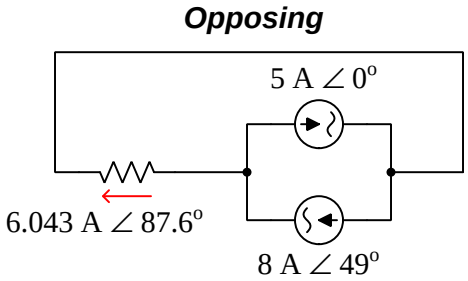


The direction of our total current arrow is not arbitrary. In both circuits the red arrow points from left to right because this is the direction assumed by our current calculations: regarding the 5 Ampere source as the "positive" quantity in both equations.

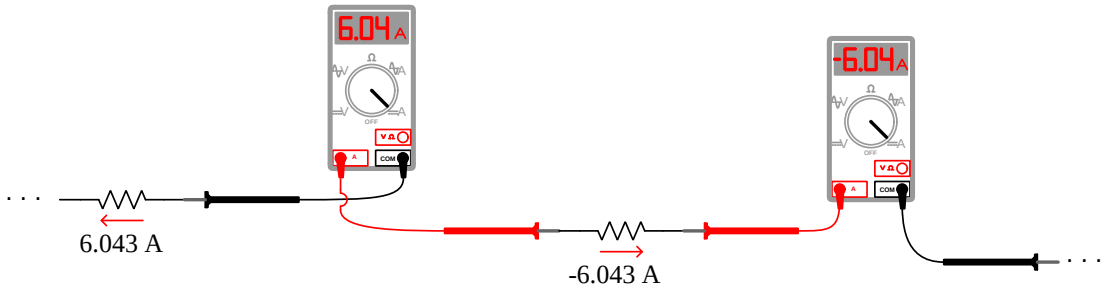
For the sake of illustration, we may re-write the second equation with the two current values in reverse order and re-calculate total current. Here, we are regarding the 8 Ampere source as the "positive" current and the 5 Ampere source as the "negative" (subtractive) current:

$$(8\ \text{A}\angle 49^o) - (5\ \text{A}\angle 0^o) = 6.043\ \text{A}\angle 87.6^o$$

Our result of 6.043 Amperes at an angle of 87.6^o would be correct for a total current arrow drawn from right to left, honoring the 8 Ampere source's arrow as the definition for a "positive" current value:

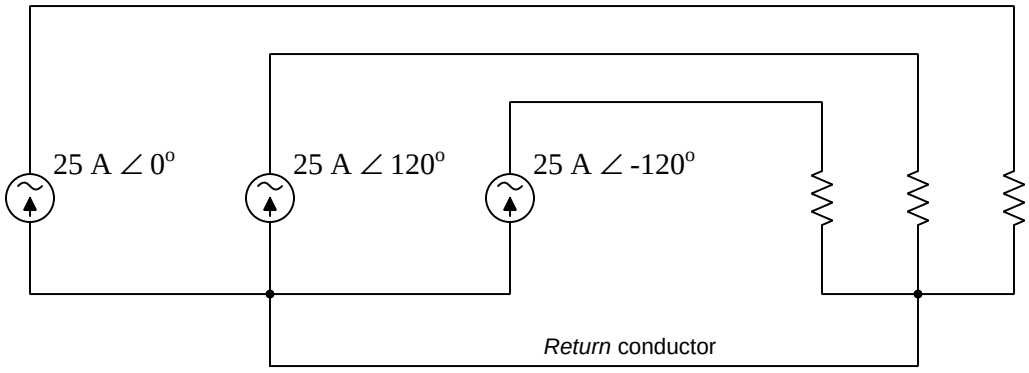


This is mathematically equivalent to the previous result: 6.043 Amperes $\angle$ 87.6^o with arrow pointing from right to left is identical to 6.043 Amperes $\angle$ -92.4^o with arrow pointing from left to right. If this were a DC circuit rather than an AC circuit, it would be analogous to a 6.043 Ampere current flowing from right to left versus the *exact same current* represented as -6.043 Amperes flowing from left to right, as shown in the following partial diagram:

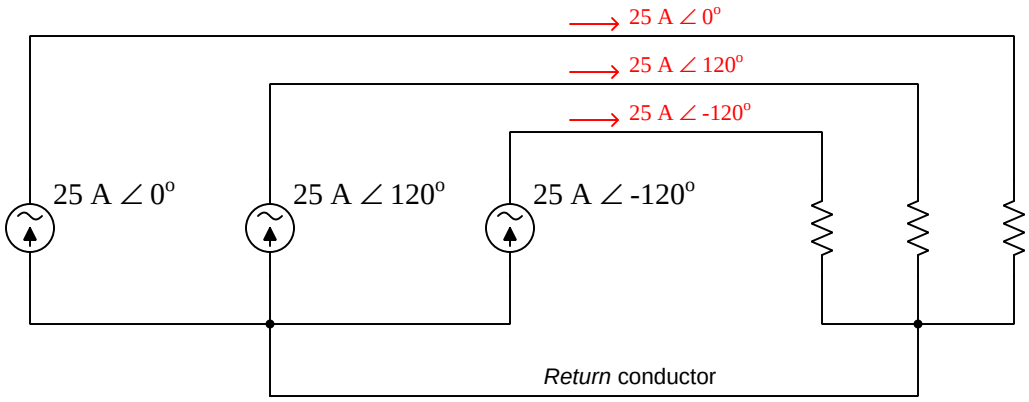


As with voltage measurement, the fundamental principle at work here is that there always exists two different ways in which a meter may be connected to a circuit, and therefore two different ways in which we may annotate the polarity of the voltage or the direction of the current in question. If we assign a mathematical value to a measurement, the sign (or phase angle) of that value only has meaning when paired with the polarity or direction of the annotation symbol(s), just as the sign of a DC meter's reading or an AC waveform's phase shift value as shown on an oscilloscope depends on how that instrument's test leads are connected to the circuit.

