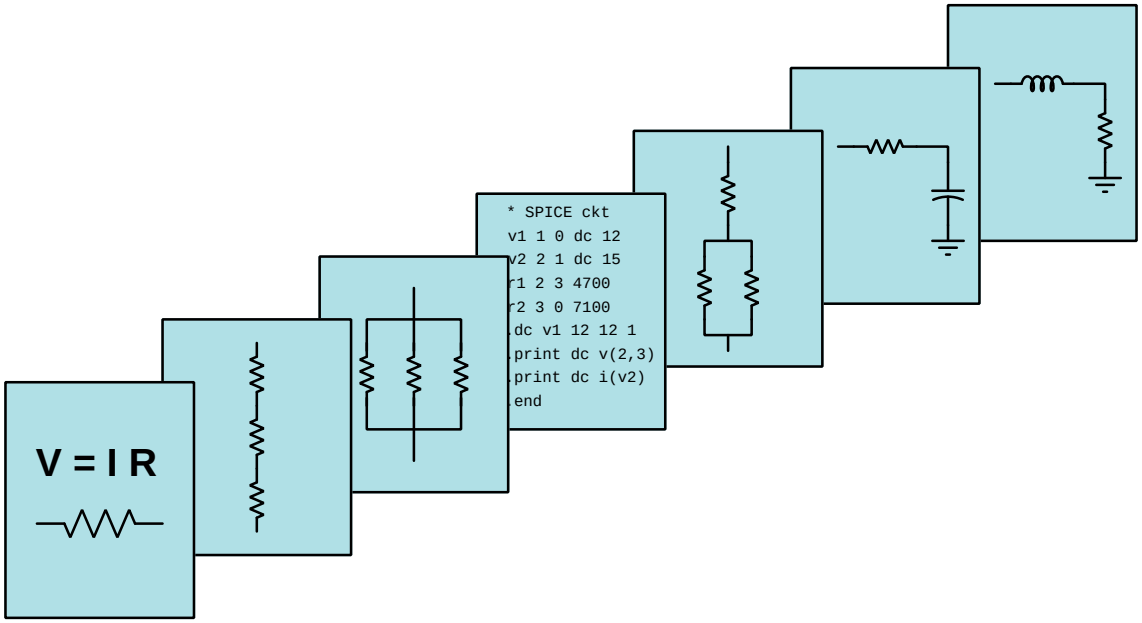
Modular Electronics Learning (ModEL) project



Series-Parallel Circuits

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Chapter 1

Introduction

1.1 Recommendations for students

The analysis of series-parallel resistor circuits naturally relies on a firm understanding of both series networks and of parallel networks. For this reason, this tutorial begins with a review of both network types. Kirchhoff's Laws are also very useful, and in some cases indispensable, for series-parallel analysis and so are reviewed at the start of the tutorial as well. Simply put, series-parallel circuit analysis brings many principles together for the first time. A great many practical circuits also lend themselves well to analysis as collections of simple-series and simple-parallel networks, so learning to identify those configurations and to analyze more complex circuits consisting of nested sub-networks is a very useful skill.

Like so many other forms of analysis, *practice makes perfect*. Good sources of practice problems for series-parallel analysis include the *Case Tutorial* chapter as well as modules in this series focused on SPICE computer analysis (where the circuit-analysis results may be used as an "answer key" to check one's work).

Important concepts related to series-parallel circuits include **Conservation of Electric Charge**, **Conservation of Energy**, **sources** versus **loads**, **series** networks and their properties, **parallel** networks and their properties, **steady-state** condition, **Ohm's Law**, **Kirchhoff's Laws**, **voltmeter** usage, **ammeter** usage, **equivalent** networks, **insertion resistance**, and **divider** networks.

A very important problem-solving strategy applied throughout the Tutorials is that of *annotating diagrams* with labels showing voltages, polarities, and currents (with direction). Mapping calculated values onto these diagrams is a helpful way to avoid confusion and to maintain proper context for all the quantities. Another important point about problem-solving is the need for *patience*, and the willingness to proceed with calculations even if the final strategy for solving the problem eludes one's immediate grasp.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve to confirm what you are reading, but it is also an excellent way to practice those mathematical

techniques.

Another useful reading strategy is to write your own summary page of important principles, especially when those principles mirror each other. For this module I would recommend writing your own summaries of series versus parallel networks: what defines each one, the properties of each, and the fundamental reasons why those properties are true.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the properties of series electrical networks? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?
- How might an experiment be designed and conducted to demonstrate the properties of parallel electrical networks? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?
- How might an experiment be designed and conducted to demonstrate the principle of Voltage Current Law? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the principle of Kirchhoff's Current Law? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What defines a *series* connection between two or more components?
- What defines a *parallel* connection between two or more components?
- How may we explain all the properties of series networks in terms of more fundamental principles?
- How may we explain all the properties of parallel networks in terms of more fundamental principles?
- Why must we connect voltmeters and ammeters to circuits in very specific ways?
- What must be done in a circuit to ensure that a set of points are equipotential to each other?
- Where do we see Kirchhoff's Voltage Law being used to tally voltage values that do *not* include the circuit's source?
- What is the significance of "equivalent" resistances in the analysis of series-parallel circuits?
- Why do we "collapse" and "expand" series-parallel networks during the process of analysis?
- How may we combine multiple sources to achieve greater levels of voltage and/or current?
- How may we combine multiple resistances to achieve novel resistance values?
- What are some of the ways you can check your work when analyzing series-parallel circuits?
- Why do voltage divider circuits "load down" when connected to other components?
- How may we build a voltage divider in such a way to anticipate and counter this "loading" effect?
- Why do meters sometimes register voltage or current inaccurately despite having no internal calibration errors?

- How much insertion resistance should an ideal voltmeter possess?
- How much insertion resistance should an ideal ammeter possess?
- How do the Laws of Energy and Electric Charge Conservation apply to series-parallel networks?
- What does it mean to "solve what you can" as an analytical strategy for electric circuits?
- Where does *patience* play a role in the analysis of series-parallel networks?

1.2 Challenging concepts related to series-parallel circuits

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Defining a series network** – series networks are defined by having a single path for current between the components. From this definition, and from the Conservation of Electric Charge, follows the conclusion that series-connected components must experience the same amount of continuous current. One challenge is that sometimes series-connected components do not physically or pictorially align to suggest a single path, even though they have but one *electrical* path for current. Another challenge is reasoning logically from this definition of "one path" to the necessary conclusion of "same current", as many students are so accustomed to memorizing facts without seeing the logical connections between those facts.

- **Definition versus properties of series networks** – again, series networks are defined by having just one pathway for current between components. Everything else we know about series networks *derives* from this definition but are not *synonymous* with this definition. For example, the fact that voltages add in a series network is a consequence of series networks but not a defining quality. The fact that series-connected components necessarily share the same current is another consequence of series networks but not a defining quality. All we need to consider in order to understand this distinction is a counter-example of some electrical network exhibiting either summed voltages or equal currents without being series. A good counter-example of the former misconception is a five-resistor bridge network where all five resistor values are unique and non-proportionate, this being a clear example of a network where voltages across resistors on either side of the network indeed add up to the source's voltage value despite none of the resistors being in series with any other. A good counter-example of the latter misconception is any network that just happens to contain two or more components having the same current that are not series to each other (e.g. two identical resistors in parallel with each other). Applying counter-examples to disprove this and similar fallacies is a rhetorical technique called *reductio ad absurdum*, and the habit of looking out for counter-examples when testing claims is a powerful tool for critical thinking.

- **Defining a parallel network** – parallel networks are defined by all components spanning the same two sets of electrically-common points. From this definition, and from the Conservation of Energy, follows the conclusion that parallel-connected components must experience the same amount of continuous voltage. One challenge is that sometimes parallel-connected components do not physically or pictorially align in a parallel fashion, even though they are in fact parallel in the *electrical* sense. Another challenge is reasoning logically from this definition of "between the same two sets of electrically-common points" to the necessary conclusion of "same voltage", as many students are so accustomed to memorizing facts without seeing the logical connections between those facts. A helpful way to both identify parallel connections as well as to conceptualize the commonality of voltage is to use color-highlighting to mark all electrically-common points with the same color. This way, all points in one set have one color, all points in the other set have another color, and all components connecting to points with those two colors must be connected in parallel with each other. This coloring approach is used in the Tutorial chapters where points A-B-C-D are all colored blue and points E-F-G-H are all colored green. The *Case Tutorial* section "Example: potential in open and shorted circuits" is a

helpful reference for using color-highlighting to denote different amounts of electrical potential in a circuit.

- **Definition versus properties of parallel networks** – again, parallel networks are defined by all components spanning the same two sets of electrically-common points. Everything else we know about parallel networks *derives* from this definition but are not *synonymous* with this definition. For example, the fact that currents divide in a parallel network is a consequence of parallel networks but not a defining quality. The fact that parallel-connected components necessarily share the same voltage is another consequence of parallel networks but not a defining quality. All we need to consider in order to understand this distinction is a counter-example of some electrical network exhibiting either divided currents or equal voltages without being parallel. A good counter-example of the former misconception is a five-resistor bridge network where all five resistor values are unique and non-proportionate, this being a clear example of a network where currents divide despite none of the resistors being in parallel with any other. A good counter-example of the latter misconception is any network that just happens to contain two or more components having the same voltage that are not parallel to each other (e.g. two identical resistors in series with each other). Applying counter-examples to disprove this and similar fallacies is a rhetorical technique called *reductio ad absurdum*, and the habit of looking out for counter-examples when testing claims is a powerful tool for critical thinking.

- **Using Ohm's Law in context.** When applying Ohm's Law ($V = IR$; $I = \frac{V}{R}$; $R = \frac{V}{I}$) to circuits containing multiple resistances, students new to circuit analysis often mix contexts of voltage, current, and resistance. That is, students often mis-apply parameters where they do not belong in a circuit. Whenever applying any equation describing a physical phenomenon, it is important to ensure each variable of that equation relates to the proper real-life value within the problem. For example, when calculating the voltage drop across resistor R_2, one must be sure that the values for current and resistance are appropriate for that resistor and not some other resistor in the circuit. When calculating V_{R_2} using the Ohm's Law equation $V = IR$, one must use the value of *that resistor's* current (I_{R_2}) and *that resistor's* resistance (R_2), not some other current and/or resistance value(s). Some students have an unfortunate tendency to overlook context when seeking values to substitute in place of variables in Ohm's Law problems, and this leads to incorrect results.

- **Voltage being relative between two points** – unlike current, which may be measured at a single point in a circuit, voltage is fundamentally relative: it only exists as a *difference* between two points. In other words, there is no such thing as voltage existing at a single location. Therefore, while we speak of current going *through* a component in a circuit, we speak of voltage being *across* a component, measured between two terminals on that component. An effective strategy for combating this misconception is to continually apply the correct definition for voltage: *the amount of energy difference between an electric charge carrier at one location versus an identical electric charge carrier at another location.* This concept also reinforces (and is reinforced by) the concept of electric circuit components behaving as sources (infusing charge carriers with energy) or loads (depleting charge carriers of energy).

- **Equivalent electrical networks** – the possibility that completely different networks of components may nevertheless *behave* identically from the perspective of two or more connection terminals is non-intuitive for many students new to electric circuits. However, there are many

analogues in life that may be used to help explain. For example, a box containing an unknown assortment of items may have precisely the same weight as a different box containing something completely different from the first: i.e. from the perspective of weight sense at the handle when lifting each box, the boxes may behave the same even though their contents may be wholly different. This concept is an important one in circuit analysis, and students often encounter it first when analyzing series-parallel networks, by systematically reducing those networks to equivalent single resistances.

- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

- **Carelessness with series-parallel analysis** – when analyzing a series-parallel circuit to determine component voltages and currents, one must be very careful to reduce the circuit step-by-step, section-by-section, into equivalent resistances. Students new to this process typically see all the work that is involved and try to save effort by taking shortcuts, which paradoxically causes *more* work and *more* confusion for them later on. Use lots of paper to document your work when you reduce series-parallel resistor networks to equivalent resistances, re-drawing the circuit for each reduction. This helps reduce the number of mistakes, and also makes it easier to transfer calculated values of voltage and current to the correct components (see the challenging concept shown above: using Ohm's Law in context).

- **Creative application of foundational concepts** – many students enter college-level study of electronics with an educational background stressing rote memorization at the expense of logical reasoning from trusted principles, and as such tend to find circuit analysis daunting where there is no single procedure or single formula always yielding the correct answer(s). The educational solution to this is for the instructor to *constantly* tie all new circuit concepts back to previously-learned principles and show the logical relationships between them, and then with every new circuit-analysis example challenge students to reason from those concepts on their own. When students construct a mental "toolbox" they may apply in a plurality of ways to any new challenge they encounter, they will be as free as any skilled mechanic who knows how and why each of their hand tools functions to freely select and use whichever of those tools is most appropriate at any given time. A word of caution for new instructors: *be very sparing in your direction on how to use those tools, instead engaging with students Socratically when they are "stuck" and cannot see a path to a solution!*

A useful resource for practicing circuit analysis is the *Case Tutorial* chapter, containing example circuits with voltage and current values provided to check your work.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how all quantitative results were obtained by the author in the Tutorial chapters' series-parallel circuit analysis examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Calculate voltage, current, and power values within reducible series-parallel circuits

  Assessment – Predict all component voltages, currents, and power dissipations in a series-parallel circuit given resistor values and source value; e.g. pose problems in the form of the "Three-resistor circuits" Quantitative Reasoning question.

  Assessment – Predict all component voltages, currents, and power dissipations in a series-parallel circuit given resistor values and a single resistor voltage or current; e.g. pose problems in the form of the "Circuit with unknown source value" Quantitative Reasoning question.

  Note – The importance of having students work through all their own calculations in class cannot be overstated!

- **Outcome** – Analyze irreducible series-parallel circuits

  Assessment – Predict all component voltages, currents, and source-vs-load modalities for a a series-parallel circuit that cannot be reduced to a single equivalent resistance; e.g. pose problems in the form of the "Mixed-source circuits" Quantitative Reasoning question.

  Note – The importance of having students work through all their own calculations in class cannot be overstated!

- **Outcome** – Diagnose a faulted series-parallel circuit

  Assessment – Predict the effect(s) of a single component failing either open or shorted in a series-parallel circuit; e.g. pose problems in the form of the "Faults in a printed circuit" Diagnostic Reasoning question.

Assessment – Determine the probability of various component faults in a series-parallel circuit given symptoms and measured values; e.g. pose problems in the form of the "Faulty electric lamp array" Diagnostic Reasoning question.

The study of series-parallel circuits presents a prime opportunity to discipline students' thinking according to first principles of electricity. A very common tendency among students is to rote-memorize rules for analyzing circuits rather than reason logically from trusted principles. A good way to encourage and reinforce sound reasoning is to have students maintain a list of all the important laws, principles, and problem-solving strategies they learn about electric circuits, and to intentionally reference this list every time they analyze a circuit. At each step in the analysis of a given circuit, ask students to explain *why* they should apply each law or principle in turn!

A great exercise for students to practice selecting laws and principles appropriate to a given scenario are the "mixed-source" circuits shown in the *Case Tutorial* and *Questions* chapters, specifically the "Quantitative reasoning" section of the latter. Contrived though they may be, these mixed-source circuits cannot all be analyzed using the same principles applied in the same order, as many single-source series-parallel circuits can, and they cannot be collapsed into equivalent networks having fewer resistors as many series-parallel networks can. Instead, students must survey each one in a unique way to identify series versus parallel connections, to identify where and when KVL and KCL may be applied, etc., and therein lies these circuits' educational value. The aforementioned list of laws and principles should be in ready view whenever mixed-source circuits are analyzed, to remind students of *all* the tools at their disposal.

Another exercise useful for honing the skill of selecting from multiple laws and principles to analyze circuits is to pose standard series-parallel networks with unlabeled source values but where at least one of the resistors' voltage or current is known.

Students who master the selection and application of universal *principles* to circuits such as these will be well-prepared later to study more advanced circuits such as transistor switching networks, transistor amplifiers, operational amplifier networks, electrical metrology networks, etc., finding them to be mere variations on common themes. Those who only memorize series-parallel network *procedures*, however, will perceive each of these later circuit types as unrelated and daunting challenges.

Chapter 2

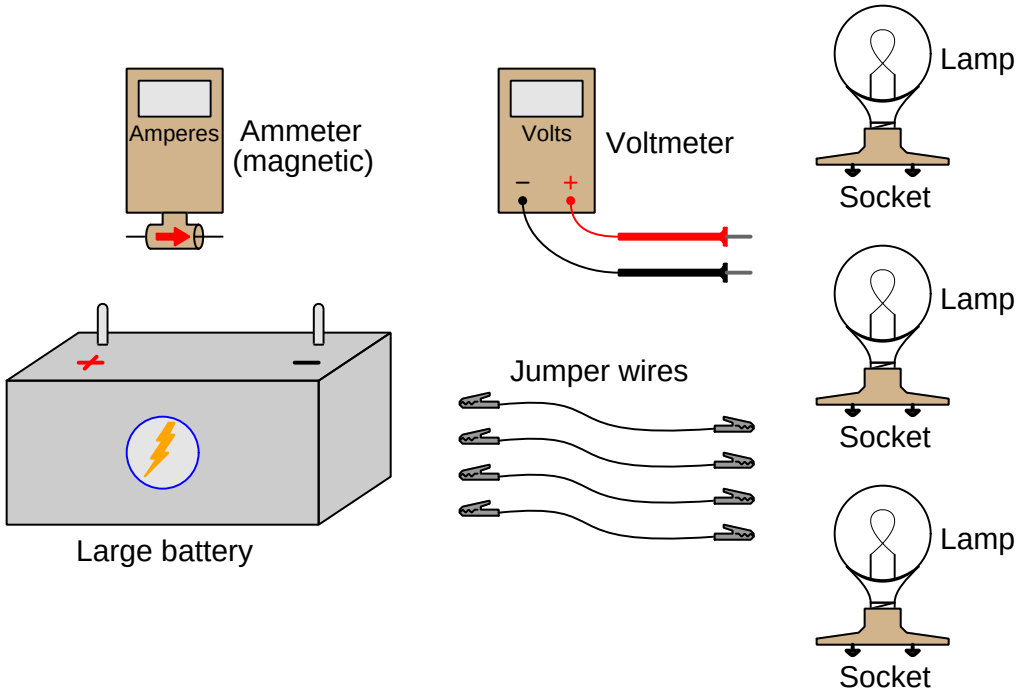
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: Battery, lamps, jumper wires, and meters

Here, a large 12 Volt battery and three 12 Volt-rated lamps are provided for experimentation, along with "jumper" wires consisting of plastic-clad stranded copper conductors terminated with spring-loaded "alligator" clip jaws at either end. A simple voltmeter and magnetic ammeter stand ready to take measurements:

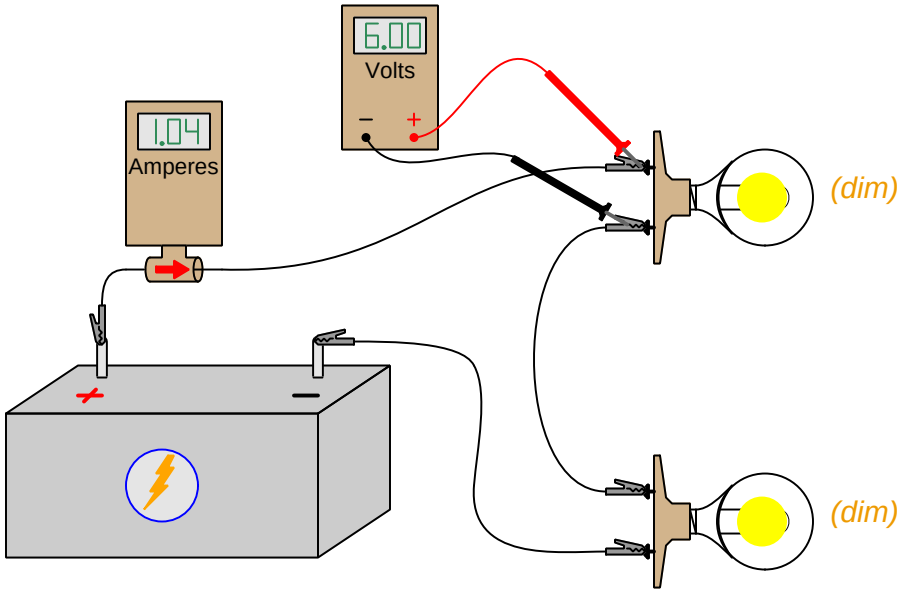


Current is the flow of subatomic particles called "electric charge carriers" through electrically conductive materials such as copper metal. We measure current in the metric unit of the *Ampere*, one Ampere (1 A) being 6.2415×10^{18} individual charge carriers moving past a point in one second of time. Our ammeter senses current by the magnetic field produced around the conductor, a positive indication meaning charge carriers moving (as interpreted by the "conventional flow" standard) with the arrow and a negative indication meaning motion in the opposite direction.

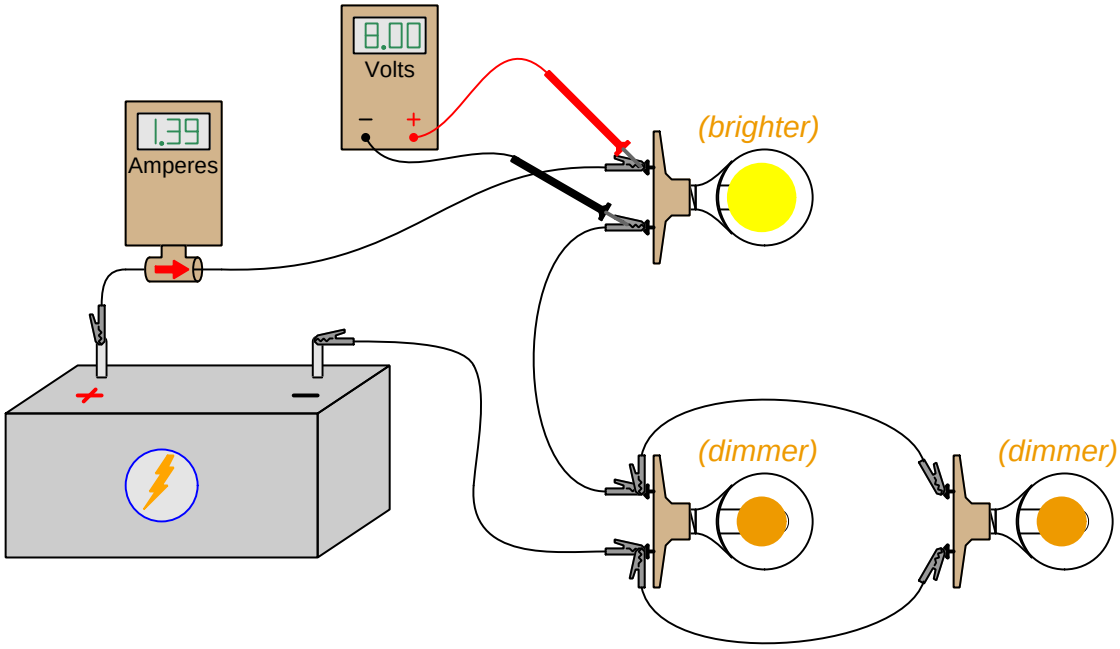
Voltage is the difference in energy for an electric charge carrier experienced at two different locations. We measure voltage in the metric unit of the *Volt*, one Volt (1 V) being one Joule of energy difference per 6.2415×10^{18} individual charge carriers. Our voltmeter's two copper-wire *test leads* will be touched to the two locations of interest, a positive indication meaning charge carriers at the red test lead have more energy than charge carriers at the black test lead; a negative indication means the opposite.

Resistance is the extraction of energy from moving electric charge carriers. We measure resistance in the metric unit of the *Ohm*, one Ohm (1 Ω) being one Volt of "drop" per Ampere of current.

Connecting two of the three lamps in *series* with each other results in each lamp glowing dimmer than it ordinarily would if connected to the battery by itself. The measured current in the circuit is also less than expected for a single lamp, and each lamp drops one-half of the battery's 12 Volts:



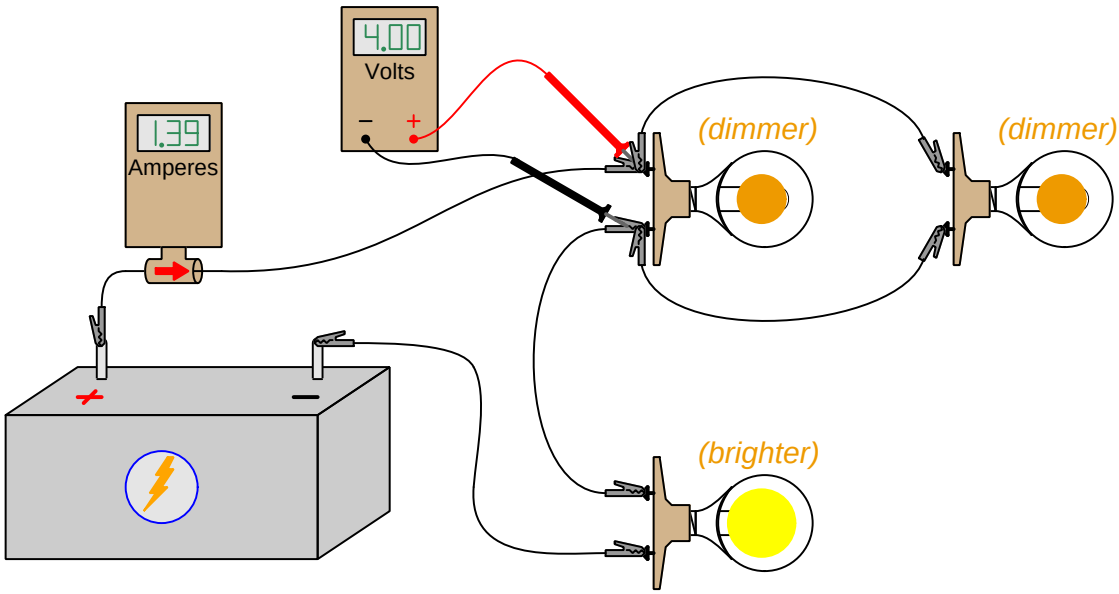
Next, the third lamp is connected in *parallel* to the lower of the two series-connected lamps. The result of this addition is that both lower lamps glow dimmer, and the upper lamp glows brighter. None of the lamps, however, glow as brightly as would be expected when connected to the battery alone:



The upper lamp now drops 8.00 Volts, while each of the lower lamps drops only 4.00 Volts. Current measured near the battery's positive terminal is greater than it was before, but still not as high as would be expected with the battery powering just a single lamp.

Disconnecting the third lamp returns the circuit to its previous state (with just two lamps connected in series): both lamp voltages return to 6.00 Volts, and current returns to 1.04 Amperes.

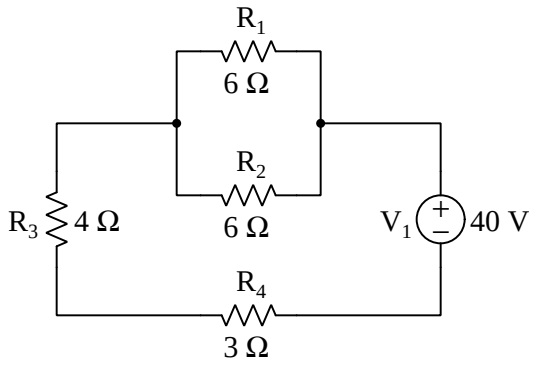
Connecting the third lamp in parallel with the *upper* of the two series-connected lamps results in the two upper lamps dropping 4.00 Volts and the lower lamp dropping 8.00 Volts, and current returning to 1.39 Amperes.



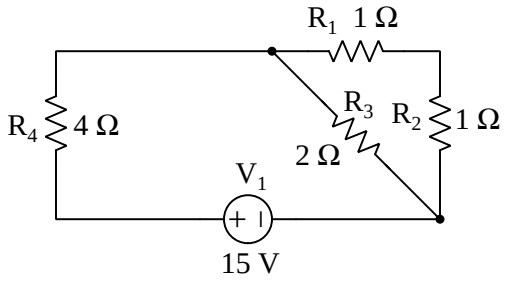
As before, the paralleled lamps are dimmest, while the remaining lamp is brighter.

2.2 Examples: series-parallel circuits with simple component values

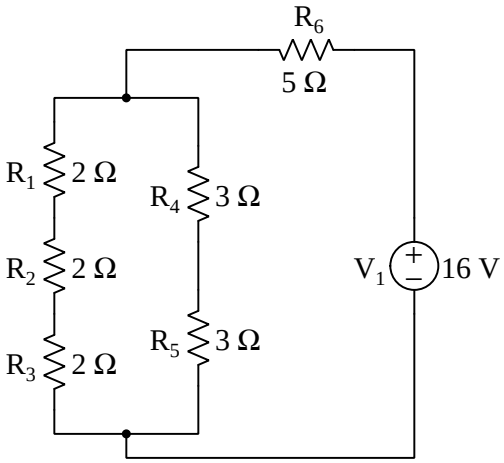
Component values in these circuits have all been chosen to make computations very easy. All voltages and currents should be computable without the aid of a calculator.



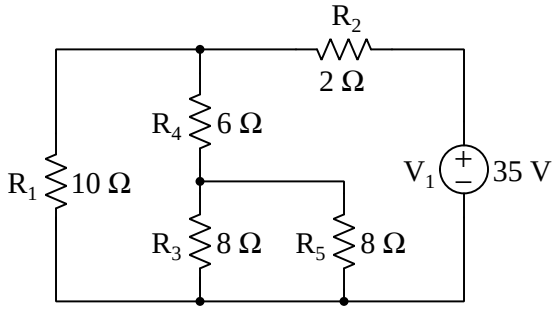
- $R_{total} = 10\ \Omega$
- $I_{source} = 4$ A
- $I_{R1} = 2$ A
- $I_{R2} = 2$ A
- $I_{R3} = 4$ A
- $I_{R4} = 4$ A
- $V_{R1} = 12$ V
- $V_{R2} = 12$ V
- $V_{R3} = 16$ V
- $V_{R4} = 12$ V



- $R_{total} = 5\ \Omega$
- $I_{source} = 3$ A
- $I_{R1} = 1.5$ A
- $I_{R2} = 1.5$ A
- $I_{R3} = 1.5$ A
- $I_{R4} = 3$ A
- $V_{R1} = 1.5$ V
- $V_{R2} = 1.5$ V
- $V_{R3} = 3$ V
- $V_{R4} = 12$ V



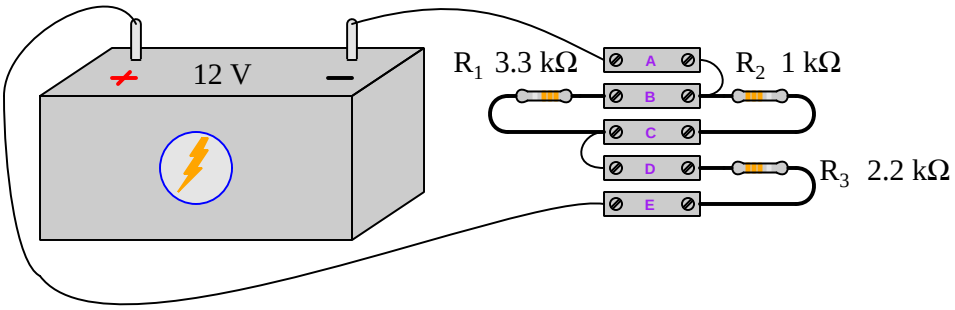
- $R_{total} = 8\ \Omega$
- $I_{source} = 2$ A
- $I_{R1} = 1$ A
- $I_{R2} = 1$ A
- $I_{R3} = 1$ A
- $I_{R4} = 1$ A
- $I_{R5} = 1$ A
- $I_{R6} = 2$ A
- $V_{R1} = 2$ V
- $V_{R2} = 2$ V
- $V_{R3} = 2$ V
- $V_{R4} = 3$ V
- $V_{R5} = 3$ V
- $V_{R6} = 10$ V



- $R_{total} = 7\ \Omega$
- $I_{source} = 5$ A
- $I_{R1} = 2.5$ A
- $I_{R2} = 5$ A
- $I_{R3} = 1.25$ A
- $I_{R4} = 2.5$ A
- $I_{R5} = 1.25$ A
- $V_{R1} = 25$ V
- $V_{R2} = 10$ V
- $V_{R3} = 10$ V
- $V_{R4} = 15$ V
- $V_{R5} = 10$ V

2.3 Example: Battery and three resistors

In this scenario we will use *terminal blocks* to neatly organize all wire connections between a battery and three resistors:

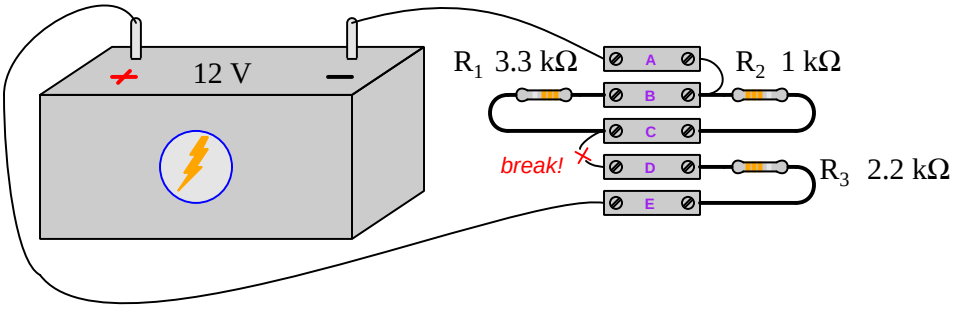


Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as V and I values[1]. With the circuit constructed as shown in the above illustration, we obtain the following measurements:

- $V_{AB} = 0.00$ Volts
- $V_{BC} = -3.10$ Volts
- $V_{CD} = 0.00$ Volts
- $V_{DE} = -8.90$ Volts
- $V_{DB} = 3.10$ Volts
- $V_{CA} = 3.10$ Volts
- $V_{EA} = 12.00$ Volts
- $I_A = 4.04$ milliAmperes (conventional flow right to left)
- $I_E = 4.04$ milliAmperes (conventional flow left to right)

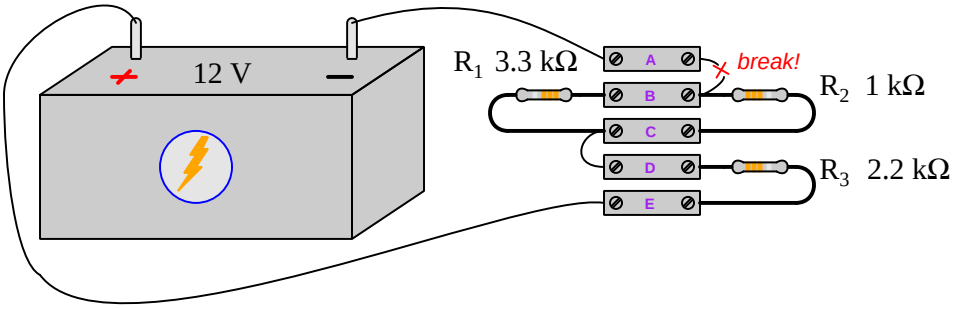
[1] Double-lettered subscripts for V denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, V_{BC} means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for V represent the voltmeter's red test lead, with the black test lead touching a defined "ground" point in the circuit. Current measurements (I) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for I. Conventional flow notation will be used to specify current direction.

Severing the wire connecting terminals C and D affects most of the measured values:



- $V_{AB} = 0.00$ Volts
- $V_{BC} = 0.00$ Volts
- $V_{CD} = -12.00$ Volts
- $V_{DE} = 0.00$ Volts
- $V_{DB} = 12.00$ Volts
- $V_{CA} = 0.00$ Volts
- $V_{EA} = 12.00$ Volts
- $I_A = 0.00$ milliAmperes
- $I_E = 0.00$ milliAmperes

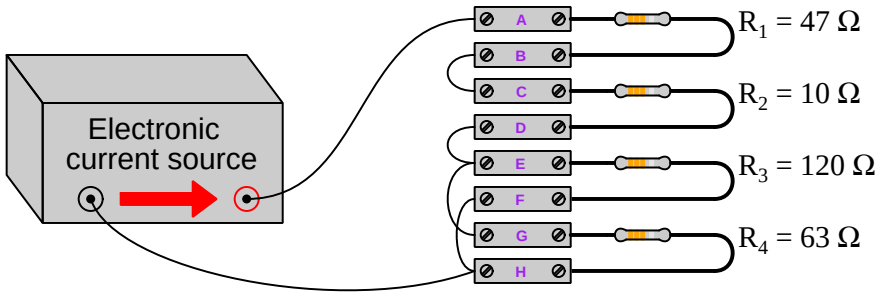
Severing the wire connecting terminals A and B results in another set of measured values:



- $V_{AB} = -12.00$ Volts
- $V_{BC} = 0.00$ Volts
- $V_{CD} = 0.00$ Volts
- $V_{DE} = 0.00$ Volts
- $V_{DB} = 0.00$ Volts
- $V_{CA} = 12.00$ Volts
- $V_{EA} = 12.00$ Volts
- $I_A = 0.00$ milliAmperes
- $I_E = 0.00$ milliAmperes

2.4 Example: Current source and four resistors

Here, an electronic *current source*[2], a set of terminal blocks, and several resistors are provided for experimentation. In this case, the current source is configured to output 150 milliAmperes of current.



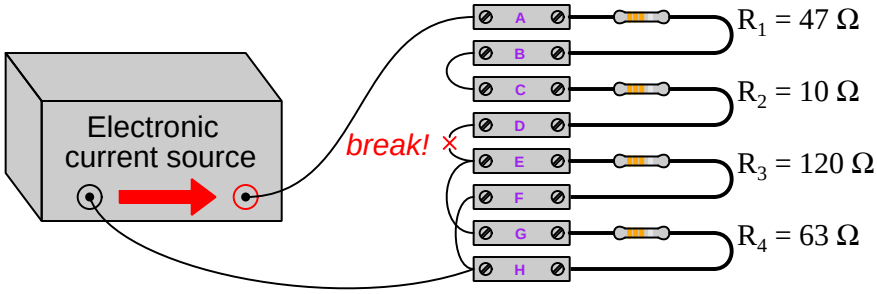
Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as V and I values[3]. With the circuit constructed as shown in the above illustration, we obtain the following measurements:

- $V_{AD} = 8.55$ Volts
- $V_{EA} = -8.55$ Volts
- $V_{GE} = 0.00$ Volts
- $V_{DH} = 6.20$ Volts
- $V_{GA} = -8.55$ Volts
- $V_{BC} = 0.00$ Volts
- $V_{DF} = 6.20$ Volts
- $I_A = 150.0$ milliAmperes (conventional flow left to right)
- $I_C = 150.0$ milliAmperes (conventional flow left to right)
- $I_D = 150.0$ milliAmperes (conventional flow right to left)
- I_H (at resistor lead) = 98.36 milliAmperes (conventional flow right to left)

[2] Unlike electrochemical *batteries* which tend to maintain a nearly constant voltage between their terminals over a wide range of current values, a *current source* maintains a nearly constant current over a wide range of voltages.

[3] Double-lettered subscripts for V denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, V_{BC} means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for V represent the voltmeter's red test lead, with the black test lead touching a defined "ground" point in the circuit. Current measurements (I) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for I. Conventional flow notation will be used to specify current direction.

