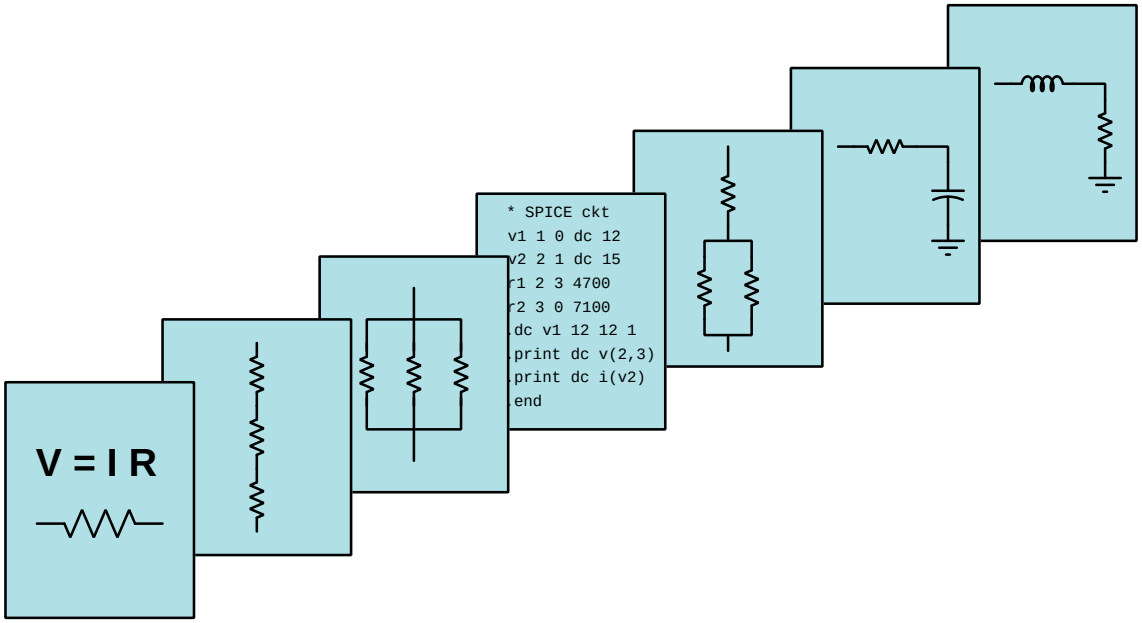
Modular Electronics Learning (ModEL) project



Smith Charts

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Chapter 1

Introduction

1.1 Recommendations for students

The measurement and expression of complex impedance values is extremely important in a wide range of AC circuits, especially radio-frequency (RF) circuitry, and a special type of graphical expression called a *Smith Chart* is designed to facilitate this. Unlike a standard Cartesian graphing plane, though, the Smith Chart uses axes that extend from zero to infinity, using curves rather than straight lines to mark major and minor divisions. The benefits of Smith Charts may not be apparent at first, but their unique geometry makes possible graphical solutions to AC calculations that otherwise might be quite tedious, especially during the era (1930's) in which Phillip Smith invented it.

In our modern era where electronic calculators have all but obsoleted graphical computing techniques, the Smith Chart still finds wide application as a mode of displaying complex-impedance information on the viewscreens of such instruments as impedance analyzers and vector network analyzers (VNAs).

Important concepts related to Smith Charts include **resistance**, **reactance**, **capacitance**, **inductance**, **impedance**, **complex numbers**, **rectangular** versus **polar** expressions of complex quantities, **Pythagorean Theorem**, **normalized** quantities, **frequency sweeps**, **admittance**, **series** versus **parallel** network properties, **transmission lines** and **termination**, **characteristic impedance**, **impedance transformation**, **impedance matching**, **standing waves**, and the **Maximum Power Transfer Theorem**.

Due to the heavy application of math implicit in Smith Charts, the reader is urged to apply those mathematical principles as soon as possible. The calculation and plotting examples contained in the tutorial serves this purpose well, as they allow you to apply the formulae discussed previously to an example and then check your work against the completed example to see if your application was correct. Do not simply read a quantitative example and assume you understand it just because nothing in the presentation seemed confusing. *Until you can perform the analysis yourself without assistance, you haven't mastered it!*

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to mathematically plot the Smith Chart response for an electrical network without the benefit of using a vector network analyzer (VNA)? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How do rectangular-form and polar-form complex number expressions differ from one another?
- Why is the impedance phase angle for an inductor always +90 degrees?
- Why is the impedance phase angle for a capacitor always −90 degrees?
- How does reactance differ in principle from resistance?
- What is *admittance*, and how does it relate to *impedance*?
- What is the unit of measurement for impedance, versus admittance?
- What does a phasor diagram represent?
- What mathematical operation(s) are represented when we stack phasors tip-to-tail?
- What is the Pythagorean Theorem useful in calculating?
- How do Smith Charts differ from normal Cartesian-coordinate charts?
- How is resistance represented on a Smith Chart?
- How is capacitive or inductive reactance represented on a Smith Chart?
- What does it mean to "sweep" the frequency of a signal during a test?
- How does an impedance-only Smith Chart differ from an impedance-admittance Smith Chart?
- How may the *addition* (i.e. series connection) of multiple impedances be shown on a Smith Chart?
- What does it mean to say that a Smith Chart is "normalized" to a particular impedance value?
- How may the values of series- and parallel-connected impedances be combined on a Smith Chart?
- What practical applications exist for matching impedances of different AC networks?
- What alternative solutions exist for creating impedance-matching networks?
- How may a Smith Chart be used to help design impedance-matching networks?

1.2 Challenging concepts related to Smith charts

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Complex numbers** – these are numerical values having both real and imaginary components, or alternatively magnitudes and angles. These are useful for expressing AC quantities where there is a shift in phase between two voltages, between two currents, or between a voltage and a current. Since impedance is defined by Ohm's Law as a quotient of a voltage and a current, the phase angle of an impedance is the phase shift between that voltage and that current.
- **Normalized values** – Smith charts use normalization to show impedance values on a simple scale with 1 (unity) being at the center of the chart. This means a normalizing quantity must be specified for each chart (typically 50 Ohms), and each real impedance value must be scaled to that normalizing factor.
- **Resistance versus Reactance versus Impedance** – these three terms represent different forms of opposition to electric current. Despite the fact that they are measured in the same unit (ohms: Ω), they are not the same concept. Resistance is best thought of as electrical *friction*, whereas reactance is best thought of as electrical *inertia*. Whereas resistance creates a voltage drop by dissipating energy, reactance creates a voltage drop by *storing* and *releasing* energy. Impedance is a term encompassing both resistance and reactance, usually a combination of both.

The *Case Tutorial* chapter contains a section showing frequency-sweep test results for a wide array of passive networks. These are well worth the time to closely study, to recognize patterns and to run "thought experiments" in your mind predicting how each network's impedance will vary as frequency sweeps from zero (DC) to some arbitrarily high value.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Plot impedance values on a Smith chart

  Assessment – Given simple RLC networks, qualitatively plot their impedances over a range of swept frequency; e.g. pose problems in the form of the "Network frequency-sweep tests" Conceptual Reasoning question.

  Assessment – Plot single impedance values as dots on a Smith chart; e.g. pose problems in the form of the "Plotting component impedances on a Smith Chart" Quantitative Reasoning question.

- **Outcome** – Design impedance-matching networks on a Smith chart

  Assessment – Use a Smith chart to assist in the design of LC impedance-matching networks for given source and load impedances at specified frequencies; e.g. pose problems in the form of the "Impedance-matching networks" Quantitative Reasoning question.

- **Outcome** – Independent research

  Assessment – Locate RF device datasheets showing Smith charts and properly interpret some of the information contained in those charts including frequency range, impedance ranges, resonant points, etc.

Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: custom inductor winding and testing

In this example we see a custom inductor made by winding 19 turns of 23-gauge enamel-coated copper wire around a Micrometals T50-66C powdered iron toroidal core. This core's datasheet specifies an inductance factor (A_L) of 51 nanoHenrys per turn-squared, so the inductor's value should be:

$$L = A_L N^2$$

$$L = (51 \times 10^{-9})(19^2) = 18.411\ \mu\text{H}$$

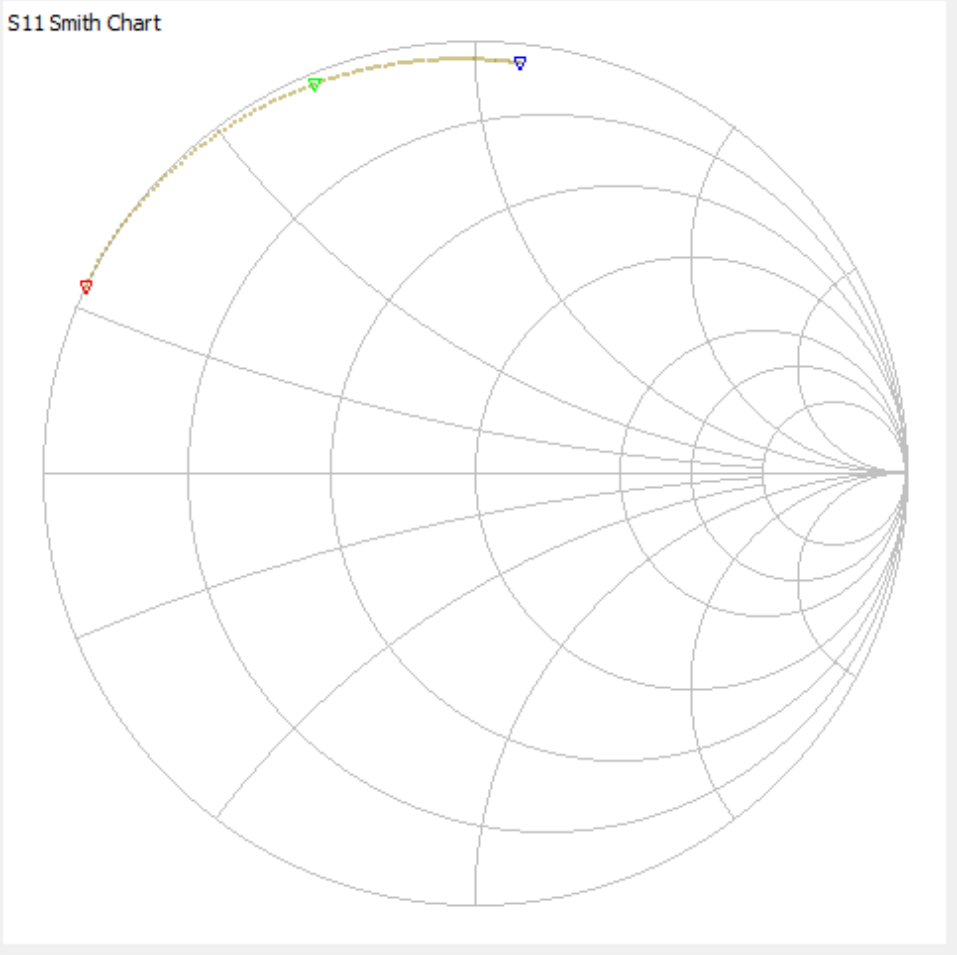
Micrometal's general catalog of core materials lists the type 66 as being "well-suited from 100 kHz to 500 kHz" and so a test was conducted using a NanoVNA vector network analyzer over that same range of signal frequencies:

Analysis from the VNA shows the measured inductance to be quite close to the predicted value of 18.411 microHenrys over the "swept" signal range of 100 kHz to 500 kHz:

Frequency	**Impedance**	**Series L**	**Deviation from predicted**
100 kHz	0.148 + j11.3 Ω	17.974 μH	−2.37%
300 kHz	1.11 + j33.5 Ω	17.782 μH	−3.42%
500 kHz	2.76 + j56.1 Ω	17.849 μH	−3.05%

The "real" portion of the complex impedance value represents series equivalent resistance in Ohms, equal in this case to the resistance of the wire in the inductor's coil as well as any connection resistances between the inductor and the VNA. This value rises slightly with signal frequency due to the *skin effect*, where electric current at very high frequency becomes excluded from traveling in the wire's center and therefore decreases the effective cross-sectional area of the conductor. The "imaginary" portion of the complex impedance value is the inductive reactance (X_L) in Ohms, relating to inductance by the formula $X_L = 2\pi f L$ where f is the signal frequency in Hertz.

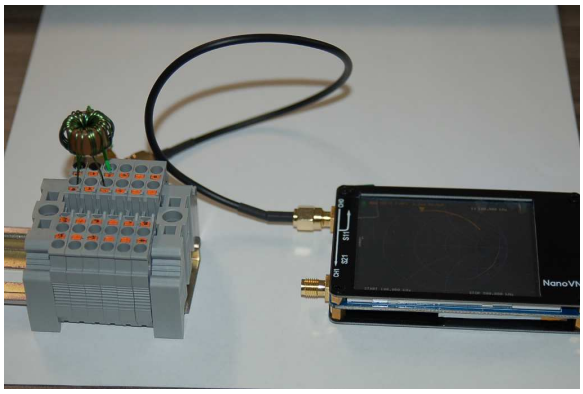
Like most vector network analyzers, the NanoVNA produces a Smith chart plot of complex impedance (normalized to 50 Ohms). As expected for a nearly-pure inductance, the path traced during the 100-500 kHz "sweep" closely follows the 0 Ohm resistive circle circumference while intersecting various inductive reactance circles as frequency rises. Here the red marker signifies 100 kHz (just above the +j0.2 circle denoting +j10 Ohms), the green marker 300 kHz (between the +j0.5 and +j1.0 circles), and the blue marker 500 kHz (just above the +j1.0 circle denoting +j50 Ohms):



Next we see the results of winding 30 turns of 23-gauge wire around another T50-66C powdered iron core with the same inductance factor of 51 nanoHenrys per turn-squared:

$$L = A_L N^2$$

$$L = (51 \times 10^{-9})(30^2) = 45.900\,\mu\text{H}$$



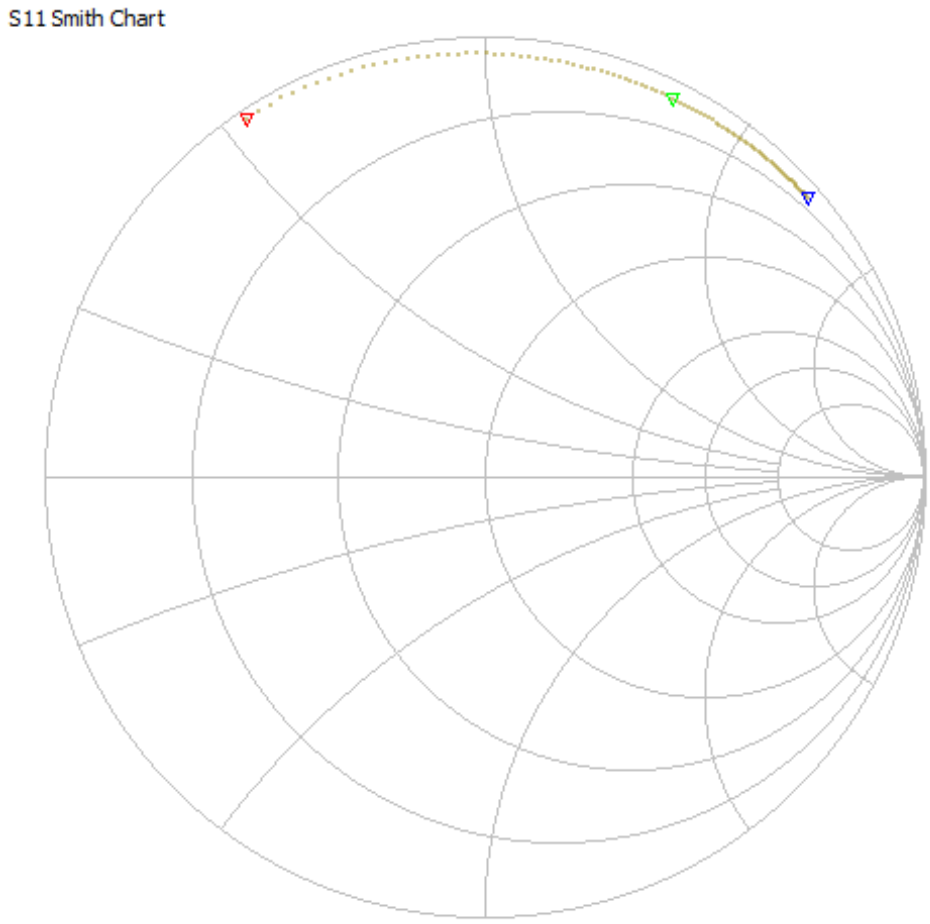
Analysis from the VNA shows the measured inductance to deviate more significantly from the predicted value of 45.9 microHenrys over the "swept" signal range of 100 kHz to 500 kHz than it did with the previous (19-turn) inductor:

Frequency	**Impedance**	**Series L**	**Deviation from predicted**
100 kHz	0.827 + j26.8 Ω	42.665 μH	−7.048%
300 kHz	3.86 + j80.6 Ω	42.771 μH	−6.817%
500 kHz	6.29 + j135 Ω	42.941 μH	−6.447%

A greater number of wire turns means an increase in two different parasitic properties: first, the wire will have greater total resistance because there is simply more of it for current to pass through; second, the greater number of turns packed onto the same size toroid means individual turns being closer together to one another than before which will increase the amount of inter-turn capacitance[1].

[1] Recall that capacitance between any two conductive surfaces is a direct function of the overlapping area of those surfaces, and an inverse function of separation distance. Here, with 30 turns of wire being packed closer together on the same toroid than 19 turns of wire, the inter-turn capacitance must be greater.

Here again we see the NanoVNA's Smith chart showing complex impedance normalized to 50 Ohms, this time for the 30-turn inductor:

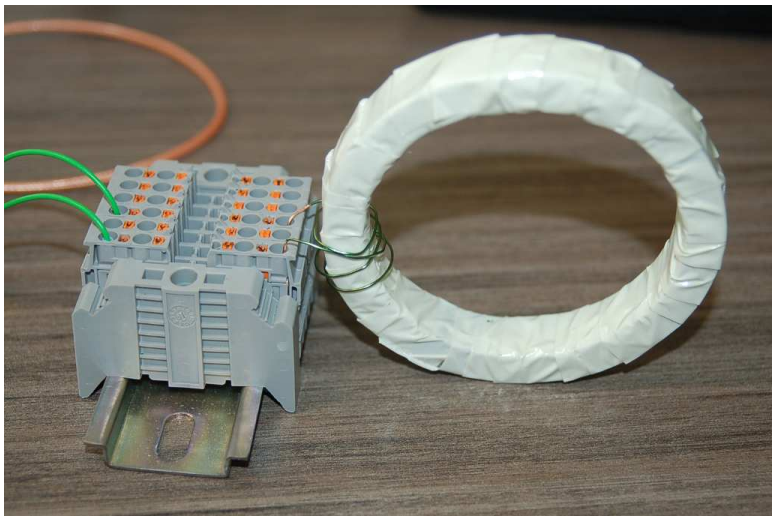


As one would expect, the curve lies farther to the right due to the increased inductive reactance of the larger inductance over the same frequency range. Increased resistance from the longer wire drives the curve closer to the middle of the Smith chart rather than remain on the periphery as before.

Using an entirely different toroidal core, I performed an experiment where I tried different numbers of turns of the same 23-gauge enameled wire and measured inductance using a different VNA (a PicoVNA model 106) sweeping from 300 kHz to 400 kHz:

Turns	**300 kHz**	**354 kHz**	**381 kHz**	**400 kHz**
2	6.06 μH	6.31 μH	6.43 μH	6.52 μH
4	22.79 μH	23.70 μH	24.13 μH	24.45 μH
5	36.36 μH	37.82 μH	38.51 μH	38.93 μH
10	149.63 μH	156.85 μH	159.24 μH	160.16 μH

The following photograph shows this iron core with four turns of wire wrapped around it, the bare iron core itself being wrapped in white electrical tape to prevent the enameled wire from chafing against the bare metal and possibly creating a coil-to-core shorted fault:



If you take any two inductance values at the same test frequency but for a different number of turns, the ratio of those two inductance values approximately equals the square of the ratio of the turns. For example, 160.16 microHenrys and 38.93 microHenrys at 400 kHz for 10 turns and 5 turns, respectively, makes an inductance ratio of nearly four to one (4.11405:1) for a turns ratio that is two to one (2:1). This confirms the general principle that inductance is proportional to the square of the number of wire turns wrapped around the core.

Computing the average A_L value at each test frequency based on the four measured inductance values and turns values ($\frac{L}{N^2}$) yields the following:

Frequency	**Average** A_L
300 kHz	1.473 μH per turn^2
354 kHz	1.535 μH per turn^2
381 kHz	1.562 μH per turn^2
400 kHz	1.579 μH per turn^2

Averaging these four A_L values yields an aggregate A_L of 1.537 μH per turn-squared for this particular toroidal iron core. This figure, of course, will be useful for creating custom inductors based on this toroid as the core material.

2.2 Example: frequency-sweeps of RLC network impedances

In this example we will use free software called `Elsie` to simulate the impedance of RLC networks, plotting the impedance of those networks on Smith Charts. In each case the frequency sweep begins at an extremely low frequency (practically DC) and ends at several megaHertz, the starting (DC) point located on the Smith Chart by a blue dot. Each Smith Chart is normalized to 50 Ohms, and the resistor value within every network containing a resistor is 50 Ohms.

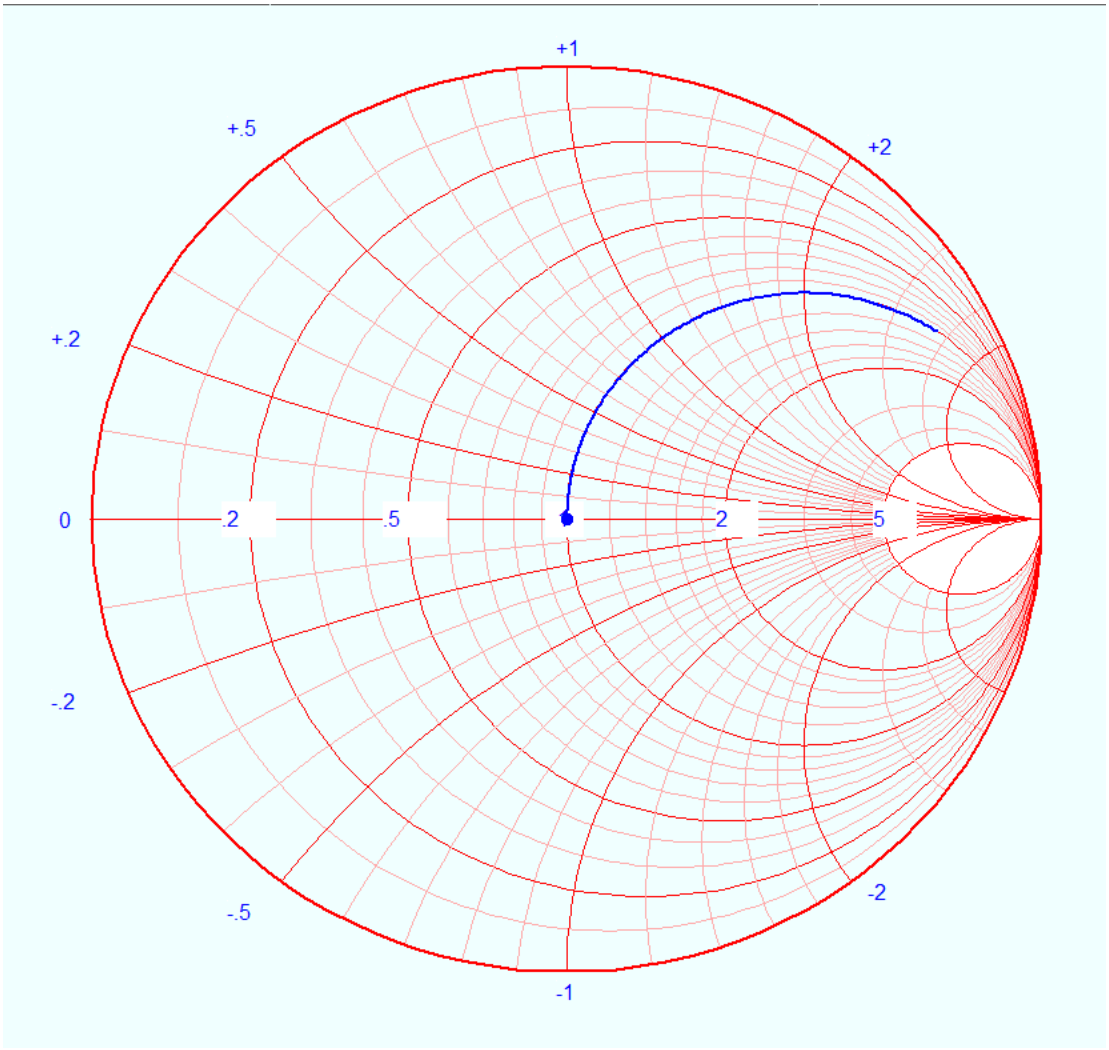
For each of these simulated frequency sweeps, the reader is urged to perform *thought experiments* on the given network to determine what the network's impedance should be at the starting frequency ($\approx$0 Hz, DC) and then what the network's impedance should be at the ending frequency (some arbitrarily high value in the megaHertz range). At some frequencies, the network's resistance (R) will dominate the total impedance value, at other frequencies a reactance (either X_L or X_C) will dominate the total impedance, and in the case of networks containing both L and C there will be a resonant point where these two reactances will cancel out. Your task is to figure out *why* the Smith Chart plot bends the way it does as frequency increases for each network.