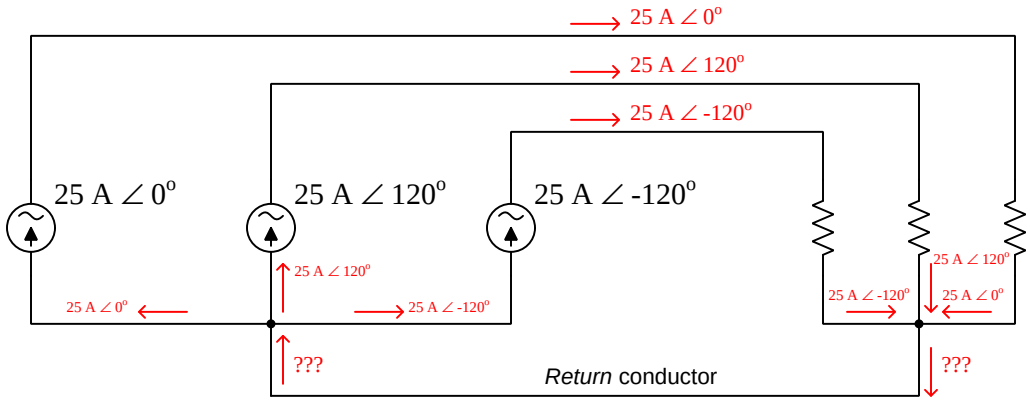
As with series AC voltage calculations, parallel AC current results may be quite counter-intuitive. A good example of this is the following *three phase* circuit, where three AC current sources having equal magnitude and equidistant phase shifts (i.e. each source is exactly 120^o shifted in phase from either of the other two sources) power three loads with a common "return" conductor. Three-phase AC circuits are very common in industrial power systems and constitutes a subject of study unto itself:



The problem before is to determine the amount of current in each of the four conductors connecting sources with loads. Current through each of the three lines connecting source to load is obvious, since these lines are each in series with their respective current source, and we know series-connected components must share the exact same current. Therefore we will annotate these three lines with arrows pointing in a direction compatible with the source arrows, and label these arrows with the same phasor quantities given as source values:



What is not so obvious is how much current exists in the return conductor. We can see below the source and below the resistors that the return conductor attaches to the source/load components at nodes, and we know Kirchhoff's Current Law therefore must apply. Sketching arrows at these points showing the three source current values, and arrows representing return current in the expected directions:



At either node, the three source current values must *add* to make the return current value. So, our Kirchhoff's Current Law equation will be as follows:

$$(25\ \text{A}\angle 0^o) + (25\ \text{A}\angle 120^o) + (25\ \text{A}\angle -120^o) = 0\ \text{A}\angle 0^o$$

Paradoxically, we see that the return conductor carries no current whatsoever because the phasor sum of these three current values is zero! As counter-intuitive as this result is, it is true for any such three-phase circuit where the individual line currents are precisely balanced (i.e. equal in magnitude and symmetrically shifted in phase). In fact, for a balanced three-phase circuit no return conductor is necessary at all – the only reason it exists is to carry current in instances where the sources' magnitudes and/or phase shifts change to become imbalanced, as may occur in a real electrical power system.

3.4 Concept summary

In summary, phasor notation for AC quantities such as voltage and current is complete only when accompanied by polarity marks (voltage) or arrows (current). Likewise, polarity marks and arrows have meaning only when accompanied by a phase angle. Either one by itself is ambiguous.

- Polarity symbols on an AC voltage source's terminals declare the potential difference between those terminals at any point in time where its voltage waveform is at the 0^o mark (i.e. when its cosine wave is at maximum positive amplitude). In other words, the voltage polarity represented by these symbols are absolutely true only when the waveform is in its positive half-cycle.
- An arrow symbol on an AC current source declares the actual direction of (conventional) charge flow at any point in time where its current waveform reaches the 0^o mark (i.e. when its cosine wave is at maximum positive amplitude). In other words, the direction of current represented by the arrow is absolutely true only when the waveform is in its positive half-cycle.
- The phase angle of the phasor (polar notation) declares the waveform's angle value at the precise moment in time when the *reference* waveform in that circuit is at its 0^o mark.
- When analyzing an AC circuit, refer to the polarity and arrow symbols *as though the circuit were DC rather than AC* to correctly set up any Kirchhoff's Laws equations, using these annotation symbols to determine when to add quantities versus when to subtract quantities. Then, as you set up your equations for calculating total voltage or current, insert the known phasor quantities associated with those symbols and trust the result given to you by the arithmetic.

As confusing as this may seem at first, the goal of associating phasor quantities with polarity and direction symbols is to enable the use of DC circuit analytical tools and techniques for the analysis of AC circuits. The alternative is to regard AC circuits as wholly different from DC circuits, and to essentially set aside[3] all the mathematical tools you mastered for DC circuits.

[3] One of the philosophical priorities of science is to economize on the number of principles – in other words, never invent a new principle when an existing principle will do. Striving to describe as many phenomena using as few principles as possible allows us to more easily see similarities in our universe which we might otherwise miss by regarding each new phenomenon as abiding by its own rules. The genius of phasors is that they allow us to see AC circuits as sharing much in common with DC circuits. Alternative approaches, which are usually employed in an effort to avoid learning new types of math, end up complicating the concepts by making it appear as though AC circuits function entirely different from DC circuits.

Chapter 4

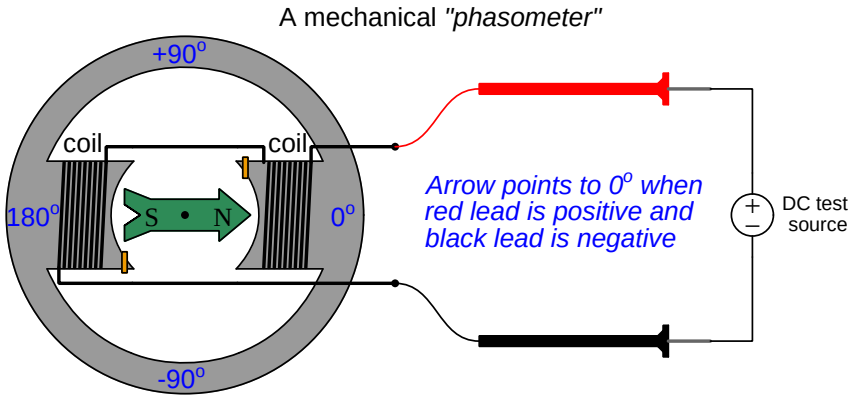
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Phasometers

The concept of assigning a fixed phase angle value to an AC voltage or current which is continuously oscillating is a confusing one. In order to make better sense of this principle, we will introduce an hypothetical test instrument designed for the purpose of showing the phase angle of any measured AC quantity, which I call a *phasometer*.