Severing the wire connecting terminals D and E affects most of the measured values:



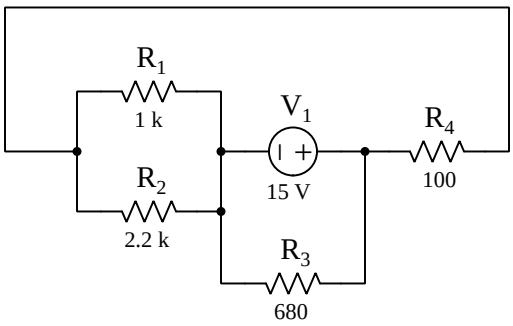
- $V_{AD} = 0.00$ Volts
- $V_{EA} = -$ (Maximum) Volts
- $V_{GE} = 0.00$ Volts
- $V_{DH} = +$ (Maximum) Volts
- $V_{GA} = -$ (Maximum) Volts
- $V_{BC} = 0.00$ Volts
- $V_{DF} = +$ (Maximum) Volts
- $I_A = 0.00$ milliAmperes
- $I_C = 0.00$ milliAmperes
- $I_D = 0.00$ milliAmperes
- I_H (at resistor lead) $= 0.00$ milliAmperes

Note: "Maximum" voltage refers to the *compliance voltage* of the electronic current source: the maximum amount of voltage it is able to muster between its terminals when faced with an open circuit. For most electronic current sources, this will be some modest limit such as 15 Volts or 30 Volts. However, for some high-energy current sources such as *current transformers* used in electric power systems, the maximum open-circuit voltage could actually reach hazardous levels: *hundreds* or even *thousands* of Volts!

2.5 Example: using VIRP tables to analyze series-parallel circuits

Tables with cells marked for holding the voltage, current, resistance, and power of each component in a circuit are useful analytical tools, especially for simple series and simple parallel circuits, where Ohm's and Joule's Laws may be applied vertically to the values in each column of the table and where the properties of the network type in question (either series or parallel) may be applied horizontally to each row of the table. Things get more complicated, though, when analyzing series-parallel combination circuits because neither the properties of series networks nor the properties of parallel networks universally apply across any row of a table holding V, I, R, and P values for all resistors in that circuit.

However, VIRP tables can be used with series-parallel combination circuits if we create a new column in the table for each reduction of multiple resistors into a series or parallel equivalent resistance. We will explore this method using a couple of series-parallel circuit examples.



	R_1	R_2	R_3	R_4
V				
I				
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω
P				

Examining this circuit, we see R_1 and R_2 are directly in parallel with each other, and so it makes sense to reduce these into a parallel equivalent (687.5 Ω). This will be our first added column to the table, and we will call it R_A. Next, we see R_4 to be in series with the R_1/R_2 equivalent we just dubbed R_A, therefore we should add another column to our table named R_B and populate that new column's resistance cell with the value 787.5 Ω. Lastly, we find that new equivalent resistance R_B to be in parallel with R_3, so collapsing those into a single equivalent resistance yields our circuit's total resistance (R_T) of 364.9 Ω. This total resistance will experience the full source voltage of 15 Volts.

Examining the new table with accompanying notes:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V							15 V
I							
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P							

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Now we are free to apply Ohm's Law and Joule's Law to each column in the VIRP table as we're able. First, calculating total current and total power:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V							15 V
I							41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P							0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Next, applying the property of equal voltage for parallel-connected components to R_B and R_3 following the last note:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V			15 V			15 V	15 V
I							41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P							0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Next, applying Ohm's and Joule's Laws to the R_3 and R_B columns:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V			15 V			15 V	15 V
I			22.06 mA			19.05 mA	41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P			0.3309 W			0.2857 W	0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Next, applying the property of equal current for series-connected components to R_A and R_4 following the note on R_B:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V			15 V			15 V	15 V
I			22.06 mA	19.05 mA	19.05 mA	19.05 mA	41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P			0.3309 W			0.2857 W	0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Next, applying Ohm's and Joule's Laws to the R_A and R_4 columns:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V			15 V	1.905 V	13.10 V	15 V	15 V
I			22.06 mA	19.05 mA	19.05 mA	19.05 mA	41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P			0.3309 W	36.28 mW	0.2494 W	0.2857 W	0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Next, applying the property of equal voltage for parallel-connected components to R_1 and R_2 following the note on R_A:

	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V	13.10 V	13.10 V	15 V	1.905 V	13.10 V	15 V	15 V
I			22.06 mA	19.05 mA	19.05 mA	19.05 mA	41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P			0.3309 W	36.28 mW	0.2494 W	0.2857 W	0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

Lastly, applying Ohm's and Joule's Laws to the R_1 and R_2 columns:

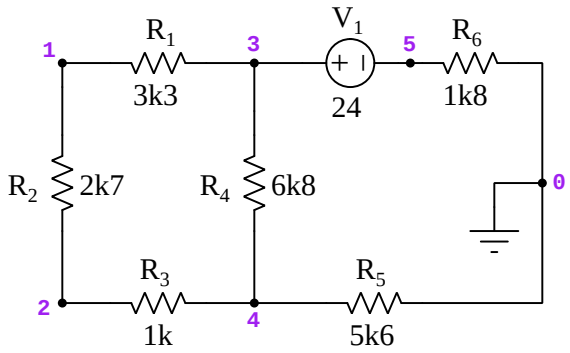
	R_1	R_2	R_3	R_4	R_A	R_B	R_T
V	13.10 V	13.10 V	15 V	1.905 V	13.10 V	15 V	15 V
I	13.10 mA	5.952 mA	22.06 mA	19.05 mA	19.05 mA	19.05 mA	41.11 mA
R	1 k Ω	2.2 k Ω	680 Ω	100 Ω	687.5 Ω	787.5 Ω	364.9 Ω
P	0.1715 W	77.95 mW	0.3309 W	36.28 mW	0.2494 W	0.2857 W	0.6166 W

R_A = Parallel equivalent of R_1 and R_2, sharing the same voltage
R_B = Series equivalent of R_A and R_4, sharing the same current
R_T = Total, being the parallel equivalent of R_B and R_3, sharing the same voltage

A good check of our work is to add up the power dissipations for each of the four resistors R_1 through R_4 and check that this power sum equals the total circuit power of 0.6166 Watts, and indeed this is the case. In summary, we may effectively use VIRP tables to manage calculations for series-parallel resistor networks by creating new columns for equivalent resistances and by adding notes reminding us where those new column resistances came from and what parameters are shared among the constituent resistances (i.e. current for series equivalents and voltage for parallel equivalents).

2.6 Example: SPICE analyzing resistor voltages

This example uses the SPICE[4] circuit simulation program to analyze a six-resistor series-parallel network powered by a 24 Volt DC voltage source. Each of the numbered points in the schematic diagram represents a *node* in the circuit:



SPICE relies on a plain-text description of the circuit called a *netlist*, where each component's value is specified and its connections defined by reference to those numbered nodes. For example, resistor R_3 is 1 kΩ and connected between nodes 2 and 4, and so the line `r3 2 4 1000` defines this resistor for SPICE:

```
* SPICE netlist
v1 3 5 dc 24
r1 1 3 3300
r2 1 2 2700
r3 2 4 1000
r4 3 4 6800
r5 4 0 5600
r6 5 0 1800
.dc v1 24 24 1
.print dc v(3,1) v(1,2) v(2,4)
.print dc v(3,4) v(4) v(0,5)
.end
```

A necessary instruction for SPICE to be able to display voltages between specified nodes is the `.dc` line, which in this case tells SPICE to analyze voltages for source V_1 beginning at a value of 24 Volts and ending at a value of 24 Volts in one step. This may seem like a very strange thing to tell the computer to do, but the `.dc` instruction was designed for the sophisticated task of "sweeping" a source over a range of values, displaying all analyzed voltages and currents for each one of those source values.

[4]This example uses NGSPICE version 26.

When SPICE compiles this netlist, the resulting voltages are displayed in scientific notation:

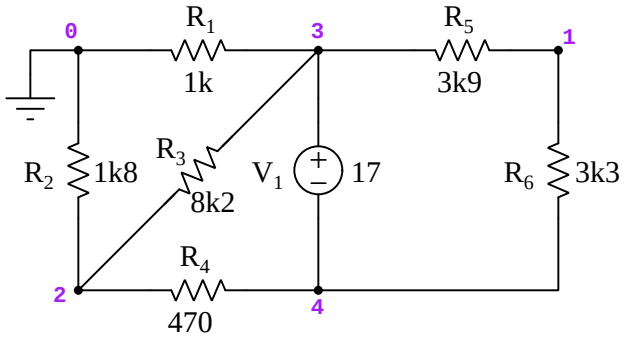
```
------------------------------------------------------------------
Index   v-sweep         v(3)-v(1)       v(1)-v(2)       v(2)-v(4)
------------------------------------------------------------------
0       2.400000e+01    3.597115e+00    2.943094e+00    1.090035e+00

------------------------------------------------------------------
Index   v-sweep         v(3)-v(4)       v(4)            v(5)
------------------------------------------------------------------
0       2.400000e+01    7.630243e+00    1.238792e+01   -3.98183e+00
```

Each of the `.print dc` lines in the netlist instruct SPICE to display voltage between the specified nodes, assuming the voltmeter's red lead is touching the first node and the black lead is touching the second. If just one node number is specified, SPICE assumes the red lead touches that node while the black lead touches node 0 (ground). For example, the voltage between nodes 3 and 1 (across resistor R_1) is 3.597115 Volts, while the voltage between nodes 5 and 0 (across resistor R_6) is −3.98183 Volts.

Summarizing these voltages according to their associated resistors:

- $V_{R1} = 3.597115$ Volts
- $V_{R2} = 2.943094$ Volts
- $V_{R3} = 1.090035$ Volts
- $V_{R4} = 7.630243$ Volts
- $V_{R5} = 12.38792$ Volts
- $V_{R6} = 3.98183$ Volts

2.7 Example: SPICE analyzing ground-referenced voltages



Rather than specify voltages between particular nodes, the `.op` instruction tells SPICE to display voltages between each of the non-zero nodes and ground (node 0):

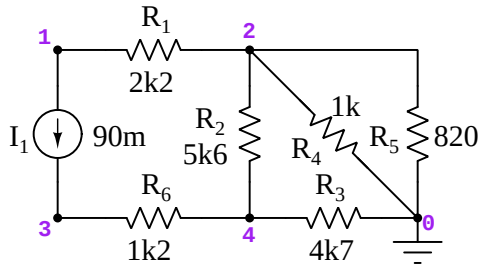
```
* SPICE netlist
v1 3 4 dc 17
r1 3 0 1000
r2 2 0 1800
r3 2 3 8200
r4 2 4 470
r5 1 3 3900
r6 1 4 3300
.op
.end
```

SPICE analysis:

```
 Node            Voltage
 V(1)          -4.25277e+00
 V(2)          -8.92001e+00
 V(4)          -1.20444e+01
 V(3)           4.955563e+00

 Source          Current
 ------          -------
 v1#branch     -9.00882e-03
```

In accordance with passive sign convention, SPICE regards the mathematical sign of any source's current as being negative in value, which is why it shows V_1's current to be −9.00882 mA.

2.8 Example: SPICE analyzing current-driven circuit



SPICE netlist:

```
* SPICE netlist
i1 1 3 dc 90e-3
r1 1 2 2200
r2 2 4 5600
r3 4 0 4700
r4 2 0 1000
r5 2 0 820
r6 3 4 1200
.dc i1 90e-3 90e-3 1
.print dc v(1) v(2)
.print dc v(3) v(4)
.print dc v(2,1) v(4,2)
.print dc v(3,4) v(3,1)
.end
```

SPICE analysis:

```
--------------------------------------------------
Index   i-sweep         v(1)            v(2)
--------------------------------------------------
0       9.000000e-02   -2.19122e+02    -2.11224e+01

--------------------------------------------------
Index   i-sweep         v(3)            v(4)
--------------------------------------------------
0       9.000000e-02    3.283422e+02    2.203422e+02

--------------------------------------------------
Index   i-sweep         v(2)-v(1)       v(4)-v(2)
--------------------------------------------------
0       9.000000e-02    1.980000e+02    2.414646e+02

--------------------------------------------------
Index   i-sweep         v(3)-v(4)       v(3)-v(1)
--------------------------------------------------
0       9.000000e-02    1.080000e+02    5.474646e+02
```

Summarizing these voltages according to their associated resistors:

- $V_{R1} = 198.0000$ Volts
- $V_{R2} = 241.4646$ Volts
- $V_{R3} = 220.3422$ Volts
- $V_{R4} = 21.1224$ Volts
- $V_{R5} = 21.1224$ Volts
- $V_{R6} = 108.0000$ Volts

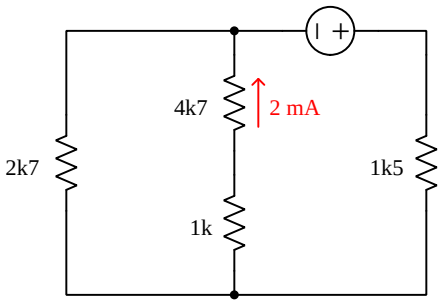
Voltage across the current source I_1 is 547.4646 Volts.

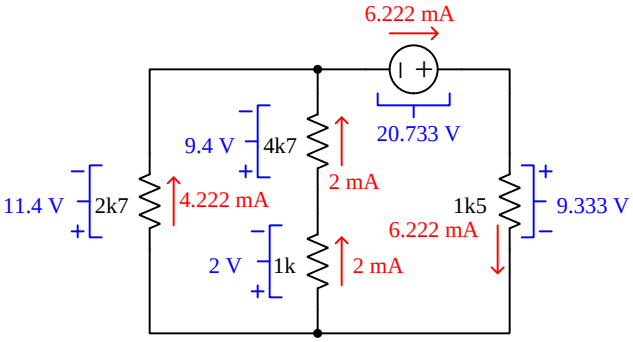
2.9 Example: circuits with unknown source values

Consider the following circuits, each one presented with just one resistor's voltage or current value and no source value given. Following each presentation is the same schematic with all values shown.

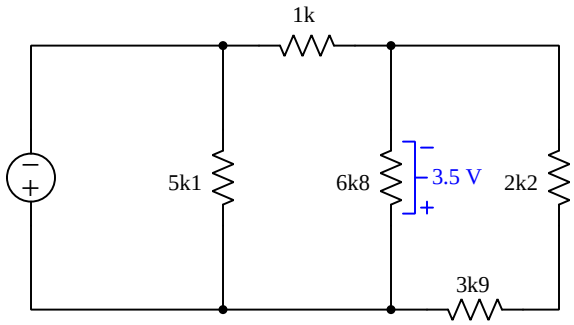
In you would like to challenge yourself again on each circuit, simply take the fully-annotated version showing all voltages and currents and erase all but one of those voltages or currents while keeping the original resistor values. Then, from this new starting point of a different resistor voltage/current, try to calculate all the rest!

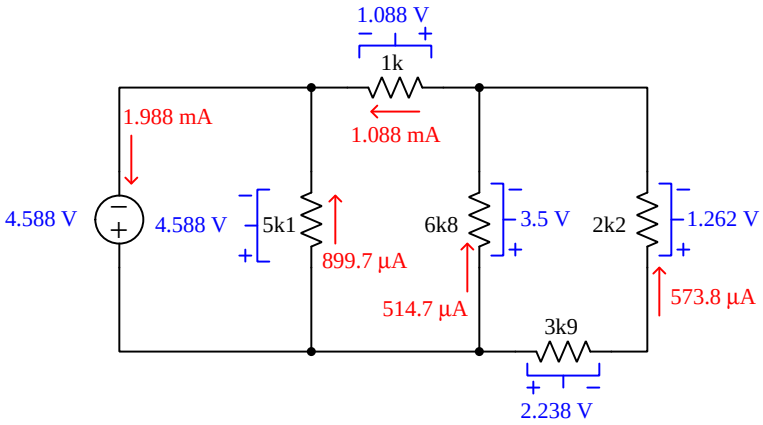
Circuit #1



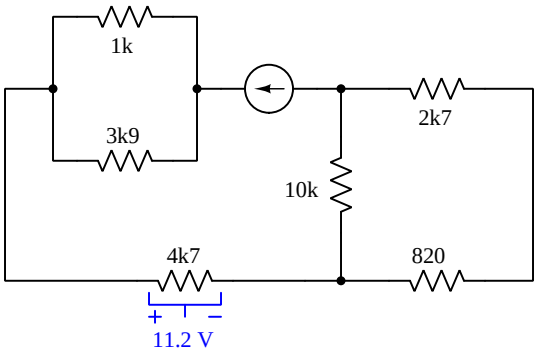


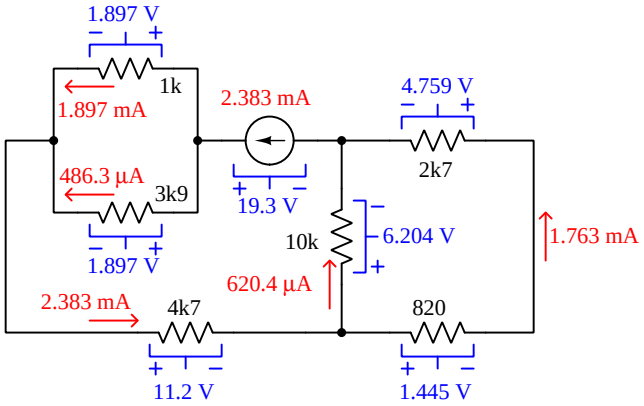
Circuit #2


1k
5k1
6k8
3.5 V
2k2
3k9


1.088 V
1k
1.988 mA
1.088 mA
4.588 V
4.588 V
5k1
899.7 µA
6k8
3.5 V
2k2
1.262 V
514.7 µA
573.8 µA
3k9
2.238 V

Circuit #3


1k
3k9
2k7
10k
4k7
820
11.2 V


1.897 V
1k
1.897 mA
2.383 mA
4.759 V
2k7
486.3 µA
3k9
19.3 V
10k
6.204 V
1.763 mA
1.897 V
620.4 µA
2.383 mA
4k7
820
11.2 V
1.445 V

Circuit #4

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

 +----------------------+
 |                      |
Vsrc                    R1
 |                      |
 +----------R2----------+
 |                      |
 |                      |
 +----------R3----------+

R[1] = 2.7 kOhms
R[2] = 6.8 kOhms
R[3] = 8.2 kOhms

Random parameter:
I_R[2] = 468.523 uA
```

Below we see all parameters for all components in this circuit:

```
  Vsrc = 5.5 V         Isrc = 857.054 uA     Psrc = 4.7138 mW
V_R[1] = 2.31405 V   I_R[1] = 857.054 uA   P_R[1] = 1.98326 mW
V_R[2] = 3.18595 V   I_R[2] = 468.523 uA   P_R[2] = 1.49269 mW
V_R[3] = 3.18595 V   I_R[3] = 388.531 uA   P_R[3] = 1.23784 mW
```

Circuit #5

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

 +----------------------+
 |                      |
Isrc                    R1
 |                      |
 +----------R2----------+
 |                      |
 |                      |
 +----------R3----------+

R[1] = 1.8 kOhms
R[2] = 2.2 kOhms
R[3] = 2.2 kOhms

Random parameter:
V_R[1] = 19.8 V
```

Below we see all parameters for all components in this circuit:

```
  Vsrc = 31.9 V     Isrc = 11 mA      Psrc = 350.9 mW
V_R[1] = 19.8 V   I_R[1] = 11 mA    P_R[1] = 217.8 mW
V_R[2] = 12.1 V   I_R[2] = 5.5 mA   P_R[2] = 66.55 mW
V_R[3] = 12.1 V   I_R[3] = 5.5 mA   P_R[3] = 66.55 mW
```

Circuit #6

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

 +----------------------+-----+
 |                      |     |
Vsrc                    R1    |
 |                      |     |
 +----------R2----------+     |
 |                            |
 |                            |
 +----------R3----------------+

R[1] = 12 kOhms
R[2] = 4.7 kOhms
R[3] = 6.8 kOhms

Random parameter:
P_R[3] = 330.882 uW
```

Below we see all parameters for all components in this circuit:

```
  Vsrc = 1.5 V         Isrc = 310.409 uA     Psrc = 465.613 uW
V_R[1] = 1.07784 V   I_R[1] = 89.8204 uA   P_R[1] = 96.8124 uW
V_R[2] = 422.156 mV  I_R[2] = 89.8204 uA   P_R[2] = 37.9182 uW
V_R[3] = 1.5 V       I_R[3] = 220.588 uA   P_R[3] = 330.882 uW
```

Circuit #7

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

 +----------------------+-----+
 |                      |     |
Isrc                    R1    |
 |                      |     |
 +----------R2----------+     |
 |                            |
 |                            |
 +----------R3----------------+

R[1] = 2.7 kOhms
R[2] = 6.8 kOhms
R[3] = 5.6 kOhms

Random parameter:
I_R[3] = 629.139 uA
```

Below we see all parameters for all components in this circuit:

```
  Vsrc = 3.52318 V     Isrc = 1 mA           Psrc = 3.52318 mW
V_R[1] = 1.00132 V   I_R[1] = 370.861 uA   P_R[1] = 371.352 uW
V_R[2] = 2.52185 V   I_R[2] = 370.861 uA   P_R[2] = 935.257 uW
V_R[3] = 3.52318 V   I_R[3] = 629.139 uA   P_R[3] = 2.21657 mW
```

Circuit #8

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

  +----------------------+
  |                      |
 Isrc                    R1
  |                      |
  +----------R2----------+
  |                      |
  |                      |
  +-----R4--------R3-----+

R[1] = 4.7 kOhms
R[2] = 5.6 kOhms
R[3] = 5.6 kOhms
R[4] = 2.7 kOhms

Random component parameter:
V_R[3] = 35.8722 V
```

Below we see all parameters for all components in this circuit:

```
  Vsrc = 127.898 V      Isrc = 15.9 mA          Psrc = 2.03357 W
V_R[1] = 74.73 V      I_R[1] = 15.9 mA        P_R[1] = 1.18821 W
V_R[2] = 53.1678 V    I_R[2] = 9.49424 mA     P_R[2] = 0.504788 W
V_R[3] = 35.8722 V    I_R[3] = 6.40576 mA     P_R[3] = 229.789 mW
V_R[4] = 17.2955 V    I_R[4] = 6.40576 mA     P_R[4] = 110.791 mW
```

Circuit #9

Note: the following circuit example was created using the open-source program `sp` to randomize component values and then randomly present a single parameter (either a voltage, a current, or a power dissipation) from which all other circuit parameters may be derived:

```
SCHEMATIC DIAGRAM:

  +-----------+----------+
  |           |          |
  |           R1         R3
 Vsrc         |          |
  |           |          |
  |           R2         R4
  |           |          |
  +-----R5----+----------+

R[1] = 6.8 kOhms
R[2] = 1.8 kOhms
R[3] = 1.5 kOhms
R[4] = 1.5 kOhms
R[5] = 2.7 kOhms

Random component parameter:
I_R[3] = 1.89706 mA
```

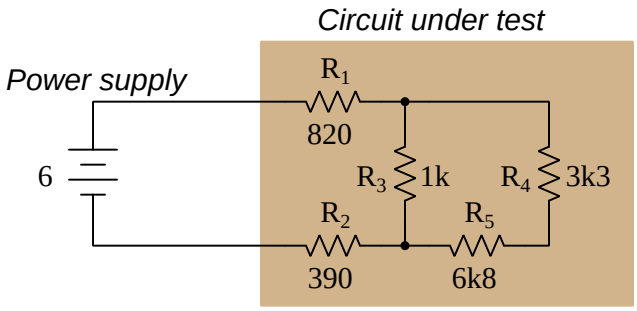
Below we see all parameters for all components in this circuit:

```
  Vsrc = 12.6 V         Isrc = 2.55882 mA      Psrc = 32.2412 mW
V_R[1] = 4.5 V        I_R[1] = 661.765 uA    P_R[1] = 2.97794 mW
V_R[2] = 1.19118 V    I_R[2] = 661.765 uA    P_R[2] = 788.279 uW
V_R[3] = 2.84559 V    I_R[3] = 1.89706 mA    P_R[3] = 5.39825 mW
V_R[4] = 2.84559 V    I_R[4] = 1.89706 mA    P_R[4] = 5.39825 mW
V_R[5] = 6.90882 V    I_R[5] = 2.55882 mA    P_R[5] = 17.6785 mW
```

2.10 Example: creating a variable-voltage supply

A *power supply* is any circuit providing electrical energy to a network of components. Some power supplies output a known amount of voltage, some a known amount of current, but in order to be most useful at least one of these two parameters needs to be a reliably-known value.

Electrochemical *batteries* naturally output a nearly-constant voltage based on the particular chemical reaction inside the battery that converts chemical potential energy into electrical potential for charge carriers passing through it. For example, lead-acid cell batteries typically output 2.2 Volts per cell. "Dry-cell" carbon-zinc batteries typically output 1.5 Volts per cell. Therefore, it is possible to use a battery as a power supply for circuits where we expect the source voltage to be constant or nearly constant:

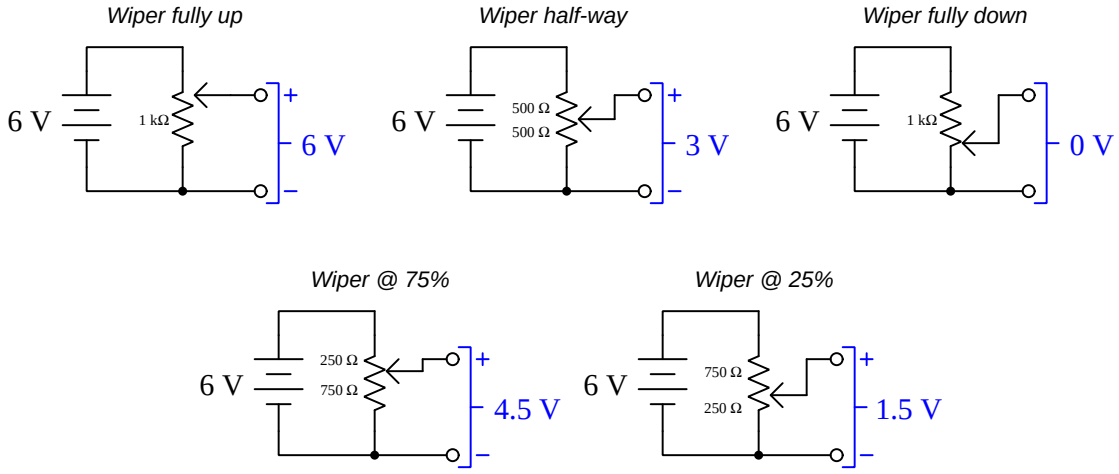


With a power supply voltage of exactly 6 Volts, the resistor voltages and currents will be as follows:

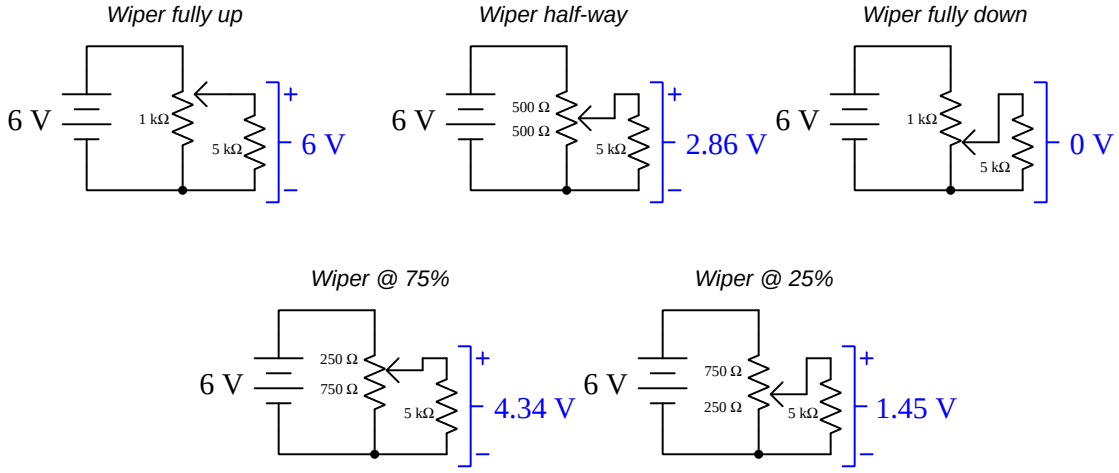
- $V_{R1} = 2.3209$ V ; $I_{R1} = 2.8303$ mA
- $V_{R2} = 1.1038$ V ; $I_{R2} = 2.8303$ mA
- $V_{R3} = 2.5753$ V ; $I_{R3} = 2.5753$ mA
- $V_{R4} = 841.44$ mV ; $I_{R4} = 254.98$ μA
- $V_{R5} = 1.7339$ V ; $I_{R5} = 254.98$ μA

However, batteries are rarely precise in their output voltage; e.g. a "6 Volt" battery will not remain at exactly 6.000 Volts over time, but instead will decrease in voltage over its life. If only battery voltage was *adjustable* . . .

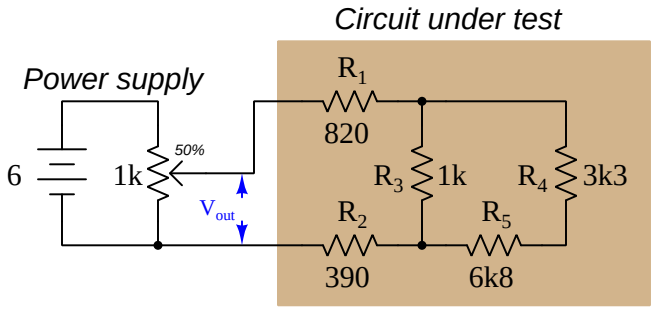
We may use a *potentiometer* to build an adjustable-voltage power supply using a battery (or any other unregulated source of DC voltage) as the energy source. The potentiometer's movable wiper forms a voltage divider network from the upper and lower portions of the resistive strip. Assuming a battery voltage of exactly 6 Volts, the voltage obtained between the potentiometer's wiper and bottom terminal will be a direct proportion of the battery's voltage. The following illustrations show how this works for a 1 kΩ potentiometer:



However, it is important to understand that potentiometer's output voltage will be directly proportional to wiper position only if its output is *unloaded*. The moment we connect a load to the output terminals of the potentiometer, its output voltage will "sag" to some lesser value at wiper positions between either extreme, as shown in these examples with a 5 kΩ load:



For example, a 1 kΩ potentiometer set at the half-way position and connecting to the original series-parallel test network will output less than 3 Volts:



- V_{out} = 2.6835 V
- V_{R1} = 1.0380 V ; I_{R1} = 1.2659 mA
- V_{R2} = 493.68 mV ; I_{R2} = 1.2659 mA
- V_{R3} = 1.1518 V ; I_{R3} = 1.1518 mA
- V_{R4} = 376.34 mV ; I_{R4} = 114.04 μA
- V_{R5} = 775.49 mV ; I_{R5} = 114.04 μA

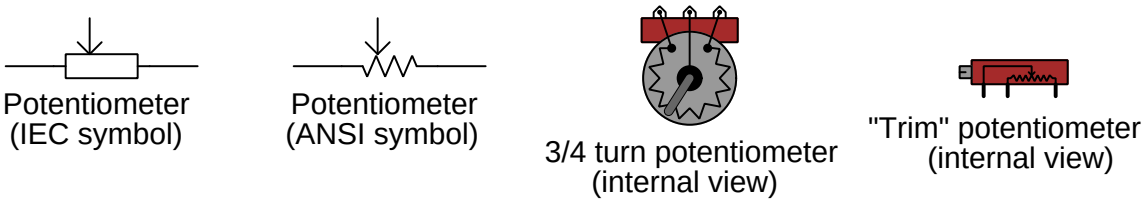
Setting the potentiometer to its 80% position should result in an output voltage of 4.8 Volts (unloaded), but with the test circuit connected we will see the following voltages and currents instead:

- V_{out} = 4.4631 V
- V_{R1} = 1.7264 V ; I_{R1} = 2.1053 mA
- V_{R2} = 821.09 mV ; I_{R2} = 2.1053 mA
- V_{R3} = 1.9157 V ; I_{R3} = 1.9157 mA
- V_{R4} = 625.91 mV ; I_{R4} = 189.67 μA
- V_{R5} = 1.2898 V ; I_{R5} = 189.67 μA

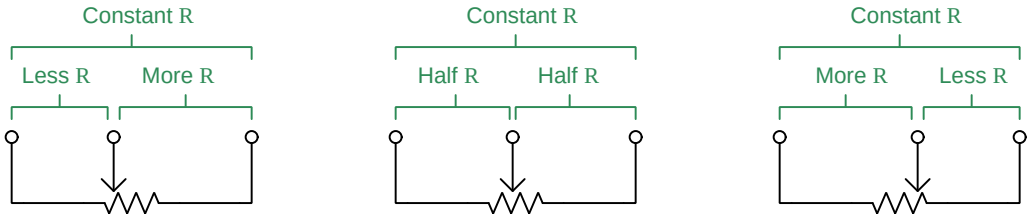
This phenomenon of "voltage sag" when using a battery and potentiometer as a simple power supply is not necessarily a problem, though, if the load resistance is constant. All we need to do is re-adjust the potentiometer's wiper position with test circuit and a voltmeter connected until we achieve the desired power supply output voltage. In other words, we may use the adjustability provided by the potentiometer to compensate for the "sag" occurring when the load is connected.

2.11 Example: limiting the range of a potentiometer

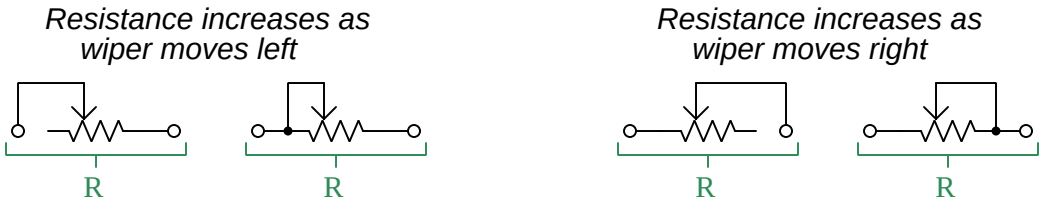
A very important electrical component designed to create a variable amount of electrical resistance is the *potentiometer*. This device has three terminals: two connecting to the ends of a resistive strip, and one connecting to a movable "wiper" that contacts the resistive strip at one point.



As the wiper forms a single point of contact with the resistive strip, it essentially makes the potentiometer appear as a pair of series-connected resistors having complementary values: when the wiper is in the exact center, each wiper-to-end resistance will be equal (each one half the total end-to-end resistance); when the wiper position is offset from center, one of those resistances will be greater than half and the other resistance will be less than half. End-to-end resistance is always constant regardless of wiper position:



Potentiometers may be used as *rheostats* where they present a simple, variable resistance as shown below:



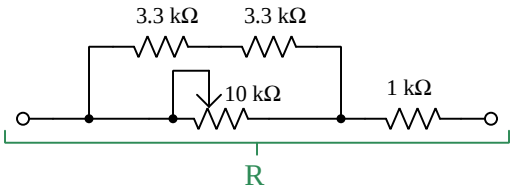
In many applications we may require a user-adjustable resistance with a range that does not begin with zero Ohms. In such cases we may achieve a low-limited adjustment range by connecting a fixed resistor in series with the rheostat:

In this network the minimum resistance value is set by $R_{minimum}$, as it constitutes the whole network resistance when the rheostat's resistance has been adjusted down to zero. For example, with $R_{minimum}$ of 4.7 kΩ and a 10 kΩ potentiometer, the full range of adjustable resistance for the network as a whole will be 4.7 kΩ to 14.7 kΩ.

Conversely, we may construct an adjustable-resistance network having a high limit less than the potentiometer's full-resistance value by connecting a fixed resistor in parallel with the rheostat:

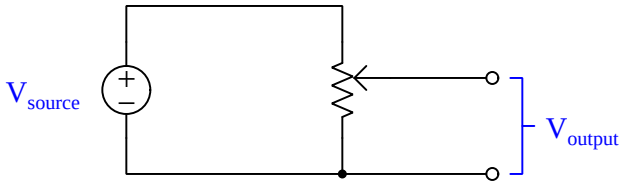
In this network the minimum resistance is zero when the rheostat has been adjusted to its minimum resistance, but at the other extreme of the rheostat's setting the maximum network resistance is limited to the parallel combination $\frac{1}{\frac{1}{R_{maximum}} + \frac{1}{R_{rheostat}}}$ which of course must be less than the full end-to-end resistance of the potentiometer owing to the behavior of resistances in parallel. For example, with $R_{maximum}$ of 4.7 kΩ and a 10 kΩ potentiometer, the full range of adjustable resistance for the network as a whole will be 0 kΩ to 3.197 kΩ.

Finally, we may combine these two techniques into one series-parallel network to achieve any desired minimum and maximum adjustment values. Consider the following example, having a minimum resistance of 1 kΩ and a maximum resistance of (very nearly) 5 kΩ using standard fixed-resistor value using standard fixed-resistor values:

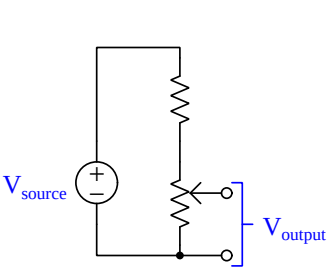


Such techniques are quite useful when constructing any other electrical network where the performance depends on some specific range of adjustable resistance values.

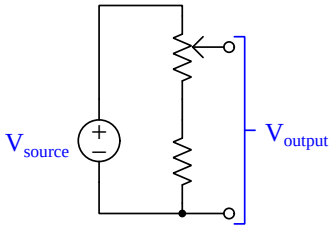
If using a potentiometer specifically within a *voltage divider* network, we typically employ all three of its terminals rather than use it as a simple rheostat:



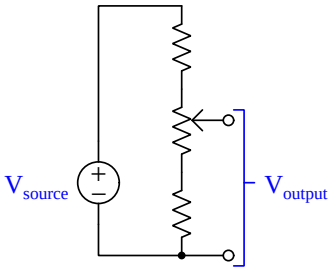
In such a circuit, the adjustable range of V_{output} extends from zero all the way to V_{source}. However, if a limited range is desired, we may modify that simple voltage divider network to include fixed-value resistors, limiting either the minimum adjustable output voltage, the maximum adjustable output voltage, or both:



$V_{output(min)} = 0$ V
$V_{output(max)} < V_{source}$



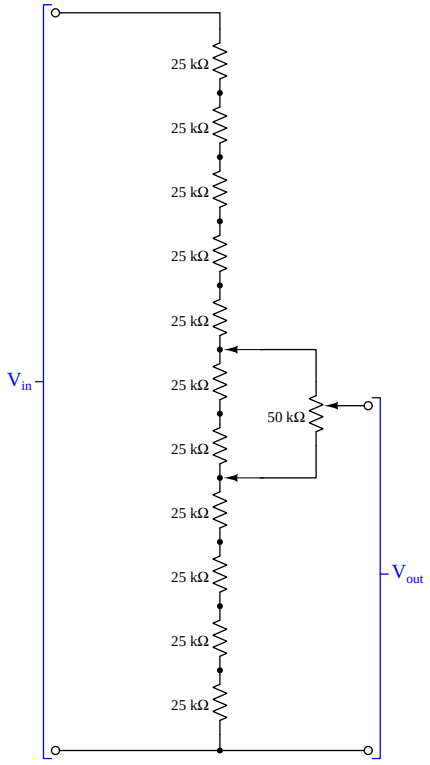
$V_{output(min)} > 0$ V
$V_{output(max)} = V_{source}$



$V_{output(min)} > 0$ V
$V_{output(max)} < V_{source}$

2.12 Example: Kelvin-Varley voltage divider networks

A special form of voltage divider network often used in precision-calibration work is the *Kelvin-Varley* voltage divider, constructed of eleven equal-rated precision resistors with a potentiometer having a total resistance of precisely *twice* that of any of the individual fixed-value resistors, the outer terminals of that potentiometer connected across any *two* of the fixed resistors in the network. An example is shown below:

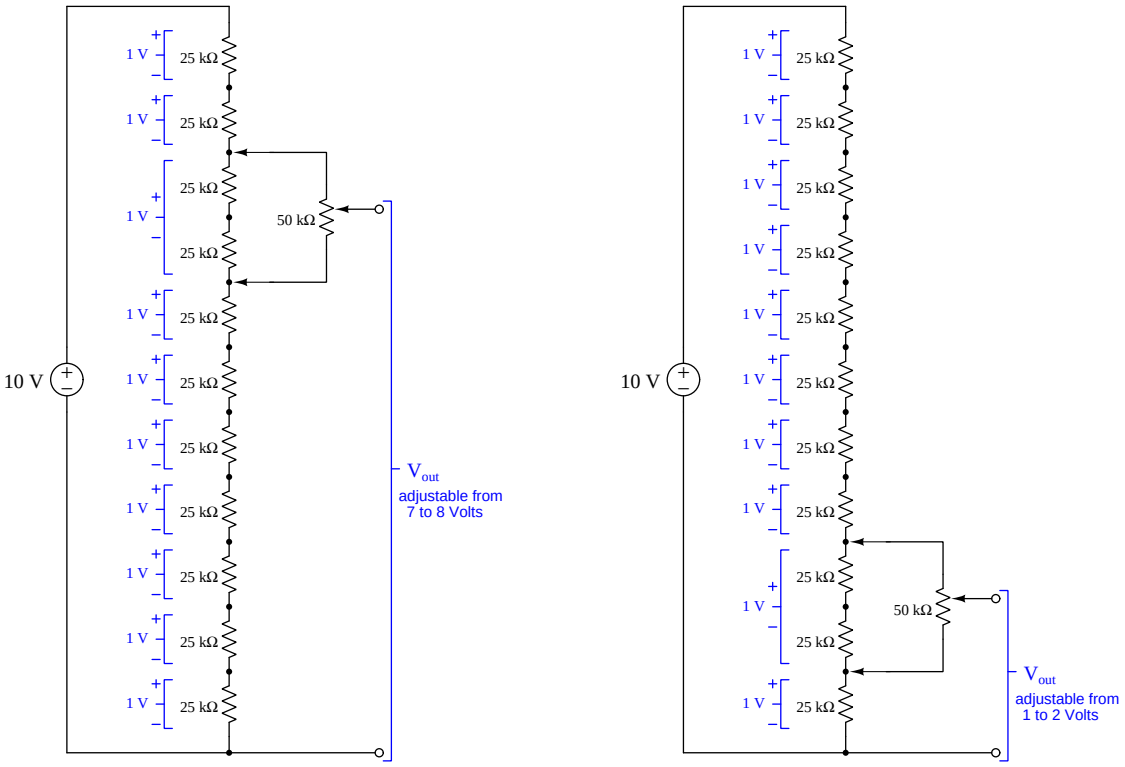


The 50 kΩ potentiometer connected in parallel with two of the series-connected 25 kΩ resistors forms an equivalent sub-network having a resistance of 25 kΩ. Essentially, connecting the potentiometer across any two of the eleven 25 kΩ resistors makes the circuit behave as if it were a series string of *ten* 25 kΩ resistors, dividing the input voltage (V_{in}) into ten equal proportions. Thus connected, moving the potentiometer's wiper position from one extreme to the other results in the output voltage (V_{out}) being adjustable between consecutive tenth-proportions of V_{in}. In the circuit shown above, if we had a 10 Volt source connected between the V_{in} terminals, moving the potentiometer wiper to the full-down position would give us an output voltage of precisely 4 Volts, and moving the wiper to the full-up position would output precisely 5 Volts.

Kelvin-Varley dividers are useful for dividing a given voltage precisely into *tenths* while providing an adjustment range also one-tenth that of the applied voltage. In the configuration shown above, moving the potentiometer's wiper from one extreme to the other yields a voltage range of 4 to 5

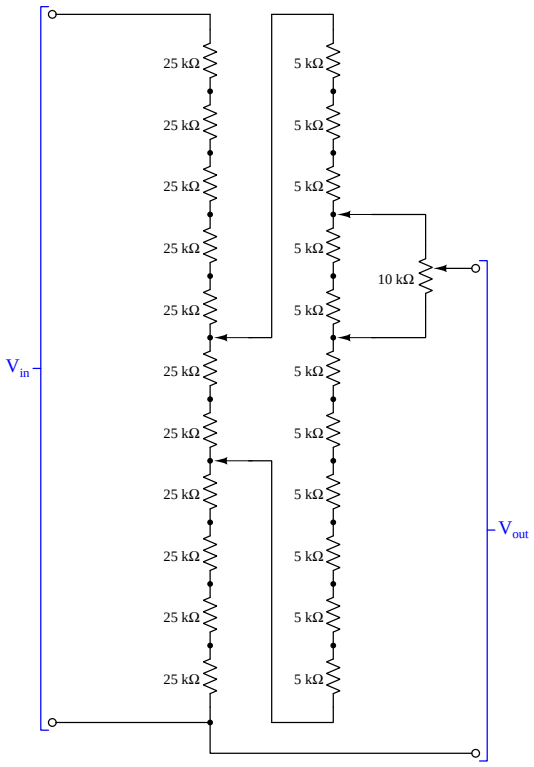
Volts (V_{out}) assuming a 10 Volt source (V_{in}). However, if the potentiometer's outer terminals are both moved one step up the series resistor chain the output voltage range becomes 5 to 6 Volts.

Just for clarity, we will examine two more examples using this same Kelvin-Varley voltage divider, the potentiometer terminals connected across different pairs of resistors in each case. The reader is encouraged to perform the necessary series-parallel analysis to confirm these voltage ranges:



One might think of the potentiometer's terminal positions along the eleven-resistor series network as being the "coarse" ratio adjustment while the wiper position of the potentiometer itself serves as the "fine" ratio adjustment.

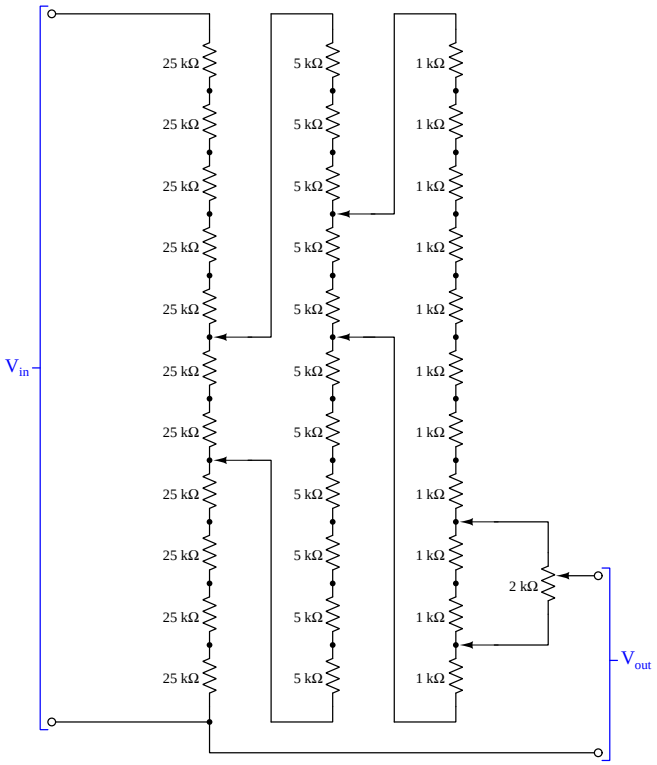
Kelvin-Varley voltage dividers used in electrical metrology[5] work consist of *cascaded* divider networks. We will gradually explore this concept by adding one more stage to the previous divider network. Just as with the single-stage divider network previously shown, where the potentiometer's total resistance value needed to be precisely *twice* that of any single fixed-resistor value, when we cascade divider stages we must ensure that the total end-to-end resistance of one divider is precisely twice that of any single fixed resistor in the other divider. Here is an example:



The eleven 5 kΩ precision resistors in tandem with the 10 kΩ potentiometer form a sub-network with a total resistance of 50 kΩ end-to-end, which in turn is connected across two of the 25 kΩ resistors in the first divider sub-network, replacing what used to be a single 50 kΩ potentiometer in the original Kelvin-Varley voltage divider circuit. In the configuration shown, V_{out} is adjustable between 46% and 47 % of the input voltage (e.g. between 4.6 Volts and 4.7 Volts if the whole network were powered by a 10 Volt source). The movable connections between the first and second divider sub-networks provides the coarsest adjustment in 10% increments, the movable connections between the potentiometer and the second divider sub-network provides less-coarse adjustment in 1% increments, and the potentiometer wiper provides fine adjustment. Those movable connections are said to be *decade adjustments* because each one provides one-tenth the adjustment range of the other.

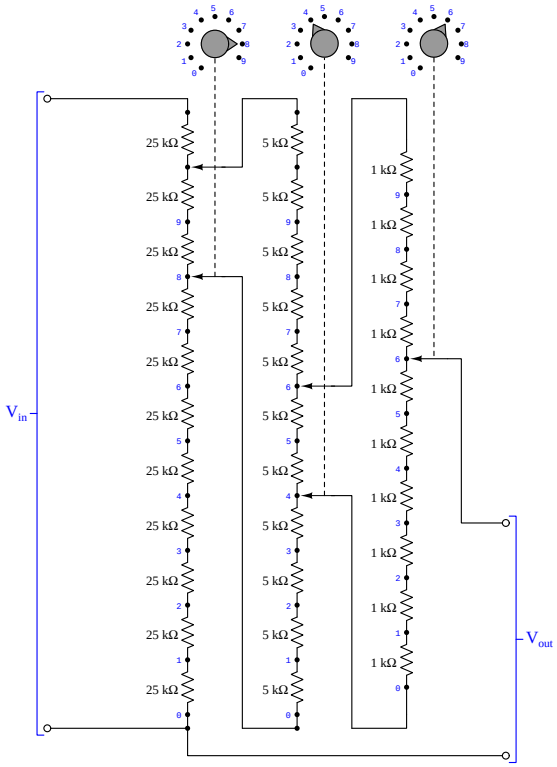
[5] Metrology is the science of precision measurement.

Extending this principle of cascaded voltage divider networks, we will now explore the behavior of a Kelvin-Varley voltage divider having *three* stages of "decade" division:



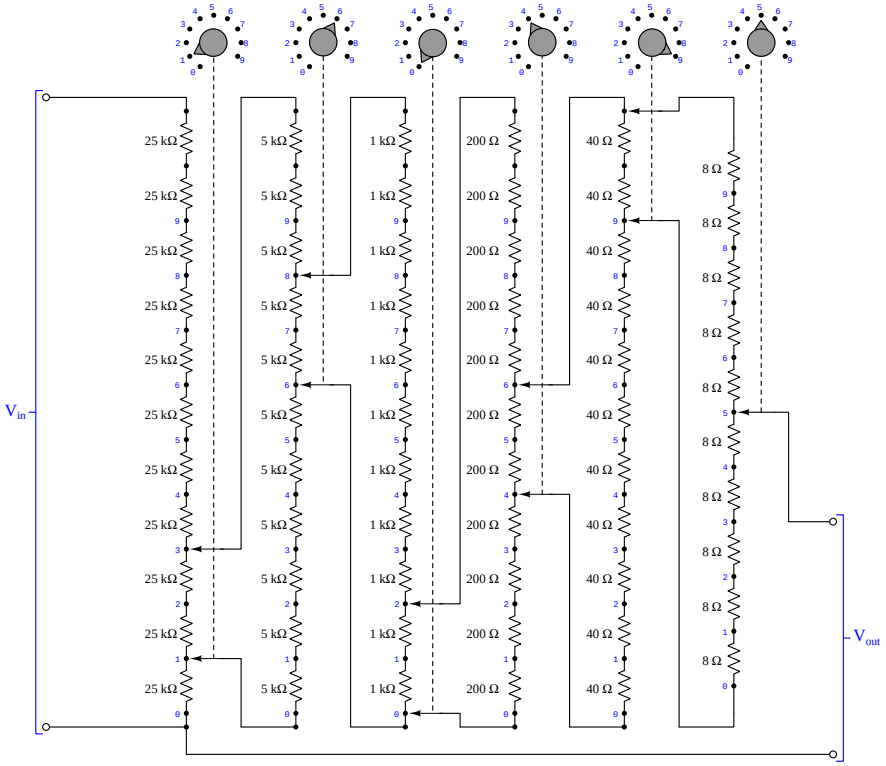
The eleven-resistor network comprised of fixed 1 kΩ resistors along with its 2 kΩ potentiometer connected across two of those forms a sub-network with an end-to-end resistance of 10 kΩ, suitable for connection across any two of the resistors in the middle sub-network (replacing the 10 kΩ potentiometer used previously). Those two sub-networks together form an end-to-end resistance of 50 kΩ suitable for connection across any two resistors of the first sub-network. Thus, the first set of movable contacts adjusts voltage in 10% increments, the second set adjusts in 1% increments, the third set adjusts in 0.1% increments, and finally the potentiometer's wiper provides fine adjustment.

As mentioned previously, Kelvin-Varley voltage divider networks are a common tool in the field of electrical metrology, where they are used to provide precise voltage-division ratios. For this type of application where we must precisely know the voltage-division ratio provided by the network in order for it to be of any practical use, it makes little sense to have a fine-adjustment potentiometer as the final stage unless that potentiometer had a graduated scale where we could accurately interpret its wiper position. Modern Kelvin-Varley voltage dividers are most often built with nothing but movable contacts forming multiple decades of discrete division ratios. Consider the following three-decade Kelvin-Varley voltage divider circuit:



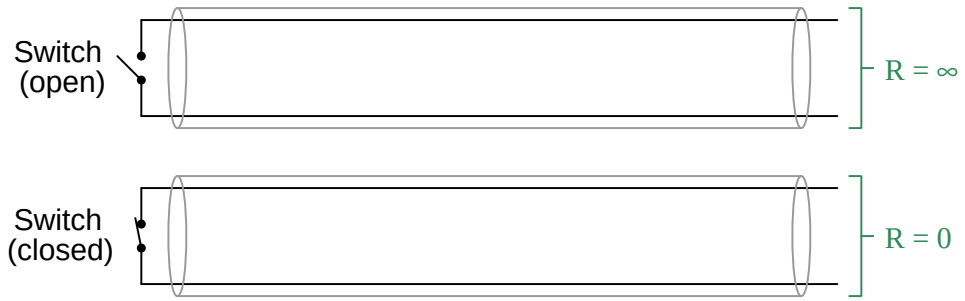
Note how the first stage has eleven fixed resistors at 25 kΩ each, the second stage has eleven fixed resistors at 5 kΩ each, and the final stage has only *ten* fixed resistors at 1 kΩ each. As before, the first set of movable contacts provide adjustment in 10% increments, the second set of contacts provide 1% increments, and the final contact provides 0.1% increments. In the configuration shown above, the voltage-division ratio is 84.6%, which means if we were to power this entire network with a 10 Volt source our V_{out} voltage would be 8.46 Volts.

Here we see a six-decade Kelvin-Varley voltage divider, with its six rotary switches set to produce a voltage-division ratio of 16.0495% or 0.160495:



2.13 Example: supervised switch circuit

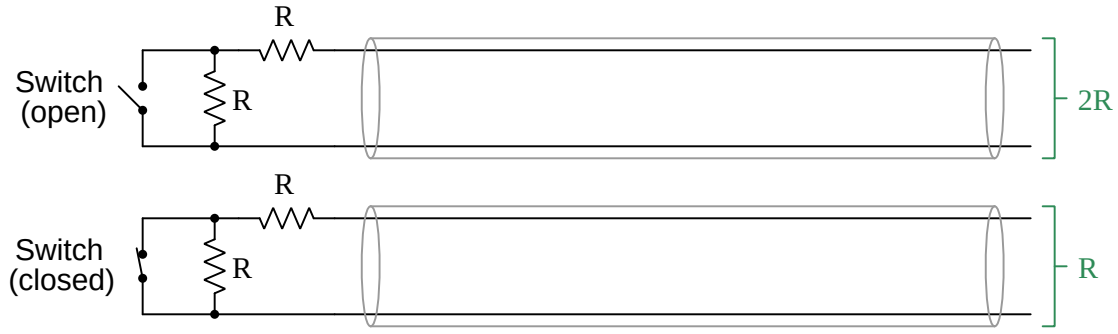
Alarm and signaling circuits often make use of remotely-located switch contacts to indicate some event such as entry into a room (e.g. door opens), detection of fire, a rise in water level, etc. Switches by their nature are either open or closed, with no in-between state:



Any circuit or device connected to the right-hand end of the cable works on the principle of resistance measurement to determine the switch's status.

However, there is a potential problem with a circuit as simple as this, which is what happens when the cable fails, either shorted or open. If the cable fails open, the sensing circuit will "think" the switch is open, whether or not it actually is. The same is true if the cable fails shorted between its two conductors: the sensing circuit will perceive the switch as being closed even if it is not. In other words, a crude switch/cable arrangement presents no way to differentiate between either of the switch's states and a cable failure.

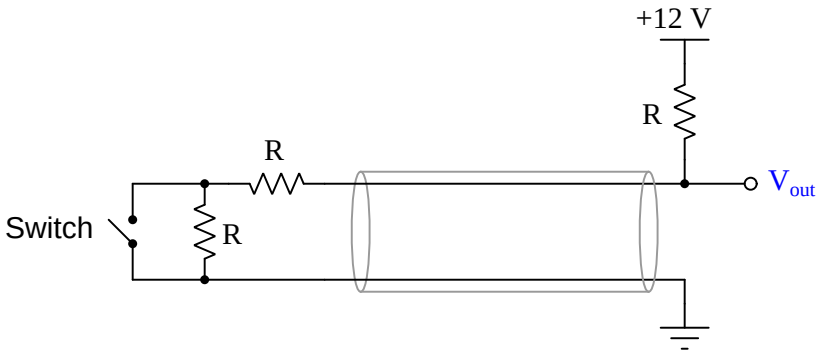
One solution to this problem is something called a *supervised* switch circuit, making use of resistors located near the switch:



When the switch is open, the amount of resistance at the right-hand end of the cable is equal to the sum of the two resistors: if those resistors' values are equal (R), then the end-resistance will be $2R$. When the switch is closed, it shorts past the parallel-connected resistor which leaves only the series-connected resistor in play: this makes the end-resistance simply equal to R.

Note how neither of these resistance values is zero or infinity, which means the amount of resistance created by legitimate switch states is differentiable from a failed cable which will still be either zero Ohms (cable shorted) or infinite Ohms (cable open).

If connected to a DC voltage source and third resistor, we find four distinct voltage values (measured between the output terminal and ground) as we analyze the circuit for its six possible conditions:

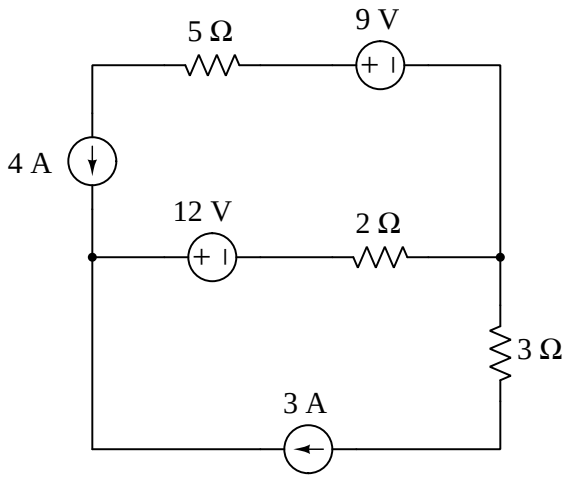


Switch state	**Cable state**	V_{out}
Open	Healthy	8 Volts
Closed	Healthy	6 Volts
Open	Failed open	12 Volts
Closed	Failed open	12 Volts
Open	Failed shorted	0 Volts
Closed	Failed shorted	0 Volts

2.14 Example: analyzing a mixed-source circuit

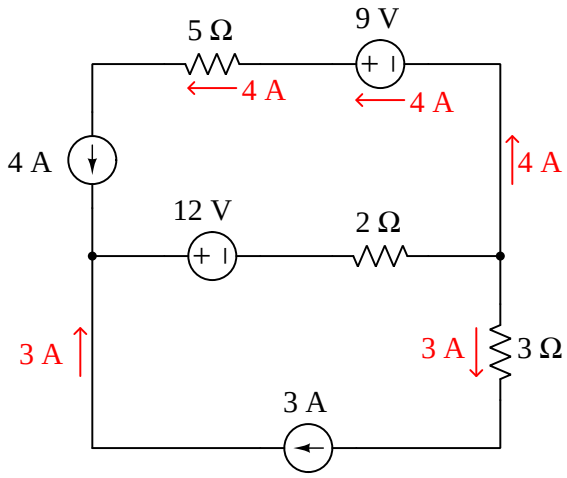
An excellent type of quantitative problem for students learning to apply Ohm's Law, Joule's Law, Kirchhoff's Laws, and the properties of series and parallel networks is where voltage and current sources coexist within the same circuit, in such a way that all currents are easily defined by those sources. This type of analytical problem forces the problem-solver to carefully consider those fundamental Laws to see where each one fits.

Take for example the following circuit, containing two constant-current elements[6] and two constant-voltage elements. Our task is to calculate all voltages and currents not specified by the component ratings, as well as to determine directions of current, polarities of voltage, and the identity of each component as being either an electrical source or an electrical load:

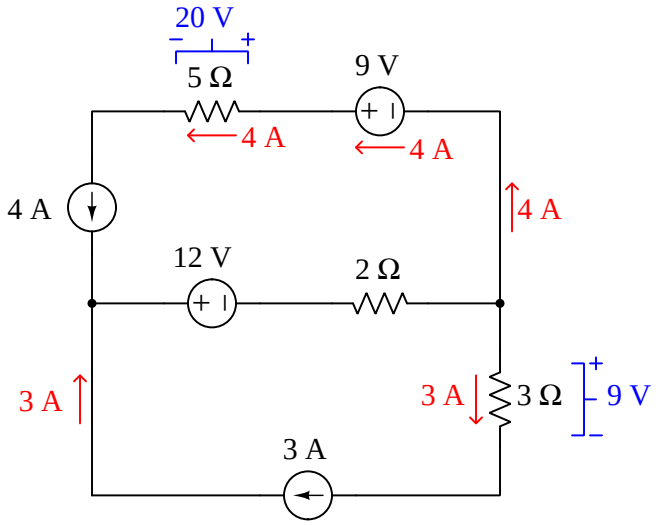


[6] I hesitate to use the term "source" to describe each, since we often discover one or more of them behaving as *loads*!

Based on the fact that series-connected elements must share the same continuous current[7], we may conclude that the 3 Ω resistor has a current of 3 Amperes, and that the 5 Ω resistor and 9 V voltage element both have 4 Amperes through them. We will notate these currents using red-colored arrows to show their directions:



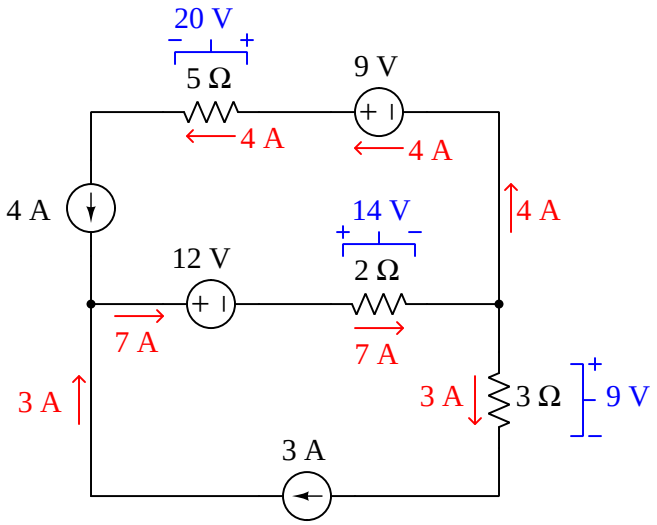
Now that we know the current value through two of the three resistors, we may apply Ohm's Law ($V = IR$) to each to calculate voltage drops, notating these in blue:



The + and − polarity marks for the resistor voltage drops are placed on the basis of knowing resistors always behave as *loads*, with charge carriers entering at higher energy (+) and exiting at lower energy (−).

[7] A consequence of the Law of Electric Charge Conservation.

Next, we see an application of Kirchhoff's Current Law[8] at the left-hand node, where the 3 Ampere and 4 Ampere currents merge. This results in 7 Amperes of current from left to right through the 12 Volt element and the 2 Ω resistor. Applying Ohm's Law again to that resistor ($V = IR$), we calculate its voltage drop to be 14 Volts:



We do not yet know the voltage across each constant-current element, but we may determine this by using Kirchhoff's Voltage Law[9]. Stepping around each of the two "loops" in this circuit and summing all the known voltages to determine the unknown voltage necessary to bring that sum to zero:

Lower loop, stepping clockwise starting from lower-left corner:

$$0 \text{ V} - 12 \text{ V} - 14 \text{ V} - 9 \text{ V} + V_{3A} = 0$$

$$V_{3A} = 35 \text{ V}$$

Upper loop, stepping counter-clockwise starting from lower-left corner:

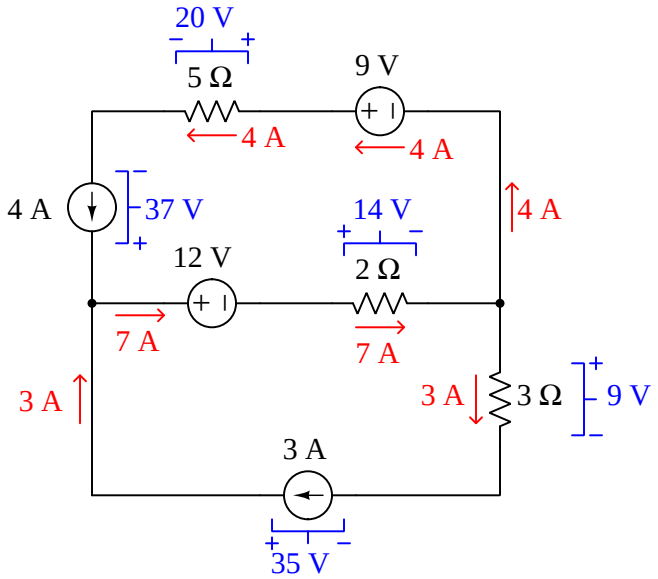
$$-12 \text{ V} - 14 \text{ V} + 0 \text{ V} + 9 \text{ V} - 20 \text{ V} + V_{4A} = 0$$

$$V_{4A} = 37 \text{ V}$$

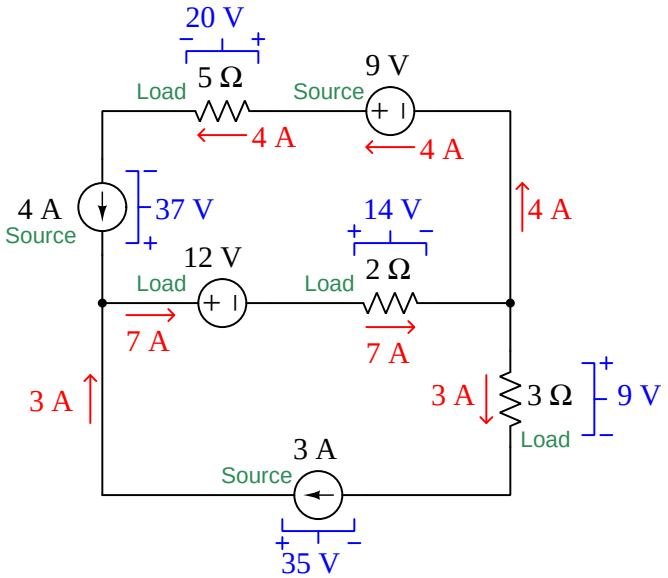
[8] Another consequence of the Law of Electric Charge Conservation.

[9] A consequence of the Law of Energy Conservation, that electric charge carriers flowing around a circuit must gain exactly as much energy as they lose.

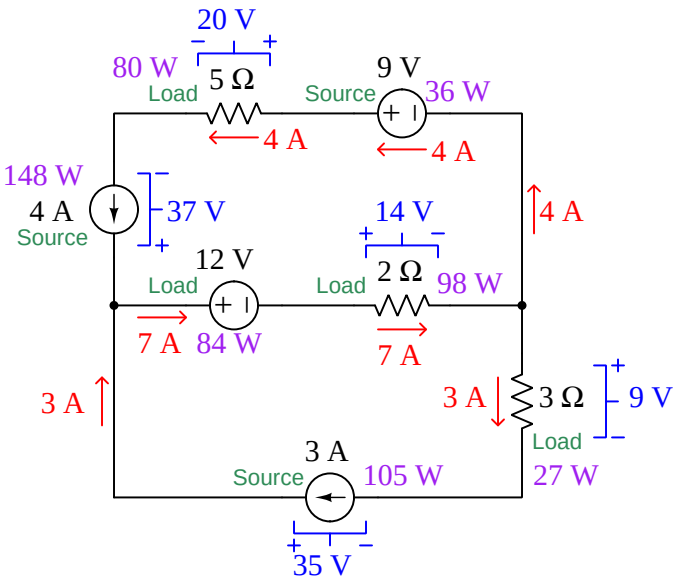
Adding these voltage notations to the schematic diagram, with polarities shown to make each loop's sum equal zero:



Identifying sources and loads, we simply label as "source" any component where charge carriers enter at lower energy (−) and exit at higher energy (+), and label as "load" any component where charge carriers go from higher to lower energy passing through:



Lastly, we may calculate power for each component using Joule's Law ($P = IV$):



A good check of our work is to verify that the sum of all source powers equals the sum of all load powers:

$$148\ \text{W} + 105\ \text{W} + 36\ \text{W} = 80\ \text{W} + 84\ \text{W} + 98\ \text{W} + 27\ \text{W}$$

$$289\ \text{W} = 289\ \text{W}$$

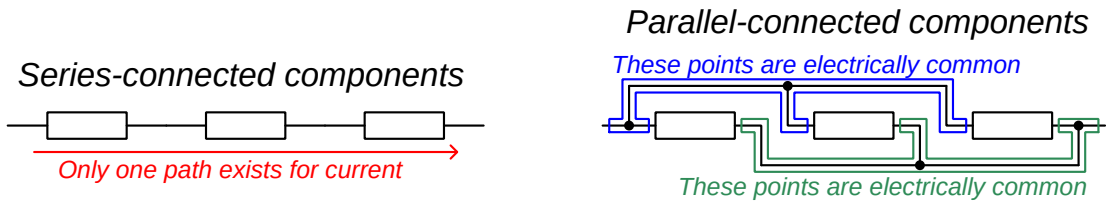
Although one may argue that circuits such as this are unrealistic, they provide an excellent practice opportunity to apply all the fundamental laws and properties of circuits: Ohm's Law, Joule's Law, Kirchhoff's Laws, and series/parallel network properties. The basic problem-solving strategy, regardless of circuit shape, is to see where any of these laws or properties apply and then annotate the results of that application, repeating the process until all parameters are solved.

It is worth noting that when students struggle with circuit analysis and/or circuit fault diagnosis, the root cause is more often than not a failure to master these foundational concepts. If your comprehension of any of these laws or principles is weak, you will invariably struggle later on, so take the time to absolutely master these!

Chapter 3

Simplified Tutorial

A *series-parallel* network is one containing both *series* and *parallel* sub-networks. A "series" network is defined by its components being connected in such a way as to form a single path for current (i.e. all components connected in a row). A "parallel" network is defined by its components sharing the same two sets of electrically common points. Both network types are shown here for comparison:



A universal property of series networks is that their components share the same amount of continuous current. This is due to the Law of Electric Charge Conservation: electric charge carriers can neither be created nor destroyed, but must always be accounted. The only way for the amount of current entering any point to differ from the amount of current exiting that same point is if charges were to somehow either vanish or come into existence at that point. Since a series circuit is nothing more than a back-to-back collection of points, this principle extends to the series circuit in its entirety.

A universal property of parallel networks is that their components share the same amount of voltage. This is due to the Law of Energy Conservation: energy can neither be created nor destroyed, but must always be accounted. The electrical commonality between connected terminals of parallel components ensures those connected points form equipotential surfaces. By definition, charge carriers are free to move along a set of equipotential points without gaining or losing any energy, and therefore any difference in energy between a charge at *any point* along that equipotential surface and an identical charge at *any point* along a different equipotential surface must be the same.

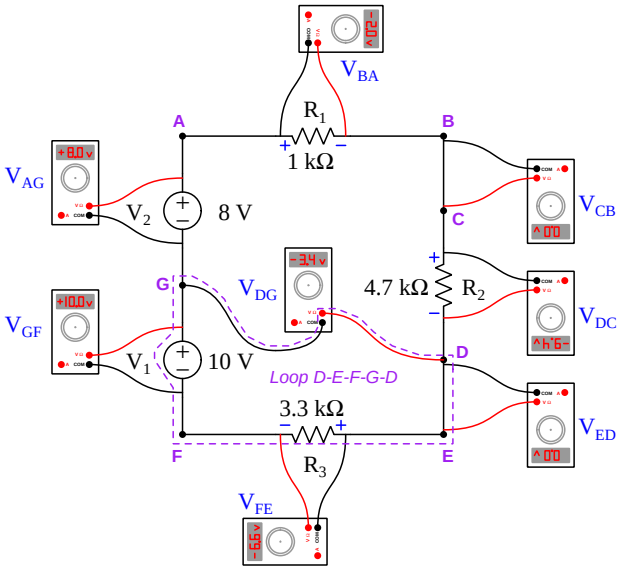
A common application of these guarantees is in the connection of electrical meters to measure current and voltage. If we desire to measure current through some component, we must connect our ammeter in *series* with that component to ensure the meter experiences the exact same current as

the component. Likewise, if we desire to measure voltage across some component, we must connect our voltmeter in *parallel* with that component to ensure the meter experiences the same voltage as the component.

For any network operating under continuous (i.e. steady-state)[1] conditions, be it series, parallel, or a combination of series and parallel, the total power dissipated by the circuit must be equal to the sum of the individual component power dissipations. This is due to the Law of Energy Conservation as well: under steady-state conditions the flow of energy into any system must equal the flow of energy out of any system.

Kirchhoff's Laws of voltage and current are important principles for analyzing series-parallel circuits as well. Kirchhoff's Voltage Law (KVL) states that the algebraic sum of all voltage rises and drops in any loop must be equal to zero. This "loop" may or may not happen to trace an actual path for current in a circuit, as KVL is true regardless of path. KVL is based on the Law of Energy Conservation: when any "test charge" is moved from location to location and eventually back to its starting point, its net energy gain/loss must be zero. This is analogous to a hiker climbing up and down a mountain, gaining potential energy when climbing and losing potential energy when descending: when the hiker returns to their starting altitude, their net total potential energy gain/loss will be zero.

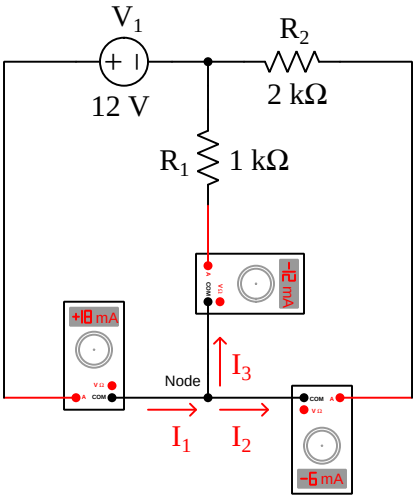
The following example shows KVL applied to a loop formed by test points D-E-F-G-D in a circuit. V_{DE} (0 Volts) plus V_{FE} (−6.6 Volts) plus V_{GF} (+10 Volts) plus V_{DG} (−3.4 Volts) equals zero:



[1] The qualification of "steady state" simply means that the system is operating with constant parameters, with nothing accumulating or depleting over time, such that it could operate like this for an indefinite period of time. This is important to note, because it is actually possible to have *temporary* conditions of $P_{in} \neq P_{out}$, for example in circuits containing energy-storing components such as capacitors, inductors, and/or secondary-cell batteries.

Kirchhoff's Current Law (KCL) states that the algebraic sum of all currents entering and exiting a point (node) in a circuit must be equal to zero. KCL is based on the Law of Electric Charge Conservation: all charges entering must eventually exit.

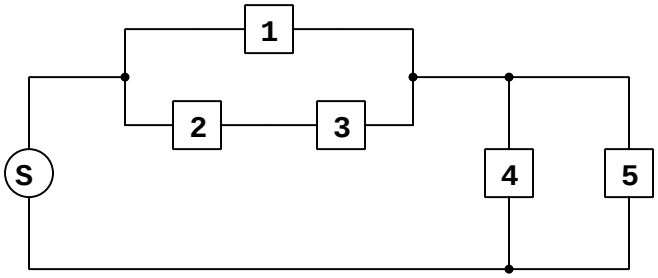
The following example shows KCL applied to a node in a circuit. By convention, currents entering a node are positive and currents exiting a node are negative, and the ammeter connections are oriented to reflect this standard. I_1 (+18 milliAmperes) plus I_2 (−6 milliAmperes) plus I_3 (−12 milliAmperes) equals zero:



Together, Kirchhoff's Voltage Law and Kirchhoff's Current Law represent a pair of powerful analytical tools for electric circuits, every bit as useful as Ohm's and Joule's Laws. You will often find certain problems solvable by KVL and/or KCL that simply cannot be solved any other way, and so these laws must be mastered in order to be fully competent in circuit analysis.

Mastery of these concepts is foundational to successful series-parallel circuit analysis, which cannot be analyzed using properties of series networks or properties of parallel networks alone. Consider the following circuit which is neither a purely series network, nor a purely parallel network. Imagine wanting to determine the voltage and current for each of the five loads based on their resistance and on the source's voltage value:

A series-parallel network with one source and five loads

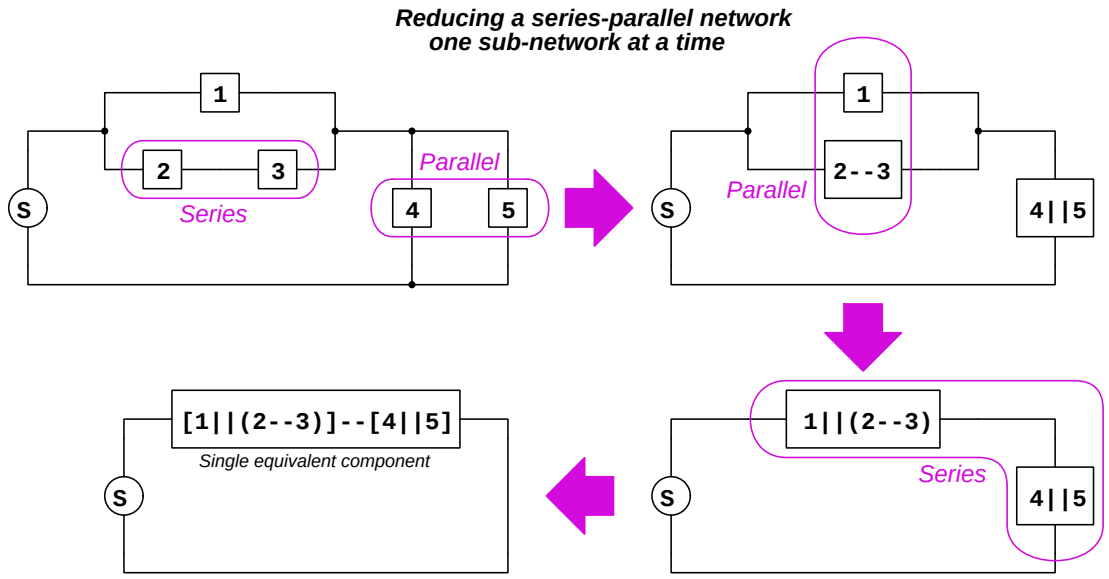


A very common mistake made by students new to series-parallel networks is to try to calculate total circuit resistance by simply adding all the resistances together. This would be appropriate if the circuit were series in nature, but this one is clearly not. Another mistake new students sometimes make when faced with a circuit such as this is to apply the source voltage to each and every load component. This would be appropriate if the circuit were parallel in nature, but this one is clearly not.

If the properties of series networks don't broadly apply to a circuit such as this, and neither do the properties of parallel networks, how then may we analyze it? If all we have are component values, it seems we cannot even apply Kirchhoff's Laws to the circuit as shown since we have only one voltage or current value (of the single source) and no others.

If we closely examine this circuit, however, we can identify *portions* of it that are series and others that are parallel. In other words, this complex circuit may be thought of as having *series sections* and *parallel sections*, and within each of those sections the properties of series and parallel networks respectively apply. Therefore, a practical strategy for analyzing a series-parallel circuit is to represent every series resistance sub-network and every parallel resistance sub-network as *equivalent resistances*, then repeating this process until the circuit is reduced to a single equivalent resistance. At that point we may take the source's given value and combine it with the single equivalent resistance value using Ohm's Law to determine the unknown (total) voltage or current, and then "expand" back to the circuit's original form as we repeatedly apply the results of these Ohm's Law calculations.

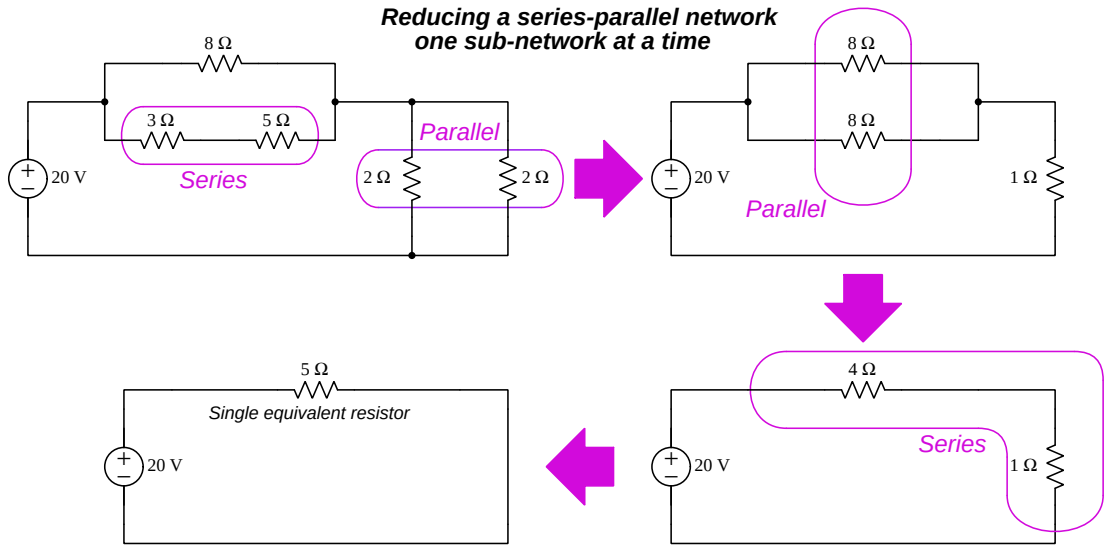
This reduction-based strategy is shown in the following illustration, showing how the series-parallel circuit may be reduced to a single equivalent component in a succession of reducing steps:



The same reduction process is applied to multiple sources, if they exist in the circuit too. At that point, Ohm's Law may be applied to that single equivalent resistance and to the single source to calculate either voltage or current. Once that is done, the equivalent circuit may be "expanded" step-by-step, transferring all voltage and current values from the simpler version as the properties of series and parallel networks dictate, re-applying various laws (Ohm's, KVL, KCL) as needed to calculate any unknown voltages or currents[2]. When complete, all voltages and currents in the circuit's original form should be known.

[2] For example, in this circuit we could take the total current calculated for the final (single-equivalent-component) version of the circuit and transfer this calculated current value to both equivalent components in the next-more-complex version. We transfer the *current* value because we see the single equivalent component being "expanded" into two *series-connected* components, and we know that series-connected components must share the same current.

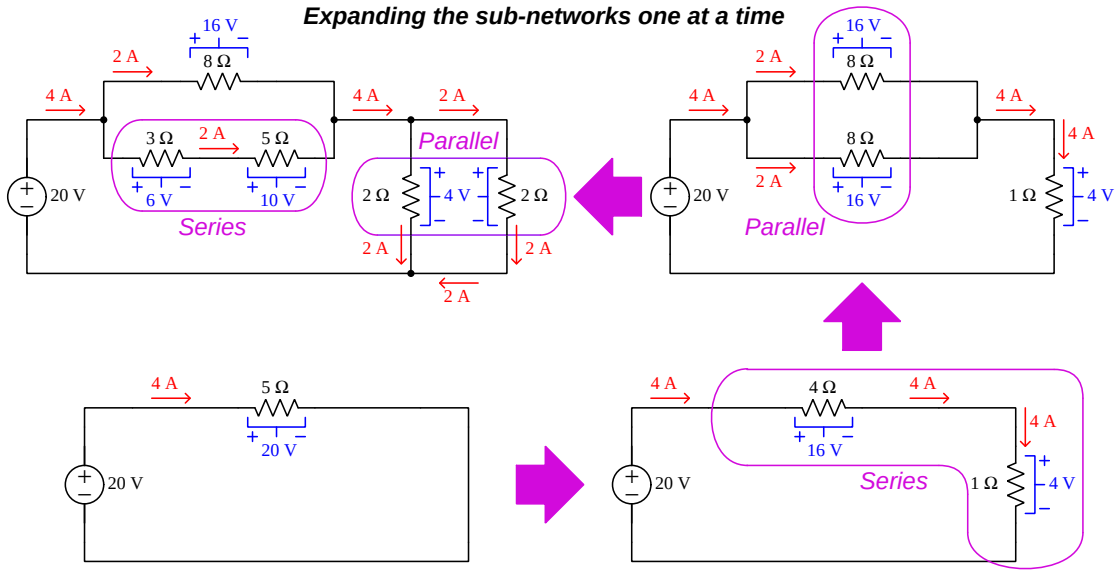
Now we will make this reduction-expansion analysis technique more concrete by showing how it applies to a series-parallel circuit with simple (round-number) component values:



Resistors connected in series combine to form a larger equivalent resistance; resistors connected in parallel combine to form a smaller equivalent resistance. This reduction occurs as often as necessary to reduce the entire circuit to a single equivalent resistance value connected to the source.

Once that single equivalent (total) resistance value is found, we may apply Ohm's Law to calculate current. In this particular example, a source voltage of 20 Volts applied across a single equivalent resistance value of 5 Ohms yields a current of 4 Amperes through the source.

Determining values for voltage and current at each of the original circuit's resistors requires we "expand" the network back to its original form, reversing each of the reductions previously made, in reverse of the order they were made:

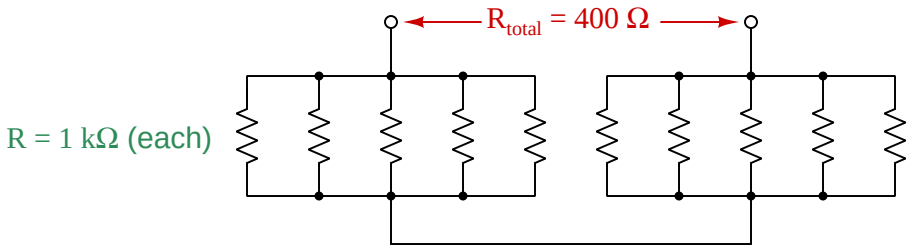


With each expansion, we take whatever electrical quantity is known to be equal among connected components (i.e. current for series-connected components, voltage for parallel-connected components) and transfer those quantities to the "expanded" component sets. For example, the 4 Ampere *current* found in the final reduction (with a single equivalent resistance) transfers to the first expansion because there we are expanding the 5 Ohm equivalent resistance into 4 Ohm and 1 Ohm resistances connected in *series*. Likewise, the 16 Volt *voltage* across the 4 Ohm equivalent resistance gets transferred to the two 8 Ohm *parallel* resistances in the next expansion. This step-by-step expansion and transfer of calculated values continues until the circuit once again attains its original form, where we end up with solved voltage and current values for all components.