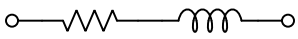
Please note that neither a real-world frequency sweep test nor a computer simulation is capable of sweeping through *infinite* frequency, so your $f = \infty$ thought experiment will never quite match the far ends of the traces shown in these Smith Charts. Hopefully, though, the trajectory of these sweep-tests will be plain enough to extrapolate in your mind, that you will be able to discern from the plots where the impedance is heading should the frequency increase more.

Important Foundational Concepts to apply in these thought experiments include:

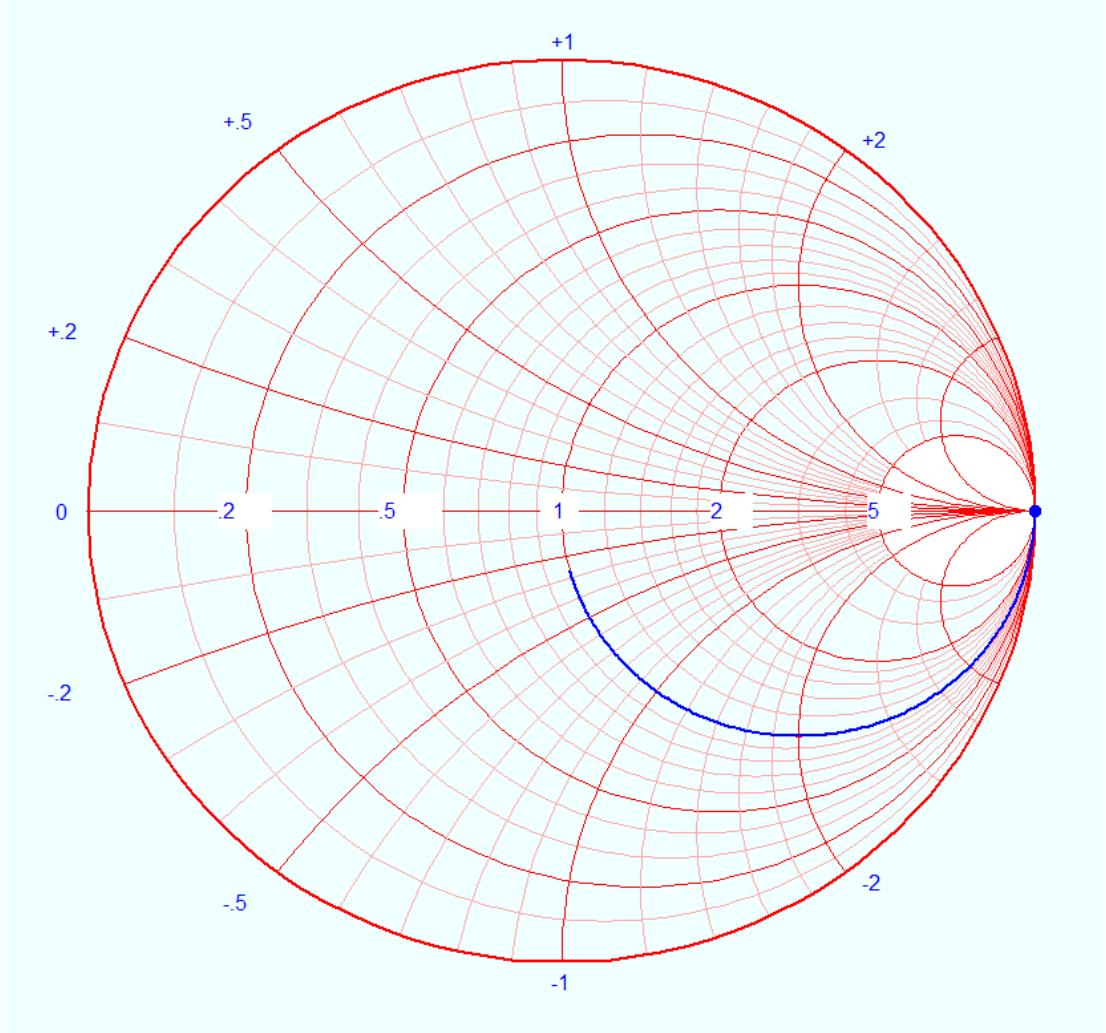
- $X_L = 2\pi f L$; inductive reactance increases with increasing frequency, and is zero at DC
- $X_C = \frac{1}{2\pi f C}$; capacitive reactance decreases with increasing frequency, and is infinite at DC
- Series impedances add, so the largest impedance in a series network will dominate total impedance
- Parallel impedances diminish, so the smallest impedance in a parallel network will dominate total impedance ; alternatively stated, parallel *admittances* add, so the largest admittance in a parallel network will dominate total admittance
- Series inductor-capacitor combinations result in nearly zero impedance (short) at resonance
- Parallel inductor-capacitor combinations result in nearly infinite impedance (open) at resonance