Imagine building a small synchronous electric motor with an arrow-shaped magnetic rotor. We will wind the stator coils and magnetize the rotor such that the arrow points toward the 0^o mark whenever the red lead is positive and the black lead is negative (connected to a DC test source). When connected to an AC source, this "phasometer" will spin in sync[1] with the AC generator powering the circuit, the rotating arrow keeping pace with the generator's shaft[2]. The intent of this arrow is to serve as a mechanical analogue of an electrical phasor, pointing in the same direction as the phasor points for any sinusoidal voltage or current signal connected to the phasometer:



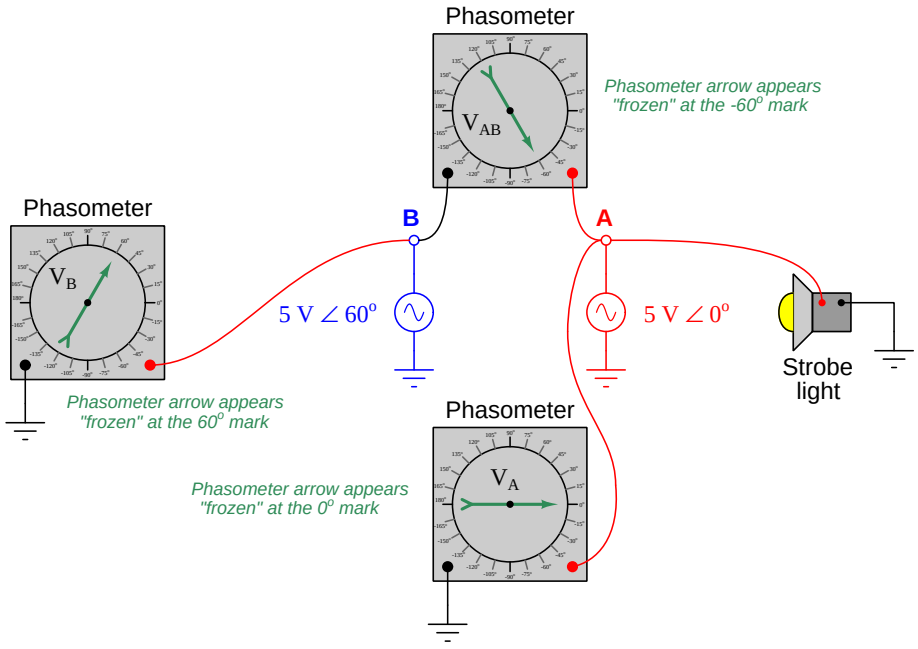
For a system frequency of 60 Hz as is standard for AC power systems in North America, the rotational speed of our phasometer's rotor will be 3600 revolutions per minute – too fast for the rotating arrow to be anything but a blur of motion to an unaided human eye. Therefore, we will need to add one more component to the phasometer to make it practical: a *strobe light* connected to an AC voltage in the system to act as a synchronizing pulse. This strobe light will emit a flash of light just once per cycle of the AC waveform, and always at the same point in time (angle) within every cycle. Just as a strobe light (also called a "stroboscope") makes a moving machine part[3] appear to "freeze" in time, this strobe light will visually "freeze" the arrow so we will be able to read its position with our eyes.

[1] An important detail is that our phasometer must always spin counter-clockwise in order to maintain proper phasor convention. We can ensure this will happen by including a pair of *shading coils* (small copper rings wrapped around one corner of each magnetic pole) in the stator structure. For a more detailed discussion of shading coils, refer to the section on AC induction motors (**??**) starting on page **??**.

[2] This, of course, assumes the generator powering the system is also a two-pole machine like the phasometer. If the generator has more poles, the shaft speed will not match the phasometer's rotor speed even though the phasometer will still faithfully represent the generator's *cosine wave* rotation.

[3] Automobile mechanics may be familiar with a tool called a *timing light*, consisting of a strobe light connected to the engine in such a way that the light flashes every time the #1 cylinder spark plug fires. By viewing the marks etched into the engine's crankshaft with this strobe light, the mechanic is able to check the ignition timing of the engine.

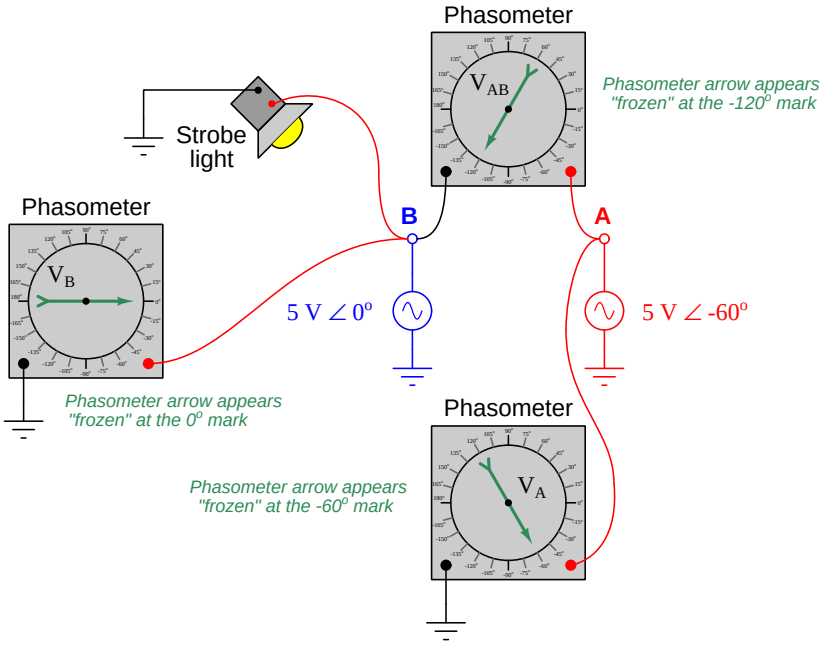
Now, imagine connecting three of these "phasometer" instruments to a pair of AC voltage sources that are at the same frequency (60 Hz) and have the same RMS voltage value (5 Volts) but are phase-shifted from one another by 60^o. We will connect the synchronizing strobe light to voltage source A so that it flashes whenever that source reaches its positive peak voltage with reference to ground, and point the strobe light at the faces of all three phasometers so that their spinning pointers will appear to "freeze" at the exact same moment in time:



The phasometer connected to source A must register 0^o because the strobe light flashes whenever source A hits its positive peak (at 0^o on its cosine waveform), and the positive peak of a waveform is the precise point at which the phasometer arrow is designed to orient itself toward the 0^o mark. The phasometer connected to source B will register 60^o because that source is 60 degrees ahead of (leading) source A, having passed its positive peak already and now headed downward toward zero Volts at the point in time when the strobe flashes. The upper phasometer registers -60^o at that same time because the voltage it senses at point A with respect to point B is lagging behind source A by that much, the V_{AB} cosine wave heading toward its positive peak.