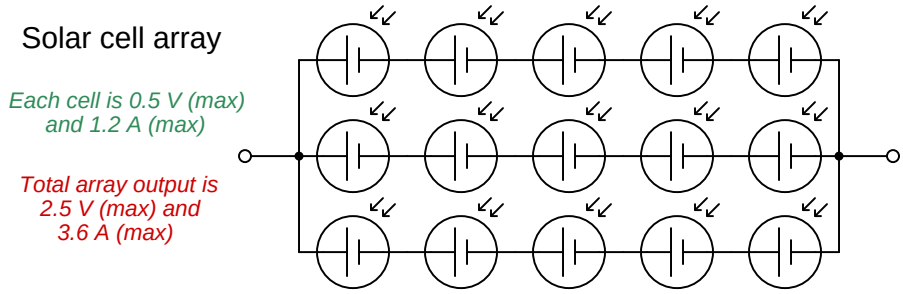
An excellent habit to develop when solving any problem is to *check your work*, with the various properties and laws you've learned about circuits being great tools for this purpose. For example, after "collapsing" and then "expanding" a series-parallel circuit to calculate all component voltages and currents, it is a good idea to apply KVL and KCL to the final result as a check of your work: no matter where you apply KVL and/or KCL, all the final results for voltage and current should correctly check.

Series-parallel networks consisting of identical components are relatively easy to analyze. Series networks of identical components exhibit a total resistance equal to one component's resistance times the number of components; identical sources connected in series (aiding) yield a total voltage equal to one source's voltage times the number of sources. Likewise, parallel networks of identical components exhibit a total resistance equal to one component's resistance divided by the number of components[3]; identical sources connected in parallel (aiding) yield a total current equal to one source's current times the number of sources. When stacked in a series-parallel fashion, these resistance, voltage, and current values compound the same: stacking parallel sub-networks in series with each other yields more resistance and more voltage; stacking series sub-networks in parallel with each other yields less resistance and more current.

We see this principle applied in the following resistor network to form a total resistance of 400 Ohms using nothing but 1000-Ohm resistors:



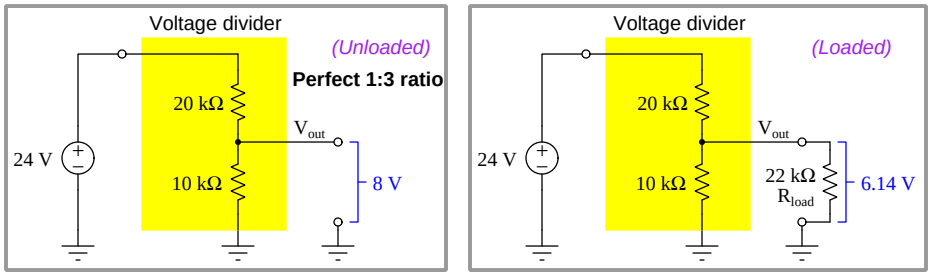
We may employ this same series-parallel "stacking" technique to construct large arrays of energy sources and energy-storage devices such photovoltaic solar cells and battery banks. For example, consider the following solar cell array where fifteen identical solar cells (rated at 0.5 Volts and 1.2 Amperes each) are stacked in a series-parallel network to output more voltage (0.5 Volts each × 5 in series = 2.5 Volts total) *and* more current (1.2 Amperes each × 3 in parallel = 3.6 Amperes total) than any one cell on its own:



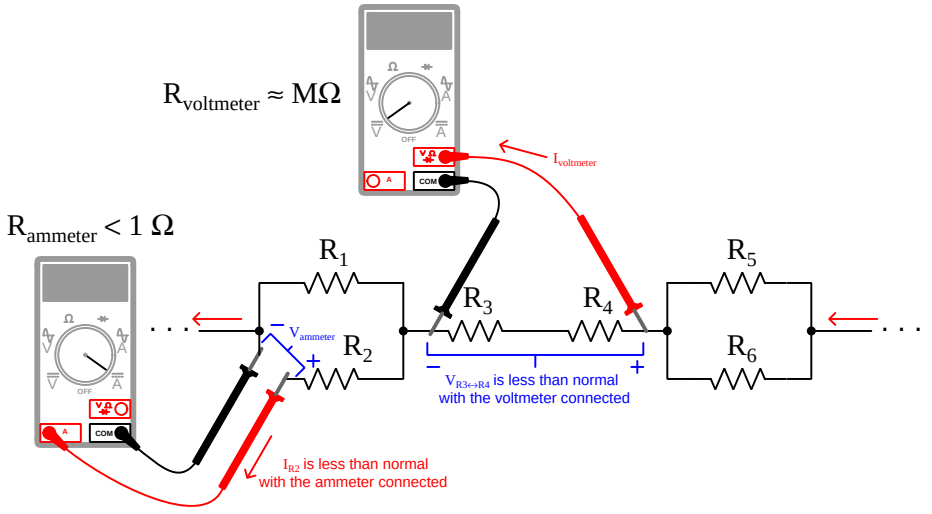
[3] We saw this in action with the previous circuit where the two 8-Ohm resistors became a 4-Ohm parallel equivalent, and where the two 2-Ohm resistors became a 1-Ohm parallel equivalent.

Voltage divider circuits are series networks, and current divider circuits are parallel networks. When any external load component is connected to either type of divider circuit, the output of that divider circuit will necessarily decrease as a result of that added load. This is due to the fact that adding a load to any simple divider circuit transforms that circuit from a simple-series or simple-parallel network into a series-parallel combination network. The resistance of the connected load combines with the resistance of the divider component connected to the load terminal(s) to form a new equivalent resistance, and this new equivalent resistance value reduces the division ratio that existed prior to the load's connection.

An example of this "loading" effect is seen in the following voltage divider circuit which ideally produces a 1:3 voltage division ratio but only when unloaded:



Connecting a meter to a circuit likewise creates either a series (ammeter) or parallel (voltmeter) sub-network that did not previously exist in the circuit, and "loads" that circuit by extracting a small amount of energy from it. If this newly-formed network converts what was once a simple series or parallel circuit into a series-parallel combination circuit (e.g. connecting a voltmeter in parallel with one component in a series circuit), the result will be that the meter reads slightly less than the true value when unconnected. Voltmeters exhibit very high (but not infinite) resistance, while ammeters exhibit very low (but not zero) resistance, and this means the loading effect should be slight. In either case, the finite resistance of a meter is called its *insertion resistance*.



Chapter 4

Full Tutorial

4.1 Concept review

First, a review of foundational circuit principles.

Three fundamental measures of electricity are *voltage* (V), *current* (I), and *resistance* (R). Voltage is defined as the amount of energy either gained or lost by electric charge carriers between two different locations. Current is defined as the rate of charge carrier flow. Resistance is defined as the amount of voltage drop (i.e. energy lost per charge carrier) for a given amount of current (flow). *Ohm's Law* relates these three variables mathematically:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

The combination of voltage and current is *power* (P): the amount of energy either gained or lost per charge carrier multiplied by the number of charge carriers passing through over time equals the rate of energy gained or lost over time. *Joule's Law* mathematically relates power to voltage, current, and resistance:

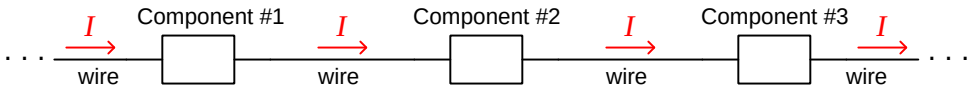
$$P = IV \qquad P = I^2R \qquad P = \frac{V^2}{R}$$

Electrical components designed to boost the energy level of charge carriers passing through are referred to as *sources*. Components extracting energy from passing charge carriers are called *loads*. The identity of any component as either a source or a load is evident by comparing the voltage polarity marks versus direction of current. If current enters a component on the "−" side and exits on the "+" side, it means the charge carriers enter at a lower energy level and exit at a higher energy level, which makes that component a source. If current enters a component on the "+" side and exits on the "−" side, it means the charge carriers enter at a higher energy level and exit at a lower energy level, which makes that component a load.

When electrical components are connected together, they form an electrical *network*. Connections between electrical components may be broadly divided into two categories: *series* and *parallel*.

Each of these network types exhibits unique properties, and must be thoroughly understood before attempting to understand more complex types of electrical networks.

When electrical components are connected together such that they form a chain providing only one path for current, they are said to be connected in *series* with each other:



With only one path for charge carriers to flow through each component, any continuous flow of electric charge carriers (i.e. current, I) through each component must be equal due to the Law of Electric Charge Conservation (electric charges can neither be created nor destroyed). Similarly, the Law of Energy Conservation – which states that energy can neither be created or destroyed – informs us that the energy gains or losses of electric charge carriers moving through each component accumulate to equal the total gain or loss in energy for the series network; that is to say, voltages across a set of series-connected components algebraically add to equal the series network's total voltage. For the same reason, the total resistance (R) of a series network (defined as the amount of energy lost per charge carrier for a given rate of flow) is equal to the sum of all the individual component and wire resistances. Mathematically stating each of these series network properties:

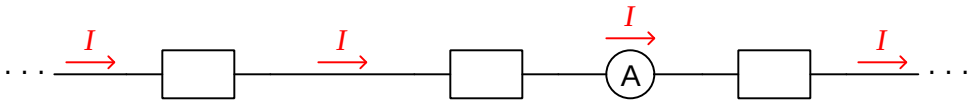
$$I_{total} = I_1 = I_2 \cdots = I_n$$

$$V_{total} = V_1 + V_2 \cdots + V_n$$

$$R_{total} = R_1 + R_2 \cdots + R_n$$

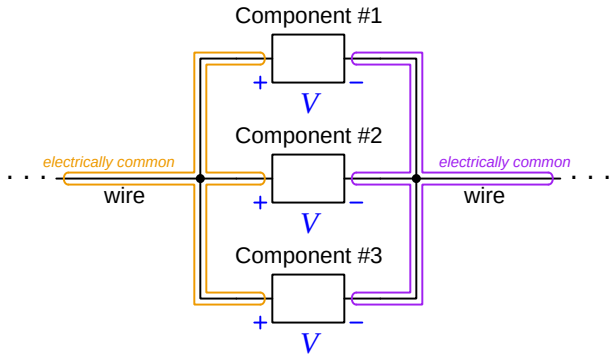
Series connections are used in the measurement of electric current, to ensure the instrument designed to measure electric current (an *ammeter*) will sense the exact same amount of current as the component being tested. A series connection between an ammeter and the component under test guarantees this equality of current:

Ammeter connected in series, so as to experience the exact same current as the other components



Note that multiple locations are possible for the ammeter – *any* location that is in-line with all components will yield the same measurement of current which is the value of current common to all components in this series network.

When electrical components are connected together such that they share the same two sets of electrically common points (i.e. their respective terminals are made equipotential by direct connection), they are said to be connected in *parallel* with each other:



The term *equipotential* means no energy is lost or gained by electric charge carriers moving between those connected points. Points connected together by a conductor of negligible resistance will be equipotential by virtue of that conductor's low resistance. In the parallel network shown above, charge carriers on all the left-hand wires of the network exist at the same potential energy level, as do charge carriers on all the right-hand wires. Therefore, any *difference* of potential between the left and right wires must be the same. Therefore every component in a parallel network experiences the same potential difference (i.e. same voltage, V). In this example, any charge carriers moving from left to right through any of the components will lose the same amount of energy, regardless of which component they happen to pass through.

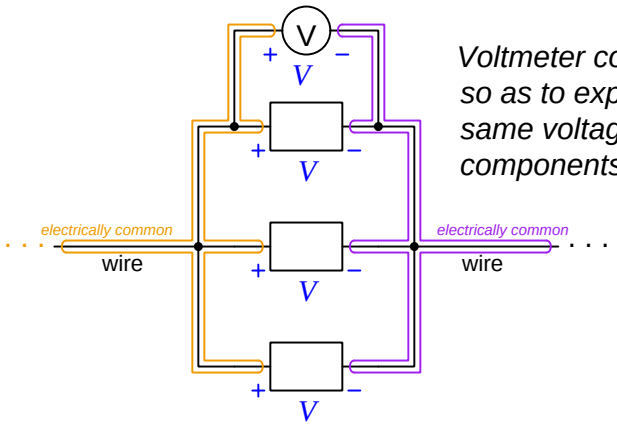
Current values through each component in a parallel network, however, may differ substantially. In fact, Ohm's Law ($I = \frac{V}{R}$) guarantees this will be the case if the components in question happen to have differing resistance values (R) but all share the same voltage value (V). Given the Law of Charge Conservation, we may conclude that the total current for a parallel network must be equal to the algebraic sum of all component ("branch") currents. Another property of parallel networks is that their total resistance is less than that of the lowest-resistance branch. Mathematically stating each of these parallel network properties:

$$V_{total} = V_1 = V_2 \cdots = V_n$$

$$I_{total} = I_1 + I_2 \cdots + I_n$$

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_n}}$$

Parallel connections are used in the measurement of voltage, to ensure the instrument designed to measure voltage (a *voltmeter*) will sense the exact same amount of voltage as the component being tested. A parallel connection between a voltmeter and the component under test guarantees this equality of voltage.



Note that multiple locations are possible for the voltmeter – touching the voltmeter's left-hand test lead to *any* point on the left-hand equipotential zone, and touching the voltmeter's right-hand test lead to *any* point on the right-hand equipotential zone, will yield the same measurement of voltage which is the value of voltage common to all components in this parallel network.

A property common to *all* networks, series and parallel alike, is that total (i.e. source) power is equal to the sum of all load powers in a circuit. This is just an extension of the Law of Energy Conservation (that energy cannot be created or destroyed and so must be accounted for in all cases), because *power* is nothing more than energy transfer per unit time:

$$P_{total} = P_1 + P_2 \cdots + P_n$$

This holds true for all circuits operating under steady-state[1] conditions, regardless of shape, because the Law of Energy Conservation is universal.

[1] By "steady-state" we mean the quantities of voltage, current, and power remain constant over substantial periods of time. This caveat is based on the fact that *energy* is always and forever conserved, but *power* is not necessarily so. Imagine a scenario where a weight is slowly hoisted up to some elevation, and then released so that it falls to the ground: the amount of energy invested in the weight's ascent is exactly equal to the amount of energy released by the weight as it falls (in the forms of air friction heating, and also energy delivered to the ground upon impact). However, the amount of *power* expended in slowly hoisting the weight far less than the amount of power delivered by the weight as it slams into the ground, simply due to the disparity in time intervals: the former happens over a long time interval, while the latter occurs over a very brief time interval. An analogous electrical scenario is that of a capacitor slowly energizing to some high voltage level, then rapidly de-energizing into a low-resistance load: the amount of energy invested in the capacitor is precisely equal to the amount of energy later delivered to that load, but the rates of power are quite unequal due to the timespan over which each action takes place. So, when analyzing any circuit operating under steady-state conditions we may regard power as a conserved quantity because everything occurs over the same span of time, but know that it is really *energy* that is conserved. If the circuit does not operate under steady-state conditions, source power and load power may in fact differ greatly.

Kirchhoff's Voltage Law (KVL) states that the net change in energy for any charge moved around in any loop – i.e. moved from point to point, returning to the same location where it started – will always be zero. That is to say, as an electric charge gains and loses energy moving between these different points, it must always return to its original energy level after returning to its original location. Since energy gains and losses for electric charges is the definition of *voltage*, we may express KVL by saying that the algebraic sum of all voltages in a loop equals zero.

If we label points within a loop by letter (e.g. A, B, C, etc.) and denote each voltage within that loop by a letter subscript pair (e.g. V_{XY}, the amount of energy gained or lost by a charge moving from Y to X), KVL becomes evident just by the subscripts. If we move a "test charge" from point A to point B and note the voltage (V_{BA}), then from point B to point C (V_{CB}), then all the way around the loop until we return to point A, the sum of those voltages must be the same as the amount of voltage moving from point A to point A (V_{AA}) – i.e. the amount of voltage between identical points. Of course, this must be equal to zero:

$$V_{BA} + V_{CB} + V_{nC} \cdots + V_{An} = V_{AA} = 0$$

KVL is true regardless of the path taken by the charge, and is applicable to any circuit configuration whatsoever, as it is an expression of the Law of Energy Conservation (i.e. that energy cannot be created or destroyed, and therefore the amount of energy carried by an electric charge cannot simply appear or vanish, but must be accounted for in every step of that charge's travels). The path traced need not be in the direction of real current in the circuit, and in fact need not follow any circuit conductors at all. It is perfectly permissible to trace a KVL loop that "jumps" between disconnected points in a circuit. In fact, this is what we often do when applying KVL to the calculation of voltage between points not directly spanned by one component.

Kirchhoff's Voltage Law (KVL) is useful for calculating unknown voltage values between points in a circuit, if all the other voltage values between points in a loop happen to be known.

Kirchhoff's Current Law (KCL) states that charge carriers entering a point must exit that point, which is a consequence of the Law of Electric Charge Conservation (i.e. that electric charges cannot be created or destroyed, but must be accounted for in all cases). That is to say, the total amount of current entering a point (or "node") in a circuit must be the same as the total amount of current exiting that same point. This is true regardless of the number of conductive paths connecting at that point, and like KVL this means KCL applies to any circuit configuration whatsoever. We may express KCL by saying that the algebraic sum of all currents at any node must equal zero:

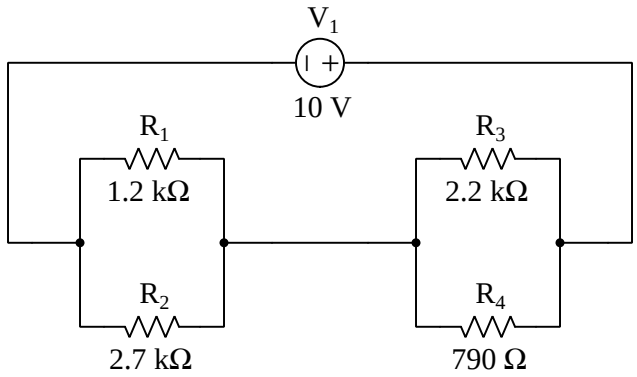
$$I_1 + I_2 + I_3 \cdots + I_n = 0$$

Kirchhoff's Current Law (KCL) is useful for calculating unknown currents in a circuit, if all the other currents entering and/or exiting a points in the circuit happen to be known.

4.2 Series-parallel network reduction

Now that we have reviewed series and parallel circuits, as well as Kirchhoff's Laws, let us move on to the topic at hand: *series-parallel circuits*. As the name implies, these circuits are a blend of series *and* parallel sections. Consider the following four-resistor circuit which is neither purely series nor purely parallel:

Original circuit:



We cannot apply the properties of series networks to this circuit in its entirety, because not all of its components are in series with each other. Similarly, we cannot universally apply the properties of parallel networks either because not all of its components are connected in parallel. However, we can see that *some* components in this circuit are connected in parallel, namely R_1 and R_2, and also R_3 and R_4. This observation suggests a way forward.

If we carefully apply the principles of series networks and of parallel networks only to those portions of the circuit that *are*[2] series and parallel, respectively, then using the results of those analyses to derive an equivalent of the original circuit using fewer resistors. This process is repeated until the circuit has been condensed to a single resistor representing that circuit's total resistance. Series and parallel connections will be represented in text form by the symbols $\leftrightarrow$ (series) and $\|$ (parallel).

After collapsing the original resistor network down to a single equivalent resistor, we may apply Ohm's Law to the calculation of current and/or voltage (depending on the nature of the source). Once those values have been determined, we may "expand" the single-resistor circuit back to its previous equivalent and apply those calculated values there. After that, we apply Ohm's Law again to the calculation of voltage and/or current and then transfer those values to the next "expansion" of the circuit and so on until we arrive back at the original circuit configuration with all voltages and currents solved.

[2] While this problem-solving technique is presented here specifically for series-parallel resistor circuits, it is an expression of a more general problem-solving strategy: *solve what you can*. A common tendency of students new to any analytical subject is to give up when presented with a problem that they don't see a *complete* solution for from the start. Instead, what one should do when presented with a complicated problem is begin by solving what you *do* know how, even if it is not clear to you how that solution might help you attain a final solution to the problem at hand. As you will see here, applying simple series and simple parallel analysis to portions of the original circuit has the effect of simplifying that circuit, and so with each step the original problem becomes less and less daunting.

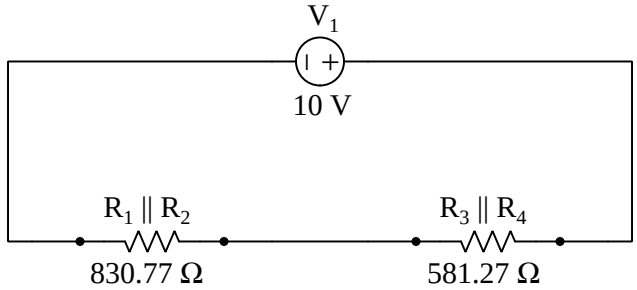
If this process sounds tedious to you, know that it certainly can be. However, there are worse problems in life than tedium. Unsolvable problems are certainly worse, and that is what this series-parallel circuit certainly appears to be without analyzing it stage by stage. You should know that a great many analytical problem-solving techniques are nothing more than substitutions of *tedium* for *complexity*. By methodically analyzing this series-parallel circuit one sub-network at a time, we exchange its original complexity for a long sequence of relatively simple steps, and that makes it solvable. Effective problem-solvers are *patient* above all else!

We will begin our analysis of this series-parallel circuit by noting which sections (i.e. sub-networks) of it are series and which are parallel, if any[3]. Here we see that R_1 and R_2 are connected in parallel with each other, as are R_3 and R_4. Knowing that parallel-connected resistances result in a diminished total resistance according to the formula $R_{parallel} = (R_1^{-1} + R_2^{-1})^{-1}$, we may calculate the equivalent resistance value for $R_1 \parallel R_2$ and for $R_3 \parallel R_4$:

$$(R_1 \parallel R_2) = \frac{1}{\frac{1}{1200\ \Omega} + \frac{1}{2700\ \Omega}} = 830.77\ \Omega$$

$$(R_3 \parallel R_4) = \frac{1}{\frac{1}{2200\ \Omega} + \frac{1}{790\ \Omega}} = 581.27\ \Omega$$

Parallel equivalents:



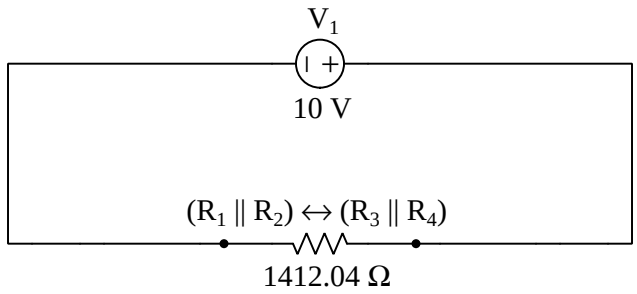
Once we re-draw the circuit to show each of the parallel sub-networks as single equivalent resistances, the circuit now appears to be a simple series network, much simpler to analyze than before. It should come as no surprise that we should be able to apply this same technique again: identify portions of the circuit that are either simply series or simply parallel and reduce those sub-networks to single equivalent resistors.

[3] It is imperative that you have a solid grasp on the definitions of series and parallel networks, so that you may readily recognize series-connected and parallel-connected components within a larger network. Recall that the definition of a series network is one where only a single path exists for current, and that the definition of a parallel network is one where the components connect between the same two points.

Reducing it one more step, knowing that series-connected resistances result in an additive total according to the formula $R_{series} = R_1 + R_2$:

$$R_{(R_1 \| R_2) \leftrightarrow (R_3 \| R_4)} = 830.77\,\Omega + 581.27\,\Omega = 1412.04\,\Omega$$

Series equivalent:

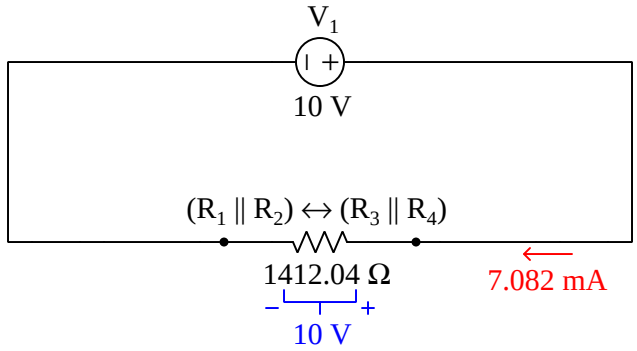


This single-resistor circuit is obviously not identical to the original four-resistor circuit, but it is *equivalent* in terms of total resistance. And, if the total resistance is the same, then the total current predicted by Ohm's Law must be the same as well:

$$I_{total} = \frac{V_1}{R_{total}} = \frac{10\text{ V}}{1412.04\,\Omega} = 7.082\text{ mA}$$

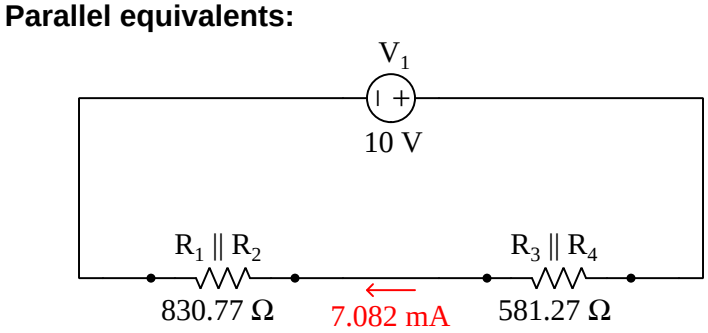
It is always a good idea during circuit analysis to annotate your schematic diagrams with calculated values, to provide clear context for those numerical values. Here, we will sketch an arrow showing the calculated total current value of 7.082 mA as well as the 10 Volts impressed across the single equivalent resistor:

Series equivalent:



4.3 Series-parallel network expansion

Now, working our way back to the original circuit configuration, we will examine the equivalent circuit prior to this one to see which values transfer. The previous equivalent circuit was a *series* configuration, and we know that it is *current* which is shared among series-connected components. Therefore, it is the 7.082 mA current value we just calculated that may be applied to the previous equivalent circuit:

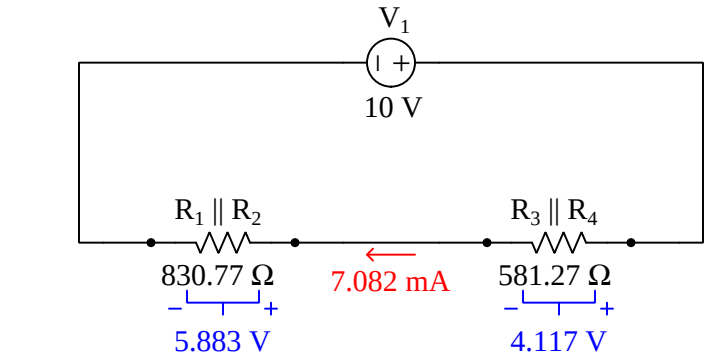


Ohm's Law may be applied here to calculate the voltage dropped by each of the two equivalent resistors:

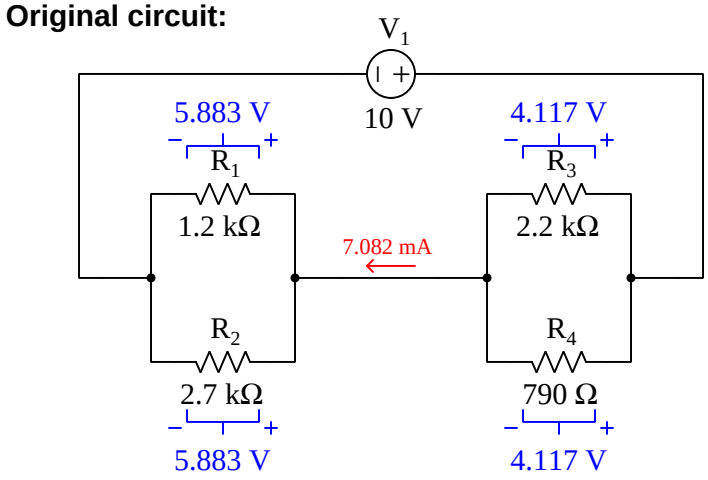
$$V_{R_1 \| R_2} = I(R_1 \parallel R_2) = (7.082 \text{ mA})(830.77\,\Omega) = 5.883 \text{ V}$$

$$V_{R_3 \| R_4} = I(R_3 \parallel R_4) = (7.082 \text{ mA})(581.27\,\Omega) = 4.117 \text{ V}$$

Annotating our diagram with these values:



Still working our way back to the original circuit configuration, we examine it to see which of the calculated values from this last analysis will transfer. The original circuit (just one step removed from the last equivalent circuit shown) is a *parallel* expansion of this circuit's two resistances. We know that it is *voltage* which is shared among parallel-connected components, and so we will transfer the 5.883 Volt and 4.117 Volt values to the original circuit:



Ohm's Law may be applied again, this time to calculate current through each individual resistor. Note how we are careful to keep all values in proper context[4] with each other, calculating the current through each resistor by dividing *that resistor's* voltage by *that resistor's* resistance:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{5.883 \text{ V}}{1200\ \Omega} = 4.903 \text{ mA}$$

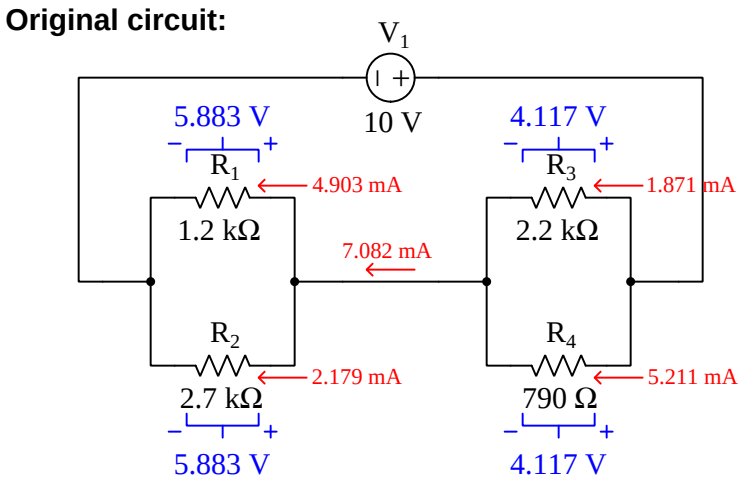
$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{5.883 \text{ V}}{2700\ \Omega} = 2.179 \text{ mA}$$

$$I_{R3} = \frac{V_{R3}}{R_3} = \frac{4.117 \text{ V}}{2200\ \Omega} = 1.871 \text{ mA}$$

$$I_{R4} = \frac{V_{R4}}{R_4} = \frac{4.117 \text{ V}}{790\ \Omega} = 5.211 \text{ mA}$$

[4] Mixing contexts is a common way in which students fail to calculate correct results when applying mathematical formulae to physical problems. It is imperative you understand what each and every numerical value *means* in a circuit, and where it fits. This is the main reason I recommend annotating schematic diagrams with calculated values: to help remind you of those values' contexts.

Annotating the original schematic diagram with all current values:



A wise habit to adopt is *checking your solutions*, and that is best done by analyzing the results in a different manner than the one by which they were obtained[5]. In this case, our calculated values consist of resistor voltage drops and resistor currents. In order to rigorously check these values, we need to apply some principles using voltages and currents other than Ohm's Law.

Kirchhoff's Laws provide an excellent means of checking these solutions. We know from Kirchhoff's Voltage Law (KVL) that the algebraic sum of all voltages in a loop must equal zero. Applying KVL to our analysis by imagining a "test charge" starting at the left-hand conductor of the circuit and progressing around it in a counter-clockwise direction, we will check to see that all the potential gains and losses indeed sum to zero:

$$(+5.883 \text{ V}) + (+4.117 \text{ V}) + (-10 \text{ V}) = 0 \text{ V}$$

We know from Kirchhoff's Current Law (KCL) that the algebraic sum of all currents entering and exiting a node must equal zero. Applying KCL to any of the nodes at the parallel sub-networks, for example at the right-hand node of $R_1 \parallel R_2$ where 7.082 mA enters, 4.903 mA exits, and 2.179 mA also exits:

$$(+7.082 \text{ mA}) + (-4.903 \text{ mA}) + (-2.179 \text{ mA}) = 0 \text{ mA}$$

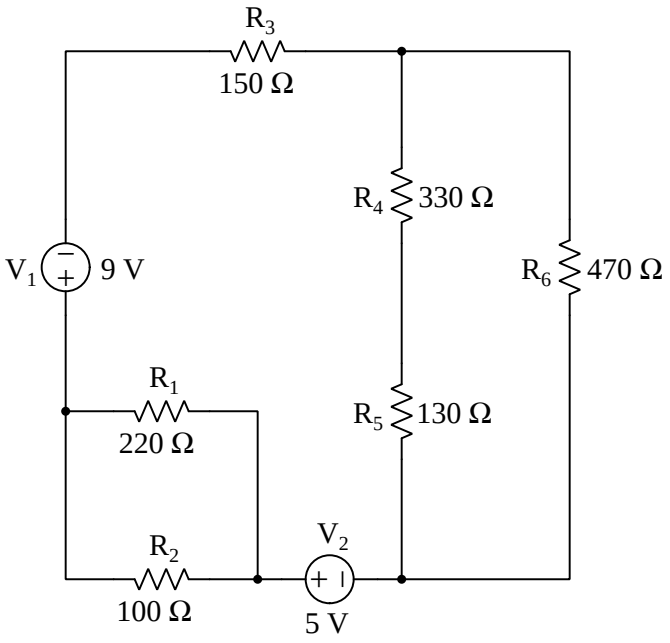
The only analysis left to do at this point is calculate resistor power dissipations, and that may be done in three different ways ($P = IV$, $P = I^2R$, or $P = \frac{V^2}{R}$). For this example I will leave this as an exercise for the reader. Once all resistor power dissipations are calculated, they may be checked by calculating source power ($P = 7.082 \text{ mA} \times 10 \text{ V}$) and verifying that the sum of all resistor powers equals this source power, according to the Conservation of Energy.

[5]The novice may see nothing wrong with repeating a set of calculations to ensure none of them were incorrectly computed. However, this naturally leads to a repeat of the same mistakes, if there were any mistakes made. When the checking is done by some means other than the one taken to obtain the original results, common mistakes such as calculator keystroke errors, procedural errors, and other errors prone to repetition are avoided.

4.4 Another series-parallel example

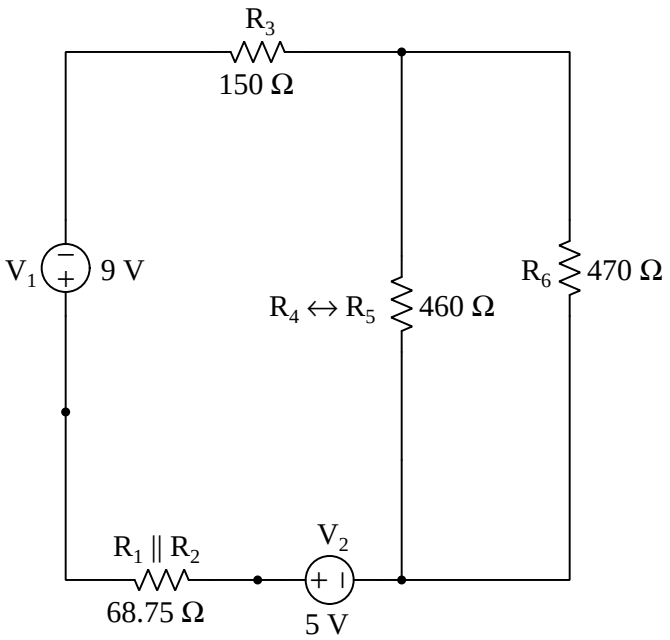
Let us analyze a more complex series-parallel circuit in order to see how these same principles may be applied. In this analysis I will omit detailed explanations of every step, and just show the circuit being condensed step-by-step into a single-source and single-resistor equivalent, then expand it step-by-step back to its original form. Each schematic diagram will be fully annotated to show the context of all calculated values.

Here is the original circuit, consisting of two voltage sources and six resistors:



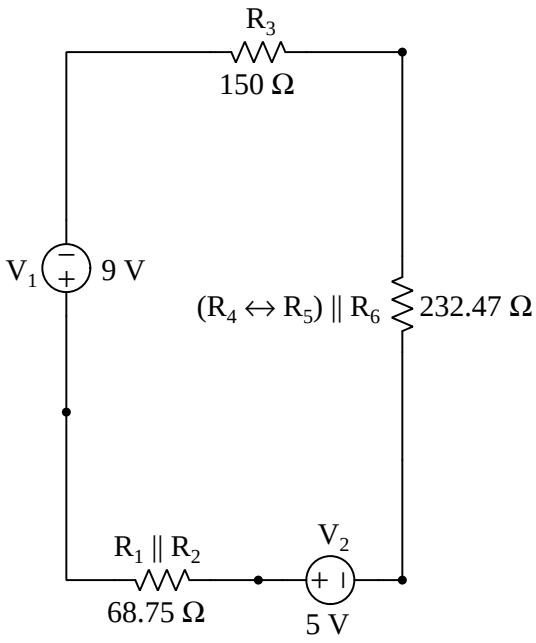
It should be evident that R_1 and R_2 are in parallel with each other, and R_4 and R_5 are in series with each other.

First equivalent circuit, condensing resistors R_1 and R_2 into a parallel equivalent, and resistors R_4 and R_5 into a series equivalent:



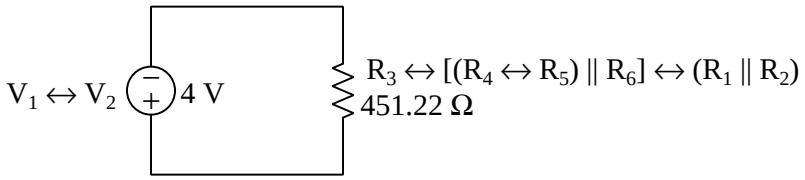
Now we see that $R_4 \leftrightarrow R_5$ is in parallel with R_6.

Second equivalent circuit, condensing resistors R_4, R_5, and R_6 into a single equivalent:



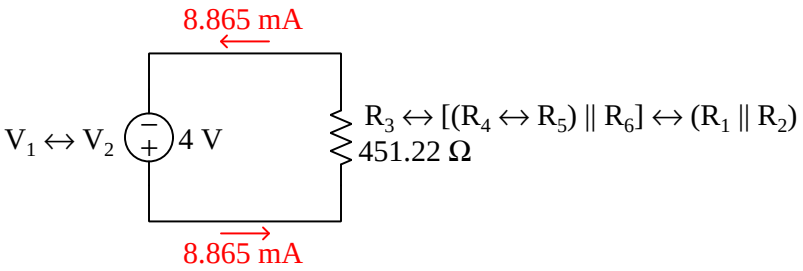
Now it is clear to see that all components are in series with each other, because there is only one path for current between them all. This means we may condense the three resistors into a single equivalent resistor, and the two sources into a single equivalent source.

Our final equivalent circuit will condense all resistors into a single equivalent and both voltage sources into a single source. The two voltage sources are opposing each other, and so the combination of them will be a single source with the stronger source's polarity and a magnitude equal to the difference in voltage between the two original sources:

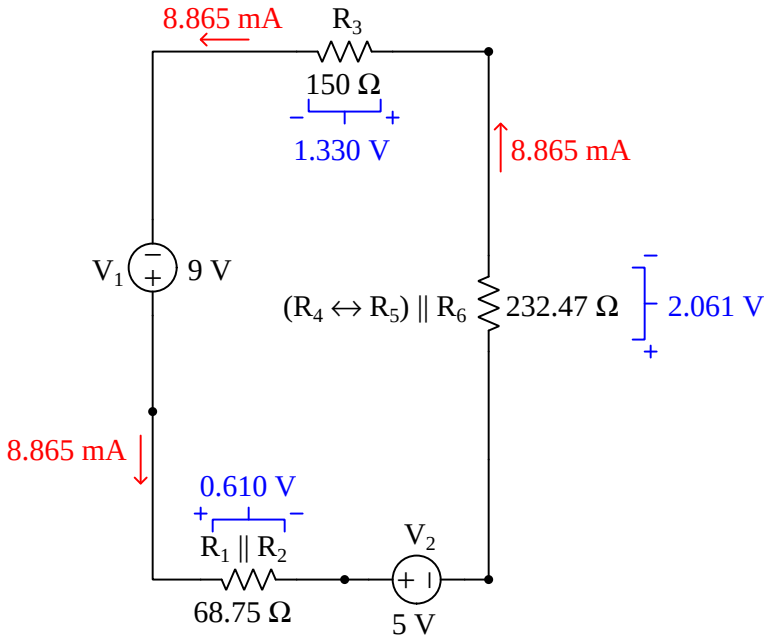


At this point we are ready to apply Ohm's Law to the calculation of current, and later to the calculation of individual resistor voltages and currents.

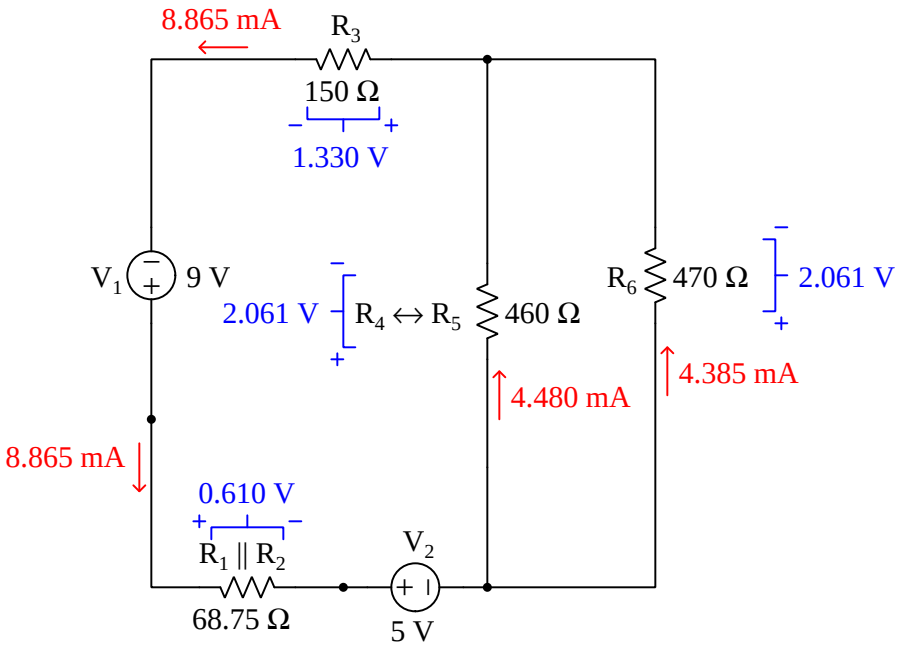
Calculating and annotating current in the final equivalent circuit:



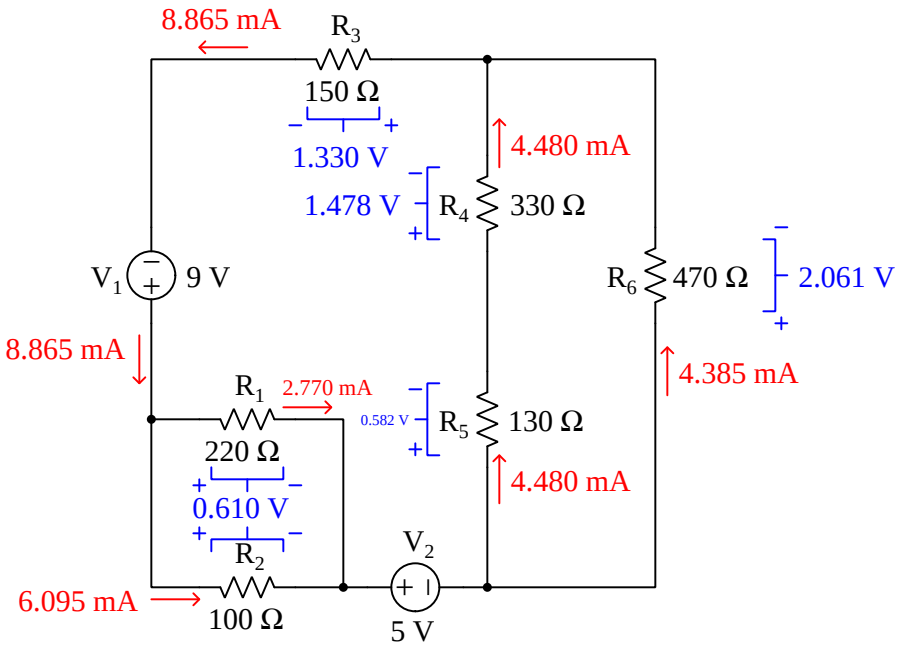
Expanding to the second equivalent circuit (series, same current), calculating and annotating resistor voltage drops:



Expanding to the first equivalent circuit (parallel $R_4 \leftrightarrow R_5$ and R_6, same voltage), calculating and annotating resistor currents:



Expanding the last resistance equivalents to their original forms, then calculating and annotating all remaining resistor voltages and currents:



As usual, KVL and KCL offer ways for us to check[6] our work. Where and how we apply Kirchhoff's Laws really doesn't matter, so long as our application verifies all calculated values of voltage and current:

$$\textbf{KVL, outer loop: } (+9 \text{ V}) + (-0.610 \text{ V}) + (-5 \text{ V}) + (-2.061 \text{ V}) + (-1.330 \text{ V}) = 0 \text{ V}$$

$$\textbf{KCL @ V1 node: } (8.865 \text{ mA}) + (-2.770 \text{ mA}) + (-6.095 \text{ mA}) = 0 \text{ mA}$$

$$\textbf{KCL @ R3 node: } (4.480 \text{ mA}) + (4.385 \text{ mA}) + (-8.865 \text{ mA}) = 0 \text{ mA}$$

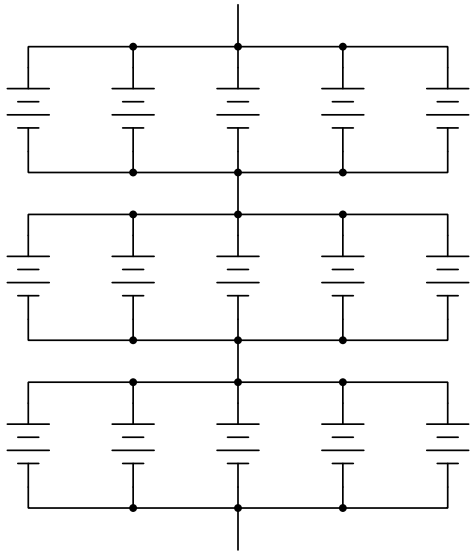
Resistor power dissipations may be calculated by applying Joule's Law in any manner desired ($P = IV$, $P = I^2R$, and/or $P = \frac{V^2}{R}$). As a final check, the sum of all resistor power values must equal the total source power (treating the two sources as a single[7] 4 Volt source carrying 8.865 mA) because the Conservation of Energy in any steady-state scenario means that *power* must be conserved as well.

[6] You will notice that the voltage values shown in the KVL check don't exact add up to zero, but that is due to rounding. If these voltage values are retained in your calculator's memory rather than written and re-typed in truncated form, the sum ends up being exactly equal to zero as it should be.

[7] Alternatively, one could calculate power sourced by V_1 and treat V_2 as a load, since you will note current is being forced "backward" through V_2.

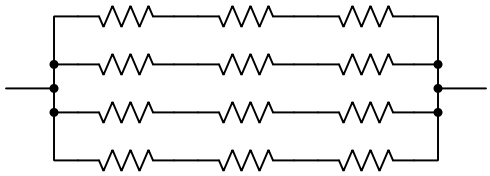
4.5 Series-parallel component banks

Some series-parallel networks are much easier to analyze than the examples previously given. If all components in the series and parallel sub-networks are equal in value, the task of calculating equivalent components in the condensed versions of the circuit becomes almost trivial. Take for instance this series-parallel network of batteries for an energy-storage system:



If each of the batteries outputs 12.6 Volts, and each one has a maximum output current of 35 Amperes, then the array as a whole (often referred to as a *battery bank*) outputs 37.8 Volts (12.6 Volts times *three* in series) and may deliver up to 175 Amperes of current to a load (35 Amperes times *five* in parallel).