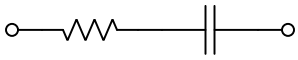
2.2.1 Series RL network



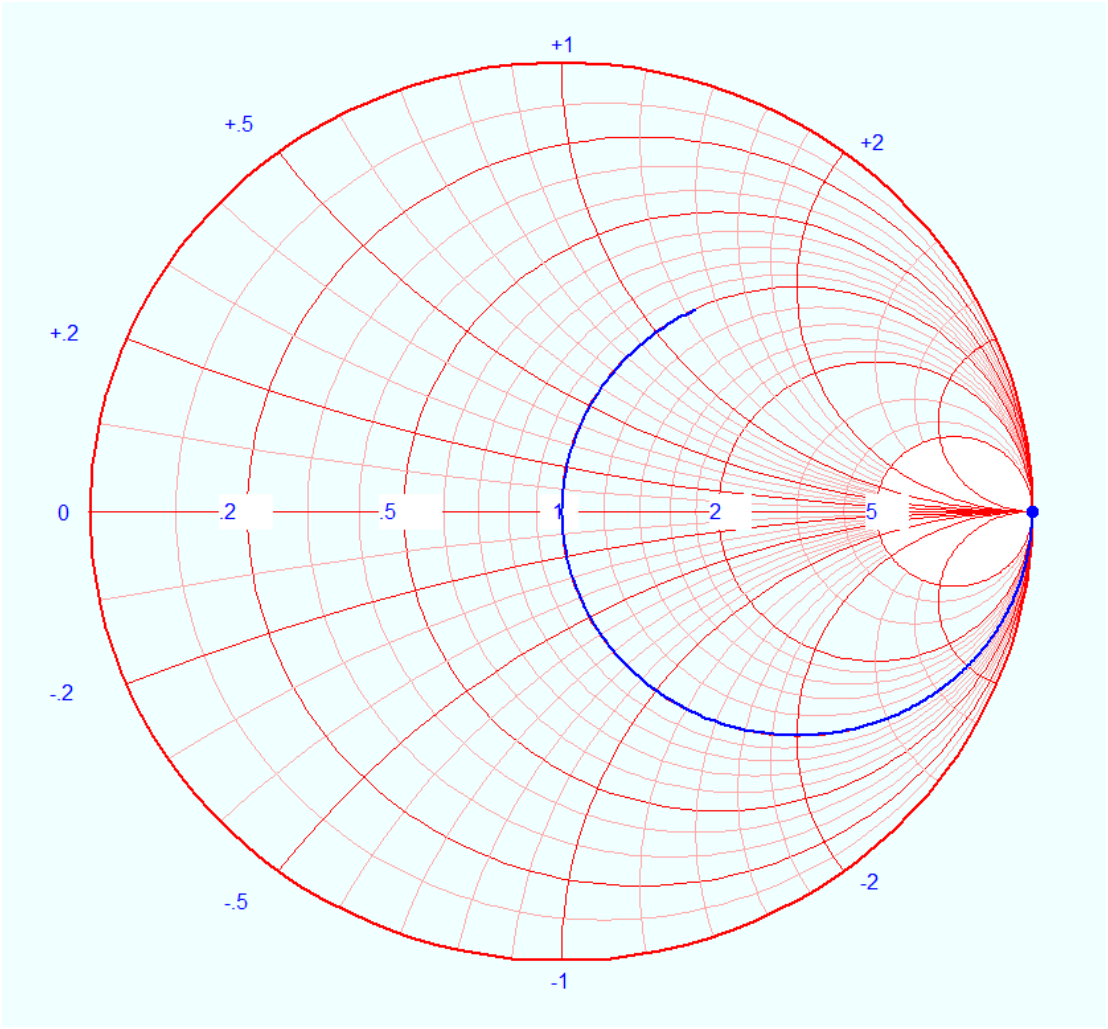


2.2.2 Series RC network

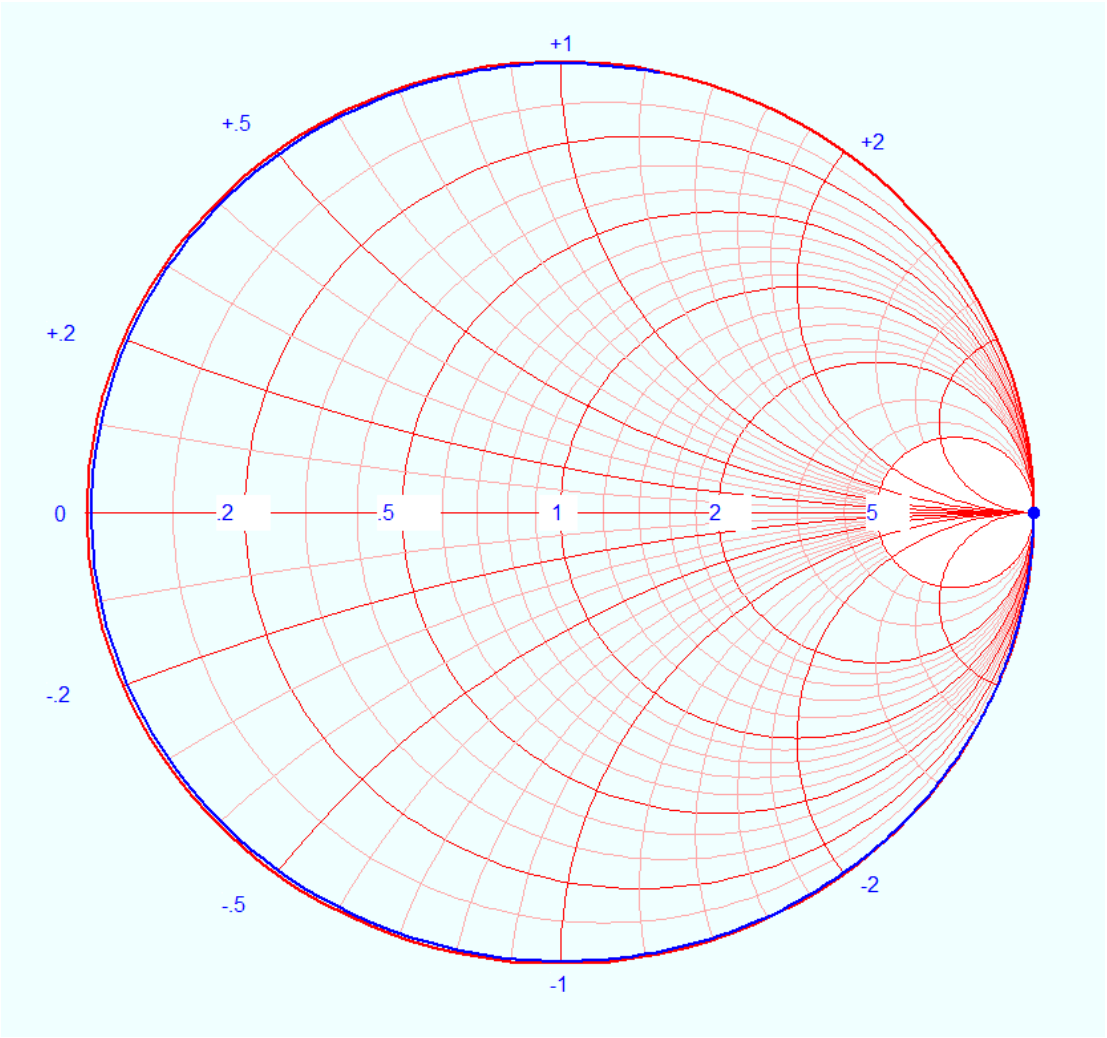


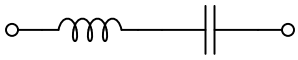


2.2.3 Series RLC network

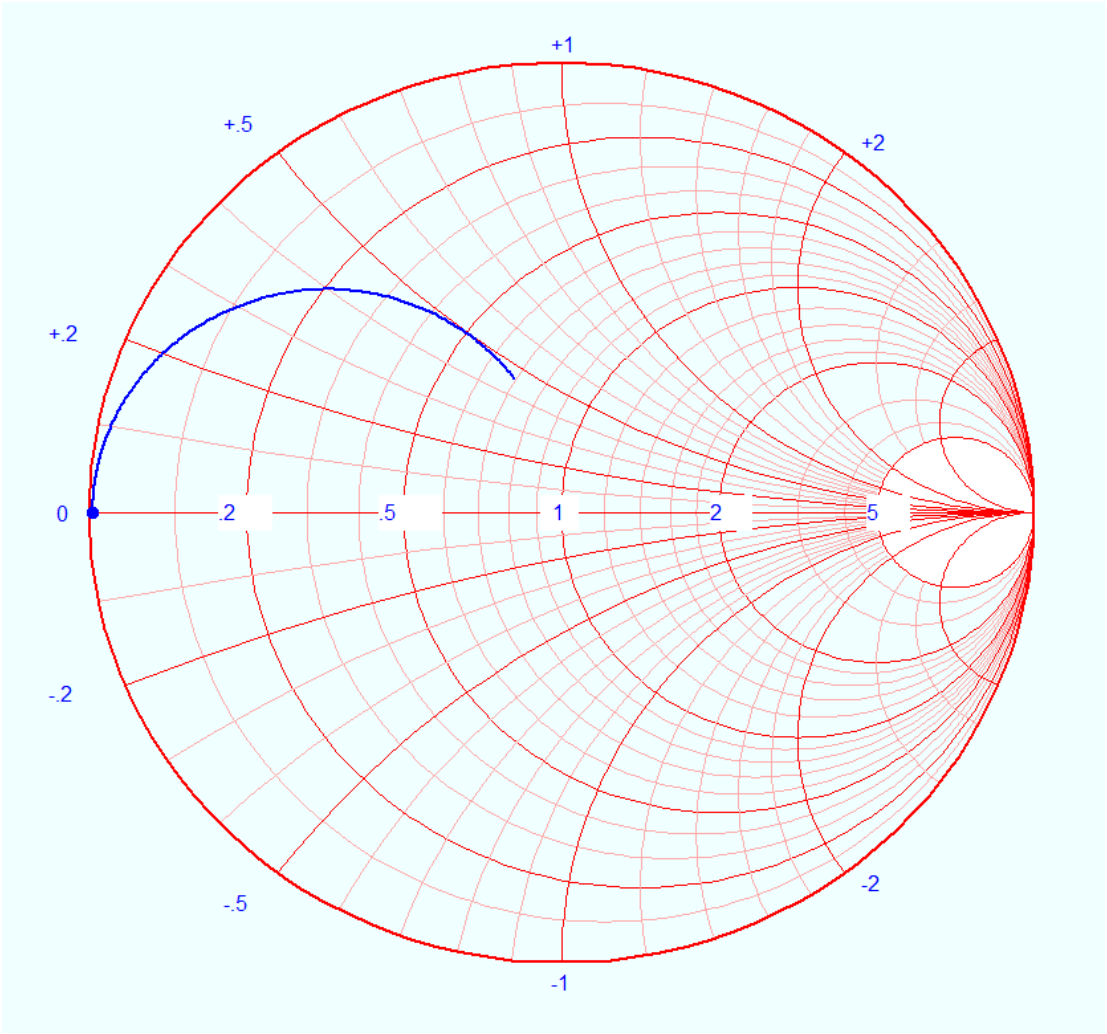


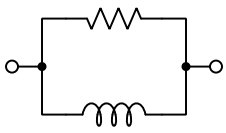
2.2.4 Series LC network



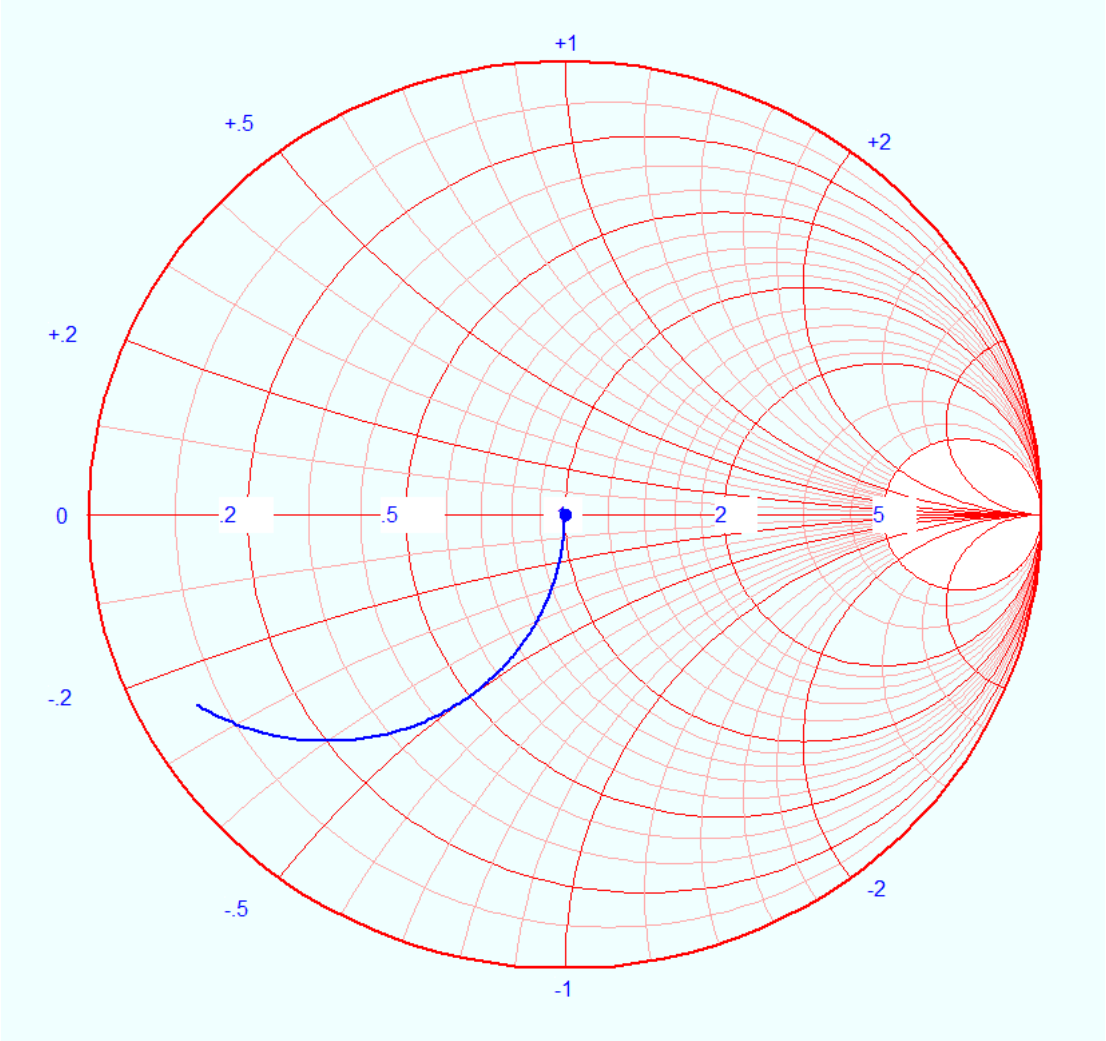


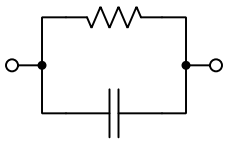
2.2.5 Parallel RL network



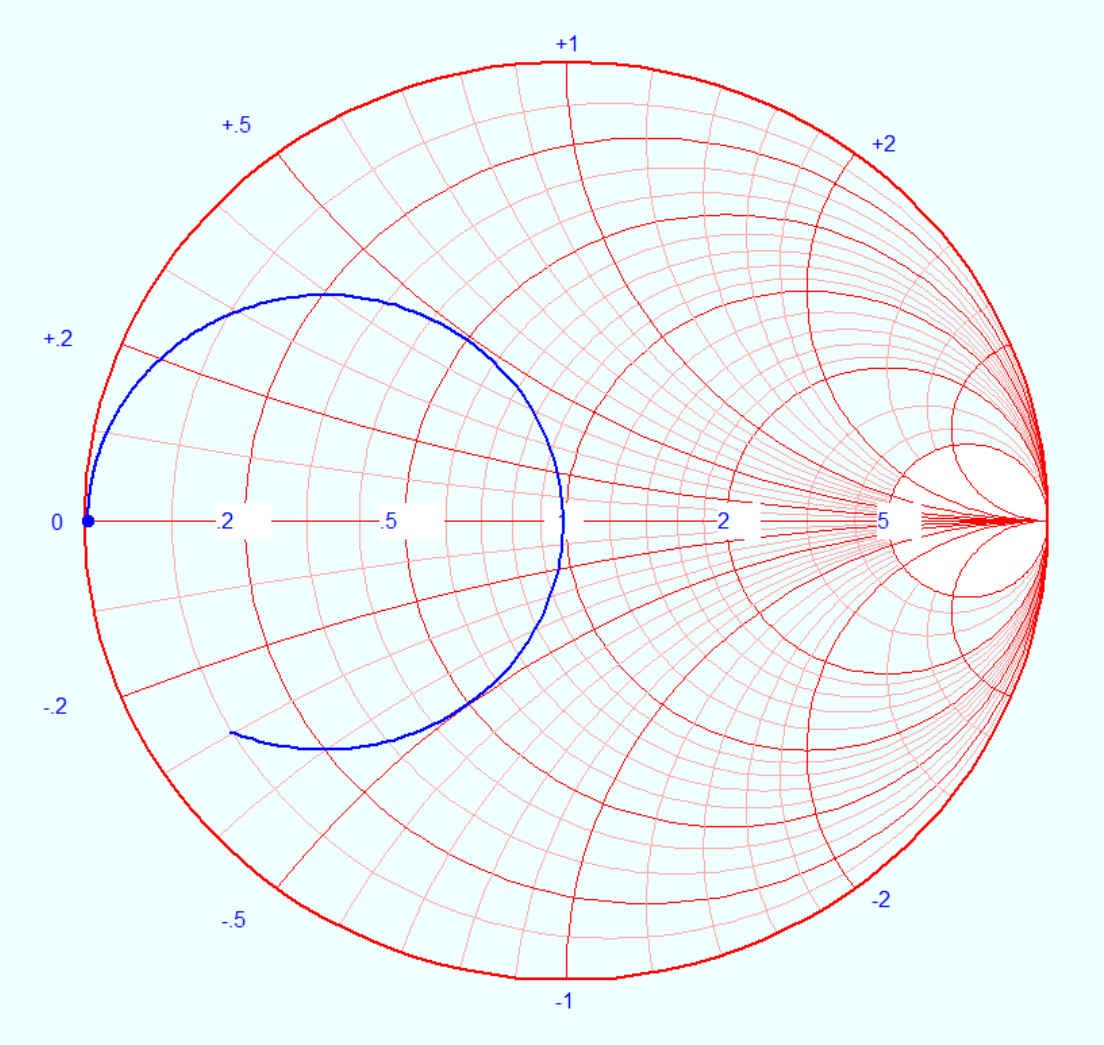


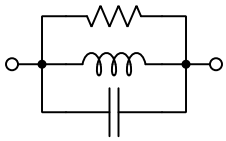
2.2.6 Parallel RC network



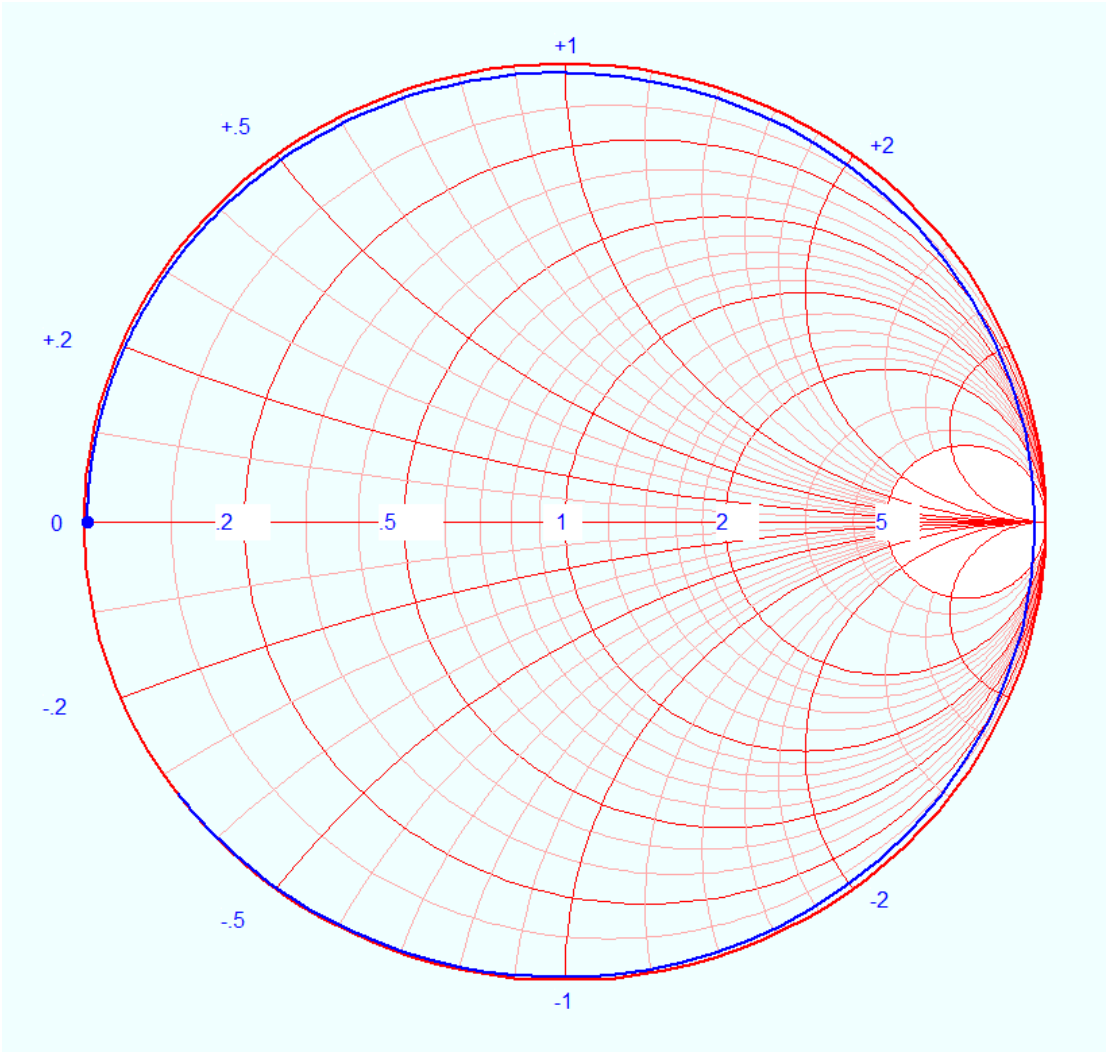


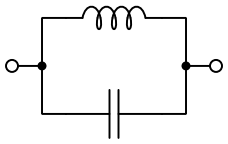
2.2.7 Parallel RLC network



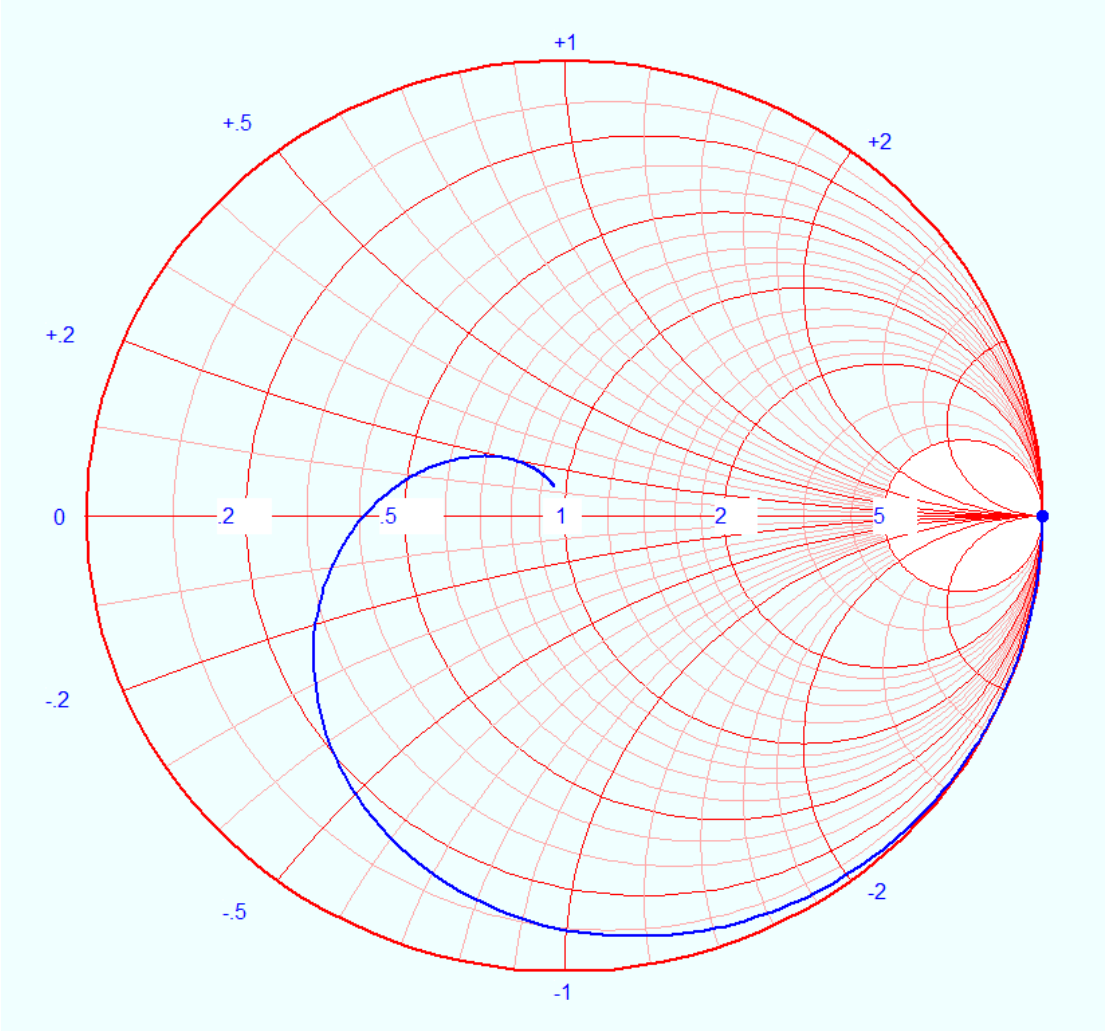


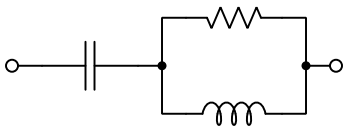
2.2.8 Parallel LC network



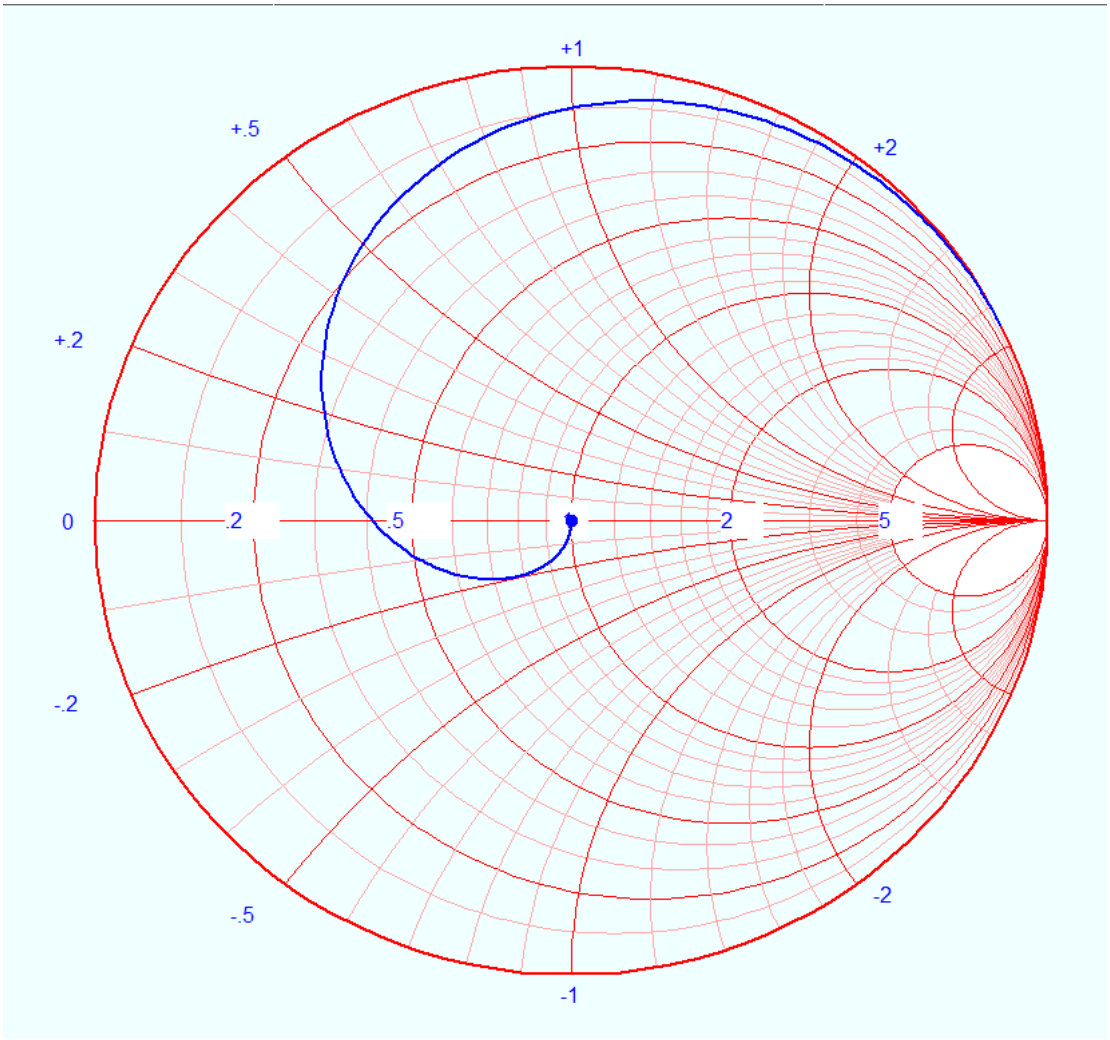


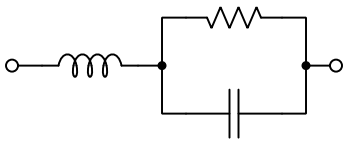
2.2.9 C in series with parallel RL network



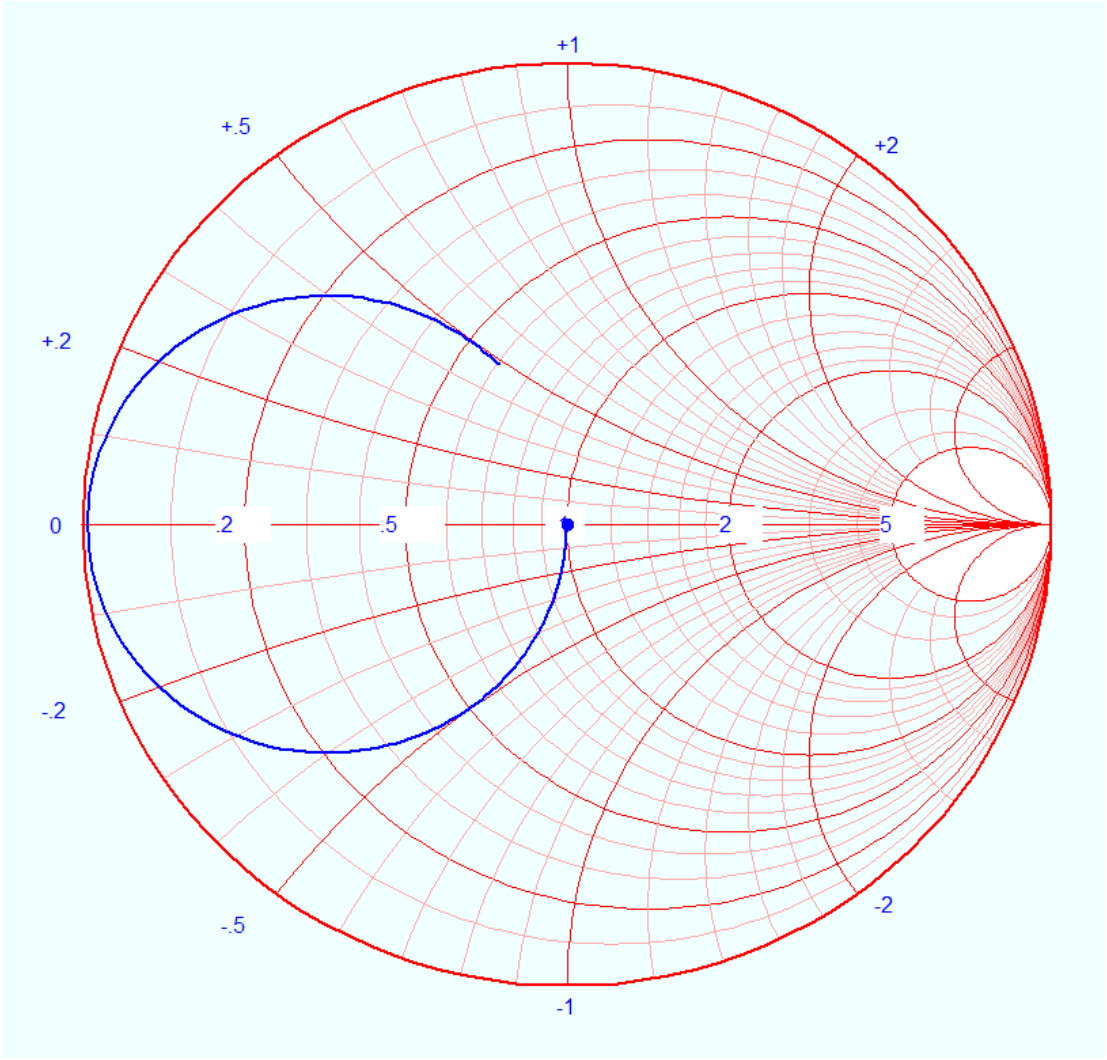


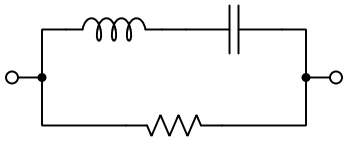
2.2.10 L in series with parallel RC network



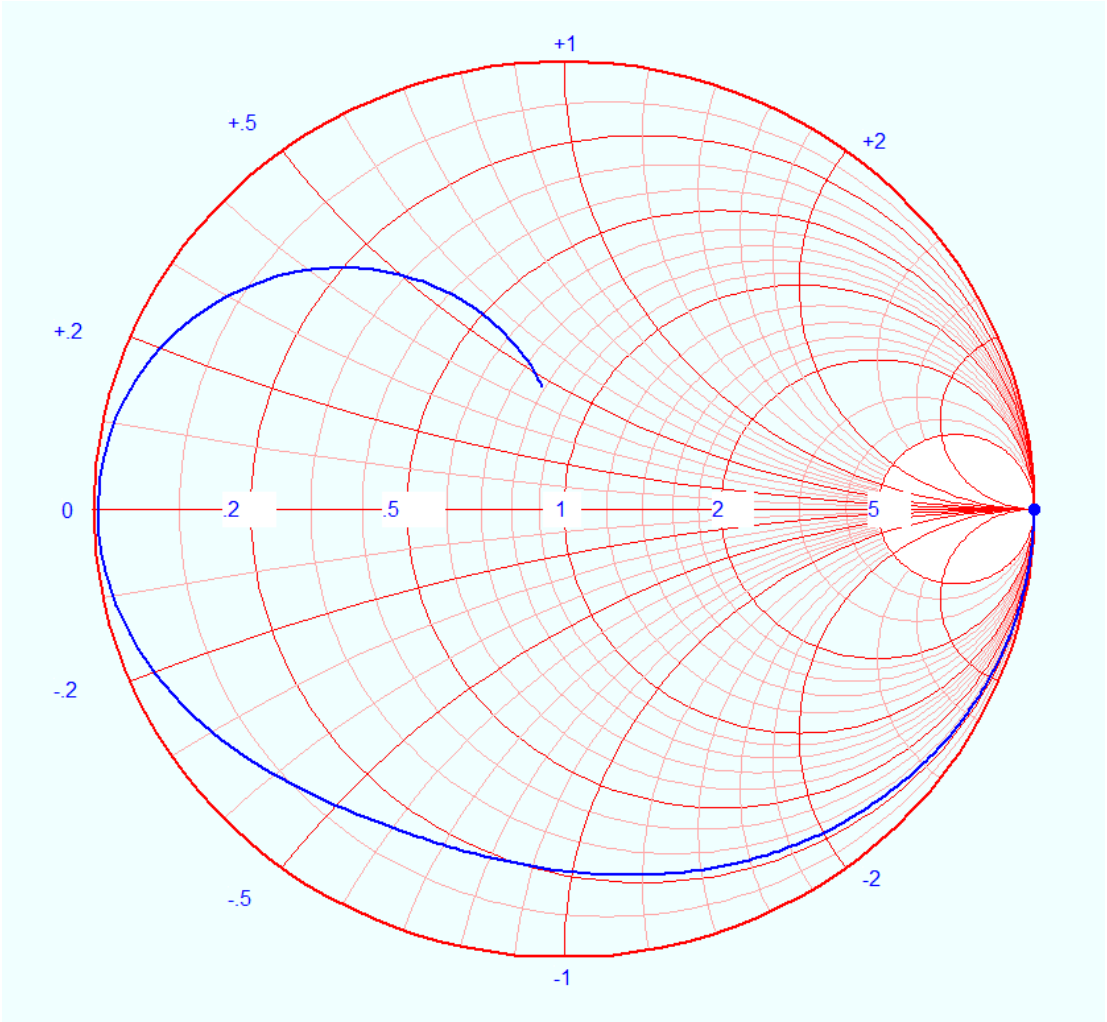


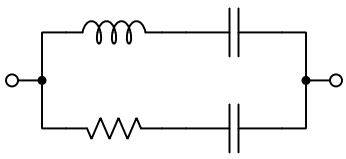
2.2.11 Series LC in parallel with R network



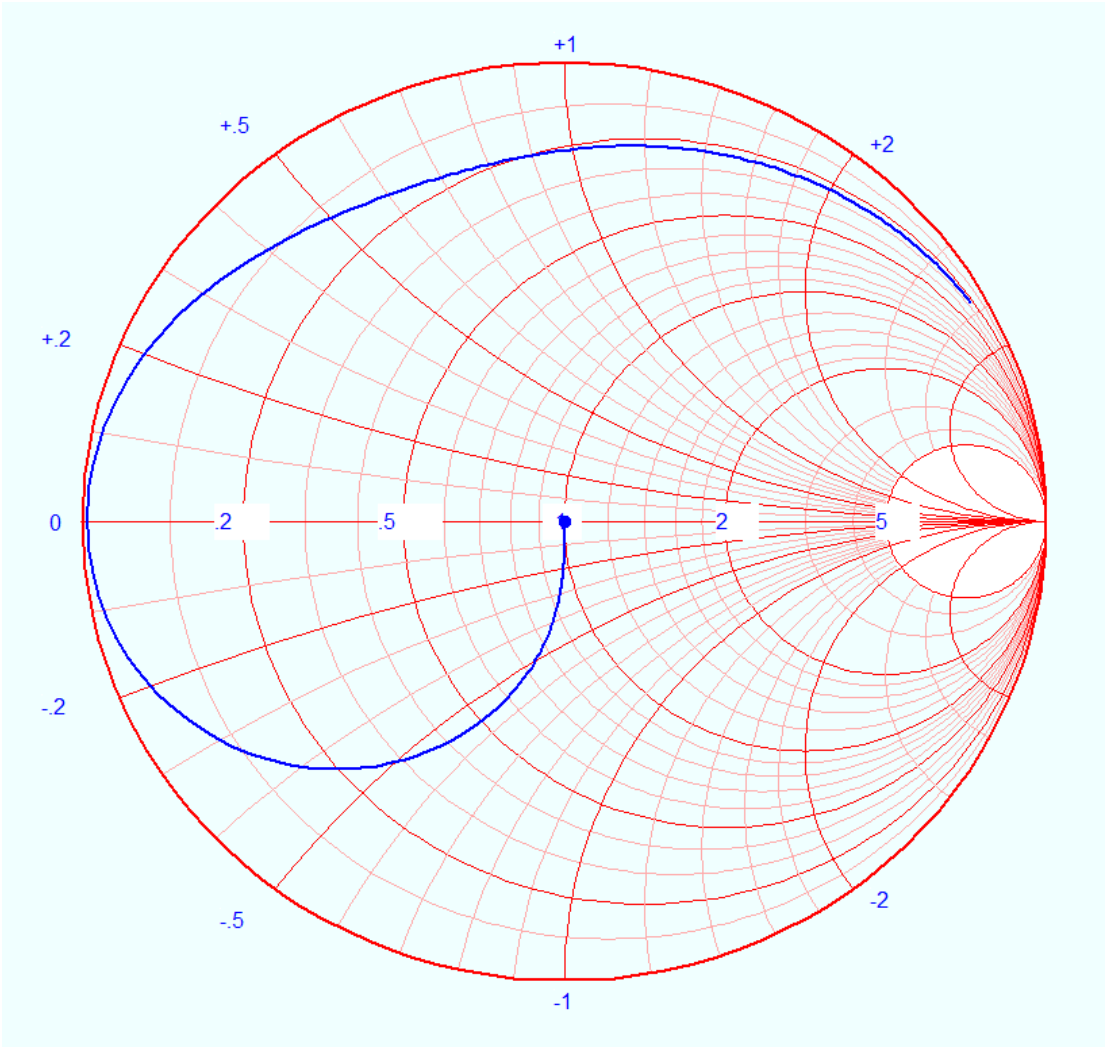


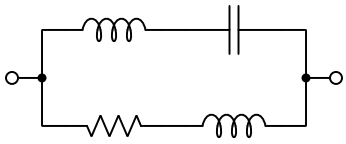
2.2.12 Series LC in parallel with series RC network





2.2.13 Series LC in parallel with series RL network





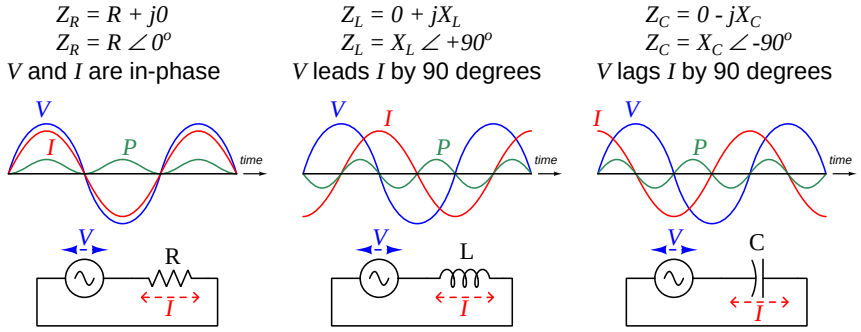
Chapter 3

Tutorial

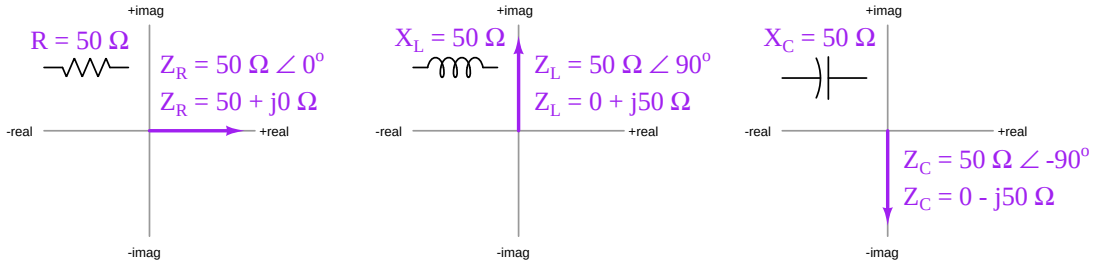
3.1 Impedance review

When analyzing any AC circuit, it is not enough to simply quantify every voltage and current in terms of amplitude (e.g. how many Volts or Amperes) as we do in DC circuits. Instead, we must consider both the amplitude of each signal as well as the amount of phase shift separating them. A mathematically elegant way of accomplishing both is to use *complex numbers* which may be expressed either in rectangular form (e.g. $a + jb$) or polar form (e.g. $m\angle\theta$). When we use complex numbers to represent any AC circuit quantity, we call that value a *phasor*.

An illustrative example is how we characterize the *impedance* of passive components. Resistors function by dissipating energy in the form of heat, with voltage and current waveforms being perfectly in-phase. Inductors and capacitors, however, function by absorbing and releasing energy rather than dissipating, and as such we find voltage and current waveforms shifted by one-quarter of a cycle (i.e. 90^o) for each, voltage leading current for an inductor and voltage lagging current for a capacitor:



Graphical expressions of complex-number resistance and reactance values are called *phasor diagrams*. Three such diagrams illustrate the difference between a 50 Ω resistor versus an inductor having 50 Ω of reactance and a capacitor also having 50 Ω of reactance. Each has 50 Ohms of impedance, but each of these impedances has a different phase angle defined by the phase shift between the component's voltage and current:



The ratio of voltage to current in a DC network is resistance ($R = \frac{V}{I}$), and in an AC circuit where phase shifts exist is impedance ($Z = \frac{V}{I}$). Both are measured in unit of the Ohm (Ω). The ratio of current to voltage is the reciprocal of these quantities: for DC networks we call it *conductance* ($G = \frac{I}{V}$) and for AC it is known as *admittance* ($Y = \frac{I}{V}$). We measure both in the unit of *Siemens*[1] (S).