If we were to substitute a constant illumination source for the strobe light, we would see all three phasometers' arrows spinning counter-clockwise, revealing the true dynamic nature of the voltage phasors over time. Remember that these phasors are all *continuously moving* quantities, because the voltages they represent are sinusoidal functions of time. Only when we use a strobe light keyed to source A's positive peak do we see fixed-angle readings of $V_A = 0^o$, $V_B = 60^o$, and $V_{AB} = -60^o$.

You may wonder what might happen if we keep the three phasometers connected to the same points, but change the strobe light's reference point. The answer to this question is that the new reference source voltage will now be our zero-degree definition, with every phasor's angle changing to match this new reference. We may see this effect here, using the same two sources and three phasometers, but moving the strobe light from source A to source B:



Having the strobe light keyed to source B makes it flash 60 degrees earlier than it did when it was keyed to source A, which in turn "freezes" the arrows on the faces of all three phasometers -60^o from where they used to be when source A was the time-reference. The following table compares the phasometer readings with both strobe light reference points:

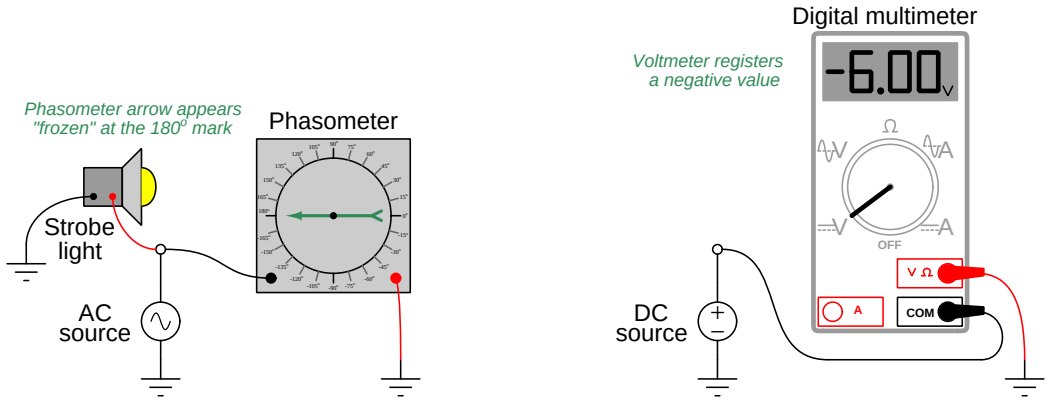
Strobe reference	V_A	V_B	V_{AB}
Source A	0^o	60^0	-60^o
Source B	-60^o	0^0	-120^o

It is even possible to imagine connecting the strobe light to a special electronic timing circuit pulsing the light at 60 Hz (the AC system's frequency), synchronized to an atomic clock so as to keep ultra-accurate time. Such an arrangement would permit phasor angle measurements based on an absolute time reference (i.e. the atomic clock) rather than a relative time reference (i.e. one of the AC voltage sources). So long as the two AC sources maintained the same frequency and phase shift, the three phasometers would still be displaced 60^o from each other, although it would only be by blind luck that any of them would point toward 0^o (i.e. that any one voltage would happen to be precisely in-phase with the electronic timer circuit's 60 Hz pulse).

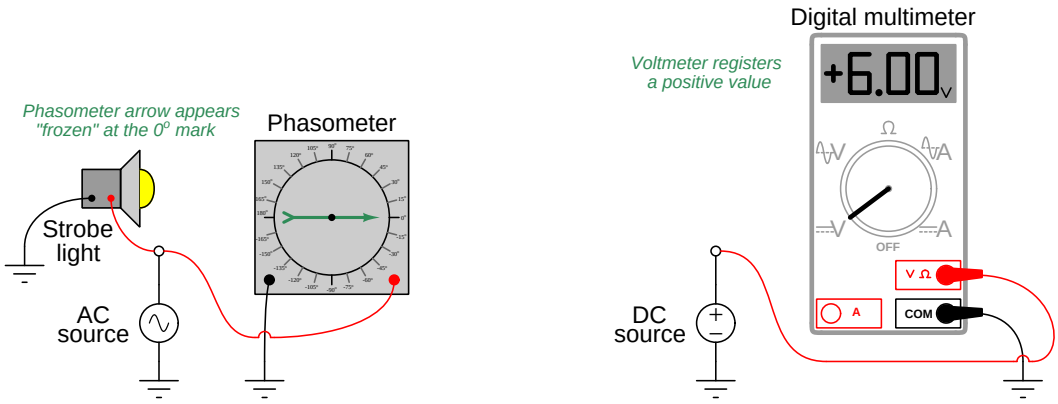
Interestingly, this technique of measuring AC power system phasors against an absolute time reference is a real practice and not just a textbook thought experiment. The electrical power industry refers to this as a *synchrophasor* measurement. Special instruments called *Phasor Measurement Units* or *PMU*s connect to various points within the power system to acquire real-time voltage and current measurements, each PMU also connected to a GPS (Global Positioning System) radio receiver to obtain an absolute time reference with sub-microsecond uncertainty. Each PMU uses the ultra-precise time signal from the GPS receiver to synchronize a "standard" cosine wave at the power system frequency such that this reference waveform is at its positive peak (0^o) at the top of every second in time.

Synchrophasor technology makes it possible to perform simultaneous comparisons of phasor angles throughout a power system, which is useful for such tasks as analyzing frequency stability in a power grid or detecting "islanding" conditions where tripped circuit breakers segment a power grid and allow distributed generators to begin drifting out of sync with each other.

Remember that *phasor angles* are to AC quantities what *arithmetic signs* are to DC quantities. If a phasometer registers an angle of 180 degrees, it means the red lead is fully negative and the black lead is fully positive at the moment in time when the strobe light flashes. If a DC voltmeter registers a negative voltage, it means the red lead is negative and the black lead is positive at the time of the measurement:



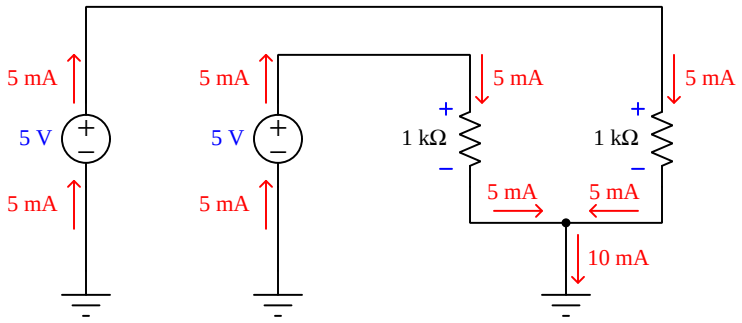
Reversing each instrument's test lead connections to the circuit will reverse its indication: flipping red and black leads on the phasometer will cause its indication to be 0^o instead of 180^o ; flipping red and black leads on the DC voltmeter will cause it to register a positive voltage instead of a negative voltage:



From these demonstrations we can see that the indication given by a measuring instrument depends as much on how that instrument connects to the circuit as it does on the circuit quantity itself. Likewise, a voltage or current value obtained in the course of analyzing a circuit depends as much on how we label the assumed polarity (or direction) of that quantity on the diagram as it does on the quantity itself.