A series-parallel resistor "bank" comprised of identical resistors likewise condenses very easily into a single equivalent resistance:

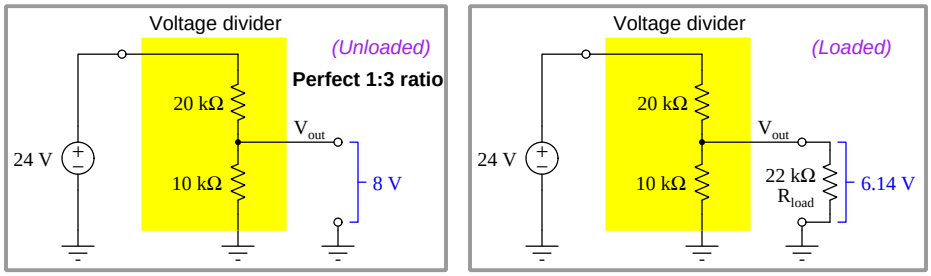


If each of these resistors is rated at 100 Ω and 2 Watts, then the combination of *three* in series yields 300 Ω for each parallel branch, and the combination of *four* parallel branches diminishes that by a factor of four for a total of 75 Ω. Power ratings simply add up to a total of 24 Watts (12 resistors times 2 Watts each), regardless of the series-parallel configuration, due to the Law of Energy Conservation which stands regardless of circuit shape.

4.6 Loaded divider networks

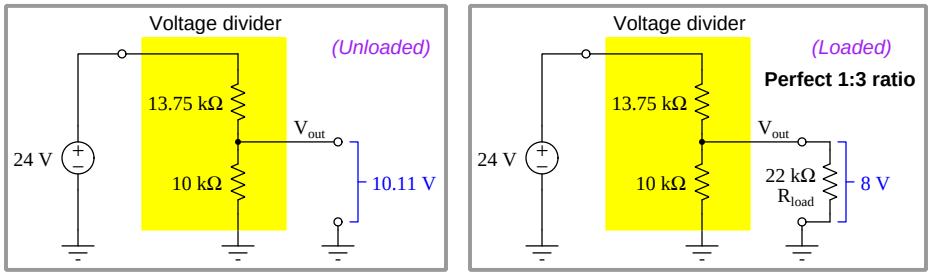
Series-parallel networks may be formed when new components are connected to existing (series- or parallel-only) circuits. One such example is that of *loaded voltage divider circuits*. As you recall, a "voltage divider" is a network of series-connected resistors intended to divide an applied voltage into smaller proportions. A common application for voltage dividers is to reduce the voltage from a source, down to some level more suitable for a load. A problem faced with such usage of a voltage divider is that once the load is connected to the divider circuit, its voltage division ratio changes by virtue of the load creating a series-parallel network where only a series network existed before.

Take the following voltage divider as an example. With no load connected, its two resistors divide the source voltage into two-thirds and one-third portion, with the lowest third constituting the "output" of the voltage divider. Once a load is connected to the output terminal, though, the output voltage "sags" to a value less than the desired $\frac{1}{3}$ proportion under the loading effect:



If this voltage divider network is to provide a precise $\frac{1}{3}$ voltage-division ratio to a load, its resistor values must be selected in anticipation of that load resistance. Only then will the division ratio be correct for the series-parallel network that is formed when the load is attached.

Applying this design strategy to the voltage divider just shown, we could replace the 20 kΩ resistor with a lower-valued resistor, equal to twice the resistance of the 10 kΩ $\parallel$ R_{load} parallel sub-network. When calculated, $2\left((10\text{ k}\Omega)^{-1} + (22\text{ k}\Omega)^{-1}\right)^{-1} = 13.75\text{ k}\Omega$. With this new upper resistor in place, the voltage divider will supply the desired $\frac{1}{3}$ ratio (8 Volts from a 24 Volt source) when loaded, but of course will output more than that when unloaded:



It should be noted that the re-designed voltage divider only provides the $\frac{1}{3}$ voltage division ratio when the load resistance is precisely 22 kΩ. A new load resistance will require another re-design.

Current divider circuits face similar challenges maintaining a desired division ratio when loaded.

Just as with voltage dividers, the addition of a load resistance converts what was once a simple (series or parallel) network into a more complex series-parallel network.

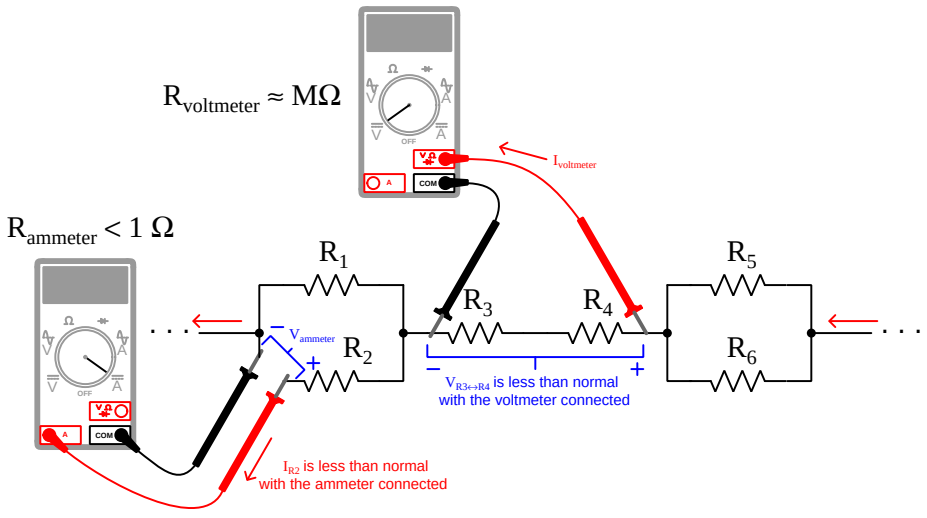
4.7 Meter loading

This same "loading" problem can occur simply by connecting a *meter* to a circuit to measure voltage or current. Recall that an ideal voltmeter possesses infinite internal resistance, and that an ideal ammeter possesses zero internal resistance. The purpose of these ideal resistance values is to minimize the amount of energy extracted from the circuit by the meter, because extracting energy alters the operation of the circuit and will yield a measured value different than the true value with the meter absent. In practice it is impossible to measure any quantity without affecting that quantity in any way, but we can minimize the impact any measurement of ours may have.

For example, a voltmeter should have infinite resistance so that no current will pass through it while sensing a voltage. If no charge carriers pass through a voltmeter, none of them will move from a position of higher energy to a position of lower energy, which means no energy will be released by charge carriers to the voltmeter – the voltmeter will simply sense the *potential* between two points without actually converting any of that potential energy into work.

Likewise, an ammeter should have zero resistance so that current passing through it will not drop any voltage. If no voltage drop occurs, it means charge carriers will have passed through without losing any of their energy – the ammeter will simply sense the flow of charge carriers without forcing them to do any work.

Real voltmeters and ammeters, though, possess finite amounts of resistance called *insertion resistance*. Digital multimeters in voltage-sensing mode typically exhibit millions of Ohms between their test leads, and DMMs in current-sensing mode typically exhibit just a small fraction of one Ohm. The following resistor network (source omitted) shows the expected loading effects of connected meters:



The voltmeter connected in parallel with $R_3 \leftrightarrow R_4$ forms a new series-parallel sub-network, $R_{voltmeter} \parallel (R_3 \leftrightarrow R_4)$ having slightly less resistance and therefore dropping slightly less voltage than $R_3 \leftrightarrow R_4$ would on their own. The ammeter connected in series with R_2 forms a new series-parallel sub-network, $R_1 \parallel (R_2 \leftrightarrow R_{ammeter})$ having slightly greater resistance in the R_2 branch which diverts more current through R_1 and therefore passes slightly less current than R_2 would on its own. In either case, the meter's loading effect results in a measurement that is *falsely low*.

Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Metric prefixes

The *metric system* of measurement specifies certain units of measurement for various physical parameters, some of them listed here:

Parameter	**Unit**	**Symbol**
Length	Meter	m
Temperature	Celsius or Kelvin	C or K
Mass	Gram	g
Force	Newton	N

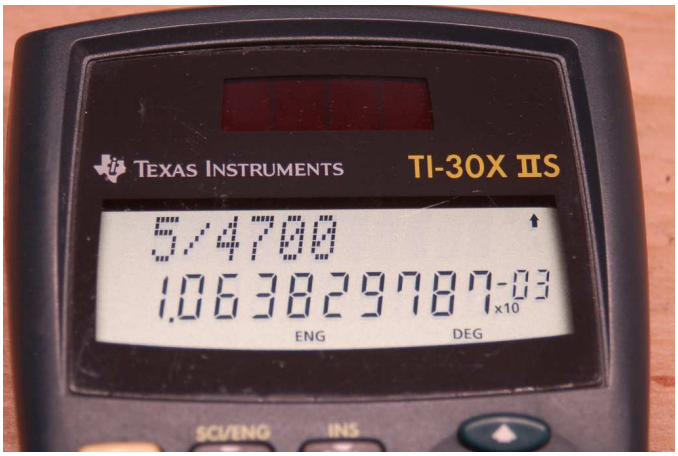
In order to make the expressions of large and small measurements more convenient, the metric system also includes *prefixes* representing power-of-ten multipliers, some of which are listed here:

Prefix	**Symbol**	**Power-of-ten**
Peta	P	$10^{15} = 1,000,000,000,000,000$
Tera	T	$10^{12} = 1,000,000,000,000$
Giga	G	$10^{9} = 1,000,000,000$
Mega	M	$10^{6} = 1,000,000$
Kilo	k	$10^{3} = 1,000$
Milli	m	$10^{-3} = \frac{1}{1,000}$
Micro	μ	$10^{-6} = \frac{1}{1,000,000}$
Nano	n	$10^{-9} = \frac{1}{1,000,000,000}$
Pico	p	$10^{-12} = \frac{1}{1,000,000,000,000}$
Femto	f	$10^{-15} = \frac{1}{1,000,000,000,000,000}$

In order to represent very large or very small quantities all we need to do is combine the right prefix with the unit. For example, a distance of 12,509 meters could be written as 12.509 *kilo*meters (12.509 km) because "kilo" simply means *thousand*. Similarly, a mass of 0.000000421 grams could be written as 0.421 *micro*grams (0.421 μg) or 421 *nano*grams (421 ng) because "micro" means *one-millionth* and "nano" means *one-billionth*.

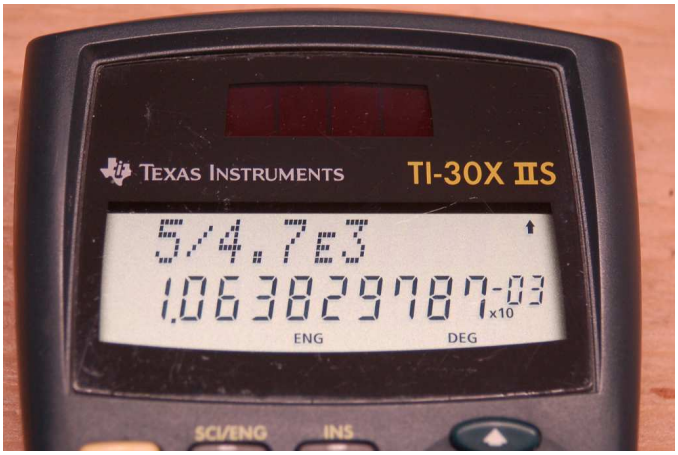
A useful way to think about metric prefixes is to consider the word *dozen*. Saying that we've collected three dozen eggs from a flock of chickens means the same thing as saying we've collected thirty-six eggs, because one "dozen" equals a quantity of twelve, and three times twelve is thirty-six. Here the word "dozen" is nothing more than a convenient multiplier for expressing some whole-numbered quantity of things. Metric prefixes constitute the same basic concept: they are simply multipliers used to help express a wide range of quantities while using numbers that are easier for us humans to handle.

Scientific hand calculators offer display modes and entry functions tailored for the purpose of expressing results and accepting input using standard metric prefix power-of-ten notation. Consider the following photograph of a Texas Instruments model TI-30X IIS calculator computing the current through a 4700 Ohm resistor energized by a 5 Volt source (using Ohm's Law, $I = \frac{V}{R}$):

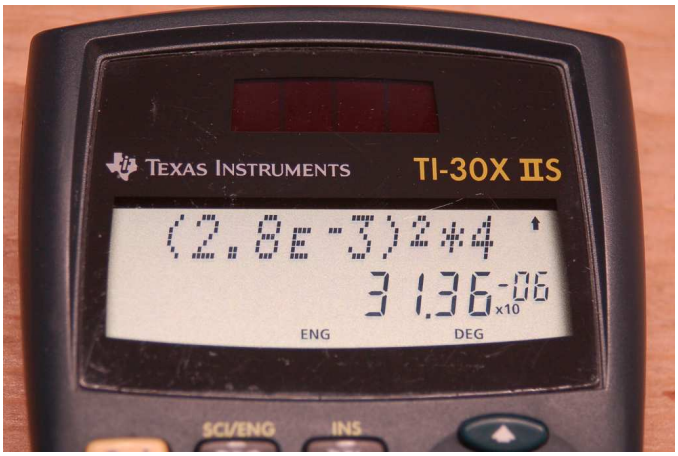


When the calculator has been set to the *Engineering* display mode, all results appear with a $\times 10^x$ label where x is an integer multiple of three to match with the metric prefixes listed previously. Here, the quotient $\frac{5}{4700}$ results in 1.0638×10^{-3}, or 1.0638 milliAmperes (1.0638 mA). It would be equally correct for the calculator to display the result as `0.001638` Amperes, but the 10^{-3} multiplier forced by Engineering mode results in a displayed number of 1.0638 that is easier for us to accurately read, write, and remember than 0.001638 with all the leading zeroes.

A button marked `EE` (sometimes $\times 10^x$ or simply `E`) allows entry of numerical values in either scientific notation or "engineering" form. Here, the same current calculation is performed entering the 4700 Ohm resistor's value as 4.7 kiloOhms (4.7 kΩ), entered as `4.7E3` in the calculator:



Another example shows the calculation of power dissipated by a 4 Ω resistor passing a current of 2.8 mA, using Joule's Law ($P = I^2R$) and once again making use of the `EE` button when entering the current value as `2.8E-3`:



Here, the resulting power dissipation is 31.36 microWatts (31.36 μW).

Some hand calculators provide the Normal/Scientific/Engineering display setting as a menu option, while others such as the TI-30X IIS shown here designate key functions to do the same. Note the `SCI/ENG` function visible in yellow-colored text above one of the buttons in these photographs.

Engineering mode and the `EE` button both require a knowledge of the powers-of-ten associated with common metric prefixes, but once those prefix multipliers are memorized they become very valuable in saving you the trouble of reading and entering numerical values filled with leading or trailing zeroes. As previously mentioned, without Engineering mode set our calculated current of 1.0638 mA would have been displayed by the calculator as 0.001638. Those leading zeroes, while mathematically accurate, are easy for humans to mis-count. The same is true for very large numbers with many trailing zeroes. *This is the chief purpose of metric prefixes: to reduce human error when reading, writing, and remembering very large and very small quantities alike.*

Mathematical formulae in science and engineering generally assume the use of *base units* in their variables. For example, *Ohm's Law* assumes voltage expressed in plain Volts (V), current expressed in plain Amperes (A), and resistance expressed in plain Ohms (Ω). However, for very large and very small quantities, it is more convenient for humans to read and write values using either metric prefixes or scientific notation rather than base units. For example, a voltage of 72 Volts applied across a resistance of 3,500,000 Ohms will result in a current of 0.00002057143 Amperes; but these quantities are much more easily written as 72 Volts, 3.5 megaOhms, and 20.57143 microAmperes, respectively.

Again, metric prefixes exist to make very large and very small numbers easier for human beings to manage. At root a metric prefix is nothing more than a particular power-of-ten multiplier. Thus, 3.5 MΩ is equal to 3.5×10^6 Ohms, or 3.5 *million* Ohms; likewise, 20.57143 μA is equivalent to 20.57143×10^{-6} Amperes, or 20.57143 *millionths* of an Ampere.

A very common error made by students new to the use of metric prefixes is to ignore them when inserting values into scientific and engineering formulae. For example, if someone tries to apply Ohm's Law to the example quantities of 72 V and 3.5 MΩ to find current ($I = \frac{V}{I}$) by simply dividing 72 by 3.5, their result will be grossly erroneous, as $72 \div 3.5 = 20.57143$. What this incorrect calculation *actually* finds is that 72 Volts applied across 3.5 Ohms yields a current of 20.57143 Amperes. When you enter "3.5" into a calculator, that calculator has no idea you really mean 3.5 *million* (3.5 *mega*) unless you specifically tell it so. This either requires you manually convert 3.5 MΩ into base units of Ohms (3,500,000 Ohms) by entering `3500000` into the calculator, or by using the calculator's power-of-ten entry key by entering `3.5E6` which means 3.5×10^6. In all cases, it is your responsibility to convert all metric-prefixed values into base-unit forms suitable for the formula(e) being calculated.

An interesting twist on the theme of base units is any formula involving *mass*, in which case the value for m must be computed in *kilograms* rather than grams. This is in accordance with the meter-kilogram-second (MKS) convention of modern[1] metric notation where the kilogram rather than the gram is considered to be the "base" scientific unit for mass.

For example, suppose we wished to calculate the kinetic energy of an 80 gram mass traveling at a velocity of 0.79 kilometers per second using the formula $E = \frac{1}{2}mv^2$. To properly calculate energy (E) in Joules, we would need to enter 0.080 for mass (m) in kilograms because 0.080 kg = 80 g and kilograms are the base unit for mass; we would also need to enter 790 or 0.79×10^3 or `0.79E3` for velocity (v) in meters per second because 790 m/s = 0.79 km/s with meters and seconds being base units. The proper result would be 24,964 Joules, or 24.964 kJ. If we were to ignore these metric prefixes by entering 80 for m and 0.79 for v, we would erroneously obtain a result of 24.964: an answer to a very different scenario where the mass is 80 kilograms rather than 80 grams and has a velocity of 0.79 meters per second rather than 0.79 kilometers per second.

So, just like the Ohm's Law example shown previously, we must first properly identify the base units required by the formula we're using, and then ensure our calculator sees all values expressed either directly in those base units or using powers-of-ten meaning the same thing. With the exception of mass where the *kilo*gram is considered the standard scientific base unit for all formulae, metric prefixes are nothing more than convenient ways to express large and small numerical values in order to avoid reading, writing, and remembering numbers containing a multitude of place-holding zeroes.

[1] The older metric convention was centimeter-gram-second (CGS) whereby all formulae using distance or displacement required casting that variable in *centi*meters rather than meters.

5.2 Resistor labeling

Resistors dissipate electrical energy in the form of heat, dropping voltage proportional to the amount of current passing through. This ratio of voltage to current is called *resistance* (R), measured in the unit of the *Ohm* (Ω), and it is the primary characterstic of any resistor. A popular method of labeling resistance values utilizes colored bands[2] to represent resistance values and tolerances. An example of this resistor style appears in the following photograph:

Four-band resistors are the most popular of the banded style, with the first and second bands representing significant digits, the third band representing a power-of-ten multiplier, and the fourth band representing tolerance.

Band color	**First digit value**	**Second digit value**	**Multiplier**	**Tolerance**
Black	0	0	10^0	
Brown	1	1	10^1	$\pm$ 1%
Red	2	2	10^2	$\pm$ 2%
Orange	3	3	10^3	$\pm$ 0.05%
Yellow	4	4	10^4	$\pm$ 0.02%
Green	5	5	10^5	$\pm$ 0.5%
Blue	6	6	10^6	$\pm$ 0.25%
Violet	7	7	10^7	$\pm$ 0.1%
Grey	8	8	10^8	$\pm$ 0.01%
White	9	9	10^9	
Gold			10^{-1}	$\pm$ 5%
Silver			10^{-2}	$\pm$ 10%
None				$\pm$ 20%

Applying this color code to the resistor shown above, we see that yellow = 4, violet = 7, red = 2, and silver = $\pm$ 10%. Therefore, this resistor's value is 47 $\times$ 10^2 Ohms (i.e. 4700 Ω) plus or minus 10%, which means it could be as low as 4230 Ω or as high as 5170 Ω.

[2] A useful mnemonic for associating these colors with decimal digits 0 through 9 and the percentages 5-10-20% is as follows: *"Better Be Right Or Your Great Big Venture Goes Wrong. Get Started Now."*.

Precision resistors require more significant digits to precisely quantify their values, and so use a *five-band* code where the first three bands represent digits, the fourth band represents the power-of-ten multiplier, and the fifth band represents tolerance. Both the four-band and five-band color codes are standardized as part of the international standard IEC 60062.

Many modern resistors are labeled with text rather than color-coded bands. For these resistors, a letter is used in place of a decimal point, the letter either being `R` (unit), `K` (kilo), `M` (mega), `G` (giga), `T` (tera), or `L` (milli). This is commonly referred to as the *RKM* code. Some labeling examples are shown in the following list:

- R82 = 0.82 Ω
- 3R9 = 3.9 Ω
- 47R = 47 Ω
- 560R = 560 Ω
- 3K3 = 3.3 kΩ
- 27K = 27 kΩ
- 100K = 100 kΩ
- 2M2 = 2.2 MΩ
- 15M = 15 MΩ
- 2L7 = 2.7 mΩ

Tolerances for text-labeled resistors use the following letter codes, the tolerance letter always being the last character of the resistor's text label:

Letter code	**Tolerance**
B	± 0.1%
C	± 0.25%
D	± 0.5%
F	± 1%
G	± 2%
H	± 3%
J	± 5%
K	± 10%
M	± 20%
N	± 30%

For example, a resistor labeled `6K8J` would be 6.8 kΩ ± 5%.

Surface-mount device (SMD) resistors are often labeled with numbers only, either a three-digit numerical code or a four-digit numerical code. These codes follow the same pattern as four- and five-band color codes (except with no specified tolerance, just **digit-digit-multiplier** or **digit-digit-digit-multiplier**). For example, `473` would be 47 kΩ and `2711` would be 2.71 kΩ.

An exception to this general rule appears in the following photograph of a 5 milli-Ohm surface-mount resistor with a tolerance of ± 1%. The `5L0` portion of the code indicates 5.0 mΩ, while the `F` portion indicates the ±1% tolerance:

This particular SMD resistor happens to be useful as a high-current *shunt resistor* developing a precise voltage drop proportional to current. Its low resistance value does little to impede the flow of charge carriers to the load, while its tight tolerance ensures its voltage drop will be a fairly good representation of current through it.

5.3 IEC standard component values

Components such as resistors, inductors, and capacitors are manufactured in several *standard values*, described by IEC standard 60063. Rather than having a single series of standard values, the IEC publishes lists called *E series* based on the number of unique values spanning a single *decade* (i.e. a 10:1 range).

The shortest of these series, called *E3* contains just three values: 10, 22, and 47. The next series is called *E6* with six unique values: 10, 15, 22, 33, 47, and 68. These values represent *significant values* for components, meaning the decimal point may be freely moved to create values spanning multiple decades. For example, "33" simply means one can expect to find components manufactured in values of 33, 3.3, 0.33, and 0.033 as well as 330, 3.3 k, 33 k, etc.

Although this may seem like a strange standard for component manufacturers to follow, there is a compelling logic to it. The terms of each series are closer-spaced at the low end than at the high end, and this allows for *series* and/or *parallel* combinations of components to achieve most any desired value. For example, in the E6 series we only have values with the significant figures 10, 15, 22, 33, 47, and 68, but this doesn't mean we are limited to *total* values with these significant figures. For example, if we needed 80 Ohms of resistance we could connect a 33 Ohm and 47 Ohm resistor together in series. 50 Ohms could be made from two 68 Ohm resistors in parallel (making 34 Ohms) plus a 15 Ohm and 1 Ohm resistor in series.

On the next page is a table showing the four most common E-series specified by IEC standard 60063.

E3	**E6**	**E12**	**E24**
10	10	10	10
			11
		12	12
			13
	15	15	15
			16
		18	18
			20
22	22	22	22
			24
		27	27
			30
	33	33	33
			36
		39	39
			43
47	47	47	47
			51
		56	56
			62
	68	68	68
			75
		82	82
			91

E48, *E96*, and *E192* series are also found in the IEC 60063 standard, used for components with tighter tolerance ratings than typical.

5.4 Ground in electric and electronic circuits

Ground and *grounding* within both electric power and electronic circuits are commonly misunderstood topics, largely because these terms can often have very different meaning depending on context. Here we will explore some of these different meanings and hopefully eliminate some of the confusion surrounding them.

The very first concept we must understand in order to comprehend what "ground" is in any circuit is that *voltage* is fundamentally a *relative* measurement of potential energy different *between two points*. This is really true of all forms of potential energy, because "potential" means energy that has not yet been put to work setting matter into motion, implying a final state yet to be realized. When we say an electrical charge carrier possesses potential energy (i.e. electrical potential), this means it *could* (potentially) release energy to do work if it were to fall into a lower state of energy. The difference in energy held by that electric charge carrier from its original (high-potential) state to some other lower-potential state will be the amount of energy released by that charge carrier to do work, with that amount of work done dependent on both the charge carrier's initial energy level and its final energy level. *Voltage* is the term we use to describe such a difference in electrical potential.

We say that voltage always exists "between two points" or "between two locations" in a circuit because voltage is the amount of potential energy either lost or gained by electric charge carriers from one position in a circuit to another. It is impossible to quantify how much energy will be gained or lost by charge carriers unless we know both the starting point and the ending point for those charge carriers. In order to quantify the absolute amount of potential energy possessed by anything, we would have to know where or how it could lose every bit of its stored energy to be left with zero capacity to do work[3]. This is really unknowable from a practical perspective, and so the best we can do is quantify gains or losses in potential energy between different states.

This fact about voltage is also why we label it with *pairs* of "+" and "−" symbols, the "+" symbol representing the point of *higher potential* and the "−" symbol representing the point of *lower potential*. We cannot place an absolute value on electrical potential at any single location because no one can say where the location of zero energy is in the universe for electric charges. The best we can do is to rate one location as having higher or lower electrical potential than another, quantifying only the difference in potential between those two locations.

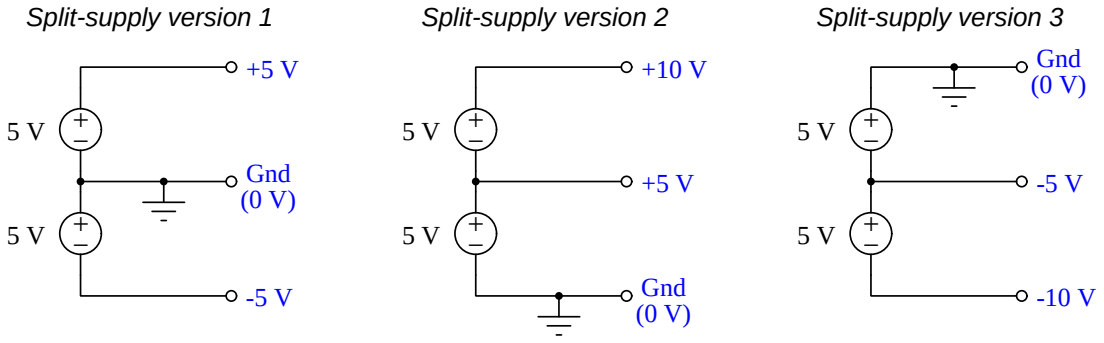
Now that we have reviewed this property of voltage and of potential energy in general, we are ready to explore what is meant by *ground* and *grounding*.

[3] Using gravitational potential energy as an example, a weight hoisted above the Earth's surface certainly possesses potential energy, and we might be tempted to quantify this potential energy by simply measuring its weight and its height above ground level and then multiplying those two quantities (i.e. potential energy equals force times distance). However, there are more places this weight could *potentially* fall than just ground level. That same weight, hoisted above the Earth's surface to the same height, will be assessed to have a far greater amount of potential energy if we consider it potentially falling down a deep hole rather than just falling to ground level. For that matter, one could even argue the weight has a *negative* quantity of potential energy with respect to what it would possess at the summit of the highest mountain! Potential energy, defined as the amount of work that *could* be done, necessarily depends on a system's initial condition *and* any number of different final conditions of which none are known to be truly zero-energy.

How do we rate the height of mountain peaks? Certainly, we may rate one mountain peak as being some number of meters higher or lower than another, but generally what people do instead is measure a mountain's peak height compared to *Mean Sea Level*. Since water is a liquid, and liquid tend to settle at equal heights under the influence of gravity when they are free to move about, the level of oceans throughout the world provide a relatively stable reference point for measurements of altitude. If we imagine a world without oceans, perhaps one with nothing but dry land and no open bodies of water anywhere, people living at different locations in this world would likely choose some flat-land reference points local (to them) for their measurements of nearby mountains, but these localized altitude measurements would be confusing if not meaningless for world-wide comparisons of mountain peaks.

Much the same is true of electrical and electronic circuits, where voltage is an important measure of electrical potential energy as well as a measure of signal strength where electricity is used as an analogue for representing other things[4]. One way to conveniently rate electrical potentials at different points in a circuit is by arbitrarily choosing one of those points to be the "zero-energy" reference point, and calling it *ground*. Ground in an electrical or electronic circuit is analogous to sea level for rating mountain peak heights or a *datum* or *benchmark* used for surveying land: simply a point of reference agreed-upon to be "zero" for the sake of ranking all other potentials to it.

A good example of this is an electrical network popularly known as a *split DC power supply*. Three different versions of this power supply network are shown with the same pair of 5-Volt DC sources, the only difference being the arbitrary location of the "Ground" (Gnd) point:

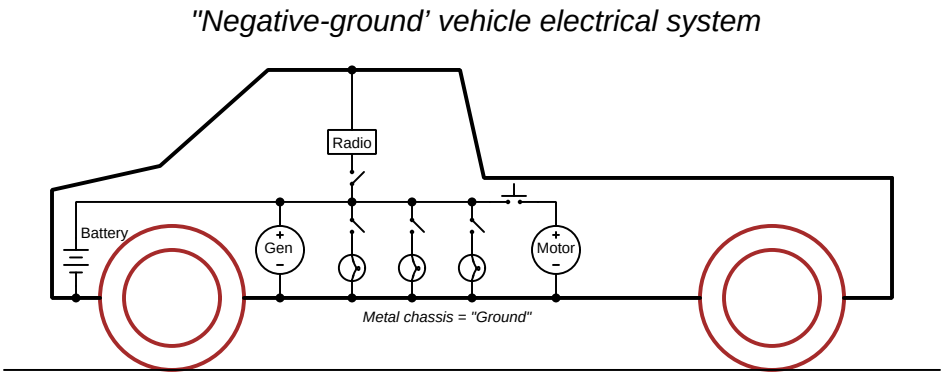


Each terminal's voltage label represents the amount of electrical potential there *relative to the "Ground" terminal*; i.e. "Ground" is simply a "sea-level" reference point against which all other potentials may be measured. The three different versions shown here illustrate just how arbitrary the location of "Ground" really is. Any load connected between the same pairs of terminals on each power supply would experience the same amount of voltage regardless of the "Ground" terminal assignment.

This is one meaning of the word "ground" in electric and electronic circuits: an arbitrary point of reference for the measurement and comparison of electrical power and signal potentials at different locations in the circuit.

[4]Many, many examples exist of voltage being used to represent non-electrical quantities. In digital computer systems, DC voltage levels represent binary "1" and "0" number states. In analog electronic sensor systems, voltage may represent temperature, speed, force, weight, sound pressure, or any number of other physical quantities.

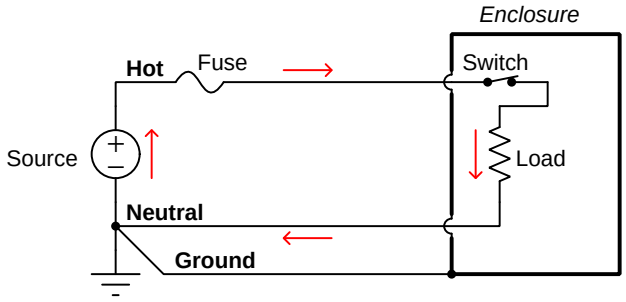
Another meaning of the word "ground" in circuits is a metallic surface used as a common conductor throughout some device or system for reasons of wiring economy. Automobiles with metal frames and chassis typically use their metallic bodies as an electrical conductor for nearly all circuits contained within. Doing so eliminates the need to install separate copper-wire conductors for every source and every load, thereby saving expense and weight:



Modern automotive electrical systems are "negative-ground" which means the vehicle chassis is always the lowest-potential point in any portion of the circuit. Interestingly, some early automobiles used "positive-ground" wiring which simply had the reverse polarity on all sources and loads: the vehicle chassis being the highest-potential point and all other points being either equal to it or at lower (−) potentials. This choice of polarity is really arbitrary, as electric circuits may be designed and built to function just as well either way. The "negative-ground" standardization we see today is really nothing more than automobile manufacturers agreeing to wiring their different vehicles the same way in order to enjoy interoperability of components.

This usage of the word *ground* – describing the metal chassis of some complex system exploited as a common electrical conductor for multiple circuits – is not incompatible with the previous usage of *ground*: a point of reference in a circuit arbitrarily defined to be "zero" potential. An automotive technician troubleshooting circuit faults in a vehicle often connects the black test lead of their voltmeter to the metal chassis, expecting to measure positive or zero voltage values on that meter when probing circuit test points with the red test lead. In a negative-ground electrical system, the chassis' electrical potential is *defined* as zero for the purpose of specifying voltages at other points in the circuit(s).

Yet another electrical meaning of the word "ground" is associated with *safety*, in which one metallic conductor of a power circuit is made electrically common to the Earth through a *grounding rod* buried in the soil. In the following circuit, the ground symbol in the lower-left corner is not merely a reference marker intended for anyone viewing the schematic diagram, but actually represents a metal rod driven deep into the soil and connected to one terminal of the voltage source with intent to achieve equipotentiality with the Earth:



Here the purpose of "grounding" is more important than merely establishing a reference point for voltage measurements. Instead, designating the conductor attached to this metal rod as "Ground" and attaching it to any conductive surface on or around an otherwise energized electrical appliance forces that surface to be equipotential to the same Earth that any humans and animals are standing on. This greatly mitigates the risk of electric shock should there be an internal fault within that appliance by making it impossible to have a difference of potential (i.e. voltage) between the grounded surface and the soil.

The following photograph shows the grounding rod for a commercial building, a large-gauge bare copper conductor clamped to the top of the rod which leads into the building's electrical room:

Ground wires in electric power systems should never carry current during normal operation. In fact, the presence of current through a ground conductor indicates a problem in the system! Note the two current-bearing power conductors in the previous schematic, specifically how the current-carrying conductor made common to Earth ground is called the *neutral*, while the other (non-grounded) power conductors is called *hot* by comparison. The Hot/Neutral designations always refer to which is grounded and which is not, rather than polarity. For example, if we were to reverse

the polarity of the DC voltage source shown in that schematic, the upper conductor (now −) would still be considered "hot" and the lower conductor (now +) would still be considered "neutral".

Chapter 6

Programming References

A powerful tool for mathematical modeling is text-based *computer programming*. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, *C++* and *Python*.

6.1 Programming in C++

One of the more popular text-based computer programming languages is called *C++*. This is a *compiled* language, which means you must create a plain-text file containing C++ code using a program called a *text editor*, then execute a software application called a *compiler* to translate your "source code" into instructions directly understandable to the computer. Here is an example of "source code" for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer's console:

```
#include <iostream>
using namespace std;

int main (void)
{
  float x, y;

  x = 200;
  y = -560.5;

  cout << "This simple program performs basic arithmetic on" << endl;
  cout << "the two numbers " << x << " and " << y << " and then" << endl;
  cout << "displays the results on the computer's console." << endl;

  cout << endl;

  cout << "Sum = " << x + y << endl;
  cout << "Difference = " << x - y << endl;
  cout << "Product = " << x * y << endl;
  cout << "Quotient of " << x / y << endl;

  return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other "whitespace" is largely irrelevant in C++ code, and is included only to make the code more pleasing[1] to view.

[1] Although not included in this example, *comments* preceded by double-forward slash characters (`//`) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.

Let's examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }, often referred to as "curly-braces").

- `int main (void)` labels the "Main" function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an *integer* number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program's completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called *arguments* to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer's memory for two *floating-point* variables, in this case the variables' names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line's end is necessary because C++ ignores whitespace on the page and doesn't "know" otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the "standard output" stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent *toward* the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer's console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.

After saving this *source code* text to a file with its own name (e.g. `myprogram.cpp`), you would then *compile* the source code into an *executable* file which the computer may then run. If you are using a console-based compiler such as *GCC* (very popular within variants of the Unix operating system[2], such as Linux and Apple's OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the `GCC` compiler to take your source code (`myprogram.cpp`) and create with it an executable file named `myprogram.exe`. Simply typing `./myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio[3], you may simply create a new console application "project" using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on
the two numbers 200 and -560.5 and then
displays the results on the computer's console.

Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

[2] A very functional option for users of Microsoft Windows is called *Cygwin*, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

[3] Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the "Hello World" `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile ("Build") and run your new program. Upon execution a console window will appear showing the output of your program.

learn computer programming by closely examining others' example programs and modifying them than it is to write your own programs starting from a blank screen.

6.2 Programming in Python

Another text-based computer programming language called *Python* allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an *interpreted* language: a software application called an *interpreter* reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3`[4] and pressing the Enter key:

```
Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>>
```

The `>>>` symbols represent the prompt within the Python interpreter "shell", signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

[4] Using version 3 of Python, which is the latest at the time of this writing.

More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which "imports" these functions from Python's math *library* (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in *radians* rather than *degrees*, and how Python provides convenient functions for translating between the two. Logarithms assume a base of e unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person's first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.

Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using *phasors*[5] as shown in the following example. Here we see Python's interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor's impedance (`zc`) as $X_C \angle -90^o$ with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values ($0 - jX_c$ Ω and $0 + jX_l$ Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω $\angle -25.102^o$). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in *radians*; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python's interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

[5] A "phasor" is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.

assignment of variables as well as a convenient text record[6] of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter's shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

```
x = 200
y = -560.5

print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
```

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace *is* significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console's prompt, at which point it would display the following result:

```
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
```

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

[6] Like many command-line computing environments, Python's interpreter supports "up-arrow" recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.

If you are interested in learning more about computer programming in *any* language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.

6.3 Modeling a series-parallel circuit using C++

Here is an example C++ program intended to display a crude representation of a three-resistor series-parallel circuit and then calculate all relevant voltages, currents, and powers:

```
#include <iostream>
#include <cmath>
using namespace std;

int main (void)
{
  float v1, r1, r2, r3, r23, rtotal, ir1, ir2, ir3, vr1, vr2, vr3;
  float pr1, pr2, pr3, ptotal;

  v1 = 240.00;
  r1 = 1.8e3;
  r2 = 2.2e3;
  r3 = 3.3e3;

  cout << "+---V1---R1---+---R2---+ " << endl;
  cout << "|             |        | " << endl;
  cout << "|             +---R3---+ " << endl;
  cout << "|                      | " << endl;
  cout << "+----------------------+ " << endl;

  cout << "V1 = " << v1 << " Volts" << endl;
  cout << "R1 = " << r1 << " Ohms" << endl;
  cout << "R2 = " << r2 << " Ohms" << endl;
  cout << "R3 = " << r3 << " Ohms" << endl;

  r23 = 1 / ((1 / r2) + (1 / r3));

  rtotal = r1 + r23;

  ir1 = v1 / rtotal;

  vr1 = ir1 * r1;
  vr2 = v1 - vr1;
  vr3 = v1 - vr1;
  ir2 = vr2 / r2;
  ir3 = vr3 / r3;

  cout << "VR1 = " << vr1 << " Volts" << endl;
  cout << "VR2 = " << vr2 << " Volts" << endl;
```

```
   cout << "VR3 = " << vr3 << " Volts" << endl;

   cout << "IR1 = " << ir1 << " Amperes" << endl;
   cout << "IR2 = " << ir2 << " Amperes" << endl;
   cout << "IR3 = " << ir3 << " Amperes" << endl;

   pr1 = pow(vr1,2) / r1;
   pr2 = pow(vr2,2) / r2;
   pr3 = pow(vr3,2) / r3;
   ptotal = pr1 + pr2 + pr3;

   cout << "PR1 = " << pr1 << " Watts" << endl;
   cout << "PR2 = " << pr2 << " Watts" << endl;
   cout << "PR3 = " << pr3 << " Watts" << endl;
   cout << "PTOTAL = " << ptotal << " Watts" << endl;

   return 0;
}
```

When compiled and executed, this program generates the following output:

```
+---V1---R1---+---R2---+
|             |        |
|             +---R3---+
|                      |
+----------------------+
V1 = 240 Volts
R1 = 1800 Ohms
R2 = 2200 Ohms
R3 = 3300 Ohms
VR1 = 138.462 Volts
VR2 = 101.538 Volts
VR3 = 101.538 Volts
IR1 = 0.0769231 Amperes
IR2 = 0.0461538 Amperes
IR3 = 0.0307692 Amperes
PR1 = 10.6509 Watts
PR2 = 4.68639 Watts
PR3 = 3.12426 Watts
PTOTAL = 18.4615 Watts
```

Let's explore the design of this program in detail, during which we will elaborate on the following programming principles:

- Reading/execution order
- Preprocessor directives (`#include`)
- The `main` function
- Arguments
- Delimiter characters (e.g. `{ } ;`)
- Variable declarations
- `float` and `int` data types
- Variable names
- Variable initialization
- The `cout` instruction
- Assignment (e.g. `=`)
- Basic arithmetic (`+`, `-`, `*`, `/`)
- Mathematical functions (e.g. `pow`)
- Return

Generally speaking text-based computer programming languages are read and executed in the same order as English documents: character by character left-to-right, and line by line top-to-bottom.

The `#include` statements are called *preprocessor directives*, and simply tell the compiler software to read the contents of certain standardized files included as part of the C++ compiler installation on your computer, in this case the `iostream` `cmath` *class templates* and code libraries. Without these in place, your code would have to be much longer and much more complicated because you would have to define for the compiler what certain functions like `cout` are supposed to do.

Following the `#include` statements is a line declaring which `namespace` this program will use. Namespaces are one of those concepts distinguishing C++ from its predecessor language C, and for the scope of this discussion all you need to know is that this line instructs the compiler to use the *standard* (`std`) definitions of C++ functions such as `cout`.

Our simulation code must be encapsulated inside the `main` function. Every C and C++ program has a `main` function where execution begins. In simple programs such as this, there is only one function, and that is the `main` function. In more complicated programs there will be one `main` function as well as other functions of different names.

The "`int`" preceding `main` tells the computer to expect an integer number value produced by the function upon completion. This is not strictly necessary, but it is considered good programming

practice to have every function "return" some value such as this, which might be read by other portions of the program as a positive indication of that function's completion.

The parentheses "`()`" immediately following `main` exist to contain *arguments* to the function, i.e. data "passed" to the function for it to process. In this case there are no arguments at all to be passed to `main` and so the parentheses contain the word `void`.

Boundaries for the `main` function are defined by the placement of {} characters, called *braces* or sometimes *curly braces*. Indentation of code lines helps the reader see where each pair of braces is located in the page(s) of code, but this indentation is not strictly necessary in C or C++. It is the braces, akin to parentheses for mathematical functions, which define how much code is contained within any function.

Immediately following the opening brace { of the `main` function we find our first custom lines of code. Up front are two lines, each beginning with `float` and each ending with a semicolon (`;`). These lines *declare* variables we will use in our program, analogous to variables we might used in a mathematical formula (x, y, z). Many programming languages require you to "declare" each of the variables you intend to use before you actually use them. This gives the computer opportunity to reserve right-sized locations in its memory to store the variable data. The term `float` is short for *floating-point* which is a form of binary notation useful for expressing numerical values in scientific notation. Floating-point variables are good for expressing numbers ranging from very small to very large, and they are necessary for expressing fractional (i.e. between integer) values. Integer data types are also offered within C and C++, the most common being called `int`.

Unlike mathematics where variables are customarily denoted by single characters, in computer programming variable names are spelled out as *words*. We are allowed to make up any variable names we wish using English alphanumeric characters, with no spaces between the characters.

After the variable declaration lines, we have some lines *initializing* some of these variables with numerical values. Like the declarations, each of these lines ends with a semicolon character, which tells the compiler where the line ends (and therefore where the next line may begin). C and C++ are languages where blank space ("whitespace") is ignored, which is why we need braces and semicolons as delimiters. Some languages (e.g. Python) don't use these special characters, instead relying on indentation and end-of-line characters. Programmers new to C/C++ will often find themselves accidently omitting braces and/or semicolons as they try to write functioning programs, not being accustomed to writing text like that.

There are ways to write code which will prompt the user to enter numerical values at run-time rather than having variables' values set in code as they are here. For the writing a simpler program we have chosen to initialize our variables in-code rather than have the user do it.

You will notice multiple lines in this program starting with `cout`, containing double-less-than symbols (`<<`), and of course ending with semicolons (`;`). These instruct the computer to send text characters to *standard output*, which will be a text "console" appearing on the computer's screen when this program runs. Anything enclosed in quotation marks (`" "`) will be printed to the screen verbatim, while any other "arguments" to `cout` are interpreted as variables, the result being the *values* of those variables displayed rather than their literal names. For example, `cout << "x";` will print the letter `x` to the screen, while `cout << x;` will print the numerical value of the variable x to the screen.

In addition to literal text (enclosed in quotes) and variables, other arguments may be passed on

to the `cout` instruction. An important one is `endl` (end-line), which tells the computer to end that line of text when printing to the console's "standard output". Without these `endl` inclusions, the text output by our program would be a messy run-on statement with no breaks between lines.

Perhaps the best way to understand what how `cout` works is to compare the code with the output generated by the program as it executes. Both have been included in this example for your edification.

Any line of code containing a single "equals" character (`=`) *assigns* a value to some variable. Whatever is on the left-hand side of the `=` gets assigned the value of whatever is on the right-hand side of the `=`. This is how we instruct the computer to perform arithmetic. For example, the statement `y = 4 + x;` would tell the computer to add 4 to the value stored in x, and then assign the resulting sum to y. If we were to include the statement `x = 4 + x;` in a C or C++ program, it would increment the value stored in x by 4 and then save this new value back into x.

All the basic arithmetic functions (addition, subtraction, multiplication, division) are available to you in the *iostream* class. Addition and subtraction use the same symbols as in a mathematical formula (`+` and `-`), while multiplication uses a "star" symbol (`*`) and division uses a forward-slash symbol (`/`). Parentheses are used just as in mathematical formulae to force groups of terms.

Some mathematical functions such as "power" (`pow`) require the `cmath` class in order for the compiler to know how to interpret them. Mathematical functions in C and C++ resemble function notation in standard mathematics $f(x)$ which is clearly evident in the `pow()` function used here. For example, `pow(x,2)` means x^2, the first argument being x and the second being 2.

Finally, the last line of code within the `main` function *returns* an integer number value. This is in agreement with the `int` preceding `main` near the top of the code listing: we told the compiler that the `main` function would generate an integer-number value upon completion, and the `return` line is where this happens. As mentioned earlier, this is not strictly necessary, but it is considered good programming practice to have every function, main included, return some sort of data upon completion.

Chapter 7

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

7.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Conservation of Electric Charge

Voltage

Current

Resistance

Electrical source

Electrical load

Equipotential points

Electrically common points

Ohm's Law

Joule's Law

Properties of series circuits

Properties of parallel circuits

Kirchhoff's Voltage Law

Kirchhoff's Current Law

Series-parallel circuit

Strategy for analyzing a series-parallel resistor circuit

Strategies for checking your work

Voltage divider loading

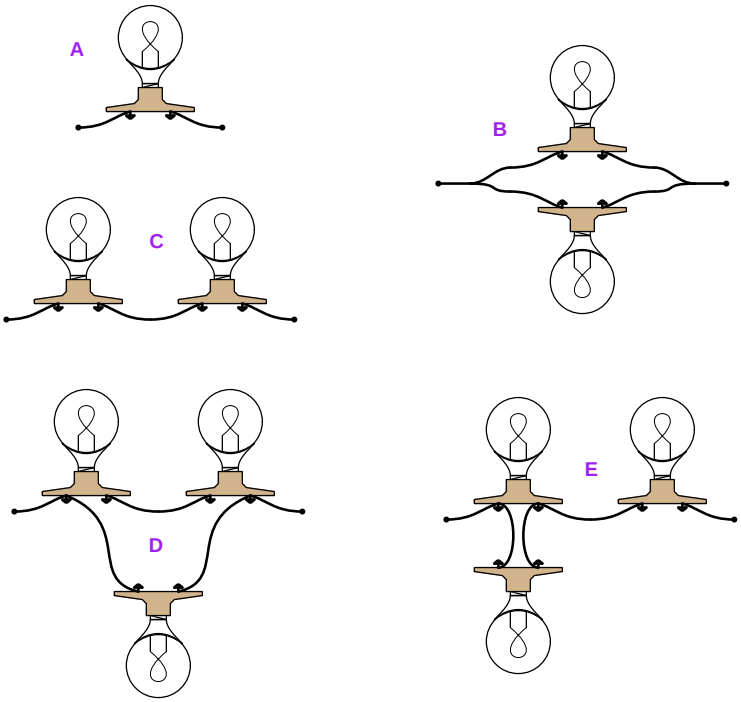
Current divider loading

Voltmeter resistance and loading effect

Ammeter resistance and loading effect

7.1.3 Electric lamp arrays

Rank these five electric lamp assemblies according to their *total* electrical resistance (in order from least to greatest), assuming each of the lamps is the same type and rating. Be sure to explain your method of solving this problem:

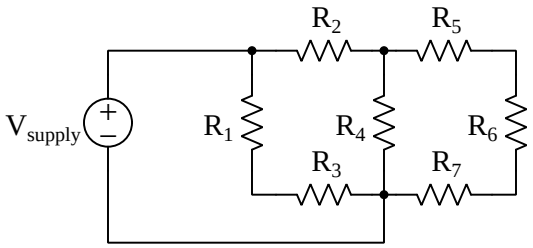


Challenges

- Identify how the two lamps in assembly C could be joined in *parallel* with just the addition of one wire.
- Which assembly(ies) will result in the brightest-glowing lamp(s), assuming all are powered by a voltage source of the same value, connected to the two wire ends.
- Identify the brightest-glowing lamp(s) within the three-lamp assemblies D and E.

7.1.4 Identifying series and parallel sub-networks

Examine the following series-parallel resistor circuit and identify components that are connected in series with each other as well as components that are connected in parallel with each other:



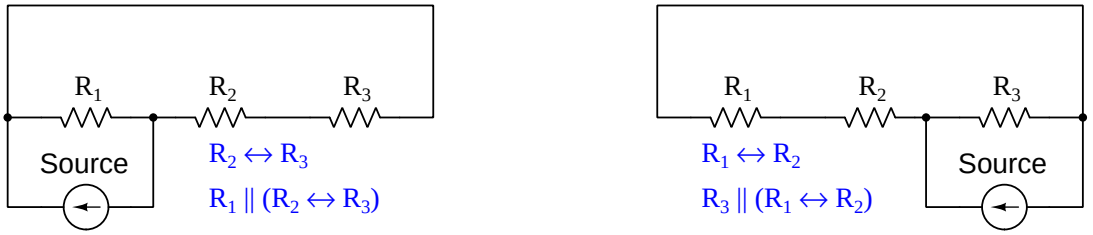
Write an expression using series ($\leftrightarrow$) and parallel ($\|$) symbols representing the total resistance of this circuit as seen from the perspective of the voltage source.

Challenges

- Re-write the total resistance expression for a condition where R_4 is failed shorted.
- Assuming all resistors are of equal value, which resistor(s) experience the greatest voltage drop?
- Assuming all resistors are of equal value, which resistor(s) experience the least voltage drop?
- Identify at least one resistor fault that would cause current to halt through R_6.

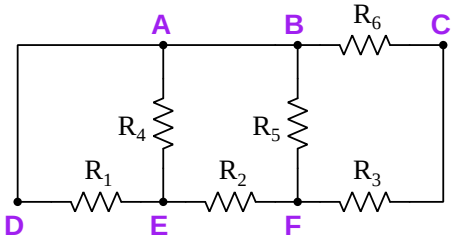
7.1.5 Series and parallel sub-networks with a movable source

The series- or parallel-connectedness of components may change depending on how a power source is connected to the network. Take this for example:



In the left-hand circuit, resistors R_2 and R_3 are connected in series with each other as one sub-network, while resistor R_1 is in parallel with that series sub-network. In the right-hand circuit, resistors R_1 and R_2 are connected in series with each other as one sub-network, while resistor R_3 is in parallel with that series sub-network.

Write an expression using series ($\leftrightarrow$) and parallel ($\|$) symbols representing the total resistance of this circuit as seen from the perspective of the voltage source, for each of the following scenarios:



- Source connected across points A and E =
- Source connected across points F and C =
- Source connected across points D and F =

Challenges

- A very useful problem-solving technique in analyzing series-parallel circuits is to *re-draw* each circuit with all resistors arranged in the same orientation (i.e. either all horizontal or all vertical). Apply this technique to any of the scenarios described in this problem.
- Explain how it is possible for a pair of resistors to be connected in series with one power supply configuration, and then for those same two resistors to no longer be in series with each other simply by virtue of having moved the *power supply* and not moving the resistors at all.

7.1.6 Extension cord

A construction crew is working in the basement of a new home, installing insulation, drywall, and other materials necessary to make the basement suitable for habitation. This basement is dark because electric power service has not yet been connected to the home. An engine-driven generator situated in the front yard supplies electricity for temporary lights and power tools used by the construction workers.

This generator has several 120 Volt AC receptacles, into which several extension cords are plugged. At the other end of each cord, multiple receptacles exist for plugging in lights and power tool cords. One of the workers happens to notice that whenever he pulls the trigger on the electric drill he's using, the portable light next to him (plugged into the same extension cord) dims slightly.

First, draw an equivalent schematic diagram of this circuit, showing the generator as a voltage source, the extension cord as two wires (each one possessing some electrical resistance), and the two loads (portable light and electric drill) connected in parallel with each other:

Next, explain why the work light dims whenever the drill is turned on.

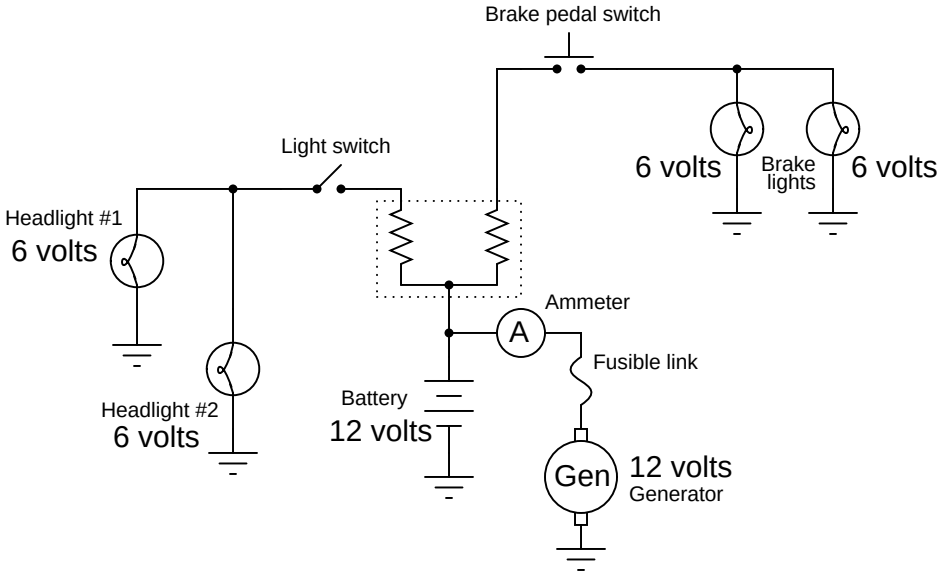
Challenges

- Assuming that the generator outputs a constant 120 Volts (AC) under all load conditions, will a work light plugged into a different extension cord also dim when the drill is turned on?
- On a real construction site, *every* load plugged into the same generator will experience some voltage "sag" whenever the drill is turned on, but some more than others. Explain why this is, and identify which load(s) will experience the greatest voltage sag when the drill energizes.

7.1.7 6-Volt to 12-Volt automotive conversion

Antique American automobiles often used 6 Volt electrical systems instead of the 12 Volt components found in more modern cars and trucks. People who restore these old vehicles may have difficulty finding old 6-Volt generators and batteries to replace the defective, original units. An easy solution is to update the vehicle's generator and battery with modern (12 Volt) components, but then another problem arises.

A 12 Volt generator and 12 Volt battery will overpower the old 6 Volt headlights, brake lights, and other electrical loads in the vehicle. A common solution used by some antique automobile restorers is to connect resistors between the 12-Volt generator system and the 6-Volt loads, like this:



Explain why this solution works, and also discuss some of the disadvantages of using resistors to adapt the new (12 volt) to the old (6 volt) components.

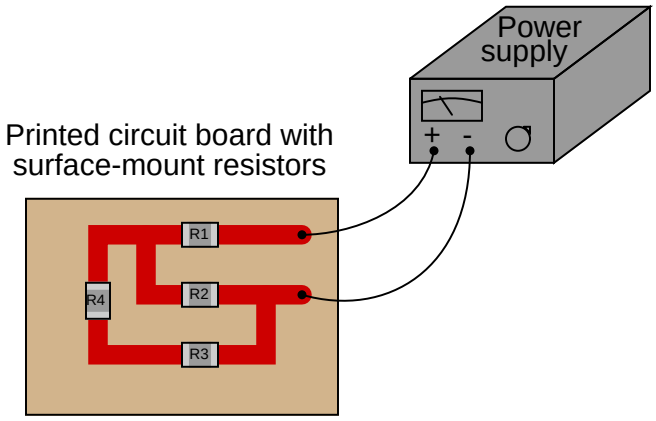
Next, devise a way to re-wire the electrical system of this old automobile so as to not require resistors between the loads and the generator/battery portion of the circuit.

Challenges

- Suppose someone mistakenly wired a resistor in series with the fusible link. What ill effect(s) might this cause?
- As it so happens, 6 Volt incandescent lamps are more rugged than 12 Volt incandescent lamps, all other factors being equal. Explain why this is so.
- Explain why using a single resistor rather than one resistor per switch would be a poor circuit design.

7.1.8 Applying foundational concepts to a series-parallel circuit

Identify which foundational concept(s) apply to each of the declarations shown below regarding the following circuit. If a declaration is true, then identify it as such and note which concept supports that declaration; if a declaration is false, then identify it as such and note which concept is violated by that declaration:



- The current through resistor R_1 will be the same as through resistor R_2
- The current through resistor R_3 will be the same as through resistor R_4
- The voltage across resistor R_3 will be the same as across resistor R_4
- The voltage flowing through resistor R_1 must be the same as through resistor R_4
- If resistor R_1 fails open, total current will decrease
- If resistor R_3 fails open, total current will decrease
- The voltage dropped by resistor R_2 will be equal to the sum of the voltages across resistors R_3 and R_4

Here is a list of foundational concepts for your reference: **Conservation of Energy**, **Conservation of Electric Charge**, **behavior of sources vs. loads**, **Ohm's Law**, **Joule's Law**, **effects of open faults**, **effect of shorted faults**, **properties of series networks**, **properties of parallel networks**, **Kirchhoff's Voltage Law**, **Kirchhoff's Current Law**. More than one of these concepts may apply to a declaration, and some concepts may not apply to any listed declaration at all. Also, feel free to include foundational concepts not listed here.

Challenges

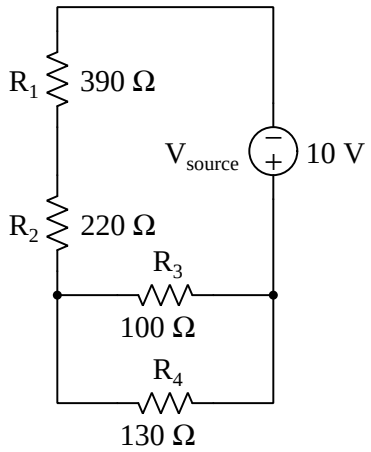
- Identify a component fault that would increase voltage across resistor R_2.

7.1.9 Explaining the meaning of calculations

An unfortunate tendency among beginning students in any quantitative discipline is to perform calculations without regard for the real-world meanings of the values, and also to follow mathematical formulae without considering the general principles embodied in each. To ignore concepts while performing calculations is a serious error for a variety of reasons, not the least of which being an increased likelihood of computing results that turn out to be nonsense.

In the spirit of honoring concepts, I present to you a quantitative problem where all the calculations have been done for you, but all variable labels, units, and other identifying data have been stripped away. Your task is to *assign proper meaning* to each of the numbers, identify the correct unit of measurement in each case, apply any appropriate metric prefixes to those values, explain the significance of each value by describing where it "fits" into the circuit being analyzed, and identify the general principle employed at each step.

Here is the schematic diagram of the circuit:



Here are all the calculations performed in order from first to last:

1. $390 + 220 = 610$
2. $(100^{-1} + 130^{-1})^{-1} = 56.52$
3. $610 + 56.52 = 666.52$
4. $\frac{10}{666.52} = 15.003 \times 10^{-3}$
5. $(56.52) \times (15.003 \times 10^{-3}) = 0.848$
6. $(220) \times (15.003 \times 10^{-3}) = 3.301$
7. $(390) \times (15.003 \times 10^{-3}) = 5.851$
8. $5.851 + 3.301 + 0.848 = 10$

9. $\frac{0.848}{100} = 8.480 \times 10^{-3}$
10. $\frac{0.848}{130} = 6.523 \times 10^{-3}$
11. $(8.480 \times 10^{-3}) + (6.523 \times 10^{-3}) = 15.003 \times 10^{-3}$
12. $\frac{5.851^2}{390} = 87.79 \times 10^{-3}$
13. $\frac{3.301^2}{220} = 49.52 \times 10^{-3}$
14. $\frac{0.848^2}{100} = 7.191 \times 10^{-3}$
15. $\frac{0.848^2}{130} = 5.532 \times 10^{-3}$
16. $10 \times (15.003 \times 10^{-3}) = 150.033 \times 10^{-3}$
17. $(87.79 \times 10^{-3}) + (49.52 \times 10^{-3}) + (7.191 \times 10^{-3}) + (5.532 \times 10^{-3}) = 150.033 \times 10^{-3}$

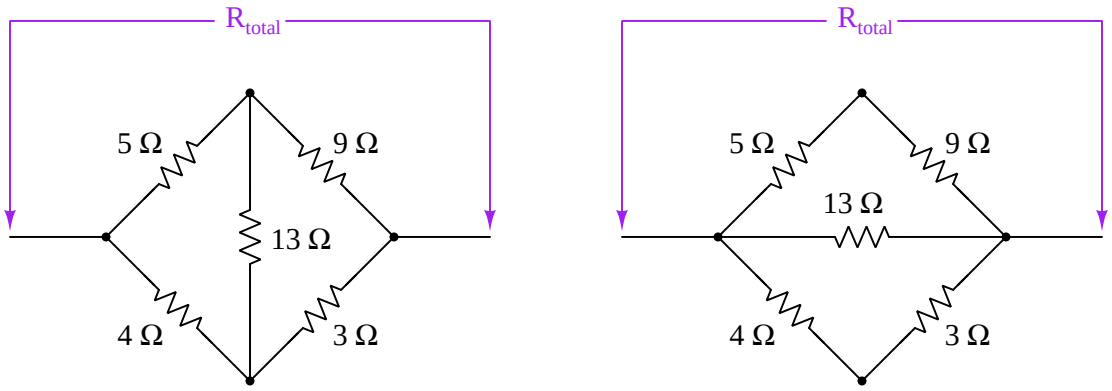
Explain what each value means in the circuit, identify its unit of measurement and appropriate metric prefix, and identify the general principle used to compute it!