Some examples[2] of component impedances and admittances are shown here:

- A 570 Ω resistor at any frequency will have the following impedance and admittance values:

  $Z = 570\ \Omega \angle 0^o$ *(polar form)* $= 570 + \text{j}0\ \Omega$ *(rectangular form)*

  $Y = 0.0017544\ \text{S} \angle 0^o$ *(polar form)* $= 0.0017544 + \text{j}0\ \text{S}$ *(rectangular form)*

- A 3.5 H inductor at a frequency of 120 Hz will have the following impedance and admittance values:

  $Z = 2.639\ \text{k}\Omega \angle +90^o$ *(polar form)* $= 0 + \text{j}2.639\ \text{k}\Omega$ *(rectangular form)*

  $Y = 0.00037894\ \text{S} \angle -90^o$ *(polar form)* $= 0 - \text{j}0.00037894\ \text{S}$ *(rectangular form)*

- A 0.01 μF capacitor at a frequency of 3 kHz will have the following impedance and admittance values:

  $Z = 5.305\ \text{k}\Omega \angle -90^o$ *(polar form)* $= 0 - \text{j}5.305\ \text{k}\Omega$ *(rectangular form)*

  $Y = 0.00018850\ \text{S} \angle 90^o$ *(polar form)* $= 0 + \text{j}0.00018850\ \text{S}$ *(rectangular form)*

The utility of phasor representation in AC circuits is that with all signal and component values expressed in phasor form we find most of the foundational principles learned for DC circuit analysis still apply in AC circuits. Quantities that add in series DC networks (e.g. voltage V, resistance R) add as phasor quantities in AC networks (e.g. voltage V, impedance Z); additive quantities in parallel DC networks (e.g. current I, conductance G) add as phasor quantities in AC networks (e.g. current I, admittance Y). With phasor quantities, Ohm's Law, Kirchhoff's Voltage Law, and Kirchhoff's Current Law still hold true in AC networks just as they do for DC.

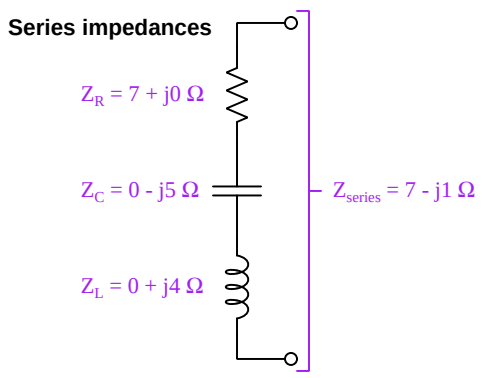
[1] Prior to the adoption of German engineer Werner von Siemens' surname as the unit of measurement for conductance and admittance, the unit of the *Mho* served quite well. This, of course, was a sort of pun on the spelling of *Ohm*, since "mho" is "ohm" spelled backwards, intended to represent the fact that the reciprocal of any Ohm value yields a value in Mhos.

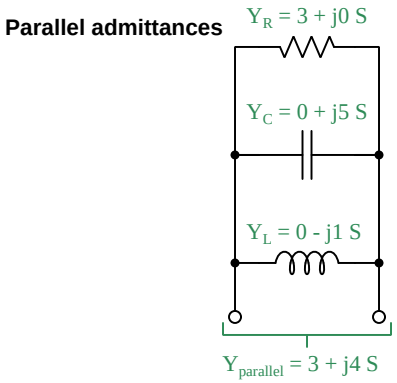
[2] Try calculating these impedance and admittance values from the given component values, to check your understanding. This is a good learning strategy to apply when reading any mathematical text: work through the presented examples on your own to see if you achieve the same results! Please note that when you apply either the $X_L = 2\pi fL$ formula or the $X_C = \frac{1}{2\pi fC}$ formula using your calculator to compute reactance, the result will *only* be a reactance value and not a (complex) impedance value. In order to attach the desired phase angle to your computed reactance value, you will have to perform the additional step of multiplying that reactance by a *unit phasor* which is nothing more than the quantity of 1 with the correct phase angle. For example, a capacitive reactance of 5.305 kΩ would be multiplied by $1 \angle -90^o$ to yield a capacitive *impedance* of 5.305 k$\Omega \angle -90^o$.

It is equally valid to express any phasor quantity in either polar or rectangular form. However, unless we have access to an electronic calculator capable of performing complex-number arithmetic, we find certain arithmetic operations much easier to perform with one notation more than the other. Specifically, addition and subtraction are simplest when phasors are in rectangular form, while multiplication and division are simplest when phasors are in polar form.

Addition of rectangular-form complex numbers consists simply of adding their real components together to find the real component of the sum, and doing the same with the imaginary components. Expressing this algebraically, $(a + jb) + (x + jy) = (a + x) + j(b + y)$. Subtraction follows much the same pattern: $(a + jb) - (x + jy) = (a - x) + j(b - y)$.

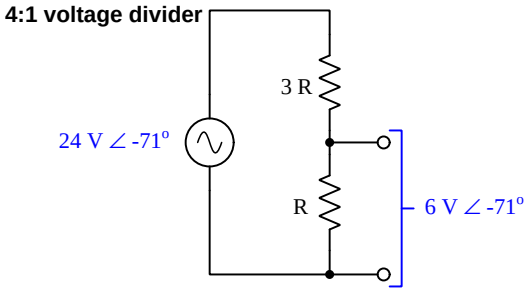
Here are some practical examples of rectangular-form phasor arithmetic where the calculations are simple enough to perform without a calculator:

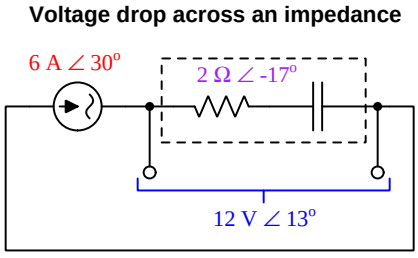




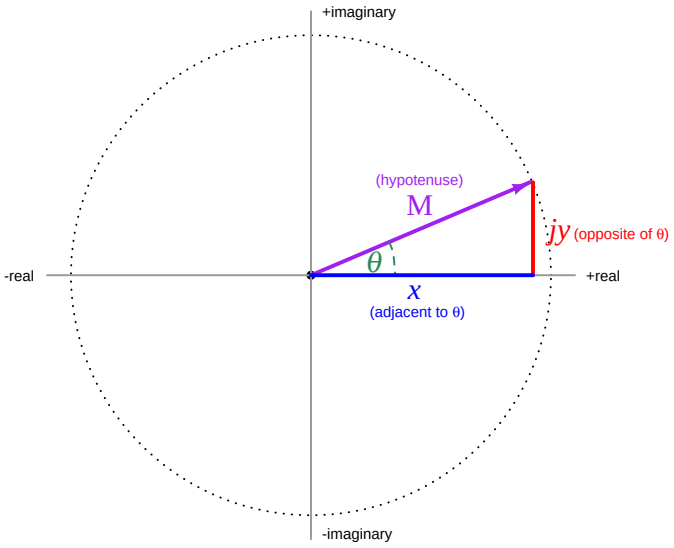
Multiplication of polar-form complex numbers consists simply of multiplying their magnitudes together to find the magnitude of the product, and adding the angles to find the angle of the product. Expressing this algebraically, $(a\angle b) \times (x\angle y) = (a \times x)\angle(b + y)$. Division follows a similar pattern: $(a\angle b) \div (x\angle y) = (a \div x)\angle(b - y)$.

Here are some practical examples of polar-form phasor arithmetic where the calculations are simple enough to perform without a calculator:





When analyzing AC circuits without the use of a complex-number calculator, we invariably must convert between rectangular and polar forms in order to prepare the phasor values for addition/subtraction or multiplication/division, respectively. Both conversions are most easily understood in terms of a right triangle, seeing the rectangular form's real and imaginary components as the adjacent and opposite sides, and the polar form's magnitude and angle as the hypotenuse:



Converting rectangular $(x + jy)$ into polar $(A\angle\theta)$:

$$A = \sqrt{x^2 + y^2} \qquad \theta = \arctan \frac{y}{x}$$

Converting polar $(A\angle\theta)$ into rectangular $(x + jy)$:

$$x = A \cos \theta \qquad y = A \sin \theta$$

Some cautionary notes are in order here. First, I highly recommend storing all computed values in your calculator's memory rather than re-entering them manually, because you will find even slight rounding errors tend to become exaggerated with trigonometric functions. Second, when computing the phase angle (θ) from real and imaginary quantities (x and jy) be careful to verify the angle against your qualitative expectations. For example, $5 + j5 = 7.071\angle 45^o$ and $-5 - j5 = 7.071\angle 225^o$, but you'll find arctan $\frac{-5}{-5}$ yields the same result (45^o) as arctan $\frac{5}{5}$ because $\frac{-5}{-5} = \frac{5}{5}$. To put it simply, the arc-tangent function does not "know" whether the phasor exists in the first or in the third quadrant of the complex plane.

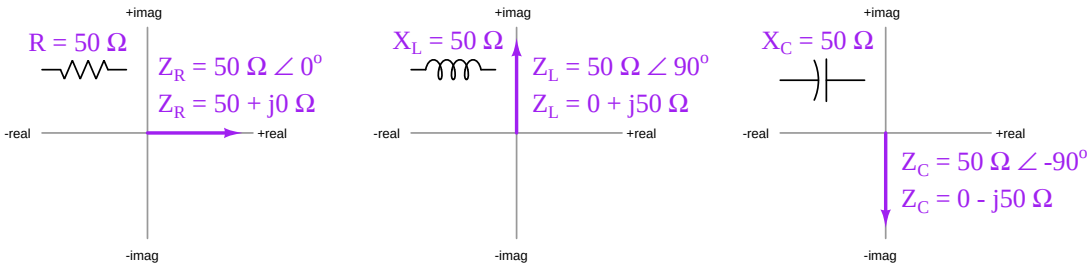
Here are some rectangular and polar equivalents, useful for practice as you master these concepts:

$$20 - j11 = 22.83\angle -28.81^o \qquad 11.49 + j9.642 = 15\angle 40^o$$

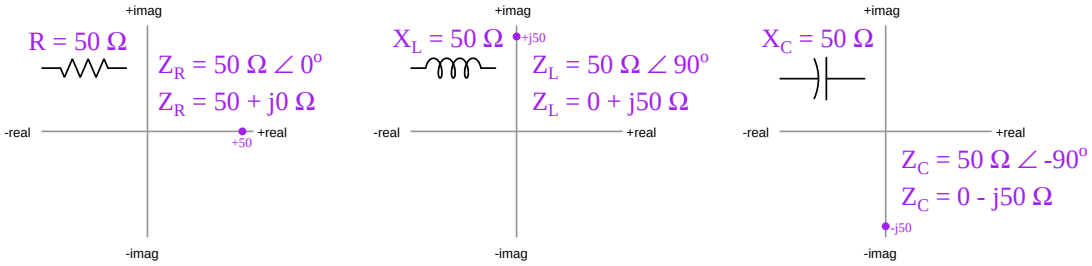
$$-10 + j2 = 10.20\angle 168.7^o \qquad -11.82 - j2.084 = 12\angle -170^o$$

3.2 Phasor diagram review

Graphical representations of complex-number AC quantities are generally known as *phasor diagrams*. Simple examples showing resistive and reactive component impedances – each one having a value of 50 Ohms but with different phase angles – appears below:



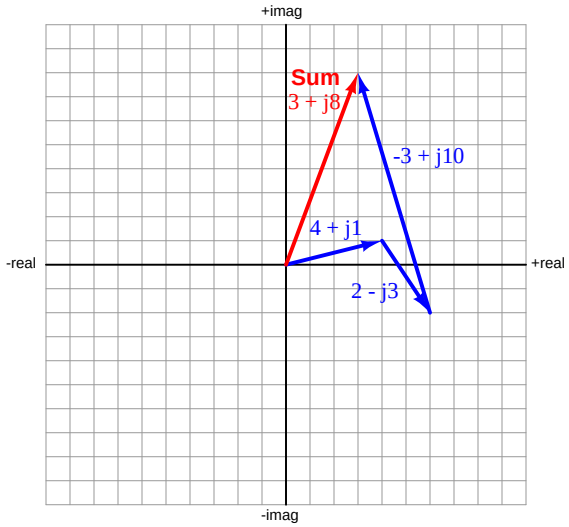
For such simple examples it is entirely appropriate to represent each complex quantity as a *point* on the complex plane rather than a *vector* (i.e. an arrow symbol) like this:



The horizontal axis represents the *real* component of the complex value, while the vertical axis represents the *imaginary*. This means the complex number in rectangular form plainly describes the horizontal and vertical coordinate values, which is why $50 + j0$ places the dot along the horizontal axis to the right of center, why $0 + j50$ places the dot along the vertical axis above center, and why $0 - j50$ places the dot along the vertical axis below center.

Vector-arrows are generally more useful than dots because the vectors allow us to easily show *addition* and/or *subtraction* of complex-number quantities. This is rather useful for any aspect of AC circuit analysis where addition and/or subtraction are appropriate, e.g. impedances connected in series or admittances connected in parallel. To graphically add two vectors together, all we need to do is stack them end-to-end, the tip of one touching the tail of the other.

In the following example we see the phasors $4 + j1$, $2 - j3$, and $-3 + j10$ (all three shown in blue) added to one another to produce the sum $3 + j8$ (shown in red):

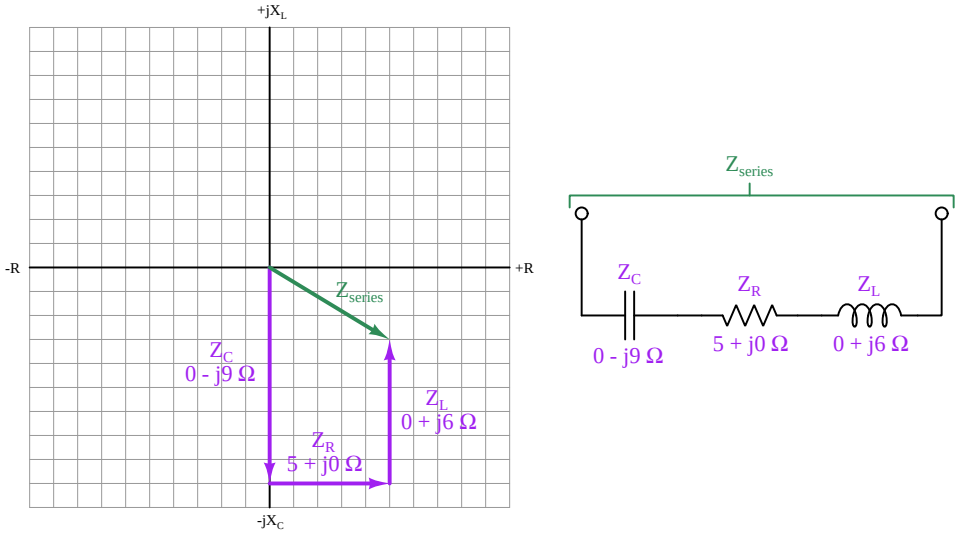


The rectangular form of these phasors makes it easy to see how the addition works. The $4 + j1$ phasor's arrow, for example, projects four units to the right and one unit up. The next phasor, $2 - j3$, begins at the tip of the $4 + j1$ phasor and then projects two units right and three units down. Lastly, the $-3 + j10$ phasor begins at the tip of the $2 - j3$ phasor and then projects three units left and ten units up.

Our sum of $3 + j8$ is easy enough to calculate simply by summing all the phasors' real components and summing all the phasors' imaginary components: $4 + 2 - 3 = 3$ and $j1 - j3 + j10 = j8$. However, it should also be clear to see that the phasor diagram provides a way for us to *graphically compute* the sum as well: simply start at the origin and count how many horizontal and vertical units are necessary to arrive at the last arrow tip. In this capacity the graph is not just a tool for displaying quantities, but actually a *calculator* for determining an arithmetic result. We just need to carefully draw each phasor's representative arrow, and then the resulting sum may simply be read from the graph.

Extending this principle to a non-trivial calculation, suppose we wished to know our sum of $3 + j8$ as a *polar*-form value rather than rectangular. Of course, we could reach for an electronic calculator to apply the Pythagorean Theorem for calculating polar magnitude ($\sqrt{3^2 + 8^2} = 8.544$), and the arc-tangent function for calculating polar phase angle ($\arctan \frac{8}{3} = 69.44^o$), but we could just as readily reach for a ruler and a protractor to measure the length and angle (from horizontal) for the sum's arrow. Such analog methods of computation are conceptually simple, and for many applications the inaccuracies incurred by sketching and measuring arrow symbols on paper are well within acceptable tolerances for circuit analysis.

Here we will use a phasor diagram to add impedances for a network consisting of a series-connected capacitor, resistor, and inductor:

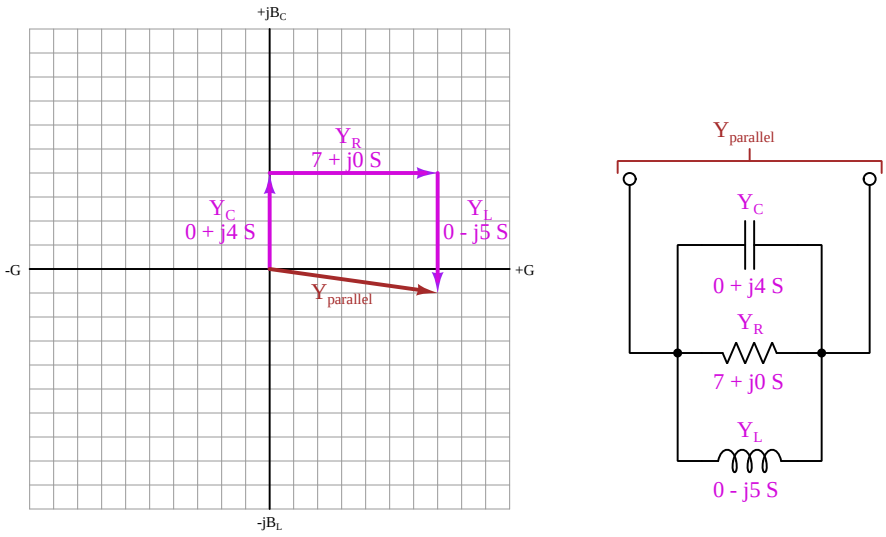


It is clear both from simple arithmetic as well as from counting squares on the graph that Z_{series} must be equal to $5 - j3$ Ohms. As before, we could also use a ruler and protractor to measure the polar dimensions of Z_{series} as an alternative to trigonometric calculations.

Another benefit of phasor diagrams is that they make certain abstract concepts more visually apparent, *series resonance* being one such example. If we imagine increasing the AC energizing frequency until the inductor's reactance rises to meet the capacitor's falling reactance[3], we shall see that those two components' imaginary (vertical) quantities will perfectly cancel each other leaving just a horizontal phasor of $5+j0$ Ohms. Seeing that those two phasors' arrows point exactly opposite each other (inductor up and capacitor down) makes it perfectly obvious that their sum must be zero at some frequency where $X_L = X_C$, leaving only the resistor to contribute to total impedance.

[3]Recall that inductive reactance is directly proportional to frequency, $X_L = 2\pi f L$ and that capacitive reactance is inversely proportional to frequency, $X_C = \frac{1}{2\pi f C}$.

Here we will use a phasor diagram to add parallel admittances[4]:



Once again we see the sum is easy to determine for this network just as it is for any application where the quantities in question are additive, either by applying simple arithmetic or by counting squares on the graph. In this particular case, $Y_{parallel} = 7 - j1$ Siemens. Perhaps the only confusing aspect of this phasor diagram is the reversal of sign and vertical direction for the capacitive and inductive phasors: inductive admittance always has a negative value (pointing down) while capacitive admittance is always positive (pointing up).

As with the series phasor diagram, the phenomenon of *resonance* becomes easy to visualize for this parallel network. If we imagine frequency increasing such that the inductor's admittance falls while the capacitor's admittance rises, there must be a particular frequency where their admittances are equal in magnitude and exactly opposite in direction. In that instance the vertical contributions of Y_C and Y_L will precisely cancel (i.e. zero Siemens for the LC combination, which is equivalent to infinite Ohms between those two components) leaving only the resistor's admittance for current to pass through from one side of the network to the other.

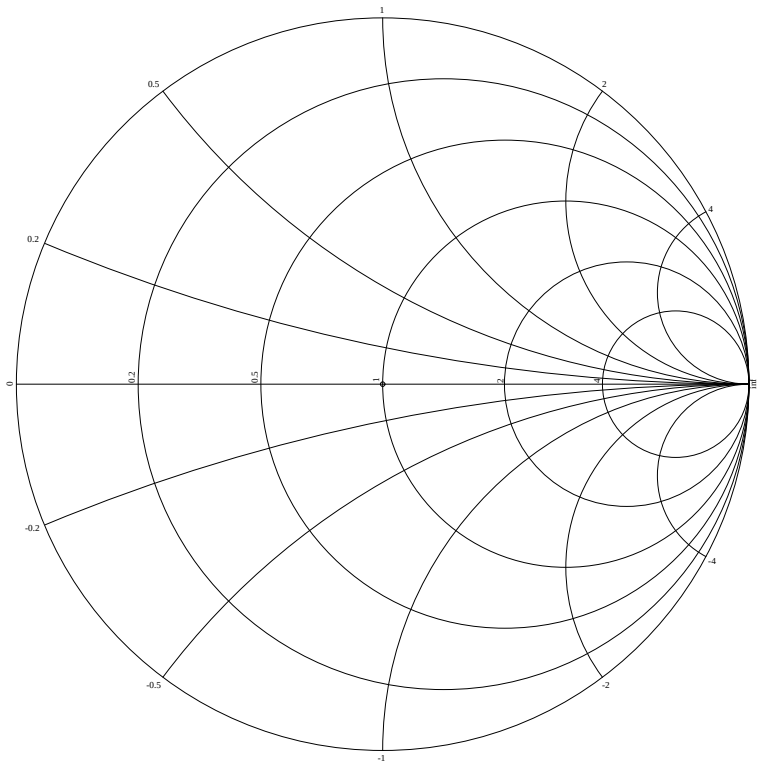
[4]Recall that *admittance* is just the reciprocal of impedance ($Y = \frac{1}{Z}$), measured in the unit of *Siemens* (or *Mhos*) rather than Ohms. In fact, if you apply this concept to the familiar parallel impedance formula ($Z_{parallel} = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_R} + \frac{1}{Z_L}}$) you will see that the parallel sum of admittances ($Y_{parallel} = Y_C + Y_R + Y_L$) is really the same formula after reciprocating all the terms.

3.3 The basic Smith Chart

The *Smith Chart* is a specialized form of graph useful for representing complex impedances in electrical and electronic networks. At root it is just an adaptation of the standard graph used to show phasor diagrams, where the horizontal axis is "real" and the vertical axis is "imaginary", but differs from that Cartesian-style graph in three principal ways:

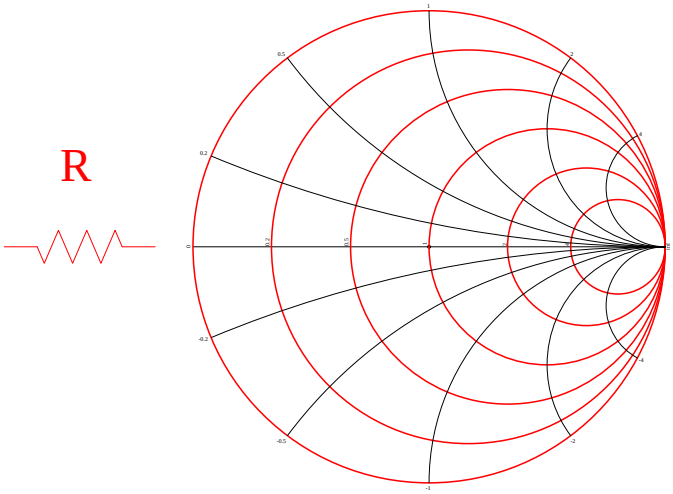
- A Smith Chart's axes nonlinearly span from zero to infinity
- Values plotted on a Smith Chart are typically *normalized* to some common standard such as 50 Ω (e.g. 50 Ω = 1.0 on the Chart, 100 Ω = 2.0 on the Chart, 10 Ω = 0.2 on the Chart, etc.)
- A Smith Chart uses *circles* rather than *lines* to form the grid

Of these differences, the last (circular grid) generally creates the most confusion when first apprehending a Smith Chart. A simple version of a Smith Chart appears below:



Here we see a set of right-aligned circles situated along the horizontal axis, as well as sets of arcs above and below that axis representing portions of circles. These circles and arcs constitute the "grid lines" of the Smith Chart, analogous to the vertical and horizontal division lines of a Cartesian graph.

The full circles situated along the horizontal axis represent real-number intervals – different degrees of resistance (R)[5]:

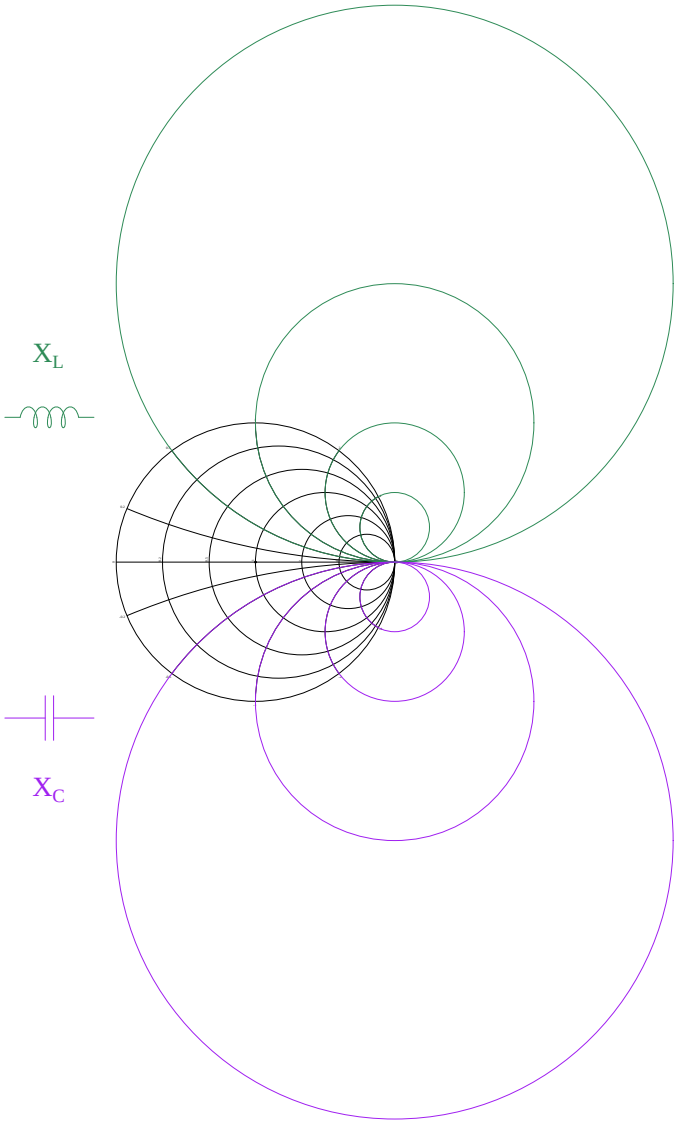


Any point falling along the outer-most circle's circumference represents zero Ohms of resistance. In a Cartesian graph the zero-resistance grid line would be the vertical line passing through the graph's origin, but here it is a circle forming the Smith Chart's outer boundary.