To illustrate the importance of signs, assumed directions, and phasor angles in circuit analysis, we will consider two multi-source circuits: one DC and one AC. In each case we will explore how Kirchhoff's Current Law relates to currents at a similar node in each circuit, relating the current arrow directions to signs and phasor angles.

First, the DC circuit:



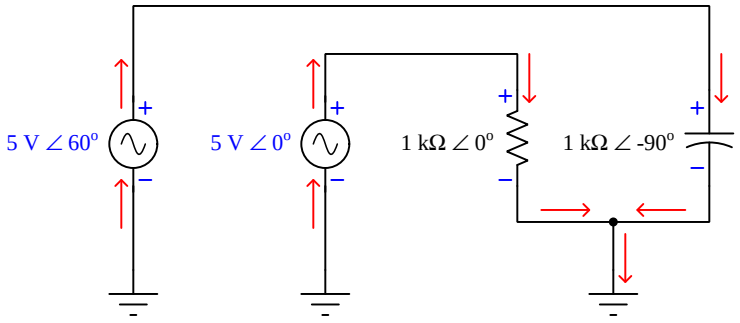
With each 1 kΩ resistor powered by its own 5 Volt DC source, the current through each of these resistors will be 5 milliAmperes in accordance with Ohm's Law ($I = \frac{V}{R}$). The direction of each current is easily predicted by examining the polarity of each source (with current represented in conventional flow notation *exiting* the positive terminal and *entering* the negative terminal of each source) and/or by examining the polarity of the voltage drop across each resistor (with conventional-flow current *entering* the positive terminal and *exiting* the negative terminal of each load). At the node below the two resistors, these two currents join together to form a larger (10 milliamp) current headed toward ground. Kirchhoff's Current Law declares that the sum of all currents entering a node must equal the sum of all currents exiting that node. Here we see this is true: 5 milliAmperes in plus 5 milliAmperes in equals 10 milliAmperes out.

The voltage polarities and current directions in this DC circuit are all clear and unambiguous because the quantities are *constant* over time. Each power supply acts as an energy source and each resistor acts as an energy load, all the time. Our application of Kirchhoff's Current Law at the node is so obvious it hardly requires explanation: the flow of charge carriers (current) in and out must be in equilibrium.

With AC, however, we know things will not be so simple. AC circuit quantities are not constant over time as they are in DC circuits. Reactive (energy-storing) components such as inductors and capacitors play alternating roles as sources and loads, complicating their analysis. The good news in all of this is that phasor representations of voltage and current allow us to apply all the same fundamental principles we are accustomed to using for DC circuits (e.g. Ohm's Law, Kirchhoff's Voltage Law, Kirchhoff's Current Law, network theorems, etc.). In order to use these principles to calculate AC quantities, though, we must be careful in how we label them in the circuit.

Here, we will modify the circuit to include two AC power sources (phase-shifted by 60^o), and replace one of the 1 kΩ resistors with a capacitor exhibiting 1 kΩ of reactance at the system frequency. As in the DC circuit, each of the two loads sees the full 5 Volts of its respective source. Unlike the DC circuit, we must represent each of the voltage and impedance quantities in complex (phasor) form in order to apply Ohm's Law to calculate load currents.

One more thing we will do to this circuit before beginning any phasor arithmetic is to label each voltage and each current as we did in the DC circuit. Placing + and − symbols at the terminals of *AC* voltage sources, and drawing conventional-flow notation arrows showing *alternating current* through loads at first may seem absurd, since we know this is an AC circuit and so by definition these quantities lack fixed polarities and directions. However, these symbols will become very important to us because they serve to define what 0^o means for each of the respective phasors. In other words, these polarities and arrows merely show us which way the voltages and currents are oriented *when each of those phasors is at its positive peak – i.e. when each phasor angle comes around to its own 0^o mark*. As such, the polarities and arrows we draw do *not necessarily* represent simultaneous conditions. We will rely on the calculated phasor angles to tell us how far these quantities will actually be phase-shifted from each other at any given point in time:



First, calculating current through the resistor[4], recalling that the impedance of a resistor has a 0 degree phase angle (i.e. no phase shift between voltage and current):

$$I_R = \frac{V_R}{Z_R} = \frac{5 \text{ V} \angle\ 0^o}{1 \text{ k}\Omega \angle\ 0^o} = 5 \text{ mA} \angle\ 0^o$$

Next, calculating current through the capacitor[5], recalling that the impedance for a capacitor has a −90 degree phase angle because voltage across a capacitor lags 90 degrees behind current through a capacitor:

$$I_C = \frac{V_C}{Z_C} = \frac{5 \text{ V} \angle\ 60^o}{1 \text{ k}\Omega \angle\ -90^o} = 5 \text{ mA} \angle\ 150^o$$

[4]When dividing two phasors in polar form, the arithmetic is as follows: divide the numerator's magnitude by the denominator's magnitude, then subtract the denominator's angle from the numerator's angle. The result in this case is 5 milliAmperes (5 Volts divided by 1000 Ohms) at an angle of 0 degrees (0 minus 0).

[5]The same arithmetic applies to this quotient as well: the current's magnitude is 5 Volts divided by 1000 Ohms, while the current's phase angle is 60 degrees minus a negative 90 degrees (150 degrees).