Challenges

- Explain how you can check your own thinking as you solve quantitative problems, to avoid the dilemma of just "crunching numbers" to get an answer.
- For each calculated step shown, identify the physical Law or electric circuit principle being applied.
- Identify any of the calculations shown above whose order could be changed. In other words, can this problem be solved in a different order?
- Identify any of the calculations shown above that are not strictly necessary to the complete analysis of the circuit, explaining why it has any merit at all.
- Do you see any alternative paths to a solution, involving specific calculations not shown above?

7.1.10 Bridge networks

Which of these resistor network's total resistance may be calculated using the analytical tools explained in this module's full tutorial, and which network is impossible for us to analyze at this stage of our learning?



For the network you can analyze, explain the steps you would take to compute its total resistance. For the network you cannot analyze, explain why it defies simplification.

Challenges

- Devise an experiment to empirically determine the total resistance of each resistor network.

7.1.11 Decade box design

A useful laboratory tool called a *decade box* allows you to "dial up" any desired resistance value over a wide range, by setting selector switches to different positions. An example of a decade box appears in the photograph below:

Sketch a plausible schematic diagram for such a device.

Challenges

- How could a decade box be superior to a potentiometer as a piece of laboratory test equipment for providing arbitrary amounts of electrical resistance?

7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

7.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

7.2.3 Building custom resistance values

Manufacturers of resistors and other electronic components often follow the *IEC 60063* standard for common component values. These are series of values (actually significant digits of values) designed to make it relatively easy to achieve *any* total value desired by connecting together common-valued components in series and/or parallel.

Suppose you have a large assortment of fixed-value resistors following the *E6* series of the IEC 60063 standard, where all your available resistor values have one of six different significant-figure pairs: either 10, 15, 22, 33, 47, or 68. For example, your assortment contains resistors with Ohm ratings of 1, 1.5, 2.2, 3.3, 4.7, 6.8, 10, 15, 22, 33, 47, 68, 100, 150, 220, 330, 470, 680, 1k, and so on. However, what you actually need for a particular project is a 350 Ω resistance. Describe how you could connect some of the resistors together from your assortment to create a single resistance of 350 Ω.

Describe how you could use resistors from the same assortment to form 390 Ω.

Describe how you could use resistors from the same assortment to form 730 Ω.

Describe how you could use resistors from the same assortment to form 910 Ω.

Describe how you could use resistors from the same assortment to form 180 Ω.

Describe how you could use resistors from the same assortment to form 260 Ω.

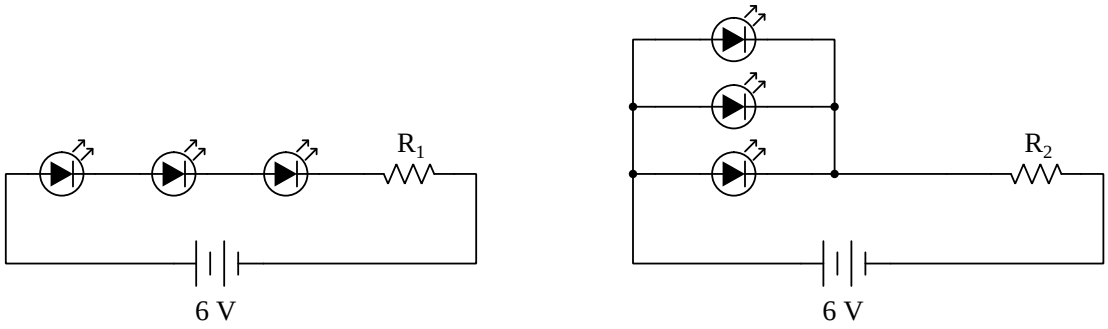
For each of these examples, try to use *as few resistors as possible* to achieve the desired resistance! Also, challenge yourself to do as much of the arithmetic *mentally* (i.e. without the aid of a calculator) as possible.

Challenges

- If each of the resistors connected to form 350 Ω is rated at $\frac{1}{4}$ Watt power dissipation, determine the maximum amount of power your proposed network will be able to dissipate without damage.
- Assuming individual resistor tolerances of ± 5%, calculate the highest and lowest possible resistance values for the 350 Ω network.

7.2.4 Two different LED circuits

Suppose we wish to power three light-emitting diodes (LEDs), each one rated at 1.6 Volts and 20 milliAmperes, with a single 6 Volt battery. We have two different ways of doing this, one way connecting the three LEDs in series and the other connecting the three LEDs in parallel with each other:



Calculate the necessary resistor value for each of these circuits to limit each LED's current to 20 mA:

$R_1 =$

$R_2 =$

Which of these two circuits will be the most energy-efficient, making the best use of the battery's energy to produce light, and wasting the least amount of energy in the form of heat?

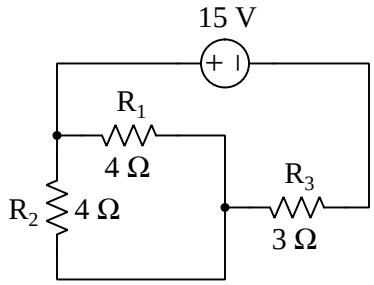
Challenges

- Calculate the power dissipated by each resistor.
- Suppose one of the LEDs fails open in each of the circuits. Predict the effect of this fault in each of the circuit configurations.
- Suppose one of the LEDs fails shorted in each of the circuits. Predict the effect of this fault in each of the circuit configurations.

7.2.5 Three-resistor circuits

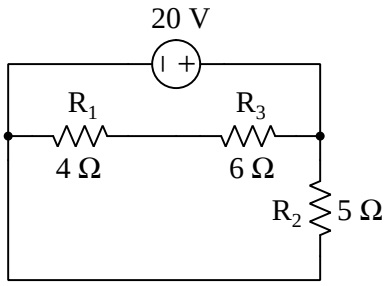
Each of these three-resistor circuits uses source and resistor values chosen to make all calculations simple, such that you should not need to use a scientific calculator.

Circuit #1



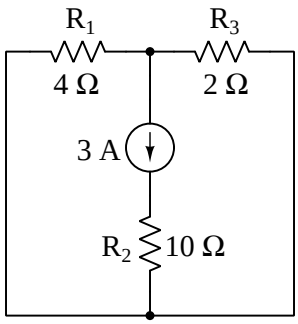
- $V_{R1} =$
- $V_{R2} =$
- $V_{R3} =$
- $I_{R1} =$
- $I_{R2} =$
- $I_{R3} =$
- $P_{R1} =$
- $P_{R2} =$
- $P_{R3} =$
- $P_{source} =$

Circuit #2



- $V_{R1} =$
- $V_{R2} =$
- $V_{R3} =$
- $I_{R1} =$
- $I_{R2} =$
- $I_{R3} =$
- $P_{R1} =$
- $P_{R2} =$
- $P_{R3} =$
- $P_{source} =$

Circuit #3



- $V_{R1} =$
- $V_{R2} =$
- $V_{R3} =$
- $I_{R1} =$
- $I_{R2} =$
- $I_{R3} =$
- $P_{R1} =$
- $P_{R2} =$
- $P_{R3} =$
- $P_{source} =$

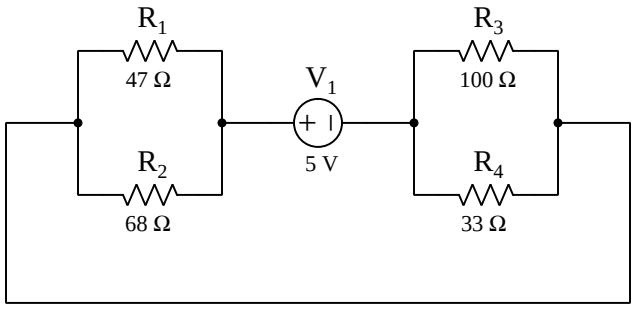
Challenges

- Demonstrate where KVL applies in each of these circuits.
- Demonstrate where KCL applies in each of these circuits.

7.2.6 Four-resistor circuits

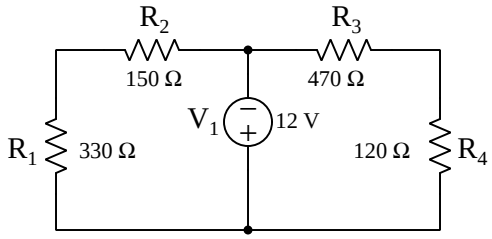
Calculate all voltages and currents in this circuit, given only the resistor values and the source value:

Circuit #1



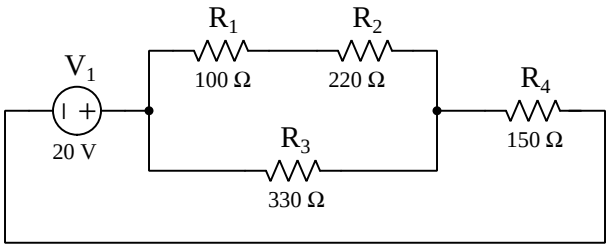
	R_1	R_2	R_3	R_4	**Total**
V					
I					
R					

Circuit #2



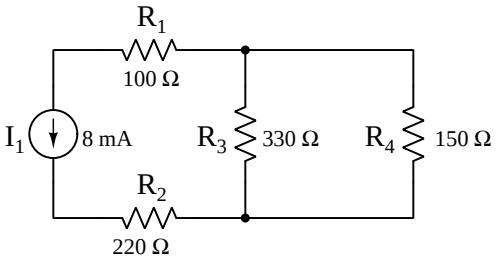
	R_1	R_2	R_3	R_4	**Total**
V					
I					
R					

Circuit #3



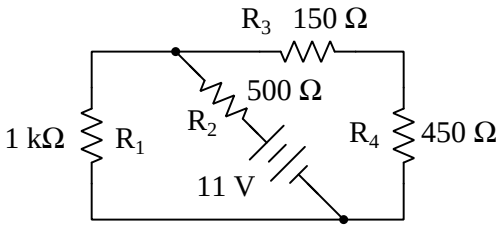
	R_1	R_2	R_3	R_4	**Total**
V					
I					
R					

Circuit #4



	R_1	R_2	R_3	R_4	**Total**
V					
I					
R					

Circuit #5



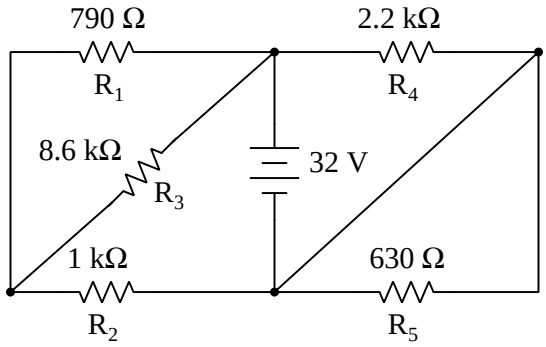
	R_1	R_2	R_3	R_4	**Total**
V					
I					
R					

Challenges

- Apply Kirchhoff's Voltage Law to the R1-R2-Source loop of the circuit, proving the sum of those voltages equals zero.
- Apply Kirchhoff's Voltage Law to the R3-R4-Source-R2 loop of the circuit, proving the sum of those voltages equals zero.
- Apply Kirchhoff's Current Law to the node below the source, proving the sum of currents entering and exiting the node equals zero.

7.2.7 Five-resistor circuit

Calculate all voltages and currents in this circuit, given only the resistor values and the source value:



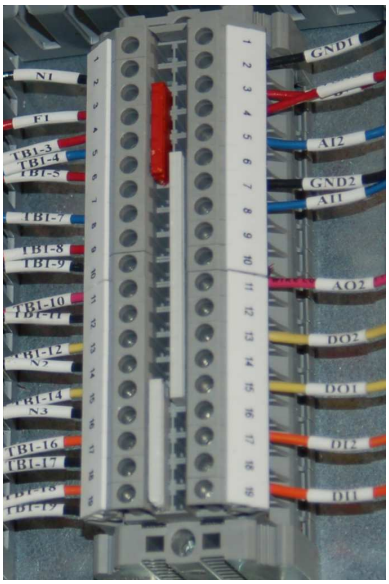
	R1	R2	R3	R4	R5	Source
V						
I						

Challenges

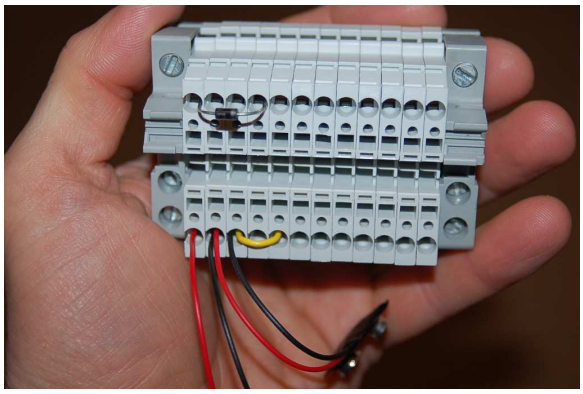
- Apply Kirchhoff's Voltage Law to the R1-R2-Source loop of the circuit, proving the sum of those voltages equals zero.
- Apply Kirchhoff's Voltage Law to the R2-R3-Source loop of the circuit, proving the sum of those voltages equals zero.
- Apply Kirchhoff's Current Law to the node lower-left, proving the sum of currents entering and exiting the node equals zero.
- Explain what would happen if the diagonal wire were severed.

7.2.8 Terminal-block resistor circuit

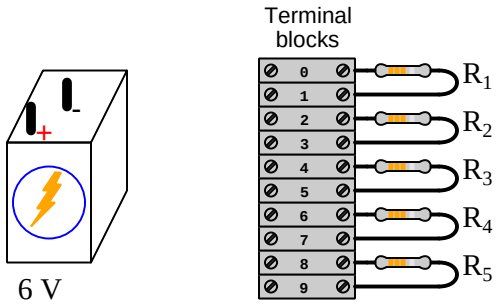
Terminal blocks are mechanical assemblies using screws or spring-clips to secure connections to metal wires and component leads, commonly used in industrial circuit to organize wire connections and facilitate quick alterations of circuit connections. The following photograph shows a typical terminal block installation in an industrial wiring panel:



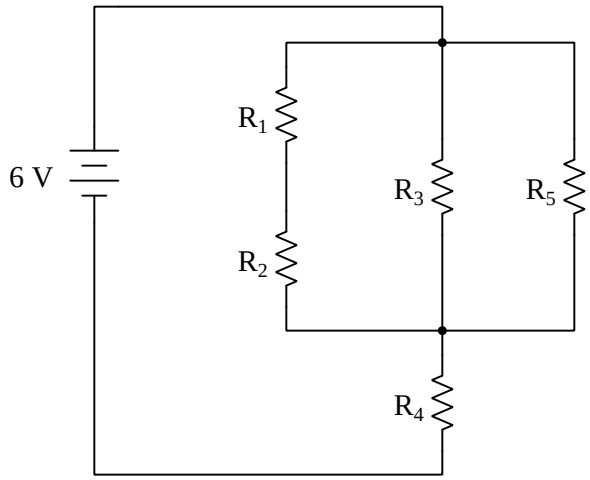
Note how each of the terminal blocks is uniquely numbered, and how many of the wires entering the left-hand side of each block bears a label identifying which terminal it should be connected to (e.g. "TB1-4" means terminal number 4 of terminal block assembly TB1). Another photograph shows a different style of terminal block, using spring clips rather than screws to clamp wire-ends to the metal bar inside of each terminal block:



In this next image, we see five resistors attached to ten terminal blocks, the left-hand side of each terminal block ready to receive "jumper" wires to connect these resistors to each other to form a network:



Write a list of necessary "jumper" wire connections to make between these numbered terminal blocks in order to create the following series-parallel circuit, then calculate all resistor voltage drops assuming each resistor is 1000 Ω in value:



Next, apply Kirchhoff's Voltage Law to the voltages within the loop represented by the terminal sequence 0-2-5-4-0. In other words, tally the voltage gains and losses as an imaginary "test charge" is moved from terminal 0 to terminal 2 to terminal 5 to terminal 4 to terminal 0 in this resistor network.

Lastly, determine V_{7-1} following the convention that the voltmeter's red test lead contacts the first subscripted test point and the black test lead contacts the second.

Challenges

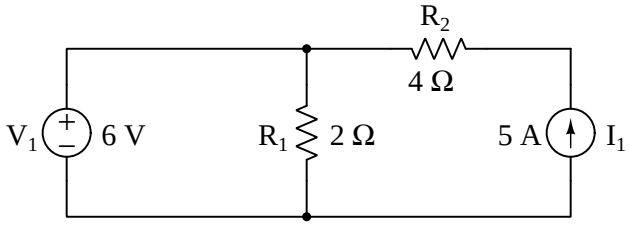
- A useful problem-solving technique when designing circuits to be constructed on a terminal block is to label the components in a schematic diagram with terminal numbers. Do that for this problem, and show how this step simplifies the process of identifying where to place connecting "jumper" wires.
- Identify a single resistor fault that will make $V_{R1} = 0$ Volts.
- Identify a single resistor fault that will make $V_{R3} = 6$ Volts.

7.2.9 Mixed-source circuits

The following circuits contain a combination of constant-voltage and constant-current components[11]. Constant-voltage components, of course, maintain the same voltage for all current conditions while constant-current components maintain the same current for all voltage conditions.

None of these circuits is "collapsible" into simpler versions of themselves with a single equivalent source and a single equivalent resistance as is possible with many series-parallel source/resistor networks. Instead, you must judiciously apply Ohm's Law, Kirchhoff's Laws, and other foundational concepts to determine all component voltages and component currents. *Hint: make a list of all the foundational concepts you know regarding circuit analysis, and use it as a check-list at each step of the problem to see what you can do next!* Be sure to specify the polarity (+, −) of each voltage and the direction (conventional flow notation) of each current, as well as explain how these fundamental circuit principles apply to each step of your analysis. *Note any opportunity to double-check your work*:

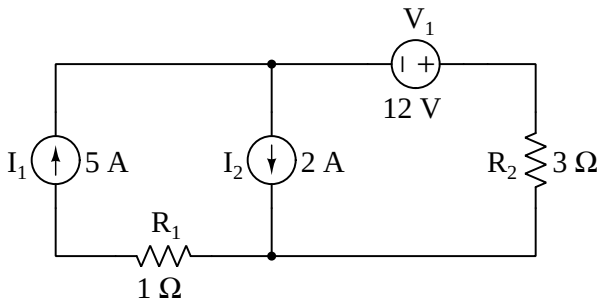
Circuit #1



- $I_{V1} =$
- $V_{R1} =$
- $I_{R1} =$
- $V_{R2} =$
- $I_{R2} =$
- $V_{I1} =$

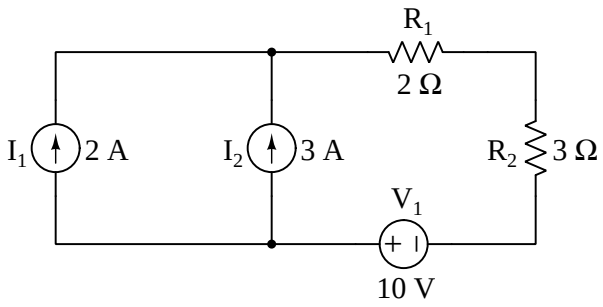
[11] While these constant-parameter components are commonly referred to as *sources*, it should be realized that they may in fact behave as loads if overpowered by another source.

Circuit #2



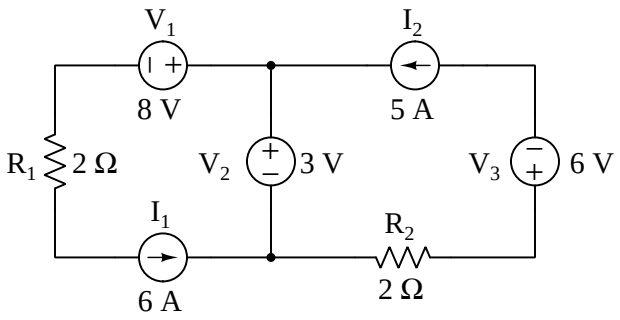
- $I_{V1} =$
- $V_{I1} =$
- $V_{I2} =$
- $V_{R1} =$
- $I_{R1} =$
- $V_{R2} =$
- $I_{R2} =$

Circuit #3



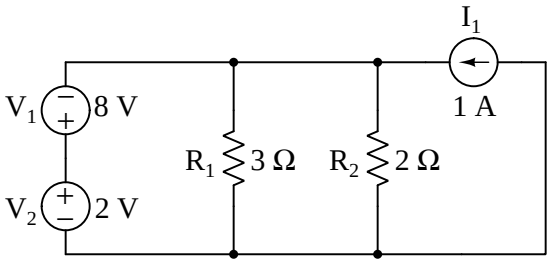
- $I_{V1} =$
- $V_{I1} =$
- $V_{I2} =$
- $V_{R1} =$
- $I_{R1} =$
- $V_{R2} =$
- $I_{R2} =$

Circuit #4



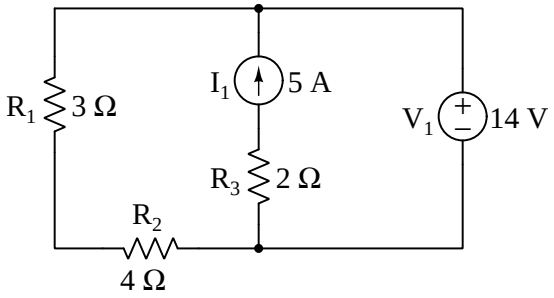
- $I_{V1} =$
- $I_{V2} =$
- $I_{V3} =$
- $V_{I1} =$
- $V_{I2} =$
- $V_{R1} =$
- $V_{R2} =$
- $I_{R1} =$
- $I_{R2} =$

Circuit #5



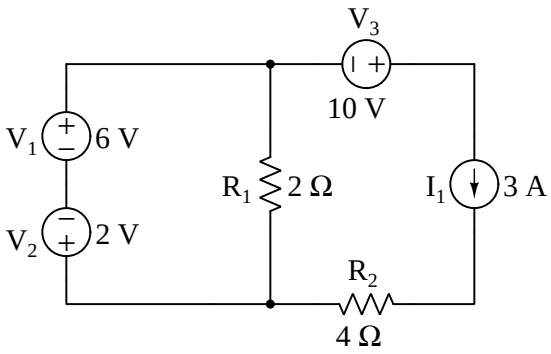
- $I_{V1} =$
- $I_{V2} =$
- $V_{I1} =$
- $V_{R1} =$
- $V_{R2} =$
- $I_{R1} =$
- $I_{R2} =$

Circuit #6



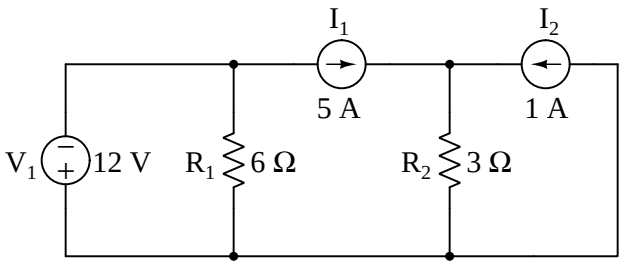
- $I_{V1} =$
- $V_{I1} =$
- $V_{R1} =$
- $V_{R2} =$
- $V_{R3} =$
- $I_{R1} =$
- $I_{R2} =$
- $I_{R3} =$

Circuit #7



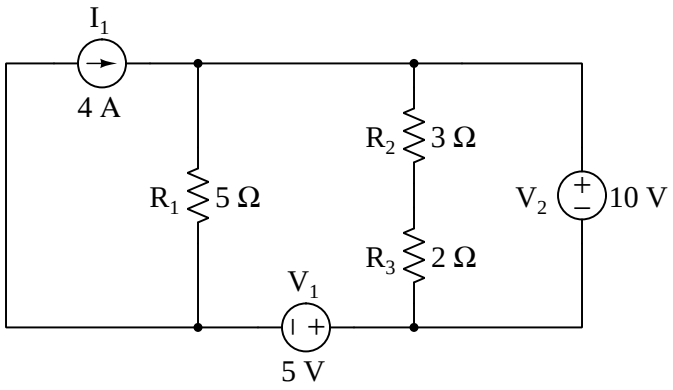
- $I_{V1} =$
- $I_{V2} =$
- $I_{V3} =$
- $V_{I1} =$
- $V_{R1} =$
- $V_{R2} =$
- $I_{R1} =$
- $I_{R2} =$

Circuit #8



- $I_{V1} =$
- $V_{I1} =$
- $V_{I2} =$
- $V_{R1} =$
- $V_{R2} =$
- $I_{R1} =$
- $I_{R2} =$

Circuit #9



- $I_{V1} =$
- $I_{V2} =$
- $V_{I1} =$
- $V_{R1} =$
- $V_{R2} =$
- $V_{R3} =$
- $I_{R1} =$
- $I_{R2} =$
- $I_{R3} =$

Challenges

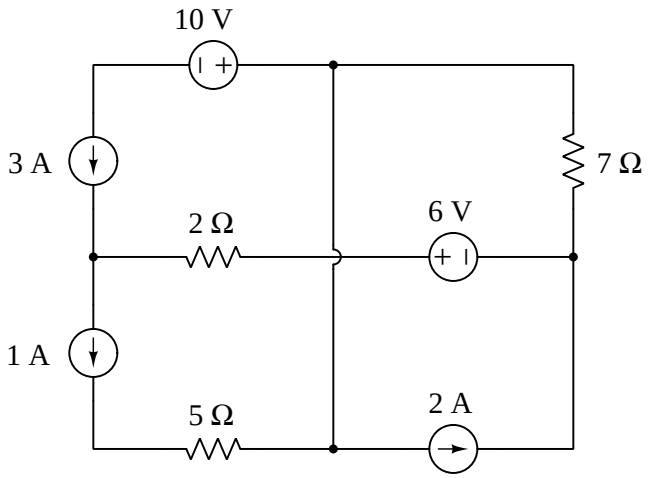
- What would happen if two voltage sources of differing voltage ratings were connected directly in parallel with each other, with matching polarities?
- What would happen if two current sources of differing current ratings were connected directly in series with each other, with matching current directions?

7.2.10 More mixed-source circuits

The following circuits contain a combination of constant-voltage and constant-current components[12]. Constant-voltage components, of course, maintain the same voltage for all current conditions while constant-current components maintain the same current for all voltage conditions.

Based on this knowledge, as well as Ohm's Law, Kirchhoff's Laws, and other foundational concepts of electric circuits, determine all component voltages and component currents in the following circuits, being sure to specify the polarity (+, −) of each voltage and the direction (conventional flow notation) of each current. Use this information to determine whether each component functions as a *source* or as a *load*. Also, explain how you used your knowledge of fundamental circuit principles to to determine each of these parameters:

Circuit #1

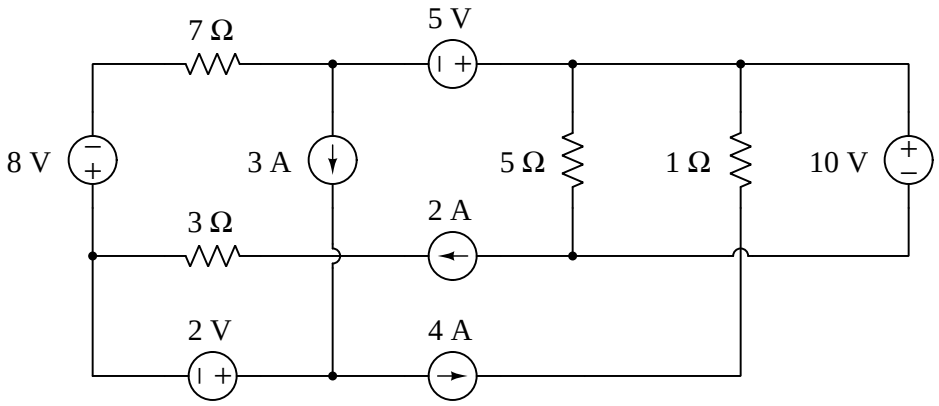


- Current through 10 V component =
- Current through 6 V component =
- Voltage across 3 A component =
- Voltage across 2 A component =
- Voltage across 1 A component =
- Current through 7 Ω resistor =
- Current through 5 Ω resistor =
- Current through 2 Ω resistor =
- Voltage across 7 Ω resistor =

[12] While these constant-parameter components are commonly referred to as *sources*, it should be realized that they may in fact behave as loads if overpowered by another source.

- Voltage across 5 Ω resistor =
- Voltage across 2 Ω resistor =

Circuit #2



- Current through 10 V component =
- Current through 8 V component =
- Current through 5 V component =
- Current through 2 V component =
- Voltage across 4 A component =
- Voltage across 3 A component =
- Voltage across 2 A component =
- Current through 7 Ω resistor =
- Current through 5 Ω resistor =
- Current through 3 Ω resistor =
- Current through 1 Ω resistor =
- Voltage across 7 Ω resistor =
- Voltage across 5 Ω resistor =
- Voltage across 3 Ω resistor =
- Voltage across 1 Ω resistor =

Challenges

- What does it mean for a constant-voltage or constant-current component to behave as a source?

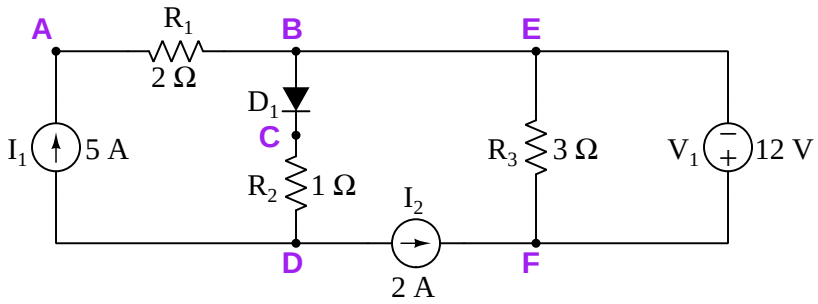
- What does it mean for a constant-voltage or constant-current component to behave as a load?
- Modify any single component value and determine the effect(s) of that change on all voltages, currents, and source/load determinations.

7.2.11 Mixed-source circuits with diodes

The following circuits contain a combination of constant-voltage and constant-current components[13] as well as ideal *diodes* which function as one-way valves for electric current. Constant-voltage components, of course, maintain the same voltage for all current conditions while constant-current components maintain the same current for all voltage conditions. An ideal diode allows free passage of electric current in the direction indicated by its arrow symbol (acting as a short) but blocks any and all current from going the other direction (acting as an open).

Based on this knowledge, as well as Ohm's Law, Kirchhoff's Laws, and other foundational concepts of electric circuits, determine all requested voltages in the following circuits, being sure to specify the mathematical sign of each requested voltage based on the convention of the first subscript letter being the voltmeter's red test lead point and the second subscript letter being the black. If only a single subscript letter appears, the assumed reference point is ground (wherever that is shown in the schematic):

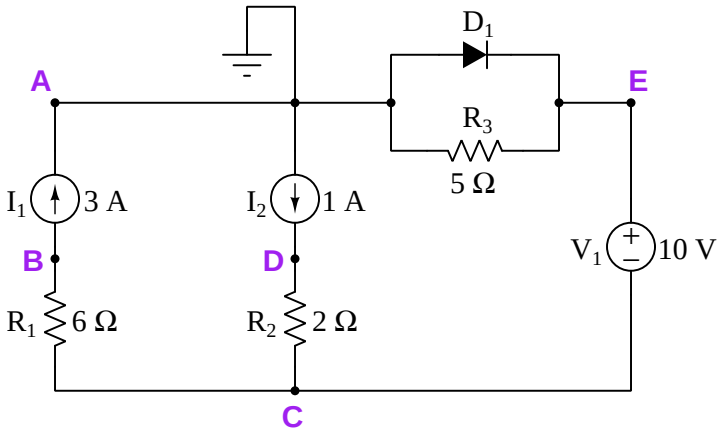
Circuit #1



- V_{AD} =
- V_{DE} =
- V_{CD} =
- V_{BF} =

[13] While these constant-parameter components are commonly referred to as *sources*, it should be realized that they may in fact behave as loads if overpowered by another source.

Circuit #2

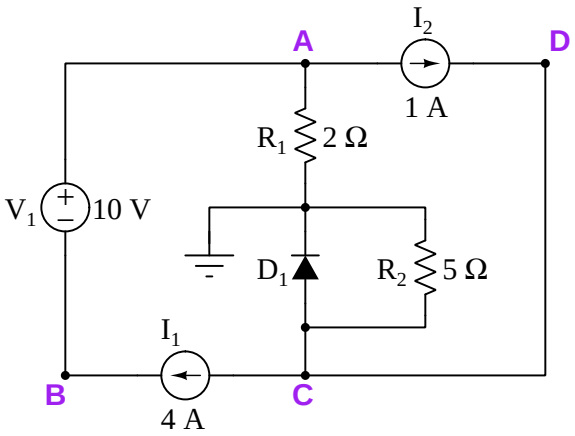


- V_{AC} =
- V_{BD} =
- V_B =
- V_C =
- V_E =

Now, re-calculate all these voltages assuming I_2's current increases from 1 Ampere to 8 Amperes:

- V_{AC} =
- V_{BD} =
- V_B =
- V_C =
- V_E =

Circuit #3



- V_{DB} =
- V_{BA} =
- V_A =
- V_B =
- V_C =

Challenges

- Identify each constant-voltage component's and each constant-current component's functional identity as either a source or a load.
- Modify any single component value and determine the effect(s) of that change on all voltages, currents, and source/load determinations.

7.2.12 Mixed-source circuits with capacitors and inductors

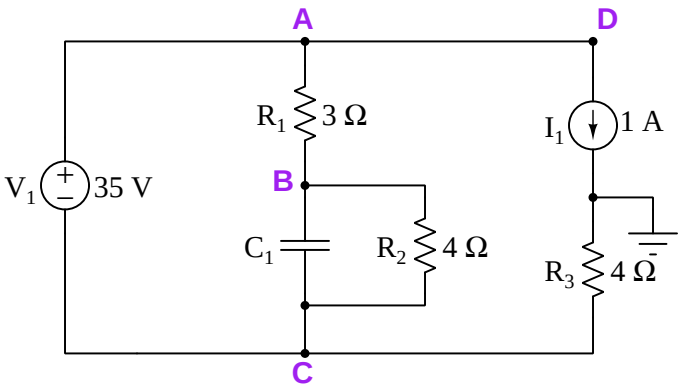
The following circuits contain a combination of constant-voltage and constant-current components[14] as well as ideal *capacitors* and ideal *inductors* which function as energy-storage devices.

Constant-voltage components, of course, maintain the same voltage for all current conditions while constant-current components maintain the same current for all voltage conditions. Capacitors store energy within electric fields and may absorb energy (i.e. act as a load) or release energy (i.e. act as a source) depending on whether their voltage is increasing or decreasing, respectively. Inductors store energy within magnetic fields and also may act as either sources or loads depending on whether their current is increasing or decreasing, respectively. Circuits containing such energy-storing devices commonly exhibit voltages and currents that *change* over time.

Based on this knowledge, as well as Ohm's Law, Kirchhoff's Laws, and other foundational concepts of electric circuits, determine all requested voltages in the following circuits, being sure to specify the mathematical sign of each requested voltage based on the convention of the first subscript letter being the voltmeter's red test lead point and the second subscript letter being the black. If only a single subscript letter appears, the assumed reference point is ground (wherever that is shown in the schematic):

[14] While these constant-parameter components are commonly referred to as *sources*, it should be realized that they may in fact behave as loads if overpowered by another source.

Circuit #1 (at a moment in time when $V_{BC} = +8$ Volts)



- $V_A =$
- $V_B =$
- $V_C =$
- $V_{CD} =$

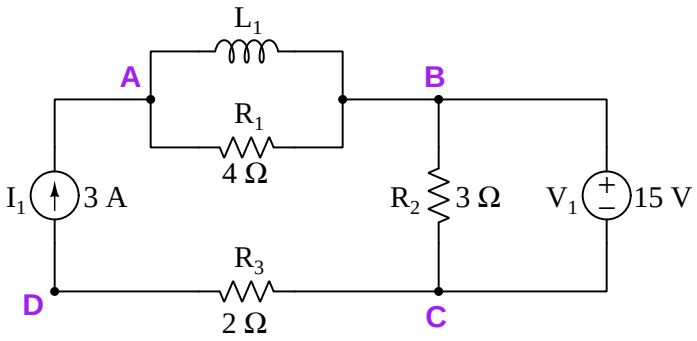
Is capacitor C_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, the capacitor will reach a state of equilibrium with the rest of the circuit, its current decreasing to zero so that it acts the same as an *open*. Re-evaluate the voltages once more for this condition:

- $V_A =$
- $V_B =$
- $V_C =$
- $V_{CD} =$

How much voltage will be across C_1's terminals in this equilibrium condition?

Circuit #2 (at a moment in time when $I_{L1} = 2$ Amperes right-to-left)



- $V_{BD} =$
- $V_{AC} =$
- $V_{CB} =$
- $V_{DC} =$

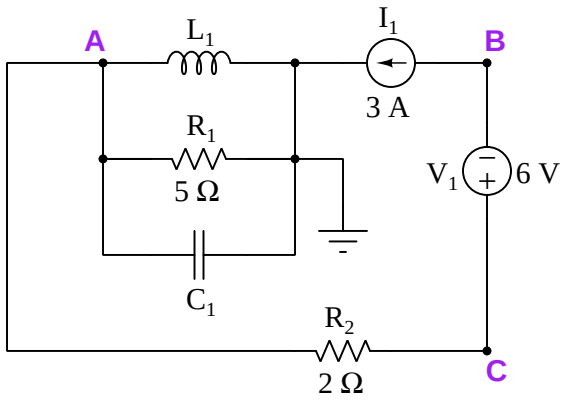
Is inductor L_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, the inductor will reach a state of equilibrium with the rest of the circuit, its voltage decreasing to zero so that it acts the same as a *short*. Re-evaluate the voltages once more for this condition:

- $V_{BD} =$
- $V_{AC} =$
- $V_{CB} =$
- $V_{DC} =$

How much current will pass through L_1 in this equilibrium condition?

Circuit #3 (at a moment in time when $I_{L1} = 4$ Amperes right-to-left and $V_A = +10$ Volts)



- $V_A =$
- $V_B =$
- $V_C =$
- $V_{BA} =$
- $V_{AC} =$

Is capacitor C_1 functioning as a source or as a load at this moment in time?

Is inductor L_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, both the capacitor and the inductor will reach states of equilibrium with the rest of the circuit, the capacitor's current decreasing to zero so that it acts like an *open*, and the inductor's voltage decreasing to zero so that it acts like a *short*. Re-evaluate the voltages once more for this condition:

- $V_A =$
- $V_B =$
- $V_C =$
- $V_{BA} =$
- $V_{AC} =$

How much voltage will there be across C_1 in this equilibrium condition?

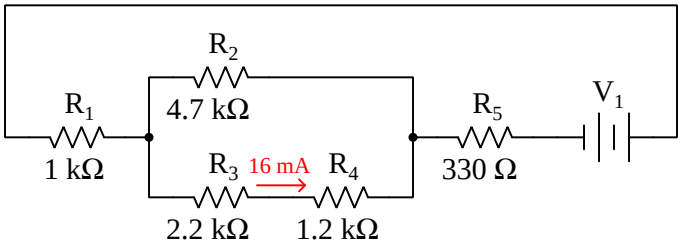
How much current will pass through L_1 in this equilibrium condition?

Challenges

- Identify each constant-voltage component's and each constant-current component's functional identity as either a source or a load.
- Modify any single component value and determine the effect(s) of that change on all voltages, currents, and source/load determinations.