As resistance increases, points move to the right of the Smith Chart until we reach infinite resistance which is at the far-right end of the horizontal axis (i.e. a circle with a diameter of zero). The circle whose circumference passes through the Smith Chart's center point represents a normalized value of 1. For example, if the Smith Chart's normalizing value is 50 Ohms, a 50-Ohm resistance would fall along that circle passing through "1" on the horizontal axis. Exactly where along that circle's circumference the point exists depends on whether or not there is any reactance along with that resistance.

[5]The radius of each of these circles may be calculated by taking the radius of the entire Smith Chart and dividing by $1 + N$ where N is the normalized value.

The arcs lying above and below the horizontal axis are actually portions of whole circles, representing imaginary-number intervals – different degrees of either inductive or capacitive reactance (X)[6]:



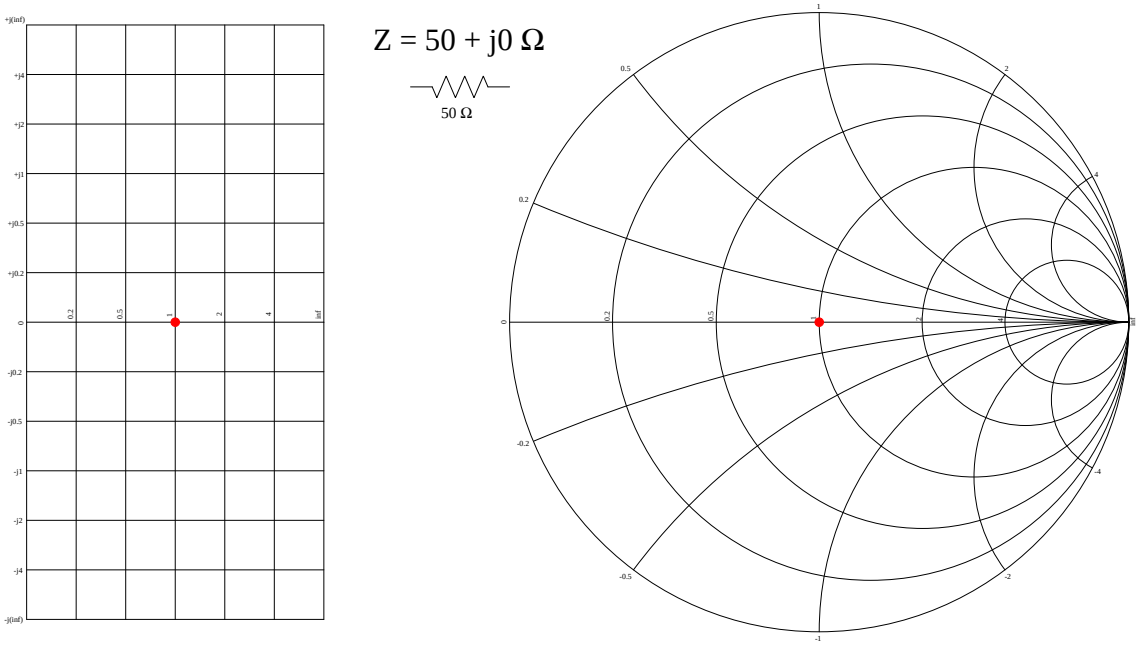
The horizontal axis itself is actually not a line, but rather a portion of a circumference for a circle with an *infinite diameter* representing zero Ohms of reactance.

For those familiar with complex impedances graphed on a Cartesian plane with real (horizontal)

[6] The radius of each of these circles may be calculated by taking the radius of the entire Smith Chart and dividing by the normalized value N.

and imaginary (vertical) axes in a rectangular-shaped grid, a good way to become oriented to the Smith Chart is to analyze side-by-side examples of each graph type for different example impedances. This is what we will do in the following subsections of this text.

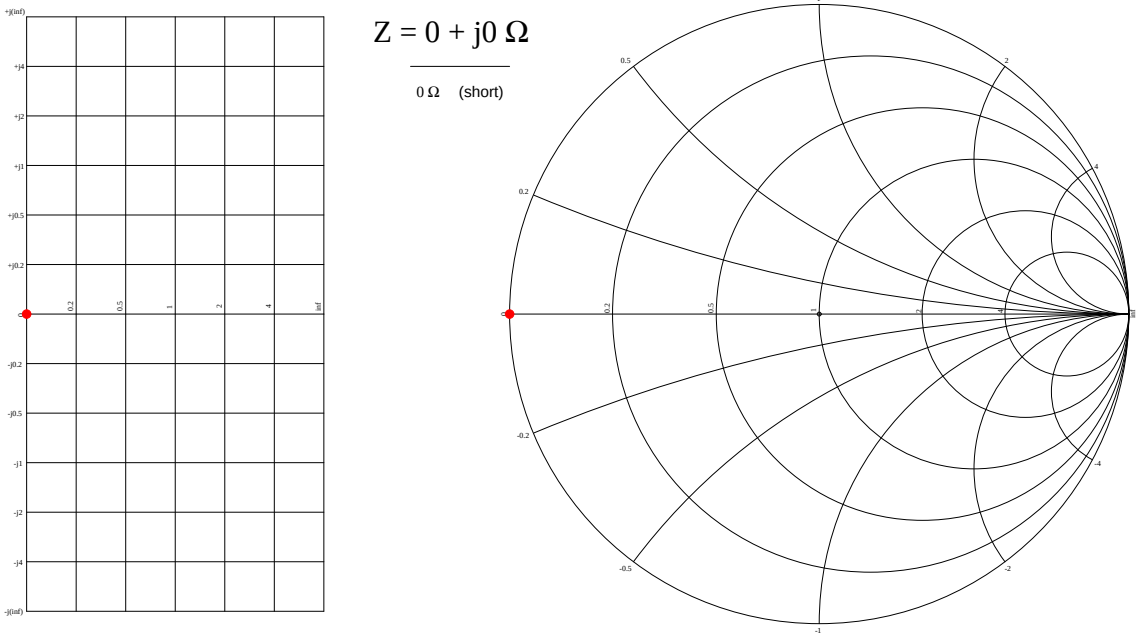
3.3.1 Plotting impedance for a 50 Ohm resistor



A pure resistance with a value of 50 Ohms results in the point (a red dot) situated along the "real" axis at a value of 1 (normalized to 50 Ohms). This happens to be at the center of the rectangular graph as well as at the center of the Smith Chart. It is the point at which the $R = 1$ resistance circle intersects the $X = 0$ reactance circle (the horizontal axis).

Increases in this resistance value will result in the plotted point moving toward the right (i.e. towards infinity), while decreases in this value move the point to the left (toward zero). There is no vertical displacement of the plotted point because in the case of a pure resistance there is no reactive characteristic. This is a circumstance where all energy is dissipated, and none stored or released. The location of the point should also remain fixed for any frequency value, since resistance (R) is independent of signal frequency.

3.3.2 Plotting impedance for a short

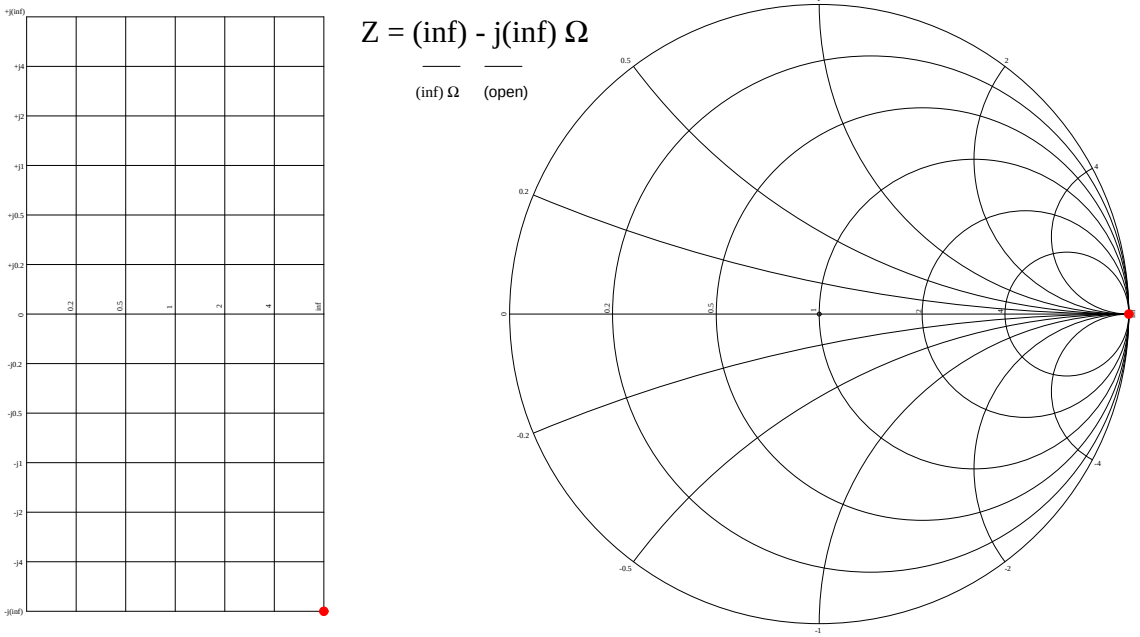


The guaranteed effect of a *short* is equipotentiality between the shorted points. This means no voltage regardless of current, which in turn means zero resistance ($R = \frac{V}{I}$). It also means no capacitance because the equipotential condition ensures no electric field can exist. Since current may exist through a short, and this will create a magnetic field which can store some energy, there may be inductance in a short. However, if the loop area is small then this inductance will be negligible, and therefore inductive reactance will for all practical purposes be zero ($X_L = 2\pi f L$).

Short-circuits, therefore, have zero Ohms of resistance and (nearly) zero Ohms of (inductive) reactance, placing the point fully to the left-hand end of the "real" axis. This is the point of intersection between the $R = 0$ resistance circle (outer circumference of the Smith Chart) and the $X = 0$ reactance circle (the horizontal axis).

As with the purely resistive example, this point's location is unaffected by changes in signal frequency.

3.3.3 Plotting impedance for an open

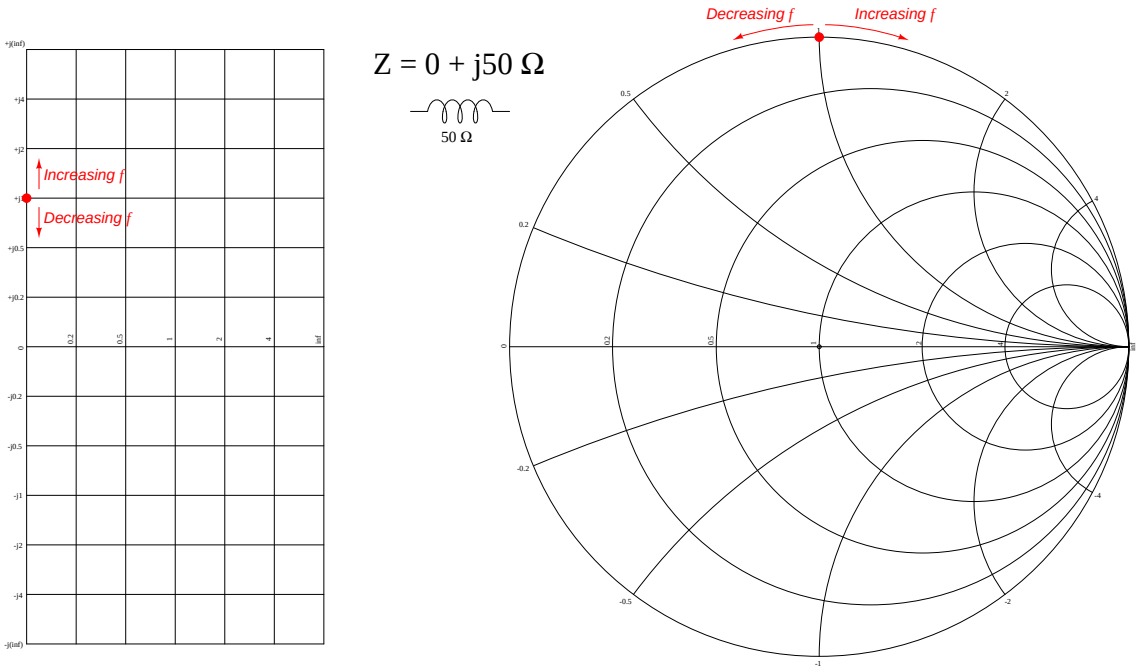


The guaranteed effect of an *open* is zero current. This in turn means infinite resistance ($R = \frac{V}{I}$). It also means no inductance because a condition of zero current is synonymous with zero magnetic field created around the opened conductor. An electric field may exist between points not connected to each other, though, and this grants the possibility of energy stored in that electric field. However, if the end-areas of the opened conductors are small this capacitance should be negligible. With negligible capacitance we know that capacitive reactance will for all practical purposes be infinite ($X_C = \frac{1}{2\pi f C}$).

Open circuits, therefore, have infinite resistance and (nearly) infinite (capacitive) reactance, placing the point fully to the right-hand end of the "real" axis. This is where the two smallest (i.e. zero-radius) resistance and reactance circles intersect.

As with the purely resistive example, this point's location is unaffected by changes in signal frequency.

3.3.4 Plotting impedance for a 50 Ohm inductor

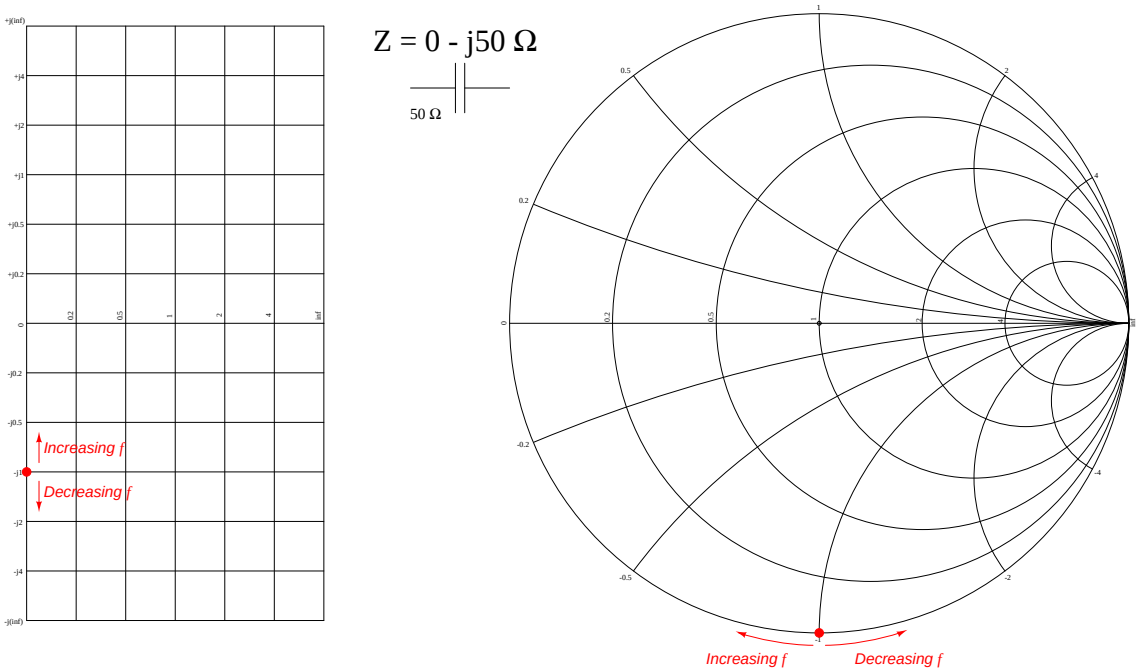


Inductors, of course, are rated in *Henrys* and as such do not have a fixed value in the unit of the Ohm. Inductive reactance is measured in Ohms, and relates to frequency by the formula $X_L = 2\pi f L$, which means for this 50 Ohm example we must suppose an inductance value (L) and frequency value (f) combining to make 50 Ohms of inductive reactance (X_L).

With a normalizing value of 50 Ohms for the entire Smith Chart, 50 Ohms of inductive reactance places the point at the intersection of the 0 resistance circle and the $+j1$ reactance circle. Mathematically, we may express this impedance as the product of 50 Ohms and a normalized complex value: $(50)(0 + j1)$ Ohms.

If frequency increases, this inductor's impedance will move clockwise along the $R = 0$ resistance circle's circumference, representing zero resistance and increasing inductive reactance. At the extreme limit-case of infinite frequency the point will move all the way to the far-right end of the horizontal axis where it is the same as an open circuit. Decreasing frequency moves the point counter-clockwise, ultimately toward zero Ohms of reactance (short).

3.3.5 Plotting impedance for a 50 Ohm capacitor



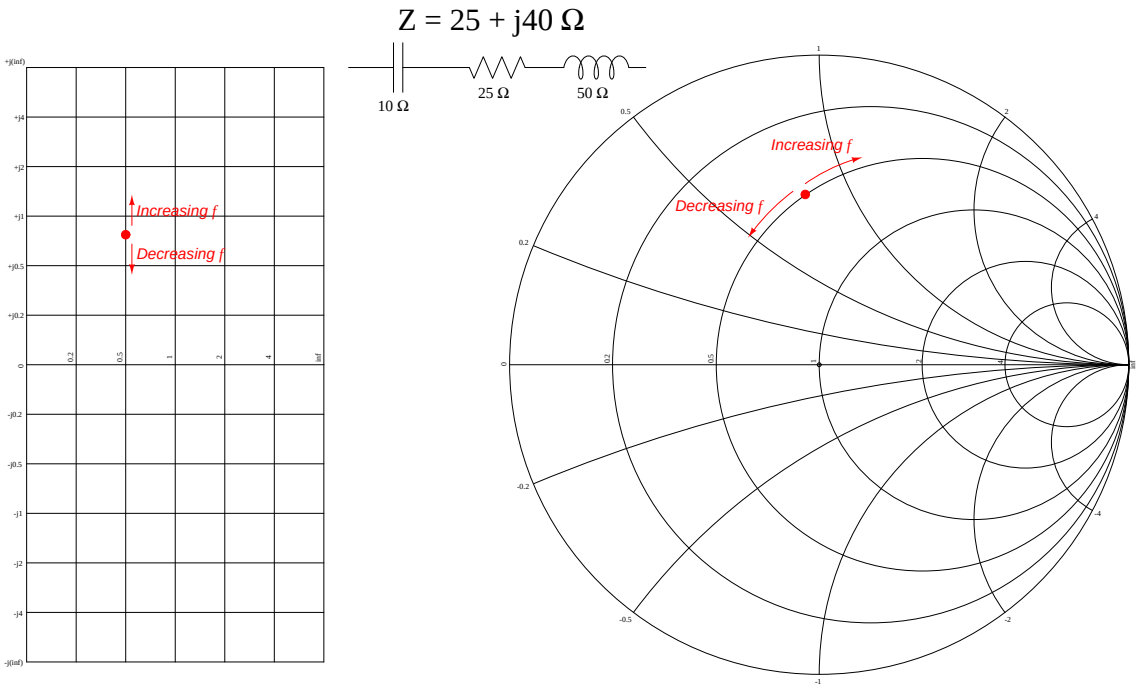
Capacitors are rated in *Farads* and just like inductors do not have a fixed value in the unit of the Ohm. Capacitive reactance is measured in Ohms, and relates to frequency by the formula $X_C = \frac{1}{2\pi fC}$, which means for this 50 Ohm example we must suppose a capacitance value (C) and frequency value (f) combining to make 50 Ohms of capacitive reactance (X_C).

With a normalizing value of 50 Ohms for the entire Smith Chart, 50 Ohms of capacitive reactance places the point at the intersection of the 0 resistance circle and the $-j1$ reactance circle. Mathematically, we may express this impedance as the product of 50 Ohms and a normalized complex value: $(50)(0 - j1)$ Ohms.

If frequency increases, this capacitor's impedance will move clockwise along the $R = 0$ resistance circle's circumference, representing zero resistance and decreasing capacitive reactance. At the limit-case of infinite frequency the point will move all the way to the far-left end of the horizontal axis where it is equivalent to a short circuit. Decreasing frequency does the opposite, moving the point toward infinite Ohms of reactance (open).

3.3.6 Plotting impedance for a series RLC network

Here, we combine a capacitor, resistor, and inductor in series to make a total impedance of $25 + j40$ Ohms:



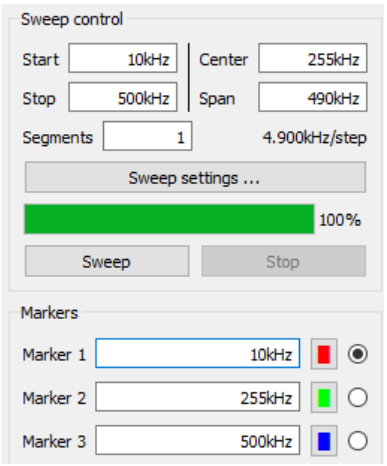
Normalizing the complex impedance $25 + j40$ Ohms against a default value of 50 Ohms yields $0.5 + j0.8$ on the Smith Chart. This places the point at the intersection of the 0.5 resistive circle and an (unplotted) $+j0.8$ reactance circle. In other words, the impedance value of $25 + j40$ Ohms is equivalent to $(50)(0.5 + j0.8)$ Ohms.

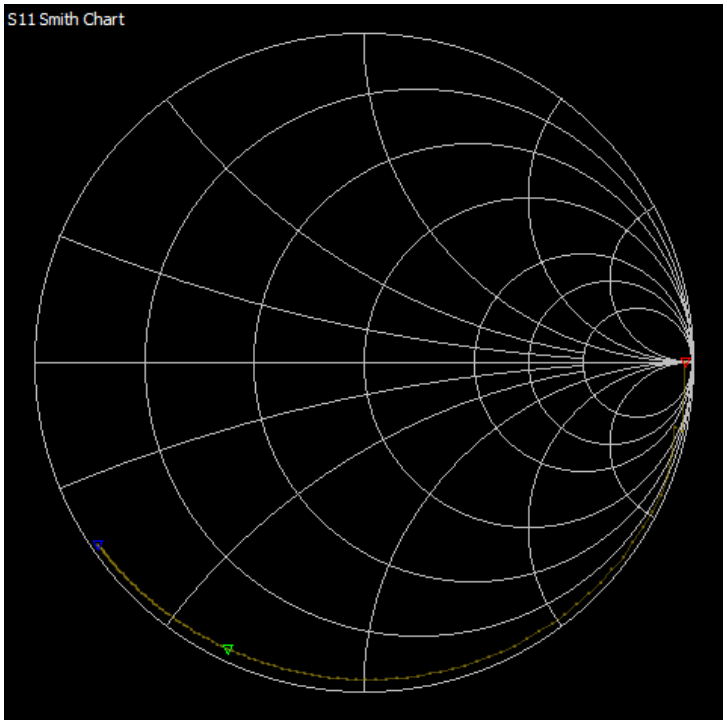
If frequency increases, the inductor's reactance will increase while the capacitor's reactance will decrease. This combined effect moves the point clockwise along the circumference of the 0.5 resistive circle. Decreasing frequency, of course, moves the point counter-clockwise along the same resistive circle. At the resonant frequency point where $X_L = X_C$ we will have a total series impedance of $25 + j0$ Ohms which will place the point at the intersection of the 0.5 resistive circle and the horizontal axis (zero Ohms reactance).

3.4 Frequency sweeps

Smith Charts are often used as a display mode on radio-frequency (RF) test instruments such as Vector Network Analyzers (VNAs) where the instrument energizes the network with a variable-frequency excitation signal and measures the response. *Sweeping* the signal frequency produces a range of results rather than a single coordinate, due to the reactance of all inductors and/or capacitors changing as a function of signal frequency.