From the arrows sketched at the node we can see the total current headed toward ground must be equal to the sum of the two currents headed into the node, just as with the DC circuit. The difference here is that we must perform the addition using phasor quantities instead of scalar quantities in order to account for phase shift between the two currents:

$$I_{total} = I_R + I_C$$

$$I_{total} = 5 \text{ mA} \angle\ 0^o + 5 \text{ mA} \angle\ 150^o$$

Representing these current phasors in rectangular mode so we may sum their real and imaginary parts:

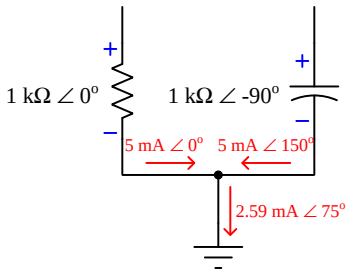
$$I_{total} = (5 + j0 \text{ mA}) + (-4.33 + j2.5 \text{ mA})$$

$$I_{total} = 0.67 + j2.5 \text{ mA}$$

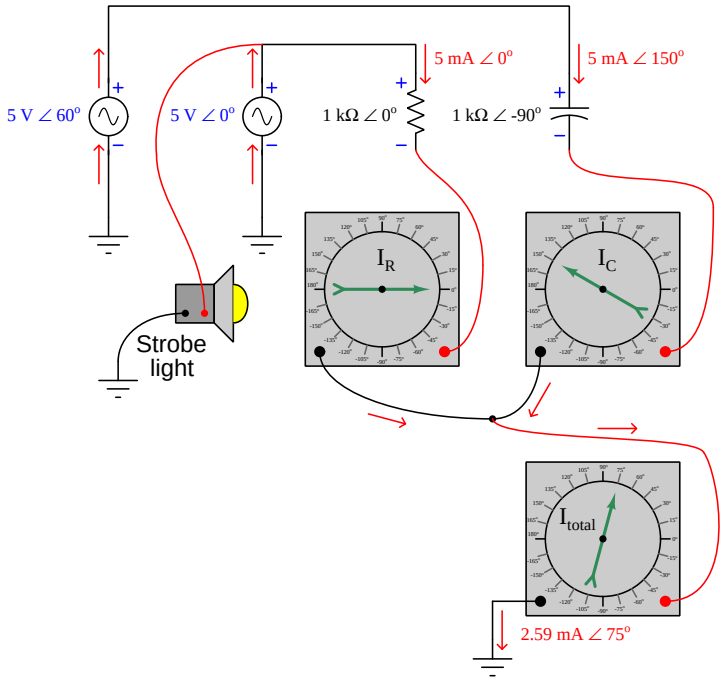
Converting the rectangular form into polar form:

$$I_{total} = 2.59 \text{ mA} \angle\ 75^o$$

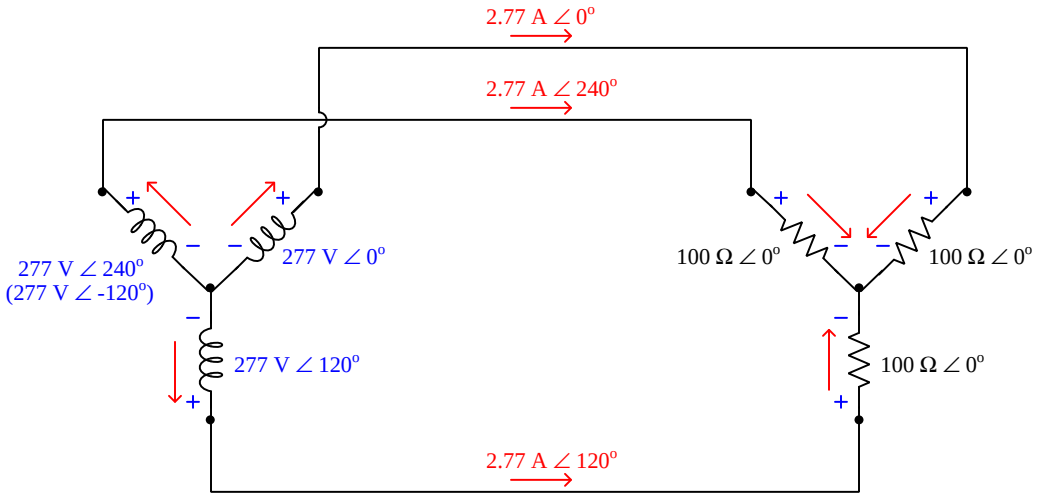
This result is highly non-intuitive. When we look at the circuit and see two 5 milliamp currents entering a node, we naturally expect a sum total of 10 milliAmperes to exit that node. However, that will only be true if those 5 mA quantities are *simultaneous*: i.e. if the two currents are 5 mA in magnitude *at the same point in time*. Since we happen to know these two currents are phase-shifted from each other by 150 degrees (nearly opposed to each other), they never reach their full strength at the same point in time, and so their sum is considerably less than 10 milliAmperes:



We may make more sense of this result by adding current-sensing phasometers to the circuit, their red and black test leads connected in such a way as to match the arrows' directions (current entering the red lead and exiting the black lead, treating the phasometer as a proper electrical *load*) as though these were DC ammeters being connected into a DC circuit:

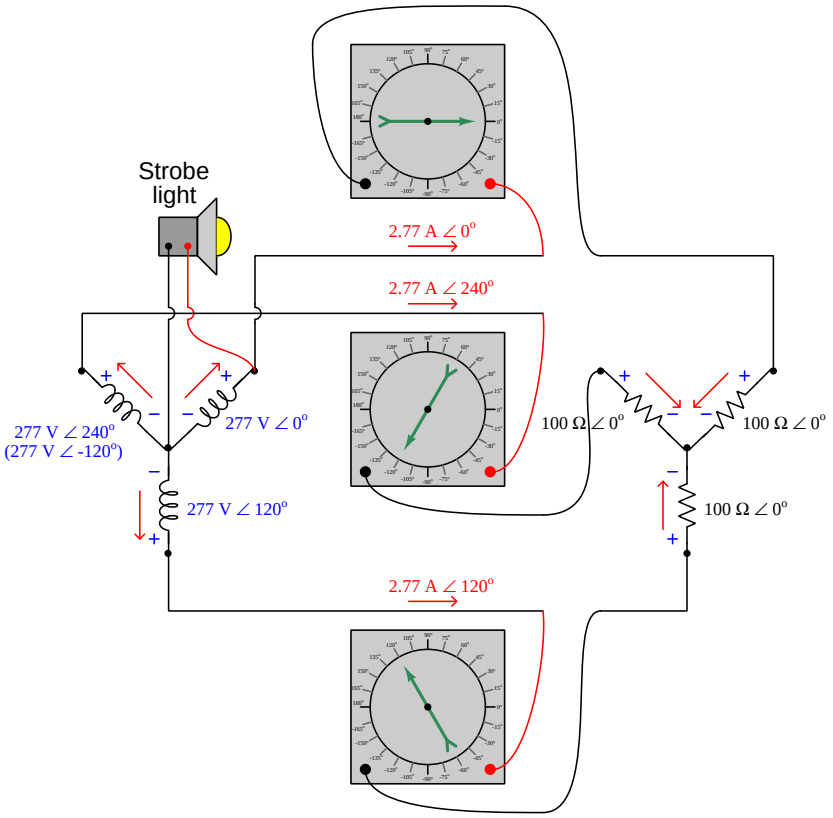


A very common and practical example of using "DC notation" to represent AC voltages and currents is in *three-phase* circuits where each power supply consists of three AC voltage sources phase-shifted 120 degrees from each other. Here, we see a "wye" connected generator supplying power to a "wye" connected load. Each of the three generator stator windings outputs 277 Volts, with the entirety of that voltage dropped across each of the load resistances:



Note the directions of the three currents (red arrows) in relation to the node at the center of the generator's "wye" winding configuration, and also in relation to the node at the center of the resistive load's "wye" configuration. At first this seems to be a direct violation of Kirchhoff's Current Law: how is it possible to have three currents *all exiting a node with none entering* or to have three currents *all entering a node with none exiting*? Indeed, this would be impossible if the currents were *simultaneously* moving in those directions, but what we must remember is that each of the current arrows simply shows which way each current will be moving *when its respective phasor comes around to the 0° mark*. In other words, the arrows simply define what zero degrees means for each current. Similarly, the voltage polarities would suggest to anyone familiar with Kirchhoff's Voltage Law in DC circuits that the voltage existing between any two of the power conductors between generator and load should be 0 Volts, reading the series-opposed sum of two 277 Volt sources. However, once again we must remind ourselves that the + and − symbols do not actually represent polarities at the same point in time, but rather serve to define the orientation of each voltage when its phasor happens to point toward 0°.

If we were to connect three current-sensing phasometers to measure the phase angle of each line current in this system (keying the strobe light to the voltage of the 0^o generator winding), we would see the true phase relationships relative to that time reference. The directions of the current arrows and the orientation of the + and − polarity marks serve as references for how we shall connect the phasometers to the circuit (+ on red, − on black; current entering red, exiting black):



We may use the three arrows at the load's center node to set up a Kirchhoff's Current Law equation (i.e. the sum of all currents at a node must equal zero), and confirm that the out-of-phase currents all "entering" that node do indeed amount to a sum of zero:

$$I_{node} = 2.77 \text{ A}\angle 0^o + 2.77 \text{ A}\angle 120^o + 2.77 \text{ A}\angle 240^o$$

Converting polar expressions into rectangular so we may see how they add together:

$$I_{node} = (2.77 + j0 \text{ A}) + (-1.385 + j2.399 \text{ A}) + (-1.385 - j2.399 \text{ A})$$

Combining all the real terms and combining all the imaginary terms:

$$I_{node} = (2.77 - 1.385 - 1.385) + j(0 + 2.399 - 2.399) \text{ A}$$

$$I_{node} = 0 + j0 \text{ A}$$

Looking closer at these results, we may determine which way the three currents were actually flowing at the time of the strobe's flash (when the upper-right generator winding is at its peak positive voltage). The first phasor has a real value of +2.77 Amperes at that instant in time, which represents 2.77 Amperes of actual current flowing in the actual direction of our sketched current arrow. The second and third phasors each have real values of −1.385 Amperes at that same instant in time, representing 1.385 Amperes each flowing *against* the direction of our sketched arrows (i.e. leaving that node). Thus, we can see that the "snapshot" view of currents at the time of the strobe's flash makes complete sense: one current of 2.77 Amperes entering the node and two currents of 1.385 Amperes each exiting the node. Viewed at any instant in time, the principles of DC circuits hold true. It is only when we sketch polarity marks and draw arrows representing voltages and currents *reaching their positive peak values at different times* that the annotations seem to violate basic principles of circuit analysis.

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Polarity

Thought experiments as a problem-solving strategy

Phasor

Phase angle

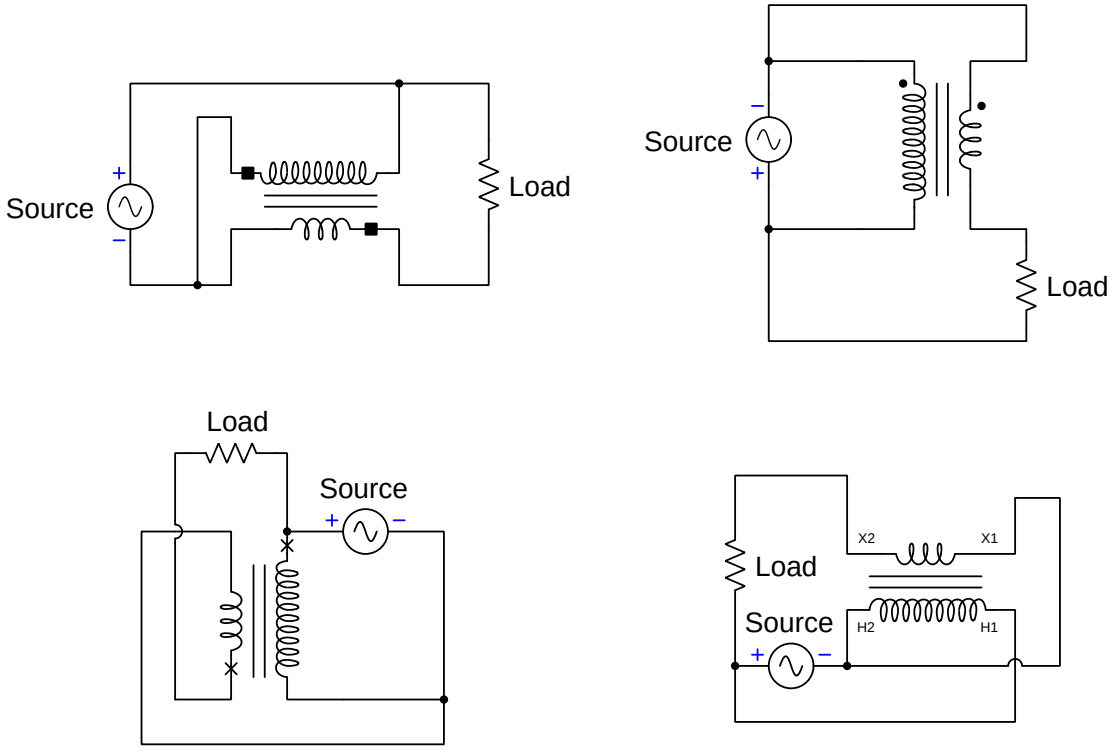
Complex numbers

Kirchhoff's Voltage Law

Kirchhoff's Current Law

5.1.3 Boost/buck transformer circuits

Identify which of these transformer circuits will *boost* voltage to the load, and which will *buck* voltage to the load, annotating each winding of the transformer as well as the load with + and − polarity marks (denoting actual voltage polarity at some instant in time):



Challenges

- A profitable problem-solving strategy for many conceptual or qualitative problems is to add some numerical quantities to it. Try that with this problem, and explain how this extra information may be helpful to finding the solution.
- After adding some numerical values, calculate the amount and direction of current through each of the transformer's windings. You should notice something very interesting!