7.2.13 Circuit with unknown source value

Calculate all voltages and currents in this circuit, given only the resistor values and one of the current values:



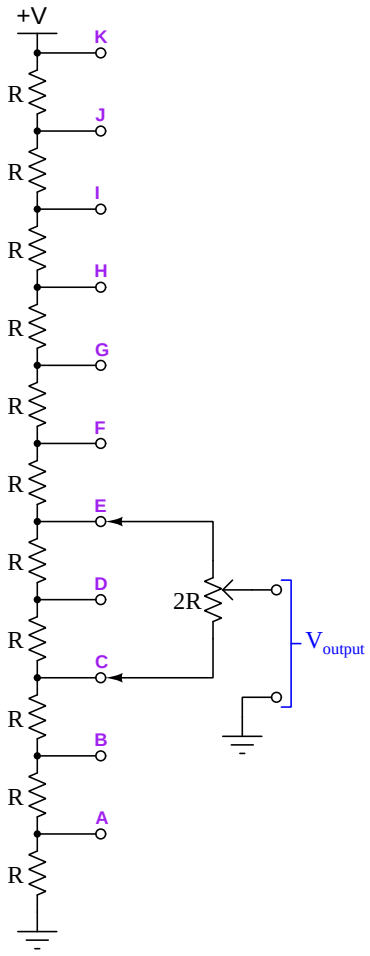
	R1	R2	R3	R4	R5	Source
V						
I						

Challenges

- Identify the effect on I_{R2} if resistor R_3 happens to fail open.
- Identify the effect on V_{R1} if resistor R_4 happens to fail shorted.
- Identify the effect on V_{R1} if resistor R_5 happens to fail open.
- Identify the effect on V_{R4} if resistor R_5 happens to fail shorted.
- Demonstrate how to apply Kirchhoff's Voltage Law in this circuit, once you have obtained values for all voltages and currents.
- Demonstrate how to apply Kirchhoff's Current Law in this circuit, once you have obtained values for all voltages and currents.

7.2.14 Kelvin-Varley voltage divider

The following network is called a *Kelvin-Varley voltage divider*, this one consisting of eleven equally-sized precision resistors along with a potentiometer having a total resistance twice that of any of those fixed resistors, connected across any *two* of the fixed resistors:



Such a network is designed to achieve fine voltage adjustment from the potentiometer by constraining the potentiometer's adjustment range to just one-tenth of the total applied voltage. This is useful in electrical *metrology*, the science and practice of precise electrical measurements.

Calculate the amount of voltage between the "Output" and ground terminals of the divider given an R size of 10 kΩ (each), a potentiometer resistance ($2R$) of 20 kΩ, a wiper position that is exactly centered (50%), the potentiometer connected as shown (to terminals E and C), and a supply voltage of 50 Volts.

Determine where to connect the potentiometer, and where to set its wiper, to output 6.3 Volts

given a power supply voltage of 10 Volts.

Challenges

- Why not just use a plain and simple potentiometer, without the eleven-resistor network? What purpose does a voltage divider this complex serve?

7.2.15 Incorrect voltmeter readings

Suppose you constructed a simple two-resistor voltage divider using identical 1 MΩ resistors, powered by a source voltage of exactly 15 Volts. Measuring voltage across each resistor using a digital multimeter (DMM), one at a time, you obtain equal measurements of 7.200 Volts.

This, of course, is less than the perfect 7.500 Volts one would expect across each resistor of such a simple voltage divider. Explain why the voltmeter gives less-than-perfect measurements of resistor voltage, and calculate the voltmeter's internal resistance based on this measurement.

Challenges

- How can we tell that the 7.200 Volt reading was not due to imperfections in the resistors?

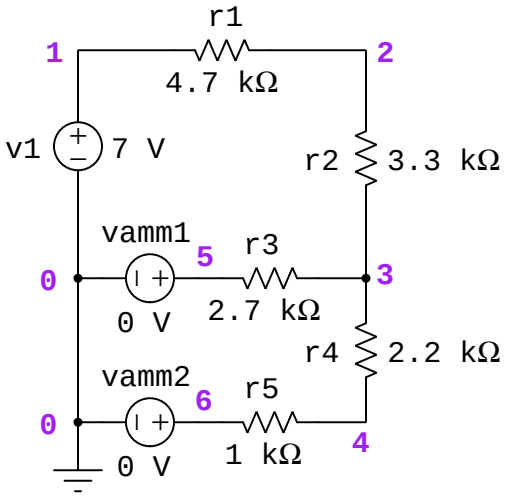
7.2.16 Interpreting a SPICE analysis

A computer program called SPICE was developed in the early 1970's, whereby a text-based description of an electric circuit (called a *netlist*) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

A *netlist* specifying the circuit to SPICE is shown here, followed by a schematic diagram (for illustrative purposes only, not for the computer running SPICE) showing all the numbered "nodes" which SPICE uses to define connection points between components:

```
* Five-resistor series-parallel DC circuit
v1 1 0 dc 7
r1 1 2 4700
r2 2 3 3300
r3 3 5 2700
r4 3 4 2200
r5 4 6 1000
vamm1 5 0
vamm2 6 0
.dc v1 7 7 1
.print dc v(1,2) v(2,3) v(3,4)
.print dc v(3,5) v(4,6) i(v1)
.print dc i(vamm1) i(vamm2)
.end
```

Note: this schematic diagram is shown only for your benefit. The computer running SPICE software needs only a text "netlist" to describe the circuit, not a graphical image!



Note the inclusion of zero-voltage sources `vamm1` and `vamm2`, used in this netlist as ammeters for the purpose of measuring current through R_3 and $R_4 \leftrightarrow R_5$, respectively. Legacy versions of SPICE

could only analyze currents through voltage sources, and so if one needed an ammeter placed in a SPICE circuit, the solution was to insert a zero-voltage source to prompt SPICE to report the amount of current through it.

Identify how each component's connecting nodes and essential parameters (i.e. values) are specified in the netlist. Based on the what you can identify in this netlist, perform your own calculations using Ohm's Law to predict all other values in this circuit.

When SPICE (software version 2G6) processes the contents of this "netlist" file, it outputs a text description of the analysis. The following text has been edited for clarity (e.g. blank lines, extraneous characters, and statistical data removed):

```
 v1           v(1,2)      v(2,3)      v(3,4)
7.000E+00     3.476E+00   2.441E+00   7.446E-01

 v1           v(3,5)      v(4,6)      i(v1)
7.000E+00     1.083E+00   3.385E-01  -7.396E-04

 v1           i(vamm1)    i(vamm2)
7.000E+00     4.011E-04   3.385E-04
```

The same netlist processed by a more modern version of SPICE (NGSPICE version 26) produces the following analysis:

```
--------------------------------------------------------------------
Index   v-sweep         v(1)-v(2)       v(2)-v(3)       v(3)-v(4)
--------------------------------------------------------------------
0       7.000000e+00    3.476182e+00    2.440723e+00    7.446275e-01

--------------------------------------------------------------------
Index   v-sweep         v(3)-v(5)       v(4)-v(6)       v1#branch
--------------------------------------------------------------------
0       7.000000e+00    1.083095e+00    3.384670e-01    -7.39613e-04

--------------------------------------------------------------------
Index   v-sweep         vamm1#branch    vamm2#branch
--------------------------------------------------------------------
0       7.000000e+00    4.011461e-04    3.384670e-04
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer's analysis agree with your own calculations.

Once you become familiar with the analysis data format of SPICE, you will be able to use the "Gallery" of SPICE simulations found in the *"SPICE Modeling of Resistor Circuits"* module as practice problems for developing your own circuit analysis skills. If you become proficient with creating your own netlists and running SPICE simulations, your freedom to *create* practice problems complete with answers will be unlimited!

Challenges

- SPICE utilizes *power-of-ten* notation for small and large values. The standard representation of this in a plain-text format is a capital letter `E` followed by the power value. Identify any such figures in the SPICE output listing, and express them in regular decimal notation.
- Modify this SPICE netlist to utilize a voltage source with a value of 16 Volts instead of 7 Volts.
- Modify this SPICE netlist to specify resistor R_1's resistance using scientific (power-of-ten) notation.
- Modify this SPICE netlist to specify resistor R_1's resistance using metric prefix ("kilo") notation.
- Explain why `i(v1)` registers as a negative value while both `i(vamm1)` and `i(vamm2)` register as positive values.
- Apply Kirchhoff's Current Law to current values in this circuit.

7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

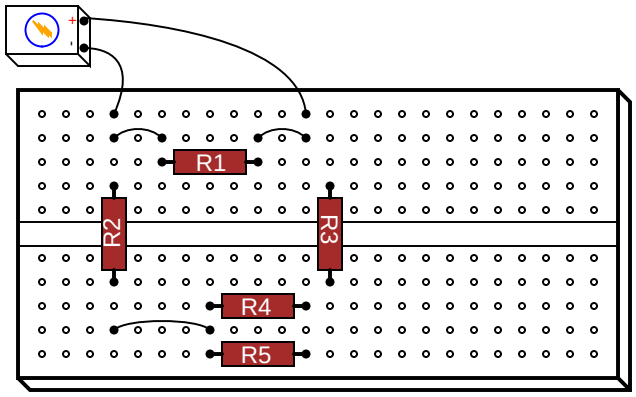
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

7.3.1 Fault in a solderless breadboard circuit

A student built this resistor circuit on a solderless breadboard, but made a mistake positioning resistor R_3. It should be located one hole to the left instead of where it is right now:



Determine the amount of voltage dropped across each resistor in this faulty circuit configuration, assuming the battery outputs 9 Volts:

- $R_1 = 2$ kΩ $\quad V_{R1} =$
- $R_2 = 1$ kΩ $\quad V_{R2} =$
- $R_3 = 3.3$ kΩ $\quad V_{R3} =$
- $R_4 = 4.7$ kΩ $\quad V_{R4} =$
- $R_5 = 4.7$ kΩ $\quad V_{R5} =$

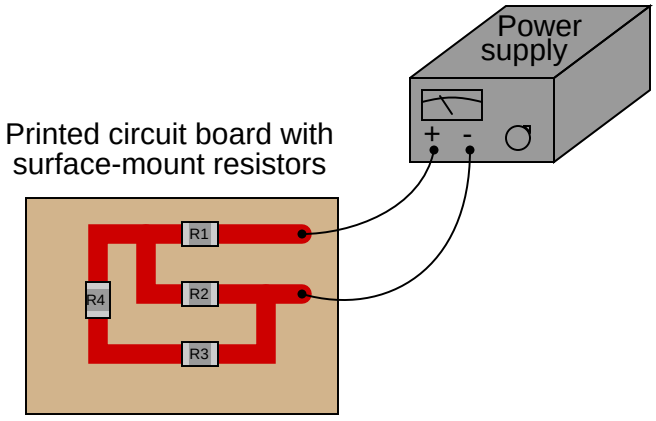
Suppose the student realizes there is a mistake, and in an attempt to correct the mistake decides to move the battery's positive (+) wire one hole to the right on the breadboard in order to align with resistor R_3. If this is the only change made to the circuit, what will be the result?

Challenges

- Identify how the student could have determined a problem existed in the circuit prior to connecting a battery to power it.

7.3.2 Faults in a printed circuit

Predict the effects of the following faults in this circuit, one fault at a time, assuming the power supply is a fixed-voltage unit:



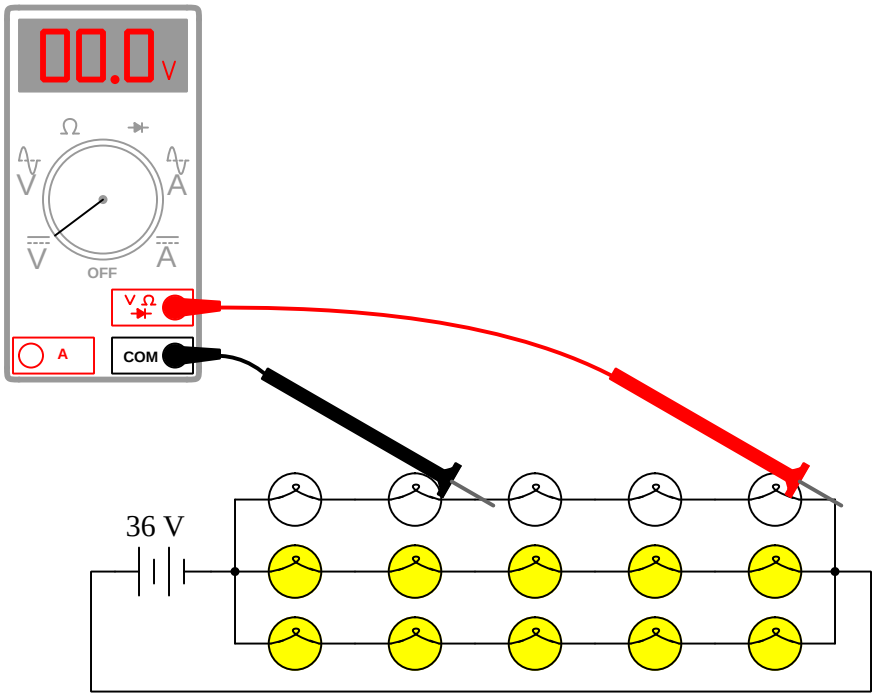
- If R_1 fails open . . .
- If R_2 fails open . . .
- If R_3 fails open . . .
- If R_4 fails open . . .
- If R_1 fails shorted . . .
- If R_2 fails shorted . . .
- If R_3 fails shorted . . .
- If R_4 fails shorted . . .

Challenges

- Components soldered to circuit boards cannot be easily removed, as de-soldering is required. This makes it difficult to test for current through any component, as there is no convenient way to break the circuit so as to insert an ammeter. Describe how it may be possible to infer current through certain circuit board components using only a voltmeter.

7.3.3 Faulty electric lamp array

Something has failed in this array of electric lamps, because only the middle and bottom strings are illuminating. None of the lamps in the upper string emit any light. A technician takes a voltage measurement (as shown):



Identify at least *two* individual faults, either one independently capable of accounting for the symptoms of this circuit.

Challenges

- Suppose the voltmeter registered 36 Volts instead of 0 Volts in this failed circuit. Identify at least two independent faults which could explain all symptoms.
- Suppose nothing was failed in this circuit. How much voltage *should* the voltmeter register when connected as shown?
- Suppose nothing was failed in this circuit. What would happen if the DMM were connected to the circuit in the same place, but configured to be an *ammeter* rather than a voltmeter?

Chapter 8

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

8.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

8.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures[1] will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. "Live" work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact*[2] *with an energized conductor, no matter what the circuit's voltage*[3] *level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

[1] Professor Charles Dalziel published a research paper in 1961 called "The Deleterious Effects of Electric Shock" detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel's subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn't require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

[2] By "electrical contact" I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

[3] Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it's generally wise (and *always* a good habit to build) to "power down" *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. "cord grips" used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use "touch-safe" terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always.* This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always.* A fuse does no good if the wire or printed circuit board trace will "blow" before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always.* Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one's body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a "hot" line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert.* If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

8.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary[4]. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards[5], and wires that are simply twisted together.

- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work**. Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

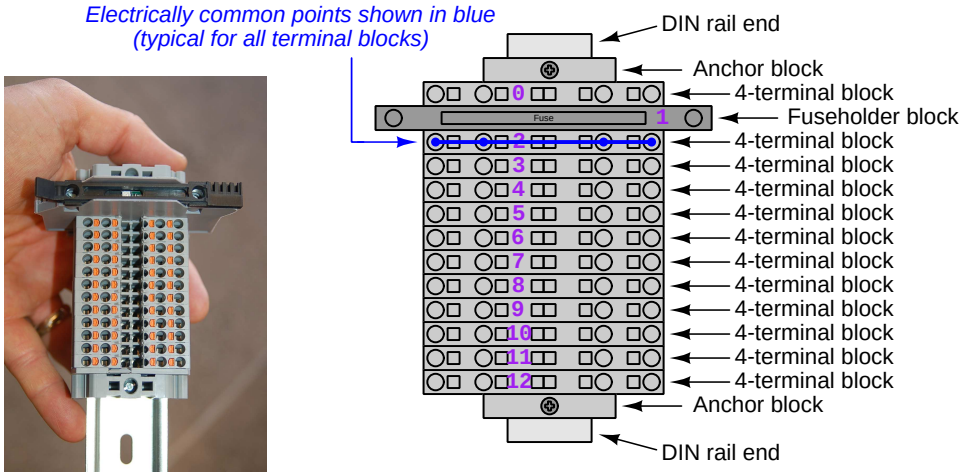
[4] An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

[5] Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

8.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems, and are also quite useful for student-built experiments, providing a robust alternative to solderless breadboards[6]. Terminal blocks provide highly reliable connections rated for significant voltage and current magnitudes, and they force the builder to think carefully about component placement which is an important mental practice. Terminal blocks designed to mount on standard 35 mm DIN rail[7] come in many types and capabilities.

Every student of electricity should build their own terminal block array consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other[8] and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a unique number starting from 0. An example is shown in the following photograph and illustration:



Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others such as the Phoenix Contact model 3211019 blocks shown in the photograph provide buttons[9] for this task which may be pressed using the tip of any suitable tool such as a small screwdriver or multimeter test probe tip.

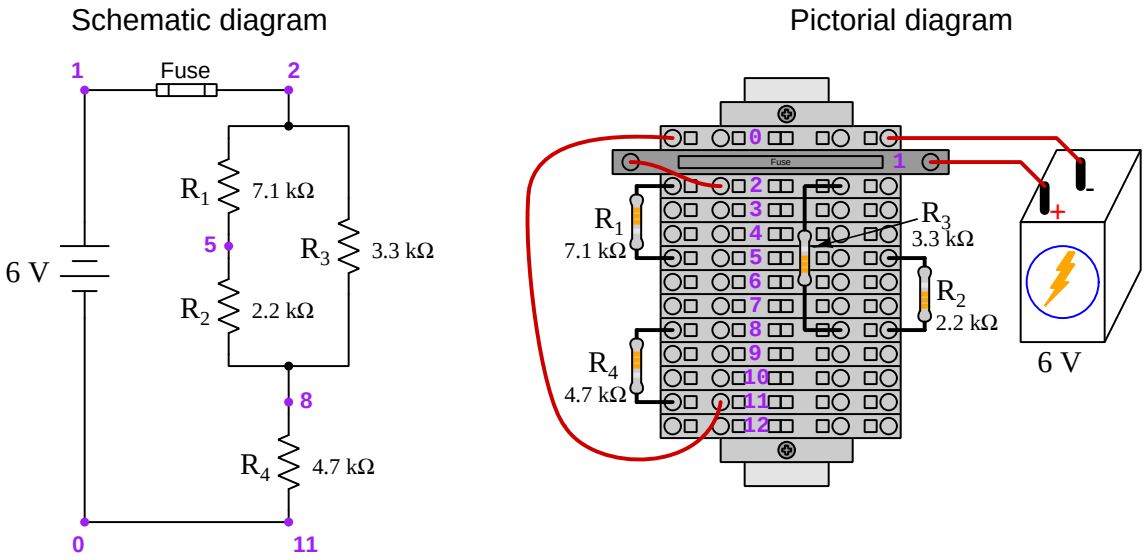
[6] Solderless breadboard are preferable for complicated electronic circuits with multiple integrated "chip" components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for "chip" circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

[7] DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

[8] Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

[9] The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:



Numbering on the terminal blocks provides a very natural translation to SPICE[10] netlists, where component connections are identified by terminal number:

```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rfuse 1 2 0.01
rwire 0 11 0.01
.op
.end
```

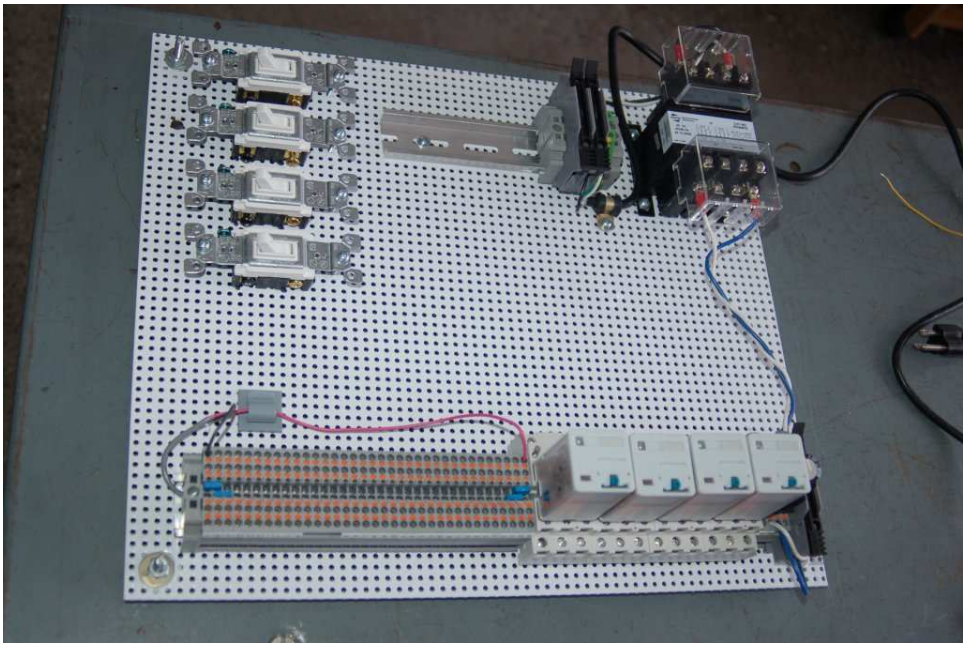
Note the use of two resistances `rfuse` and `rwire` to describe fuse resistance between terminals 1 and 2 and "jumper" wire resistance between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm appropriate to their real-life resistance values.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a

[10] SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

"wiring sequence" may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and "ice-cube" style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel[11]. This "terminal block board" hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT "ice-cube" relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed "feet" support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord's ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer's screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer's 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a "terminal block board" is an inexpensive[12] yet highly flexible means to construct physically robust circuits using industrial wiring practices.

[11] An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

[12] At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.

8.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

> An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*[13]. In order for an hypothesis to be valid, it must be testable[14], which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the "baseline" variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available[15].

[13] Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scient*ists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but scientific *method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one's hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

[14] This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

[15] A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting "data" from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even "bad" data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the "tolerance" of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of "successful" experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

```
Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional
to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage
DC power supply.  Use an ammeter in series to measure resistor current and
a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/
or pose a burn hazard, while excessive voltage poses an electric shock hazard.
30 Volts is a safe maximum voltage for laboratory practices, and according to
Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts
(P = V^2 / R), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:
  (Voltage)    (Current)         (Voltage)   (Current)
   0.000 V   =  0.000 mA          8.100    =  7.812 mA
   2.700 V   =  2.603 mA          10.00 V  =  9.643 mA
   5.400 V   =  5.206 mA          14.00 V  =  13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected
exactly one milliAmpere per Volt the actual current was usually less than
that.  The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts)
to a high of 1037.81 (at 14 Volts), but this represents a variance of only
-0.0365% to +0.0541% from the average, indicating a very consistent
proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself.  I
did not measure it, but simply assumed color bands of brown-black-red meant
exactly 1000 Ohms.  Based on the data I think the true resistance is closer
to 1037 Ohms.  Another possible explanation is multimeter calibration error.
However, neither explains the small positive and negative variances from the
average.  This might be due to electrical noise, a good test being to repeat
the same experiment to see if the variances are the same or different.  Noise
should generate slightly different results every time.
```

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

```
Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be
exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single
1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis
from 0 Volts to 25 Volts in 5 Volt increments.

    * SPICE circuit
    v1 1 0 dc
    r1 1 0 1000
    .dc v1 0 25 5
    .print dc v(1) i(v1)
    .end

RISKS AND MITIGATION: none.

DATA COLLECTED:
       DC transfer characteristic  Thu Feb 14 13:05:08  2019
    ------------------------------------------------------------
    Index   v-sweep         v(1)            v1#branch
    ------------------------------------------------------------
    0       0.000000e+00    0.000000e+00    0.000000e+00
    1       5.000000e+00    5.000000e+00    -5.00000e-03
    2       1.000000e+01    1.000000e+01    -1.00000e-02
    3       1.500000e+01    1.500000e+01    -1.50000e-02
    4       2.000000e+01    2.000000e+01    -2.00000e-02
    5       2.500000e+01    2.500000e+01    -2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely
1/1000 of the applied voltage for all values.  Anything other than perfect
agreement would have probably meant my netlist was incorrect.  The negative
current values surprised me, but it seems this is just how SPICE interprets
normal current through a DC voltage source.

ERROR SOURCES: none.
```

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

8.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

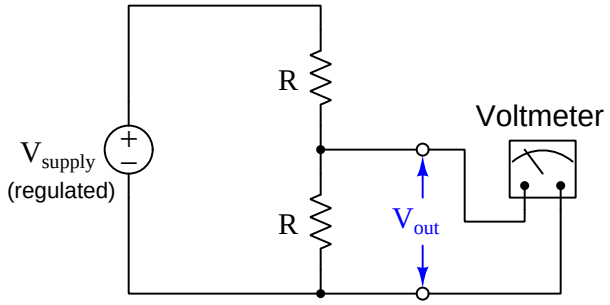
- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead.*
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

8.2 Experiment: voltmeter resistance

Ideal voltmeters possess infinite resistance, so as to draw zero current (and therefore pose no energy burden) when connected to a circuit. Real voltmeters fall short of this ideal goal. Your task is to set up an experiment whereby you determine the actual resistance of a voltmeter under realistic operating conditions.

One way to do this is to build a circuit with a known output voltage predicted in advance, and then measure that output voltage with a voltmeter to see how closely the voltmeter's measurement agrees with the predicted (ideal) voltage. This circuit should be built in such a way that the loading effect of the voltmeter will be clearly evident.

A simple circuit fitting this requirement is a 2:1 ratio voltage divider, constructed of two identical-valued resistors connected in series, powered by a regulated voltage source (e.g. laboratory-style power supply). The resistor values should be rather large, in order that the paralleled resistance of the voltmeter makes a significant different to the division ratio of the circuit:



The power supply's output voltage may be measured by a voltmeter with little or no loading effect, because regulated power supplies have very little internal resistance and so will not be appreciably affected by the miniscule current drawn by a directly-connected voltmeter. The loading effect on the voltage divider's output (V_{out}), however, will be much more pronounced because the voltmeter's current must pass through the upper resistor of the divider which has been chosen to be fairly large in value.

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Choose two resistors of equal value, chosen such that the meter's loading effect will be noticeable. It is up to you to select two resistors which are as close in value to each other as possible, and also to demonstrate the means by which you have determined their equality. Document the resistance values.

Next, use your DMM to measure the power supply's voltage. Document this value as well.

Calculate the voltage divider's output voltage (V_{out}) with no meter connected. Document this "ideal" voltage value.

Connect your DMM (set to measure voltage) and measure the voltage divider's output voltage (V_{out}). Compare this measured voltage value with the predicted voltage value from the previous step.

Calculate the voltmeter's internal resistance based on the discrepancy between these two voltages: V_{out} (ideal) versus V_{out} (empirical). Be sure to explain your reasoning at every step in the process.

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

Challenges

- An alternative to using a voltage divider is to simply measure the power supply's voltage through a high-valued resistor (i.e. connecting a resistor in series with one of the voltmeter test leads). Explain how this experiment could work as well.
- Predict the effect of the upper resistor failing open.
- Predict the effect of the upper resistor failing shorted.
- Predict the effect of the lower resistor failing open.
- Predict the effect of the lower resistor failing shorted.

8.3 Experiment: SPICE modeling of a series-parallel circuit

SPICE is a powerful computer-based simulation tool for circuit analysis, and if mastered provides a way for students to create their own practice problems. In this experiment, we will do just that: create a series-parallel resistor circuit practice problem, and use SPICE to give us the answer key!

Your hypothesis may simply be the predicted values for voltage and/or current in any series-parallel resistor circuit of your choosing. Feel free to take any series-parallel circuit found in this learning module – especially one with an accompanying SPICE netlist – modify it, and use this as the basis of your experiment. The *SPICE Modeling of Resistor Circuits* learning module is another resource for example circuits and SPICE netlists, which you may sample and modify.

EXPERIMENT CHECKLIST:

- Prior to experimentation:
  - ☑ Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - ☑ Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
  - ☑ Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.
- During experimentation:
  - ☑ Safe practices followed at all times (e.g. no contact with energized circuit).
  - ☑ Correct equipment usage according to manufacturer's recommendations.
  - ☑ All data collected, ideally quantitative with full precision (i.e. no rounding).
- After each experimental run:
  - ☑ If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - ☑ Identify any uncontrolled sources of error in the experiment.
- After all experimental re-runs:
  - ☑ Save all data for future reference.
  - ☑ Write an analysis of experimental results and lessons learned.

Challenges

- Science is an *iterative* process, and for this reason is never complete. Following the results of your experiment, what would you propose for your *next* hypothesis and *next* experimental

procedure? Hint: if your experiment produced any unexpected results, exploring those unexpected results is often a very good basis for the next experiment!

- How can you alter the value of a resistor in a SPICE netlist?
- How can you reverse the polarity of a voltage or current source in a SPICE netlist?

8.4 Experiment: demonstrate KVL in a series-parallel circuit

Kirchhoff's Voltage Law (KVL) is a fundamental principle of electric circuits and must be well-understood in order to analyze circuit networks and diagnose circuit faults. Your task is to devise and execute an experiment to demonstrate the validity of Kirchhoff's Voltage Law within a series-parallel DC resistor circuit of your choice.

In order to make this demonstration of KVL thorough, you should run multiple tests. In other words, choose multiple "loops" (or "paths") within your circuit, and prove that KVL is true in every instance.

EXPERIMENT CHECKLIST:

- Prior to experimentation:
  - ☑ Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - ☑ Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
  - ☑ Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.
- During experimentation:
  - ☑ Safe practices followed at all times (e.g. no contact with energized circuit).
  - ☑ Correct equipment usage according to manufacturer's recommendations.
  - ☑ All data collected, ideally quantitative with full precision (i.e. no rounding).
- After each experimental run:
  - ☑ If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - ☑ Identify any uncontrolled sources of error in the experiment.
- After all experimental re-runs:
  - ☑ Save all data for future reference.
  - ☑ Write an analysis of experimental results and lessons learned.

Challenges

- Science is an *iterative* process, and for this reason is never complete. Following the results of your experiment, what would you propose for your *next* hypothesis and *next* experimental procedure? Hint: if your experiment produced any unexpected results, exploring those unexpected results is often a very good basis for the next experiment!
- How can you alter the value of a resistor in a SPICE netlist?
- How can you reverse the polarity of a voltage or current source in a SPICE netlist?

8.5 Experiment: demonstrate KCL in a series-parallel circuit

Kirchhoff's Current Law (KCL) is a fundamental principle of electric circuits and must be well-understood in order to analyze circuit networks and diagnose circuit faults. Your task is to devise and execute an experiment to demonstrate the validity of Kirchhoff's Current Law within a series-parallel DC resistor circuit of your choice.

In order to make this demonstration of KCL thorough, you should run multiple tests. In other words, choose multiple nodes within your circuit, and prove that KCL is true in every instance.

EXPERIMENT CHECKLIST:

- Prior to experimentation:
  - ☑ Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - ☑ Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
  - ☑ Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.
- During experimentation:
  - ☑ Safe practices followed at all times (e.g. no contact with energized circuit).
  - ☑ Correct equipment usage according to manufacturer's recommendations.
  - ☑ All data collected, ideally quantitative with full precision (i.e. no rounding).
- After each experimental run:
  - ☑ If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - ☑ Identify any uncontrolled sources of error in the experiment.
- After all experimental re-runs:
  - ☑ Save all data for future reference.
  - ☑ Write an analysis of experimental results and lessons learned.

Challenges

- Science is an *iterative* process, and for this reason is never complete. Following the results of your experiment, what would you propose for your *next* hypothesis and *next* experimental procedure? Hint: if your experiment produced any unexpected results, exploring those unexpected results is often a very good basis for the next experiment!

- How can you alter the value of a resistor in a SPICE netlist?
- How can you reverse the polarity of a voltage or current source in a SPICE netlist?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

22 September 2026 – deleted the "Circuit #1" reference from the "Example: using VIRP tables to analyze series-parallel circuits" Case Tutorial section because that was the only circuit shown! Also corrected a few tyopgraphical errors throughout the module. Also added more circuit examples to the "Example: circuits with unknown source values" Case Tutorial section.

31 January 2026 – minor formatting edits made to the "Example: circuits with unknown source values" Case Tutorial section.

28 December 2025 – added more example circuits to the "Example: circuits with unknown source values" Case Tutorial section.

15-17 September 2025 – typographical error corrections courtesy of Wyatt Gleason. This included edits made to image_0274 where I did not *fully* expand the simplified circuit back to its original form. Also corrected one error in some instructor notes.

24 August 2025 – added new bullet-points to the list in the "Challenging concepts related to parallel circuits" section of the Introduction chapter regarding the confusion of definitions versus properties of both series and parallel networks.

26 March 2025 – added comments to the Introduction chapter urging instructors to have students show their in-class work solving problems.

13-19 February 2025 – added a new Case Tutorial section demonstrating a way to effectively apply VIRP tables to the analysis of series-parallel resistor circuits. Also changed the name of the "Four-resistor circuit" Quantitative Reasoning question to "Four-resistor circuits" and added more circuit examples for students to solve. Also made minor edits to image_7328 and image_7329.

2 October 2024 – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

29 August 2024 – added another section to the Introduction chapter specifically for challenges related to this module's topic. Also added another Case Tutorial section with series-parallel circuits having no given source value.

26 June 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures.

6 November 2023 – added a new Case Tutorial section showing how to constrain the resistance range of a potentiometer.

12-20 September 2023 – edited image_0037 and image_0038 contained within review_03 to standardize on colors representing higher versus lower electrical potential (orange for higher, violet for lower).

19 July 2023 – added more circuit problems to the "Mixed-source circuits" Quantitative Reasoning question.

6 July 2023 – minor edit to image_????.

2-3 July 2023 – added more explanatory text to the tutorial chapters on *why* we must reduce series-parallel networks to solve for their component voltages and currents and powers, as well as added new Quantitative Reasoning questions with mixed-source circuits containing other types of components such as diodes and capacitors.

25 June 2023 – added Technical Reference section on grounding.

22 June 2023 – added a requirement in the "Explaining the meaning of calculations" Conceptual Question to also express each quantity using appropriate metric prefixes.

30 March 2023 – trivial change to the name of a Case Tutorial section title to reflect its multiple (rather than single) circuit example.

23 February 2023 – added a Case Tutorial section on Kelvin-Varley voltage dividers.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

26 September 2022 – edited image_6004 to more clearly show where output voltage is measured on a Kelvin-Varley divider network.

21 September 2022 – fixed a mathematical typo in the Full Tutorial where a 20 kΩ resistance should have been listed as 22 kΩ, courtesy of Konstantin Kolesnikov.

21 September 2022 – fixed an omission by adding 150 Ω to the list of resistor values in the

"Four-resistor circuit" Quantitative Reasoning problem, this fix courtesy of Gavin Koppel.

20 September 2022 – minor edits to wording in some of the questions, as well as to one of the schematic diagrams in the "Mixed-source circuits" Quantitative Reasoning question.

11 July 2022 – changed text label from "point" to "node" in image_0681. Also fixed a minor omission in one of the instructor notes.

10 July 2022 – divided the Full Tutorial chapter into sections, and also added a quantitative series-parallel analysis example to the Simplified Tutorial.

20 July 2022 – added an "Applying foundational concepts..." question to the Conceptual Reasoning section of the Questions chapter.

7 May 2022 – added more text to the Quantitative Reasoning question on Kelvin-Varley voltage divider networks.

11 April 2022 – minor additions to the Introduction chapter.

16 March 2022 – added a Quantitative Reasoning problem on Kelvin-Varley voltage divider network.

15-16 February 2022 – minor additions to the Introduction chapter.

7 February 2022 – made minor edits to the text in the "Mixed-source circuits" and "More mixed-source circuits" and "Building custom resistance values" Quantitative Reasoning problems.

15 December 2021 – added another Quantitative Reasoning question, on three-resistor series-parallel circuits using values simple enough that no calculator should be necessary to compute all voltages and currents.

22-23 September 2021 – expanded the Simplified Tutorial chapter to show illustrations of series-parallel "stacking" as well as voltage divider and multimeter "loading" effects. Also moved the "Problem-solving example: mixed-source circuit" to the Case Tutorial chapter which is a better place for it.

5 August 2021 – added Case Tutorial chapter showing a series-parallel circuit with component values chosen to be easy enough to calculate using simple arithmetic.

22 June 2021 – added technical reference on resistor labeling.

31 May 2021 – added a Case Tutorial section on supervised switching circuits.

17 April 2021 – added a Conceptual Reasoning question regarding the design of a resistive decade box.

17 February 2021 – added instructor notes.

16 February 2021 – minor edits to the Simplified Tutorial, regarding "steady-state" conditions. Also added some content to the Introduction chapter and edited the text of the "Mixed-source circuits" Quantitative question.

26 September 2020 – added another Case Tutorial example, showing how to use a potentiometer to create an (unregulated) variable-voltage power supply for energizing resistor networks.

24 September 2020 – added more content to the Introduction chapter, including the suggestion to make a summary page of all properties of series and parallel networks.

19-22 September 2020 – added more Case Tutorial examples, added more text to the Introduction chapter, fixed some minor typographical errors, and added more instructor notes to Questions.

3 September 2020 – added Technical Reference section on metric prefixes.

26 August 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

22 April 2020 – corrected error in instructor answers for a Quantitative Reasoning problem, courtesy of Ron Felix.

17 April 2020 – added a Technical Reference section on IEC standard component values (file techref_IEC60063.latex).

29 January 2020 – added some instructor notes.

2 January 2020 – removed from from C++ code execution output, to clearly distinguish it from the source code listing which is still framed.

31 December 2019 – added short comment on namespaces in C++ programming example.

30 December 2019 – added explanations to C++ programming example, describing in detail what the various instructions do.

23 September 2019 – explicitly added the term *insertion resistance* to the already-exiting Tutorial discussions on meter loading effects.

19 September 2019 – minor edit to footnote in the Case Tutorial chapter.

16 September 2019 – added a Case Tutorial chapter.

26 August 2019 – added mention of difficult concepts to the Introduction.

8 June 2019 – added sub-questions asking students to apply KVL to series-parallel circuits.

22 May 2019 – added Quantitative Reasoning problem calculating the resistance of a voltmeter based on its loading effects on a high-resistance voltage divider circuit.

28 April 2019 – added problem-solving example showing a multi-source circuit completely worked through.

3 April 2019 – minor edits.

30 March 2019 – noted the existence of the "SPICE Modeling" learning modules as resources for experimenting with SPICE simulations of series-parallel circuits.

26 March 2019 – minor edits to the "Interpreting a SPICE analysis" Quantitative problem, noting at the end that SPICE is useful as a tool for creating practice problems.

16 March 2019 – added new Experiments (demonstrate KVL/KCL in series-parallel circuits).

12 February 2019 – added a new Experiment (SPICE modeling of a series-parallel circuit).

10 February 2019 – added requirement in "Explaining the meaning of calculations" Conceptual Question to identify the principle applied in each step.

17 December 2018 – added an example C++ program showing simple series-parallel circuit calculations.

October 2018 – added explanatory diagram to Simplified Tutorial chapter showing how a series-parallel circuit may be reduced to a single equivalent component by successive reductions of series and parallel subnetworks.

September 2018 – added content to the Quantitative Reasoning section.

August 2018 – added content to the Introduction chapter.

June 2018 – finished Simplified Tutorial.

May 2018 – added more Quantitative Reasoning questions, added Simplified Tutorial. Minor edit to "parallel" illustration, annotating connected points as electrically common to each other, not just equipotential to each other.

November 2017 – elaborated on Energy versus Power conservation, and added more questions.

June 2017 – added more quantitative questions.

October 2016 – created changelog for future use.

September 2016 – document first published.

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