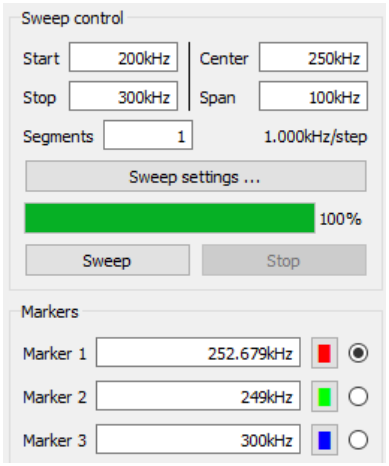
The following screenshot shows the results of testing the impedance of a 20 nanoFarad capacitor across a range of frequencies starting with 10 kHz and ending with 500 kHz using a vector network analyzer:

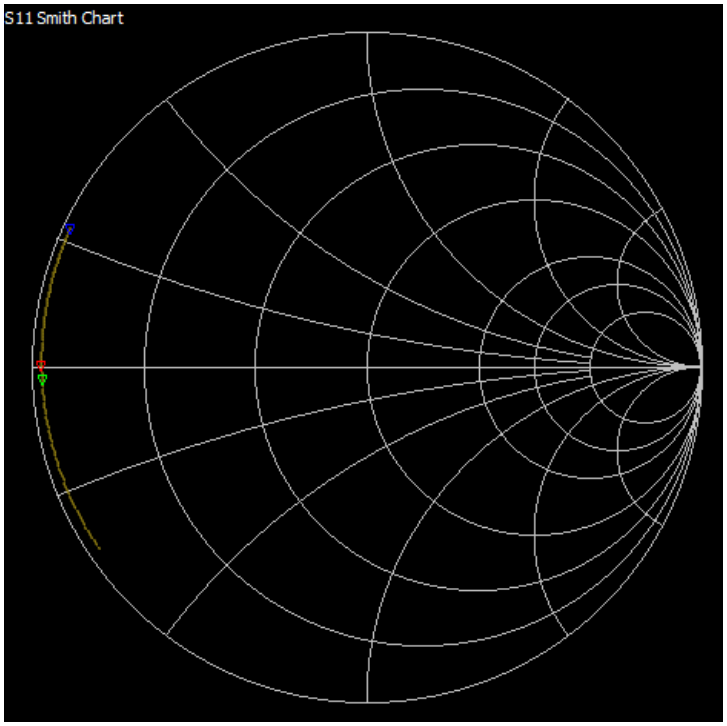




Three colored *markers* indicate specific frequency values within the 10-500 kHz sweep range: red at the starting frequency of 10 kHz, green at 255 kHz, and blue at the ending frequency of 500 kHz. Note that the resulting curve remains fairly close to the outer circumference of the Smith Chart, which is what we would expect for any component having reactance but little resistance. At 10 kHz we would expect a 20 nF capacitor to exhibit approximately 796 Ohms of reactance (i.e. 15.9 normalized to 50 Ohms), which is why the red marker appears very near the "infinite" reactance circle. At 500 kHz that same 20 nF capacitor should exhibit close to 15.9 Ohms of reactance (i.e. 0.318 normalized to 50 Ohms), placing the blue marker between the $-j0.2$ and $-j0.5$ reactance circles.

Next we see a series inductor-capacitor network tested with a VNA over a range of 200 kHz to 300 kHz. The inductor in this case has a nominal value of 20 μH and the capacitor a nominal value of 20 nF, yielding a theoretical resonant frequency of 251.6 kHz. The blue marker rests at the ending sweep frequency of 300 kHz, but the red and green markers have been moved away from their original positions in order to close in on the resonant point where the sweep curve crosses the horizontal axis of the Smith Chart (i.e. resistance with no reactance). Here the green marker is at 249 kHz and the red marker at 252.679 kHz:



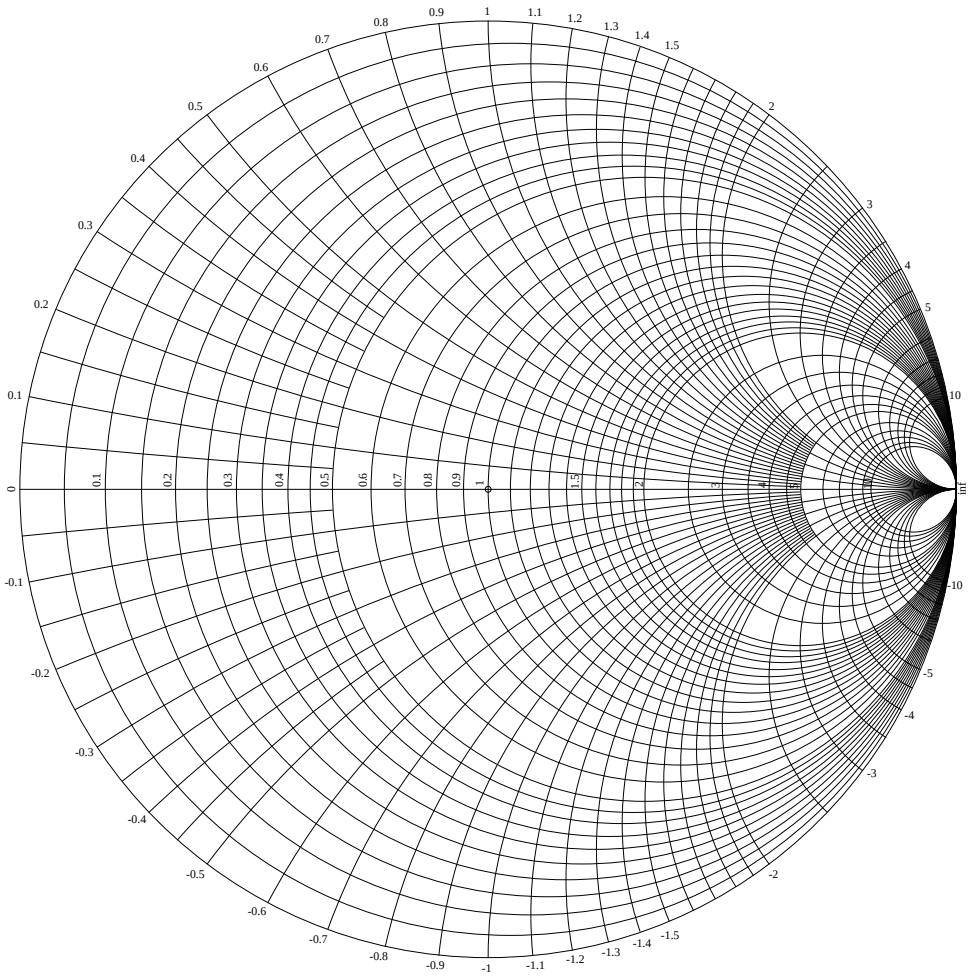


In case you were wondering, the `S11 Smith Chart` label appearing on the display of this VNA shows us the *S-parameter* perspective of this particular test. In network testing parlance, every network has different *ports* into which signals may be injected and/or signals may be measured from. "S11" simply means[7] the signal was applied to port 1 and the result of that signal was measured at port 1.

[7] By convention, the second subscript refers to the port where the signal is injected or applied, while the first subscript denotes the port where the signal is measured. For example, S21 on a two-port network means the VNA injected the excitation signal into port 1 and measured the resulting output on port 2.

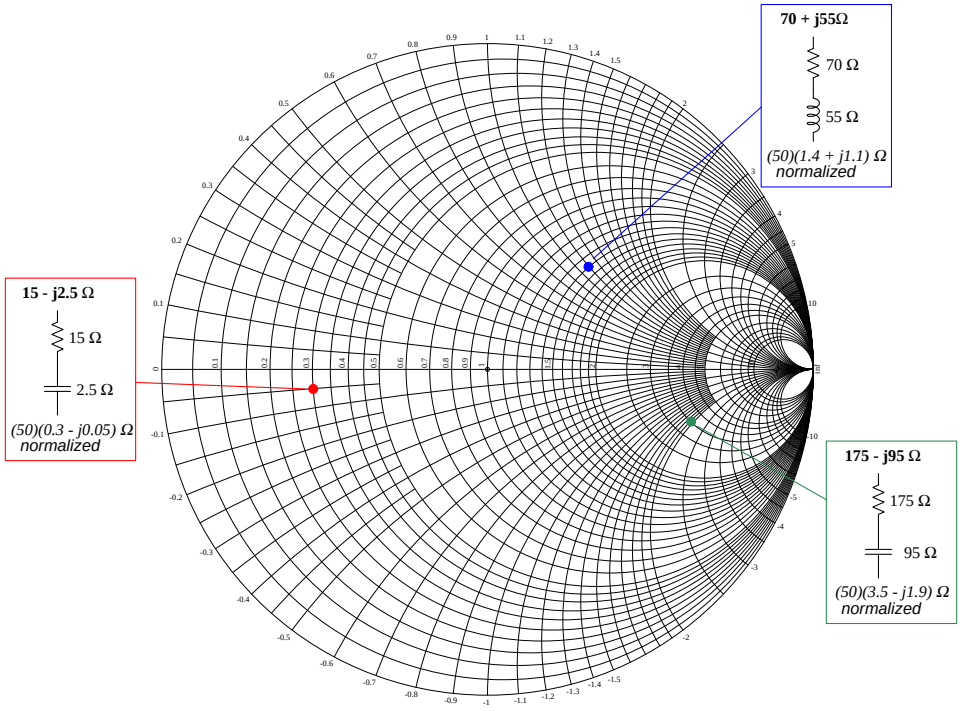
3.5 Fine-resolution Smith Charts

The simple Smith Chart format used in the previous section is suitable for conveying basic concepts, but is generally too coarse for many practical applications. In order to provide finer resolution, we generally find Smith Charts with many more circles, such as the following example:



Note how many of the reactance circles do not pass through the infinite ("inf") point at the far-right end, solely to prevent all the curves from becoming so tightly converged that they would create a solid mark on the page rather than individually-readable curves. As the linear scaling becomes more compressed toward the right-hand side of the Smith Chart, we eliminate more of the circles to create more "whitespace" on that portion of the page.

To illustrate the utility of these finer-resolution circles, we will plot a few more complex-impedance network values with greater precision than we could using the simpler Smith Chart introduced earlier. As usual, all impedance values are normalized to a 50-Ohm standard:

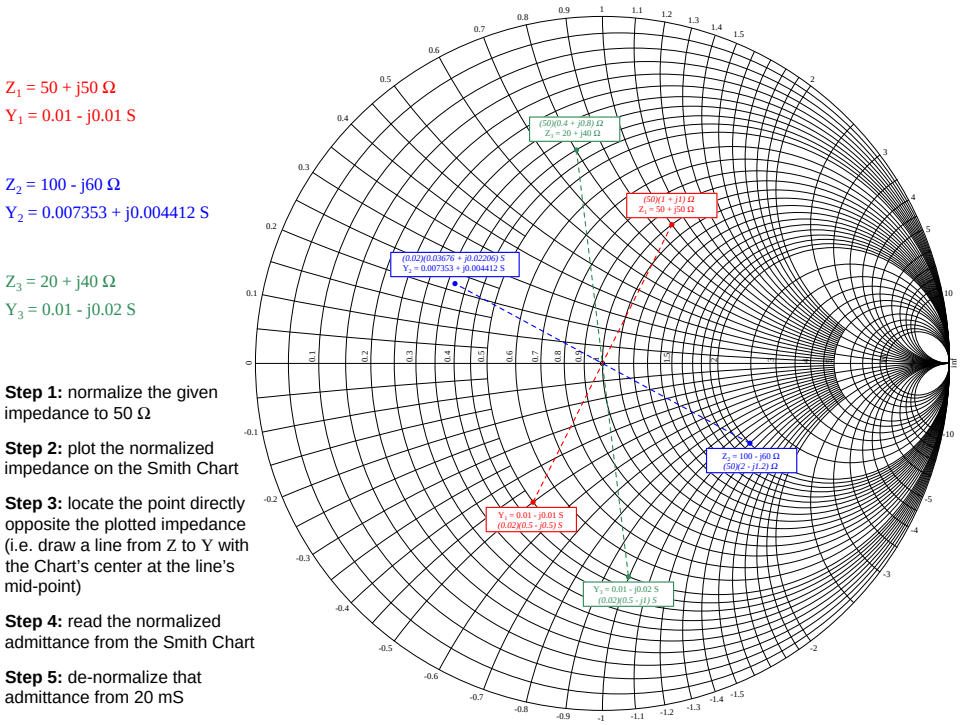


3.6 Impedance and admittance Smith Charts

A Smith Chart, at its most basic, is nothing more than a specialized graph for plotting complex-number coordinates. Although we usually see Smith Charts used to plot impedance (Z), they are perfectly appropriate for plotting *admittance* values (Y) as well.

Recall that admittance is nothing more than the reciprocal of impedance, and just like *conductance* (the reciprocal of resistance), admittance is measured in the unit of *Siemens*. For example, a 50 Ohm impedance is equivalent to 20 milliSiemens of admittance. This means that if an impedance Smith Chart is normalized to 50 Ohms, then the corresponding admittance Smith Chart must be normalized to 20 milliSiemens.

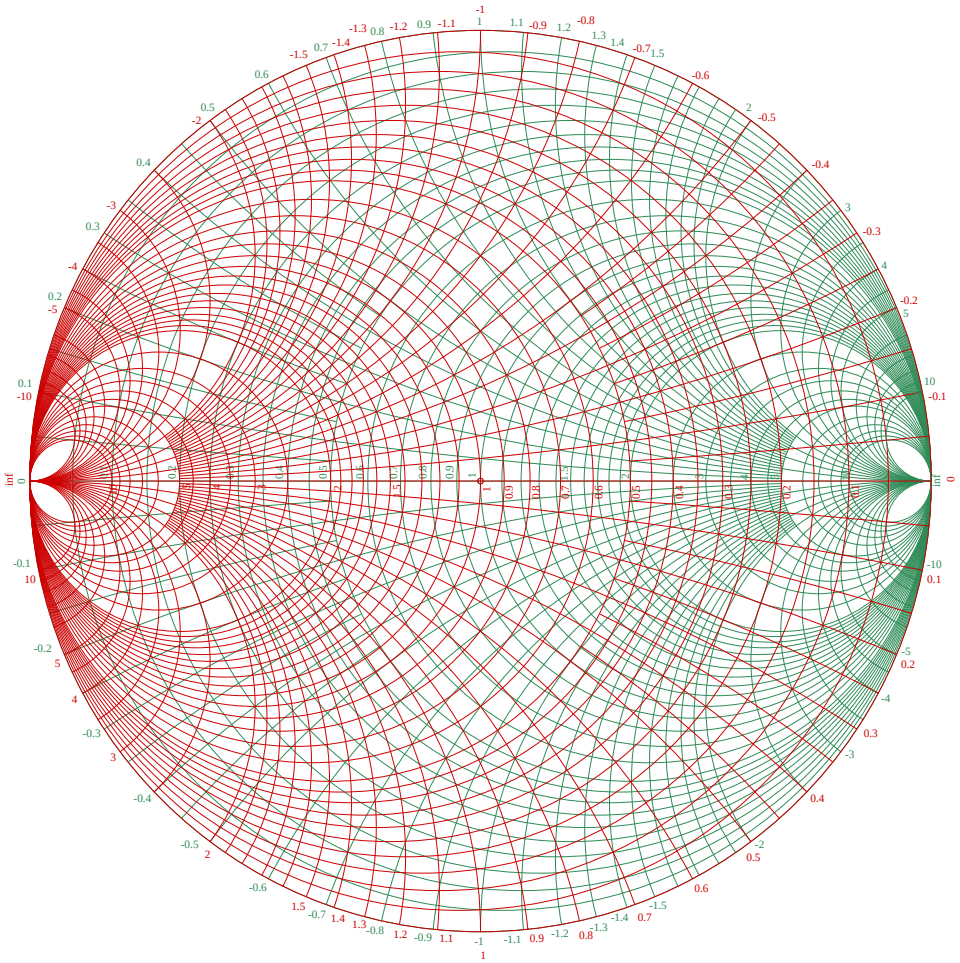
If we plot arbitrary impedance values on a Smith Chart, and then compute[8] the corresponding admittance values on the same Chart, we notice an interesting phenomenon:



Note how each impedance-admittance pair has points equidistant from the Smith Chart's center. The Chart's nonlinear scales and circular graduations are key to this property, allowing the Smith Chart to be used as a *graphical calculator* for conversions between impedance and admittance. In fact, the most troublesome aspect of these conversions are the normalization and de-normalization steps. As with all demonstrations of mathematical techniques, it is highly advisable for you (the reader) to work through these examples to make sure you can compute them yourself.

[8] Either use a calculator capable of complex-number arithmetic to enter and then reciprocate each impedance value; or alternatively convert each rectangular-form impedance value into polar form, reciprocate the magnitude and reverse the angle's sign, and then convert back into rectangular form.

Another way to manage impedances and admittances on the same Smith Chart is to superimpose two of these Charts, one of them being flipped both horizontally and vertically such that the two Charts are symmetrical about their shared center-point. Color-coding helps to keep the impedance (green) distinct from the admittance (burgundy) circles and labels:

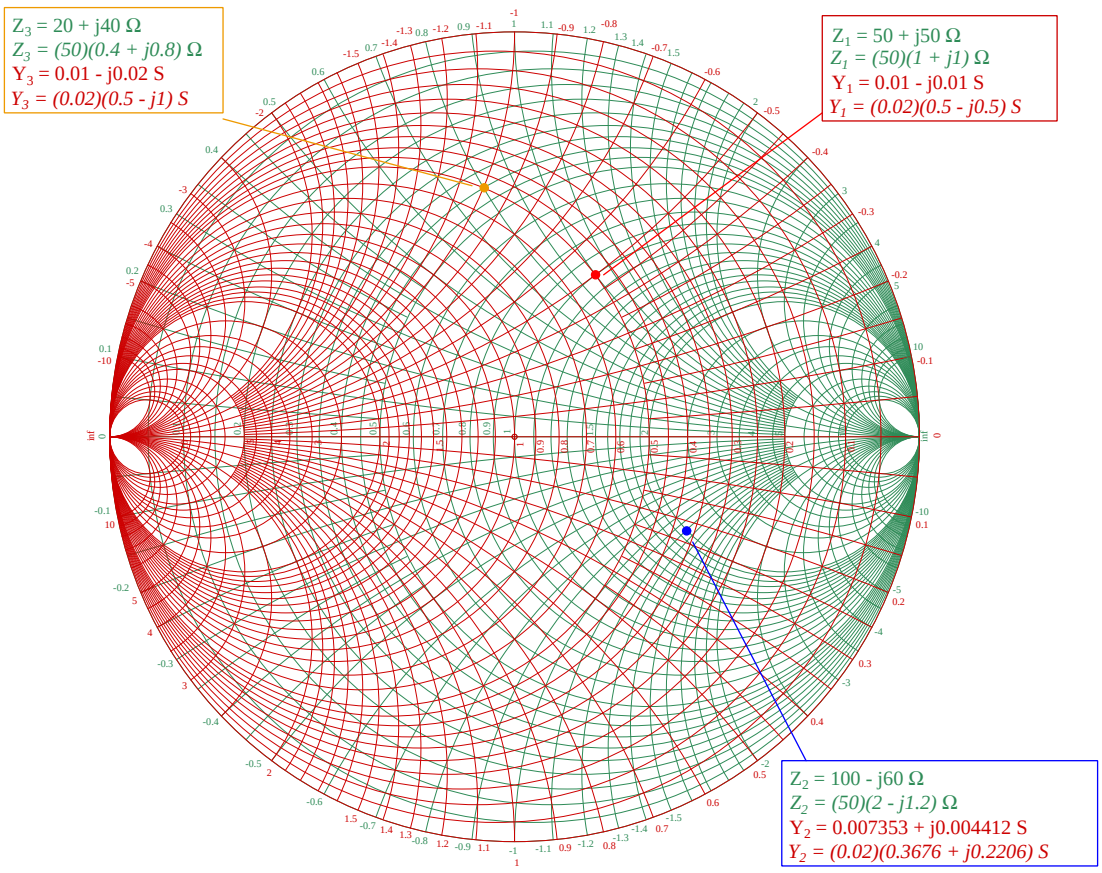


To convert an impedance into an admittance using the Impedance-Admittance Smith Chart, simply locate the impedance value on the green (impedance) Chart and directly read the corresponding admittance value from the burgundy (admittance) Chart.

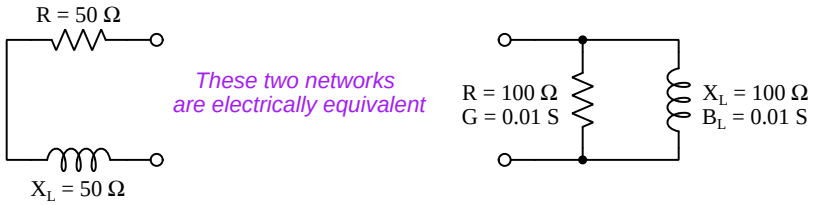
Note how the impedance circles of this Smith Chart (green) arc upward for positive impedance values and downward for negative impedance values, but that the admittance circles (burgundy) arc downward for positive admittance values and upward for negative admittance values. This is one of the prices we must pay for the convenience of being able to read equivalent impedance and admittance values from the exact same point: we must pay close attention to the imaginary sign depending on which value we intend to read. At least the Z-Y Smith Chart maps all inductive

values in the top half and all capacitive values in the bottom, unlike the Z-only Smith Chart where capacitive admittances share the same chart space as inductive impedances, and vice-versa.

To illustrate, we will examine the same three impedance values plotted on a Z-Y Smith Chart, shown here as both impedance and admittance values:



A practical application of impedance-admittance equivalents is graphically determining electrically-equivalent series versus parallel networks, because we know impedances add in series while admittances add in parallel. For example, the complex sum $Z_1 = 50 + j50$ Ohms describes a 50 Ohm resistor in series with an inductor exhibiting 50 Ohms of reactance. Its mathematical equivalent $Y_1 = 0.01 - j0.01$ Siemens is a sum describing a resistor having a conductance of 0.01 Siemens (100 Ohms resistance) in parallel with an inductor exhibiting 0.01 Siemens of susceptance (100 Ohms reactance). If $50 + j50$ Ohms is mathematically equivalent to $0.01 - j0.01$ Siemens, then a series combination of 50 Ohms resistance and 50 Ohms inductive reactance must be electrically equivalent to a parallel combination of 100 Ohms resistance and 100 Ohms inductive reactance:



Of course, Smith Charts are not strictly necessary to convert between series and parallel equivalent networks. All we need to do is express the impedance or admittance in question as a complex-number value in rectangular form, and then reciprocate that value using an electronic calculator or a computer to find the other. The utility of a basic Smith Chart would be to graphically perform this reciprocation by locating the point directly opposite the Chart's center, or in the case of an Impedance-Admittance Smith Chart to simply read equivalent coordinates on the same point. Some examples help illustrate such series-parallel equivalent network conversions given reciprocation by any means:

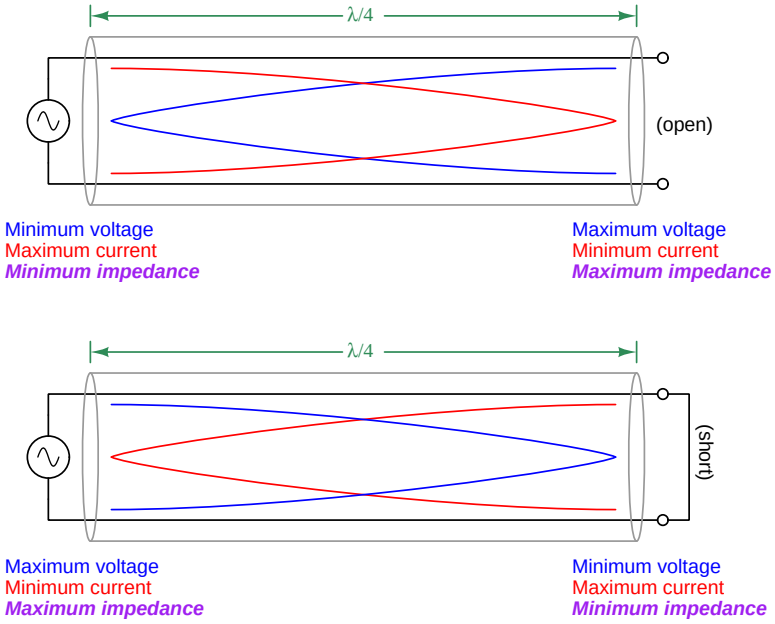
- A series network comprised of an 80 Ω resistor and a 0.47 μF capacitor will have a total impedance of 80 − j102.61 Ω at a frequency of 3.3 kHz because the resistor's 80 + j0 Ω adds to the capacitor's 0 − j102.61 Ω in series. Reciprocating this total impedance value yields a total admittance value of 0.004725 + j0.00606 S at that same frequency, representing a parallel combination of 0.004725 S conductance and +j0.00606 S capacitive susceptance. If we then reciprocate the real and imaginary terms individually, the conductance value of 0.004725 Siemens becomes 211.62 Ω for the parallel resistor, and the susceptance value of +j0.00606 Siemens becomes −j164.98 Ω for the parallel capacitor. At 3.3 kHz, that 164.98 Ω of capacitive reactance translates to 0.2923 μF of parallel capacitance.

- A parallel network consisting of a 470 Ω resistor and a 10 mH inductor will have a total admittance of 0.002128 − j0.003183 S at a frequency of 5 kHz. Reciprocating this total admittance value yields a total impedance value of 145.14 + j217.14 Ω. The real portion of this impedance is the equivalent series network's resistance of 145.14 Ω. The imaginary portion of this impedance is the equivalent series network's inductive reactance of 217.14 Ω at 5 kHz. Solving for inductance at that same frequency yields a series inductor value of 6.912 mH.

3.7 Transmission lines on the Smith Chart

Smith Charts are also very useful for computing the amount of impedance seen at one end of a transmission line, given the transmission line's characteristic impedance, the terminating impedance, and the transmission line's length compared to the signal frequency's wavelength. Ideally all transmission lines should be terminated by impedance values precisely matching the line's own surge impedance, as this will prevent signal "reflections" off of the mis-matched ends and render the line's length irrelevant, but in practice there will always be *some* degree of mis-match. The *standing waves* created by reflected signals interfering with incident signals along the line will create conditions where the amount of impedance exhibited at one end of the line does not match the amount of impedance terminating the other end of the line.