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

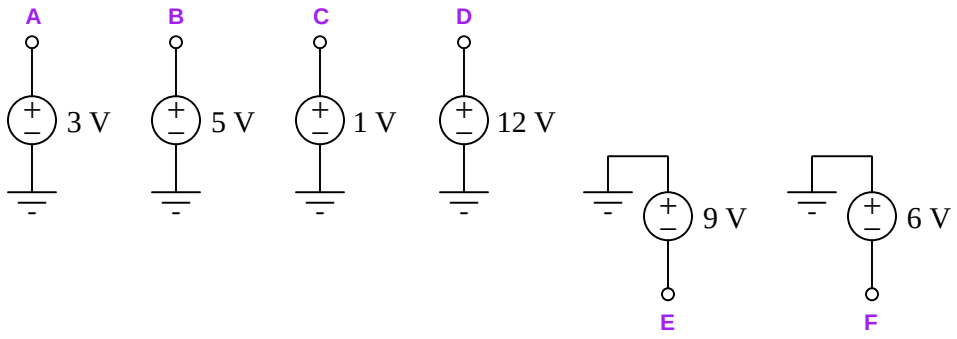
Note how the square-root term (y) is calculated in cell `C1`, and the denominator term (z) in cell `C2`. This makes the two final formulae (in cells `B1` and `B2`) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

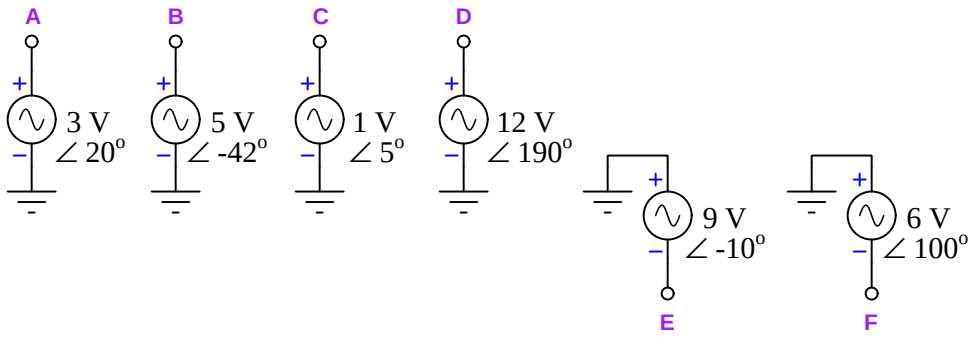
5.2.3 Voltages between specified points

Calculate the amount of voltage as registered by a digital voltmeter when its test leads are connected to the specified points (red lead on the first point and black lead on the second), being sure to specify the mathematical sign as well as the magnitude of the voltmeter's measurement. A single-subscript voltage measurement refers to one taken with the red lead on the specified point and the black lead on ground:



- $V_{DB} =$
- $V_{AE} =$
- $V_{CF} =$
- $V_{CA} =$
- $V_{FE} =$
- $V_{EB} =$
- $V_{E} =$

Now, do the same for these AC voltage sources (assuming all at the same frequency), being sure to specify the phase angle of the measured voltage as well as its magnitude:



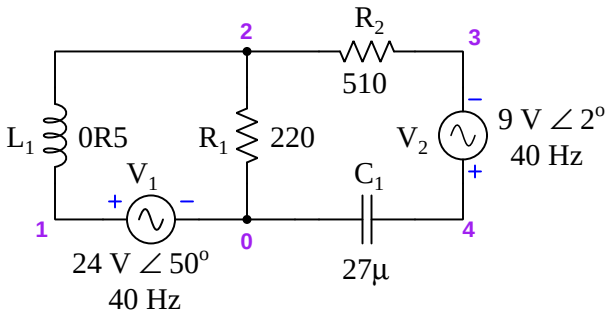
- V_{DB} =
- V_{AE} =
- V_{CF} =
- V_{CA} =
- V_{FE} =
- V_{EB} =
- V_E =

Challenges

- The black-colored test lead of a voltmeter may be thought of as the "reference" and the red test lead as the "measurement", such that the measured voltage is an expression of how much more (or less) electrical potential is at the red lead compared to the black. Relate this principle to your answers here.
- Sketch phasor diagrams showing any of these resultant voltages.

5.2.4 Incomplete SPICE simulation

Below is a schematic diagram for an AC circuit followed by an incomplete SPICE netlist:




```
* Netlist
v1 1 0 ac 24 50
v2 4 3 ac 9 2
r1 2 0 220
r2 2 3 510
c1 4 0 27e-6
l1 2 1 0.5
.control
set units=degrees
.endc
.ac lin 1 40 40
```

Add more lines to this netlist instructing the computer to calculate voltage across resistor R_1 (red lead on top, black lead on bottom) as well as voltage across resistor R_2 (red lead on right, black lead on left).

Challenges

- Run this simulation using SPICE and show the results for V_{R1} and V_{R2}.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

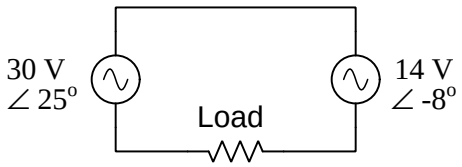
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Find the mistake(s)

Suppose a student calculates the load voltage in this circuit to be 19.787 Volts $\angle$ 47.666^o:



Explain what (may) be incorrect about this calculated load voltage.

Challenges

- Alter the given parameters of the circuit so that a definitive load voltage may be calculated.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

9 March 2026 – deleted a redundant word, courtesy of Wyatt Gleason.

7 October 2024 – minor edits to the "Phasor polarity" section of the Full Tutorial chapter.

25 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors. Also divided the Full Tutorial into sections.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

8 May 2021 – commented out or deleted empty chapters.

6 March 2021 – added Challenge question to "Boost/buck transformer circuits".

4 March 2021 – minor additions to instructor notes and also to the Full Tutorial.

7 February 2021 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

6 January 2021 – added some Questions, added a Derivations and Technical References chapter, and made minor edits to the Full Tutorial.

August 2018 – content added to the Introduction, clarified one of the examples in the Simplified Tutorial, and clarified one of the examples in the Full Tutorial.

June 2018 – document first created.

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