The most extreme example of a transmission line's ability to "transform" its terminating impedance into some other impedance value at the other end occurs when the line is precisely one-quarter of a wavelength[9] from one end to the other. In this scenario any standing wave in the line will have a node (minimum amplitude) at one end and an anti-node (maximum amplitude) at the other. Consider these quarter-wavelength transmission lines, terminated with an open versus with a short:



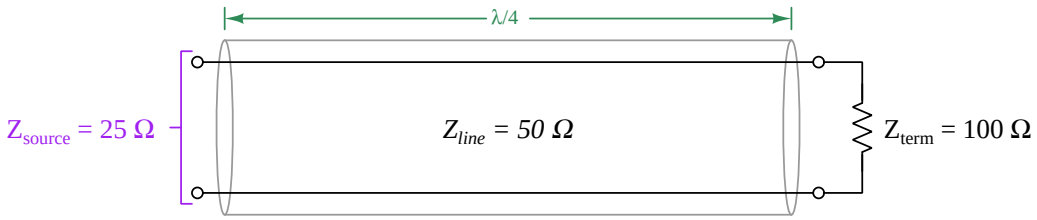
From these illustrations we see that a transmission line operating at one-quarter wavelength is able to "transform" an open-circuit termination into a short-circuit as seen by the AC source, and vice-versa.

[9] This length is based on the signal frequency as well as the speed of signal propagation within the transmission line, which may be substantially slower than the speed of light in a vacuum for plastic-insulated transmission lines. For example, one-quarter wavelength for a 150 MHz signal is 0.49965 meters for vacuum (or air) but only 0.32977 meters for a cable with a velocity factor of 0.66.

For less-extreme termination values, the transformed impedance at the source-end of the quarter-wave transmission line is computed using the following equation:

$$Z_{source} = \frac{Z_{line}^2}{Z_{termination}}$$

For example, if a 50-Ohm transmission line is terminated by a 100-Ohm resistor, the source will "see" a 25 Ohm resistive impedance at the other end of the line:



This formula does not show the full complexity of the transformation offered by the transmission line, because it only provides polar impedance magnitude (i.e. no indication of phase angle, or real vs. imaginary components) and also because it only applies to transmission lines that are exactly one-quarter wavelength from end to end. A Smith Chart, however, can show us the transformed impedance at the source-end in *full complex form* as well as for transmission lines of *any* length.

The procedure for using a Smith Chart to compute line-transformed impedance is quite simple:

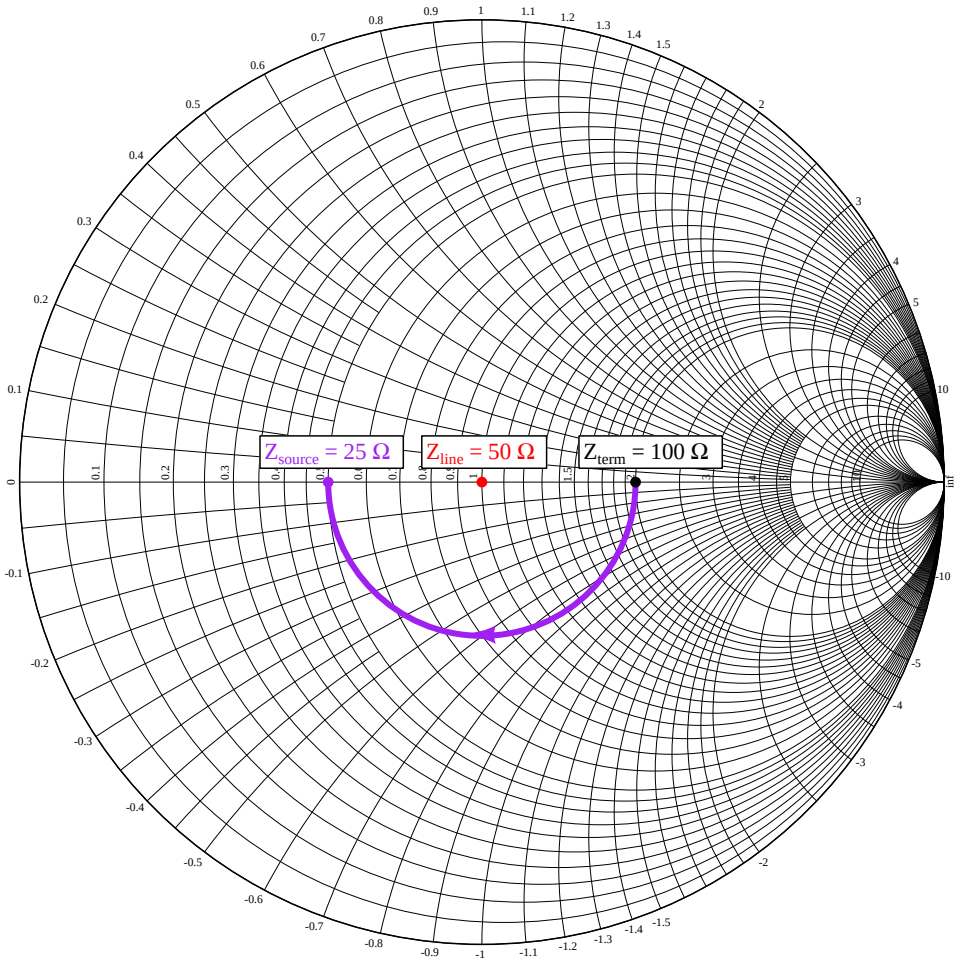
1. Mark on the Smith Chart the transmission line's characteristic impedance. This will be a point along the horizontal axis (i.e. a real number with no imaginary portion), normalized to 50 Ohms.

2. Mark on the Smith Chart the termination impedance value. This will be a point placed at the appropriate location on the Chart, normalized to 50 Ohms. It will be a complex quantity, and may be entirely real, entirely imaginary, or any combination thereof.

3. Draw a circle (using a compass) where the line's impedance point is the circle center, starting at the termination impedance, moving in a *clockwise* direction. The transmission line's length (in wavelengths) will equate to a certain angle of rotation that you draw:, a quarter-wavelength being 180 degrees, a half-wavelength being 360 degrees, an eighth-wavelength being 90 degrees, etc.

4. Note where your arc terminates, and de-normalize that complex value to determine the source-end impedance on this transmission line.

Here we see the previous example (100 Ohm resistor at the end of a 50 Ohm line) mapped on a Smith Chart:

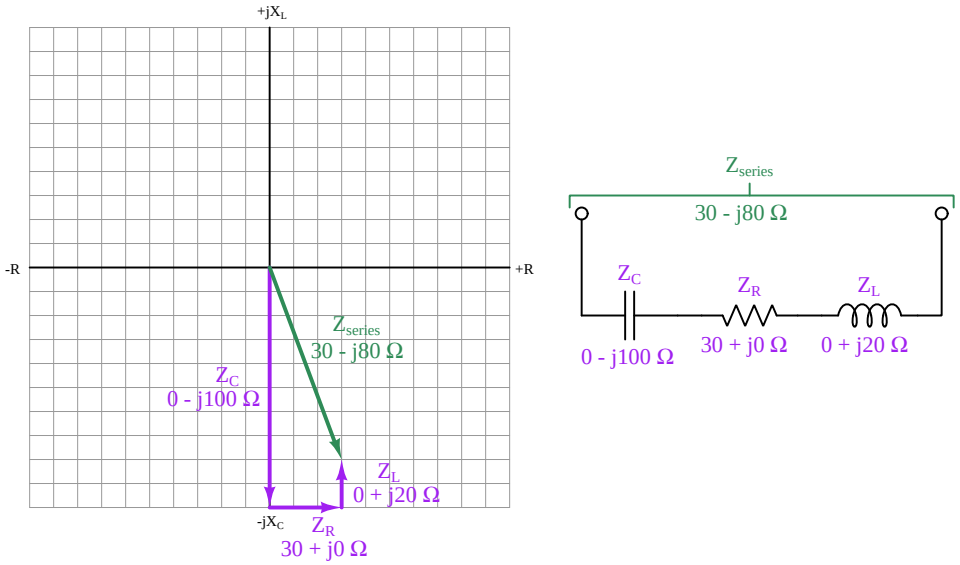


This is a fairly trivial example because the terminating impedance is purely resistive (placing the starting point on the real axis) and the transmission line is exactly one-quarter wavelength long (making the arc 180^o), but even in this simple case we can see some interesting possibilities. For example, if the transmission line were half as long (i.e. one-eighth of a wavelength), the arc would trace only 90^o which would place the source-end impedance at $(50)(0.8 - j0.6)$ Ohms, which is $40 - j30$ Ohms (de-normalized) or 50 Ohms $\angle$ 36.87^o in polar form. As non-intuitive as it may seem, a one-eight wavelength 50-Ohm transmission line would make the 100 Ohm terminating resistance "look" like a series RC network with 40 Ohms of resistance and 30 Ohms of capacitive reactance.

3.8 Series-parallel analysis using Smith Charts

Phasor diagrams are useful for any AC circuit quantities known to be additive in nature. This would include voltages in series, currents in parallel, impedances in series, and admittances in parallel. Impedance-Admittance Smith Charts are particularly useful for representing series-parallel networks as we shall soon see.

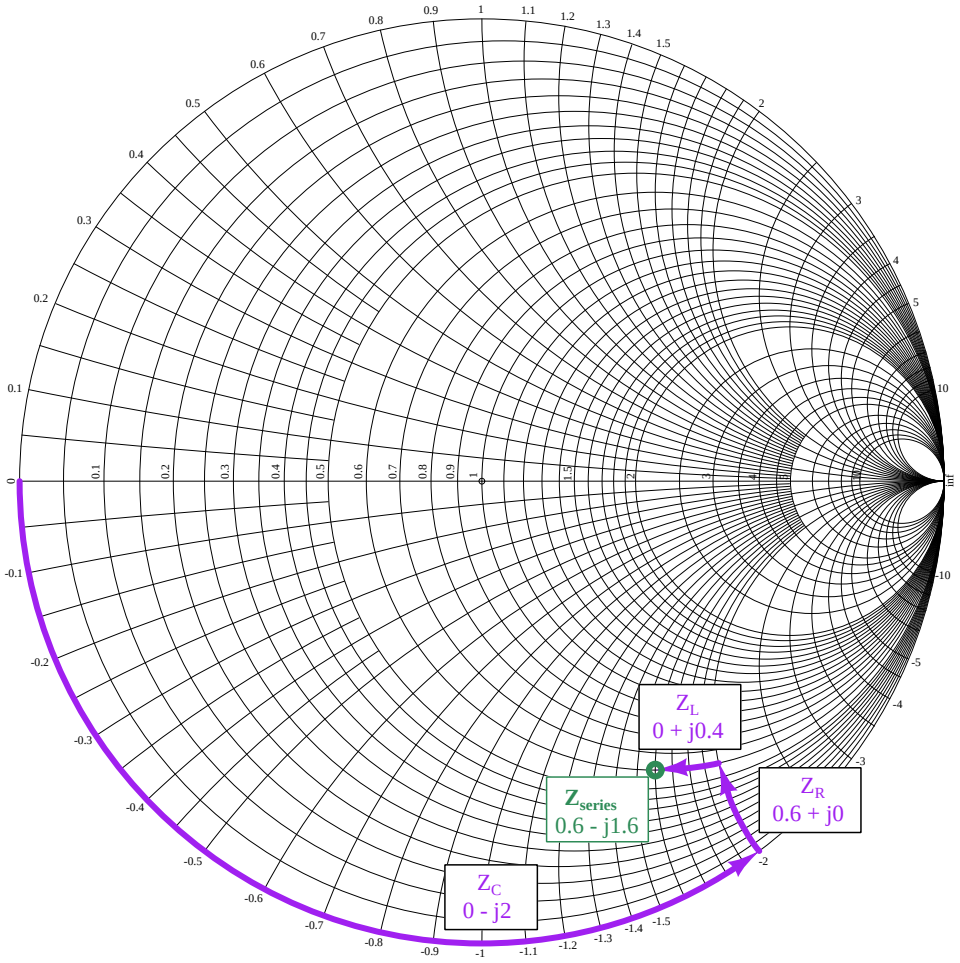
Consider the following series network with its component impedances represented in a standard phasor diagram:



From an assumed starting point of zero Ohms impedance (the graph's origin), we see the first vector pointing down 100 units, followed by the next vector pointing right 30 units, and finally the third vector pointing up 20 units. The result of this vector sum is the impedance $30 - j80$ Ohms, represented by the green-colored vector.

In order to show how these same phasor quantities may be mapped on to a Smith Chart, we must first "normalize" the given component impedances to a 50 Ohm standard:

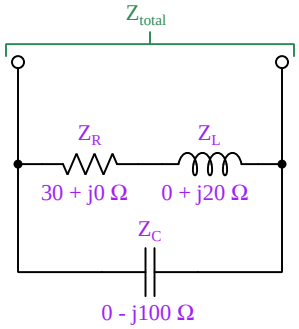
- $Z_C = 0 - j100\,\Omega = (50)(0 - j2)$ *normalized to 50 Ohms*
- $Z_R = 30 + j0\,\Omega = (50)(0.6 + j0)$ *normalized to 50 Ohms*
- $Z_L = 0 + j20\,\Omega = (50)(0 + j0.4)$ *normalized to 50 Ohms*



Like all sums, this one begins at a starting value of zero which means a short-circuit (zero impedance) at the far-left end of the real axis. From there we see each phasor impedance drawn as an arc (following the Smith Chart's circular patterns) rather than a straight arrow-shaped vector. As with the standard phasor diagram, though, the final sum of these three component impedances is still $30 - j80$, which is normalized to 50 Ohms as $(50)(0.6 - j1.6)$.

So far it does not appear that the Smith Chart offers any advantage over a standard phasor diagram plotted on a (rectangular) Cartesian coordinate grid. If anything, it's more confusing than stacking straight-arrow vectors tip-to-tail! However, the ability for Smith Charts to translate between impedances (Z) and admittances (Y) allows them to represent combinations of series *and parallel* components much easier than a conventional phasor diagram.

Consider the following series-parallel network, using the same three components we just saw connected in series:



We cannot stack the three components' phasor impedance vectors tip-to-tail in a phasor diagram like we could before, because the three impedances don't add in this series-parallel network. Z_R and Z_L certainly add because those two components are connected in series, but Z_C is connected in parallel with the $Z_R \leftrightarrow Z_L$ series subnetwork, and we know parallel impedances don't add.

Parallel *admittances* do add, however, and so what could be done is to stack the Z_R and Z_L phasor arrows tip-to-tail (because they add), then convert that phasor impedance sum into a phasor *admittance*, then add to that the capacitor's admittance to arrive at a total admittance for the whole network, then finally convert that total admittance value (Y_{total}) into a total impedance value (Z_{total}). The Smith Chart's ability to convert between equivalent impedance and admittance values will accomplish what a standard Cartesian-coordinate phasor diagram cannot.

Smith Charts actually provide two different strategies for this graphical series-parallel network analysis, each with its own advantages and disadvantages:

1. Use a standard (Z) Smith Chart, converting between impedance and admittance values by "reflecting" impedance values across from the Chart's center-point to read admittance values and vice-versa. This strategy uses the simple version of the Smith Chart but is complicated by the need to occasionally located points opposite each other from the center (coordinate 1,0).

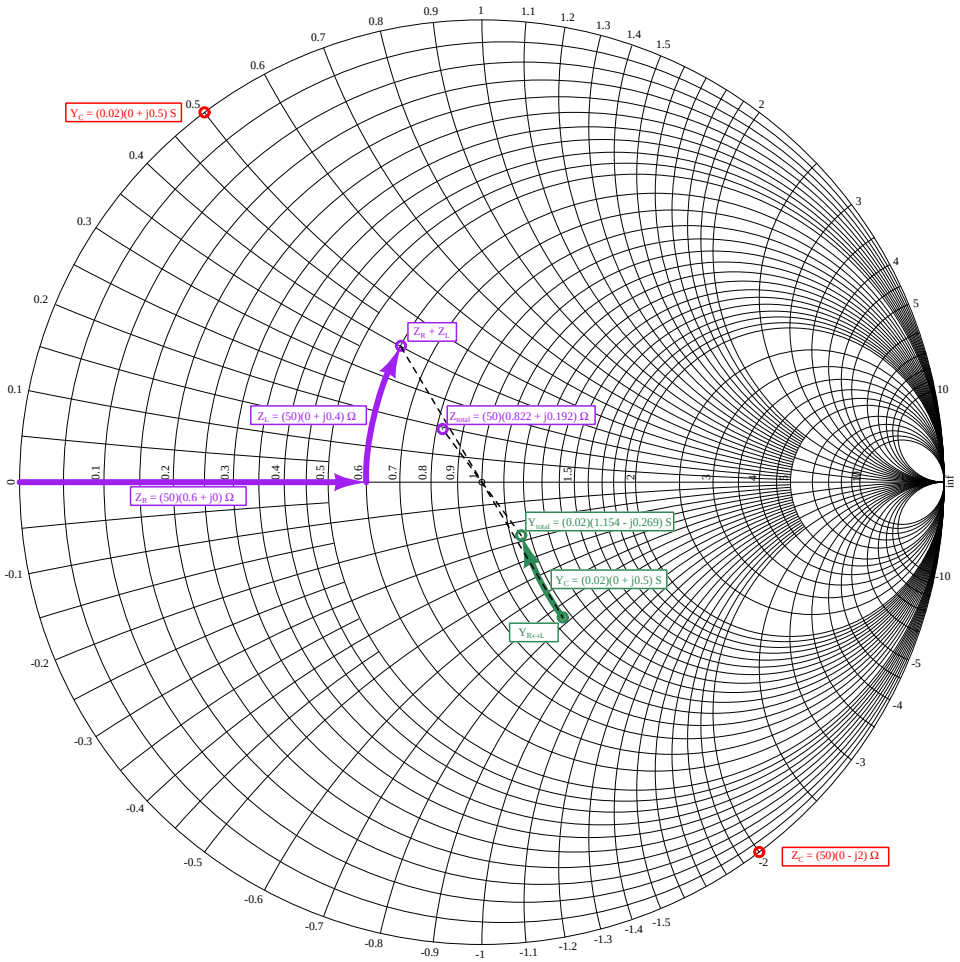
2. Use an impedance-admittance (Z/Y) Smith Chart, converting between impedance and admittance values simply by reading the same point's coordinates off of the opposing Chart's divisions. This method is much simpler in terms of impedance-admittance translation, but it requires the use of the relatively "cluttered" impedance-admittance version of the Smith Chart with its opposing sets of circles rendered in different colors.

We will apply both these methods to the RLC network shown here, in the following subsections.

3.8.1 Series-parallel network on the Z Smith Chart

First we will compute total impedance for this RLC network using an impedance-only Smith Chart. Our first step, as always, is to *normalize* each component's phasor values suitable to the Chart's standard value. Since the capacitor is in parallel with the series combination of the other two components, we will need to have its value expressed as an admittance rather than an impedance. Y_C may be read opposite the Chart's center-point from the Z_C point. In this case, the location opposite $0 - j2$ is $0 + j0.5$:

- $Z_R = 30 + j0\,\Omega = (50)(0.6 + j0)$ *normalized to 50 Ohms*
- $Z_L = 0 + j20\,\Omega = (50)(0 + j0.4)$ *normalized to 50 Ohms*
- $Z_C = 0 - j100\,\Omega = (50)(0 - j2)$ *normalized to 50 Ohms* ; $Y_C = (0.02)(0 + j0.5)$ *normalized to 20 milliSiemens*



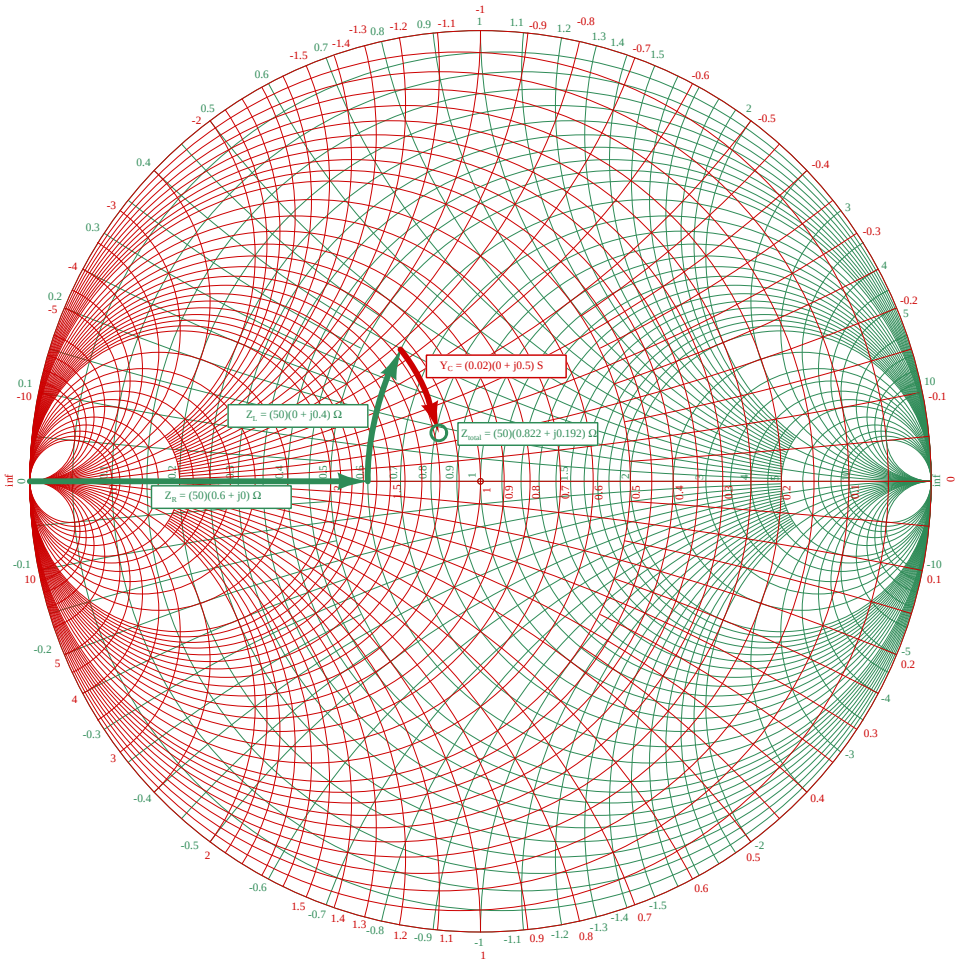
Starting at zero impedance (short) for the series $Z_R \leftrightarrow Z_L$ combination, we follow the impedance Chart (green) to plot curved arrows for Z_R and Z_L. At the sum of those two phasors we must then translate that series impedance value into an admittance by locating the point exactly opposite the Chart's center. There, we plot another curved arrow representing Y_C arcing upward $+j0.5$ normalized units. At this arrow's tip we have a total normalized admittance value ($Y_{total} = (0.02)(1.154 - j0.269)$ Siemens) for the whole series-parallel network, which we then go across from the Chart's center-point to find a whole-network normalized impedance value of ($Z_{total} = (50)(0.822 + j0.192)$ Ohms). De-normalizing these, we get $0.0231 - j0.00538$ Siemens for total admittance and $41.1 + j9.59$ Ohms.

It should be obvious to anyone reading this and trying to map these values on the Smith Chart for themselves that it is impossible to achieve that many digits of precision merely by visually inspecting the Chart, no matter how much care is taken in doing so. In full disclosure I admit to using a calculator to compute these precise impedance and admittance values. However, even if we round the final impedance value to the nearest divisions on the impedance (green) Chart to $(50)(0.8 + j0.2)$ Ohms we get a value that is fairly close to the correct result: $40 + j10$ Ohms. It should impress you, the reader, that the Smith Chart is able to graphically estimate this network's series-parallel impedance using nothing more than simple arithmetic to count along its divisions, and even with access to a calculator capable of complex-number arithmetic the Smith Chart has value in being able to check for any major calculated errors against the approximate result it provides.

3.8.2 Series-parallel network on the Z-Y Smith Chart

Next we will compute total impedance for the same RLC network using an impedance-admittance Smith Chart. Our first step, as always, is to *normalize* each component's phasor values suitable to the Chart's standard value. Once again, we need to know the capacitor's value as an admittance rather than an impedance because of its parallel relationship to the other two components. Y_C may be read directly off the Chart's admittance (burgundy) coordinates after locating the Z_C point on the impedance (green) coordinates:

- $Z_R = 30 + j0\,\Omega = (50)(0.6 + j0)$ *normalized to 50 Ohms*
- $Z_L = 0 + j20\,\Omega = (50)(0 + j0.4)$ *normalized to 50 Ohms*
- $Z_C = 0 - j100\,\Omega = (50)(0 - j2)$ *normalized to 50 Ohms* ; $Y_C = (0.02)(0 + j0.5)$ *normalized to 20 milliSiemens*

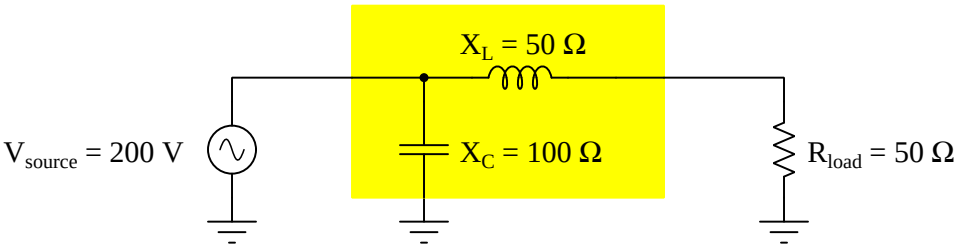


Starting at zero impedance (short) for the series $Z_R \leftrightarrow Z_L$ combination, we follow the impedance Chart (green) to plot arrows for Z_R and Z_L. At the sum of those two phasors we then regard that point as an admittance (on the burgundy Chart) and add to it Y_C by arcing downward $+j0.5$ normalized units. At the final arrow's tip we may read the network's total normalized admittance ($Y_{total} = (0.02)(1.154 - j0.269)$ Siemens) and total normalized impedance ($Z_{total} = (50)(0.822 + j0.192)$ Ohms). De-normalizing these, we get $0.0231 - j0.00538$ Siemens for total admittance and $41.1 + j9.59$ Ohms.

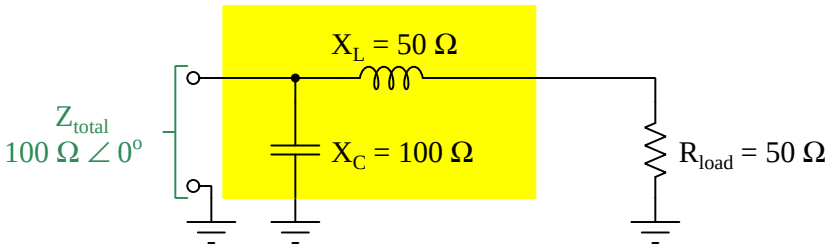
Again, it should be obvious to anyone reading this and trying to map these values on the Smith Chart for themselves that it is impossible to achieve that many digits of precision merely by interpreting the Chart. These figures were actually computed using a complex-number calculator to achieve three significant digits of precision. However, even the rounded value read from the nearest divisions on the impedance (green) Chart of $(50)(0.8 + j0.2)$ Ohms is quite close to the correct answer: $40 + j10$ Ohms. Once again the Smith Chart proves its worth as a means to quickly estimate series-parallel impedance.

3.9 LC impedance-transformation networks

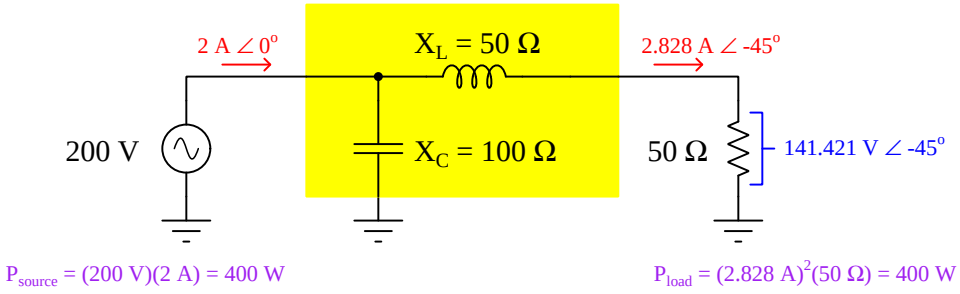
An interesting and useful property of inductor-capacitor networks is their ability to make the impedance of a load "look" different to a connected source. As an example of this, consider the following network connected between a 200 Volt AC source and a 50 Ohm resistive load. In order to keep this example as simple as possible, we specify reactance values for the inductor and capacitor rather than actual inductance and capacitance values (along with a specified source frequency):



If we perform the necessary series-parallel impedance calculations for the LC network and resistive load, we find that the total impedance (as calculated from the perspective of the input terminal where the source would connect) is precisely 100 Ohms resistive (i.e. $100\ \Omega \angle\ 0^o$ or $100 + j0$) which is twice the impedance of the load with the same phase angle.

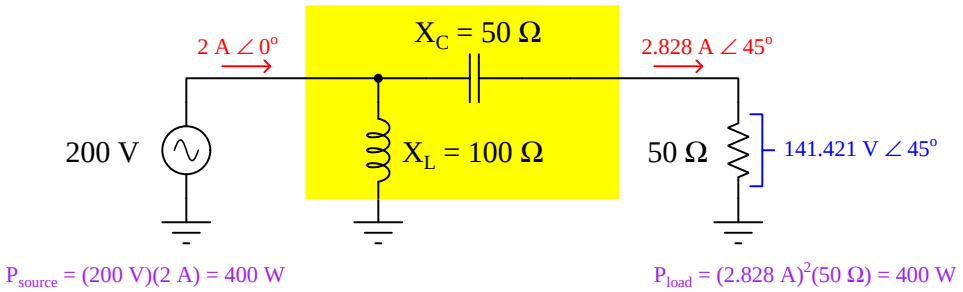


Therefore, when this circuit is energized we find the following voltage, currents, and powers at the source and load:



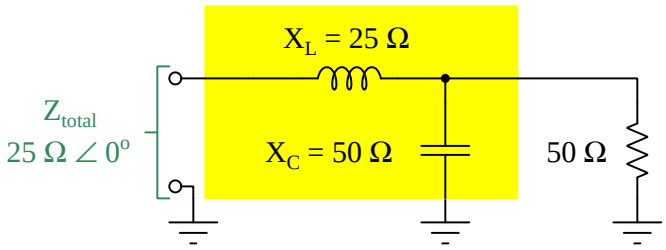
Note how with the simple LC network the source "sees" a 100 Ohm load impedance even though the actual load is 50 Ohms, and also how load power and source power are precisely equal (at least given an ideal inductor and an ideal capacitor having no dissipative losses).

We may achieve the same load impedance transformation by exchanging the placement of the inductor and capacitor, but maintaining the original reactance values (50 Ohms in series, 100 Ohms in parallel). The only difference is in the phase shift of current and voltage at the 50 Ohm resistive load:

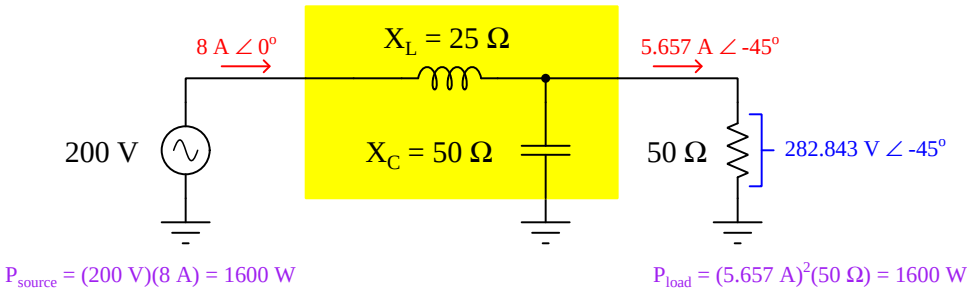


One way to view these LC impedance-transformation networks conceptually is to view the series-connected component (with the load) as adding to the load's impedance, and the parallel-connected component (with the source) correcting that increased impedance to become purely resistive as seen by the source.

It is also possible to transform a resistive load's impedance *down* rather than up, by altering the series-parallel relationship of the reactive components and suitably altering their reactance values. In this case, the LC network transforms the 50 Ohm resistive load into 25 Ohms as "seen" by the AC source:



Therefore, when this circuit is energized we find the following voltage, currents, and powers at the source and load:

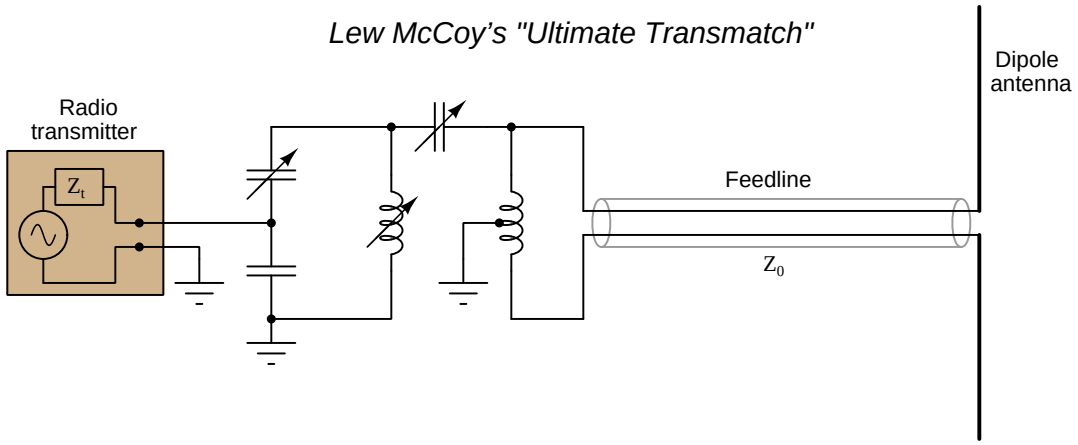


Again, we may conceptually understand these LC impedance-transformation networks by considering the parallel-connected component as a means to diminish the load's apparent impedance, and considering the series-connected component as a means to cancel out any remaining reactance so that the source sees a purely resistive load. The reason this network decreases the load's apparent impedance is because the component connected directly to the load is in *parallel*, while the former network increased the load's apparent impedance because the component connected directly to the load was in *series*.

As before with the impedance-boosting network, we may swap the positions of L and C and achieve the same effect if the reactance values remain the same (i.e. 50 Ohm inductive reactance in parallel with the 50 Ohm load resistance and 25 Ohm capacitive reactance in series with the source).

Impedance transformation occurs because capacitors and inductors are (ideally) non-dissipative components. Stacking inductance and capacitance in a series-parallel network with a load obviously alters the total impedance value, thus making the load "appear" to have a different impedance than it does from the perspective of the source, but the non-dissipative nature of the LC network means no energy is "wasted" therein and the resistive load must receive all of the source's power. Just as a transformer with a turns ratio other than 1:1 clearly presents a different voltage to the load than what the source supplies, and given the non-dissipative nature of the transformer and the Law of Energy Conservation this *must* mean current gets transformed in the inverse ratio.

A common application of this impedance-transformation method is matching the impedance of an antenna and feedline system to the output impedance of a radio-frequency power amplifier, such as that found in a radio transmitter system. The principle involved here is the Maximum Power Transfer Theorem, which states the amount of power delivered to a load from a source is maximized when the load's impedance is adjusted to equal that of the source[10]. Proper impedance-matching between load and source is also necessary to minimize signal reflections and *standing waves* along transmission lines such as the feedlines connecting radio transceivers to their respective antennas. Often referred to as *transmatch* networks, an example of impedance transformation by an LC network[11] is found in Lew McCoy's "ultimate transmatch" designed for amateur radio operations:



Adjustable capacitors and inductors are typically provided in transmatch networks so that the radio operator may "align" the unit to the operating frequency of the transmitter as well as to the exact impedances of the amplifier and antenna/feedline system. McCoy's general instructions for using this particular network (as written in his July 1970 *QST* magazine article "The Ultimate Transmatch") include setting both variable capacitors to maximum capacitance, adjusting the variable inductor until minimum reflected power is obtained (i.e. as close to a perfect standing wave ratio as possible), and then adjusting variable capacitors C_1 and C_2 by trial-and-error until the best match is obtained, as indicated by the SWR reaching unity or reflected power decreasing

[10] This would be the source's equivalent Thévenin or Norton impedance.

[11] In fairness, McCoy's design also incorporates a wideband transformer at its output, accounting for a 1:4 impedance transformation of its own.

to zero. Although multiple combinations of C and L values are possible for matching any given transmitter to its line and antenna at a particular frequency, McCoy recommends optimizing for a solution using maximum capacitance[12].

A photograph of an impedance-matching network for a 50 kW AM broadcast transmitter operating at 550 kHz is shown in the following photograph, with silver-coated inductor tubes to minimize resistance at high frequencies[13]. The entire network is housed inside of a small shielded room, visible through the open door:

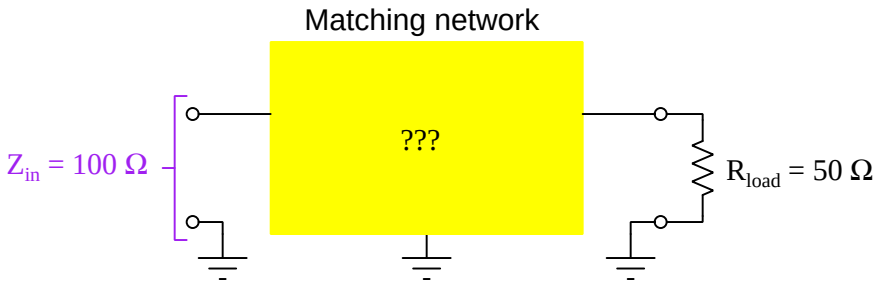
A caveat to this method of impedance transformation is that it only functions properly at (or near) one frequency value. Other impedance-transformation techniques such as *transformers* and *amplifiers* work over much broader frequency ranges.

[12] For example if a combination of high L and low C results in a good match, but another combination consisting of low L and high C also happens to result in a good match, the latter combination is preferable.

[13] At radio frequencies, the *skin effect* causes the majority of the current to travel on the outside surface of the conductor rather than the interior. Therefore, the electrical conductivity of the conductor's surface material matters more than its interior bulk. This is also why most RF conductors are hollow rather than solid metal: having a solid RF conductor would just be a waste of metal since nearly all of the current travels along the exterior surface.

3.10 Matching network design using Smith Charts

As described in the previous section, a very useful application of inductor-capacitor networks is to match impedances between otherwise mis-matched sources and loads. Smith Charts are a useful tool for designing such impedance-transformation networks. Let us begin with the first application from the last section, an LC network intended to "transform" a 50 Ohm load impedance into a 100 Ohm impedance for connection to a source. This time, however, we will use a Smith Chart to determine necessary reactance values within the network to achieve the desired impedance doubling:

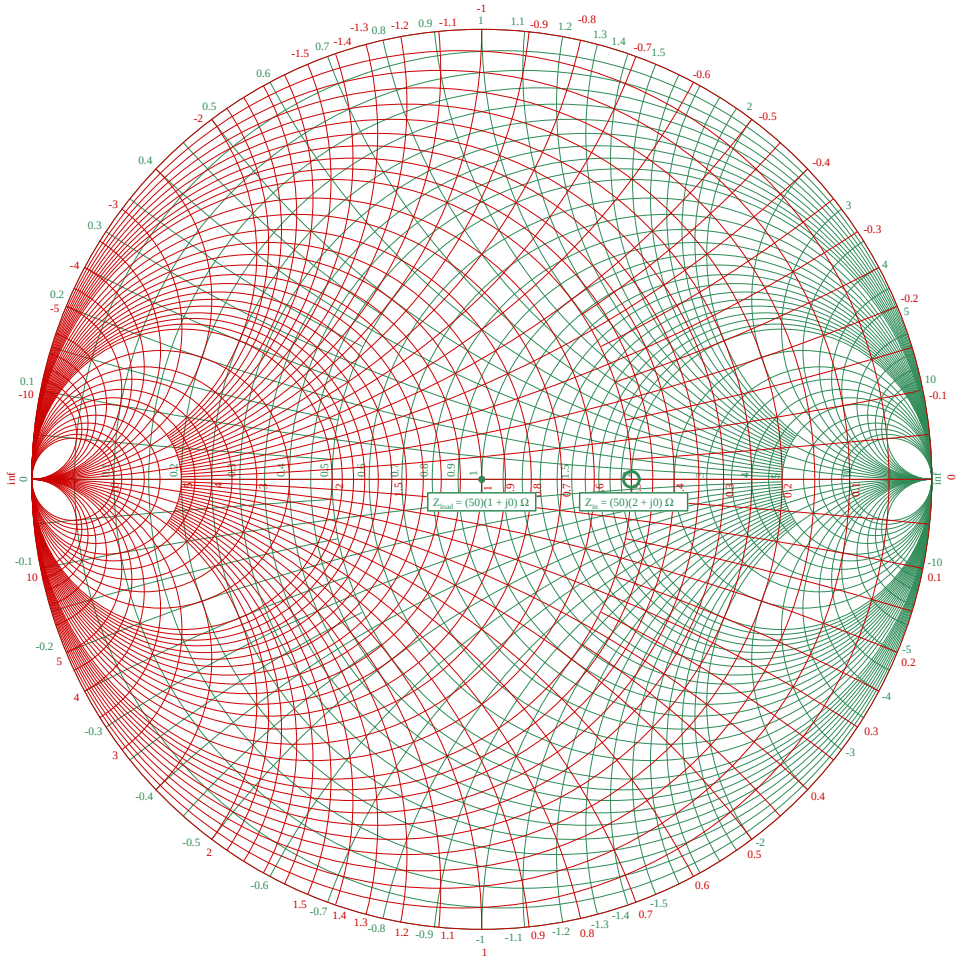


Our goal is for the matching network to boost the load's impedance value of 50 Ohms to a value of 100 Ohms for the source to power. Thus, 50 Ohms will be the "starting point" in our Smith Chart analysis and 100 Ohms will be our "ending point".

A mathematically obvious way to make a 50-Ohm load "look like" 100 Ohms to a source would be to simply make the matching network be a series 50 Ohm resistance with the load. However, this solution would not be good for any application requiring the efficient use of energy because that extra 50 Ohms of resistance would dissipate energy uselessly in the form of heat. This is why we always use inductors and capacitors to perform impedance transformation rather than resistors: inductors and capacitors are *reactive* components, meaning they store and release energy rather than dissipate energy. This results in much more of the source's energy arriving at the 50 Ohm load where we want[14] it. The task of selecting appropriate inductors and capacitors to perform this impedance transformation is not as mathematically simple as adding resistance, however, hence the use of a Smith Chart as a useful guide.

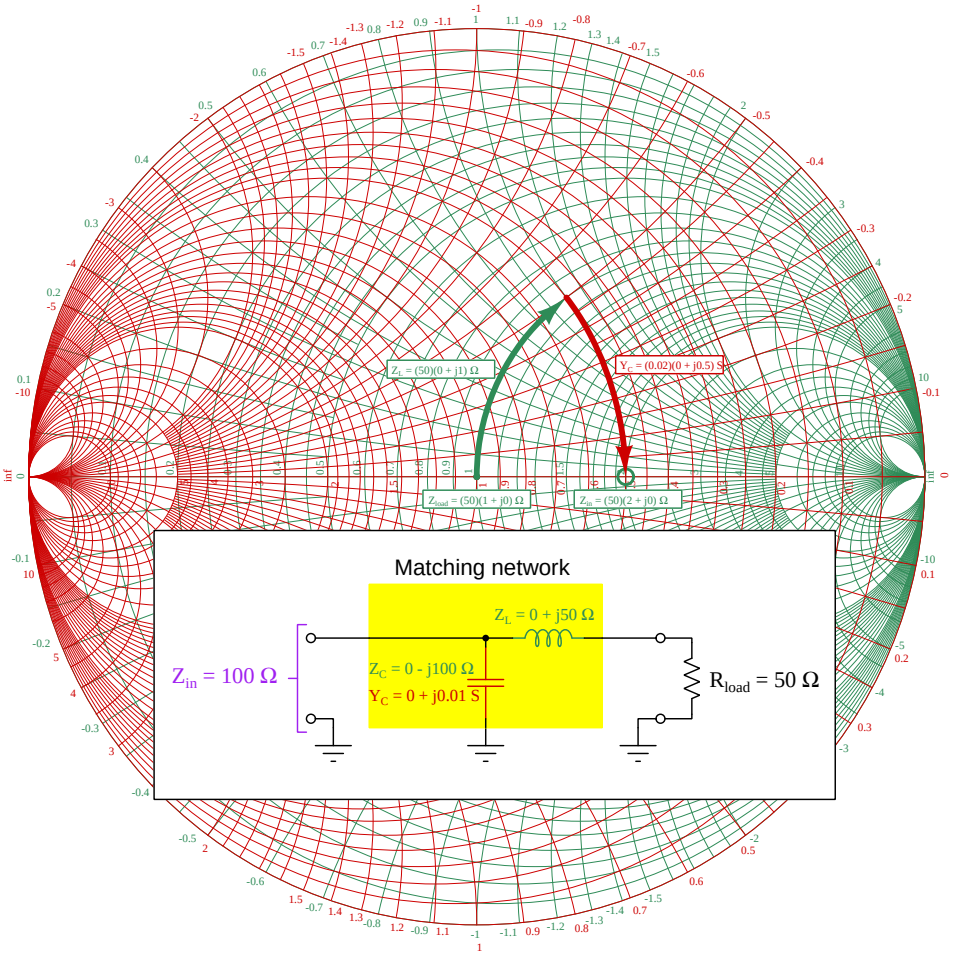
[14] In a realistic application, the load would not be a resistor either but rather something like a radio transmitting antenna where the AC power could do useful work.

To begin, we will plot the two known impedances, Z_{load} (50 Ohms resistive) marked by a solid dot, and the desired Z_{in} (100 Ohms resistive) marked by a circle:



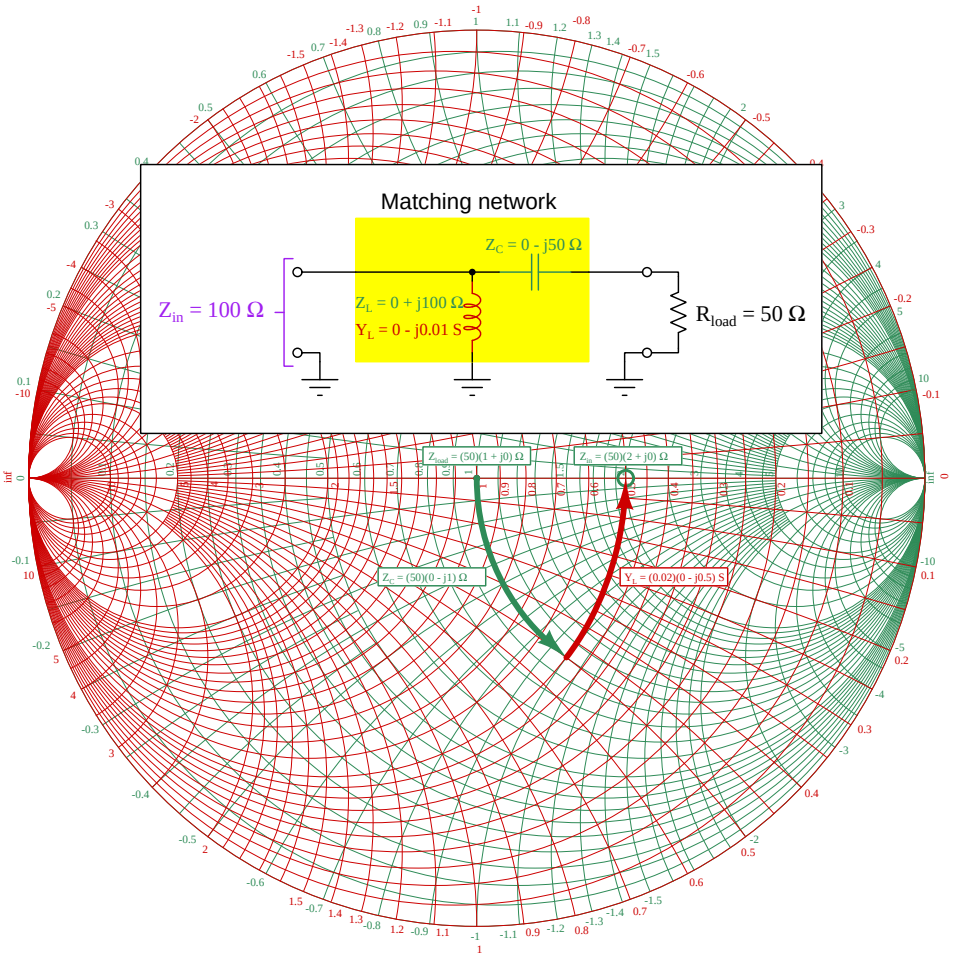
Somehow we need to find a pathway from the starting point (dot) to the ending point (circle) by adding reactive impedances and/or admittances to the load's 50 Ohm value, plotting these added quantities as vectors following the curvature of the Chart's resistance and conductance circles. Again, the mathematically simplest solution would be to draw a straight-line vector from the starting point to the ending point, but that would mean adding 50 more Ohms of resistance to the circuit, which we already ruled out as a valid solution because it would dissipate energy wastefully. In order to not dissipate energy needlessly, we must limit ourselves to adding inductive and capacitive reactances, in the form of series impedances (following the Z portion of the chart) or parallel admittances (following the Y portion of the chart).

One solution consists of adding 50 Ohms of Z_L then adding 10 milliSiemens of Y_C. The addition of $Z_L = 0 + j50$ Ohms means tracing an arc from the starting point (50 Ohms resistive) along the circumference of the $(50)(1 + j0)$ Ohm resistance circle until we reach the $(50)(1 + j1)$ location on the impedance Smith Chart. Then, we trace a second arc following the circumference of the $(0.02)(0.5 + j0)$ Siemens conductance circle down to our ending point of 100 Ohms (resistive). This second arc stretches from the $-j0.5$ admittance curve to the $j0$ admittance curve, an addition of $+0.5$ (normalized) admittance:

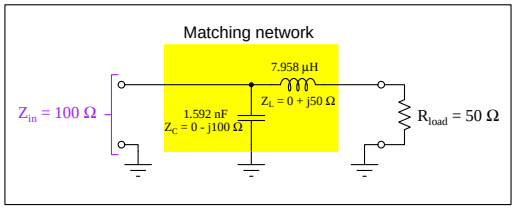


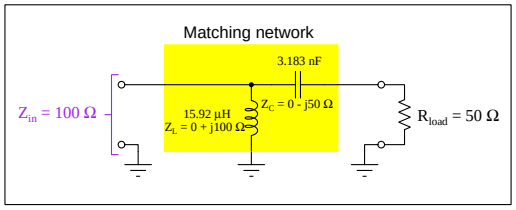
In schematic form, this network consists of 50 Ohms of inductive reactance in series with the 50 Ohm load, then 100 Ohms of capacitive reactance (i.e. 10 milliSiemens of capacitive susceptance) in parallel with that.

A similar solution uses another two-step arc approach to reaching the 100 Ohms resistive ending point from the 50 Ohms resistive starting point, but this time along the lower half of the Smith Chart:



Now, with either solution, determining actual component values is as simple as solving for L and C based on the impedances obtained from the Smith Chart as well as the known signal frequency (f). For example, at a frequency of 1 MHz the component values would be as follows:





Chapter 4

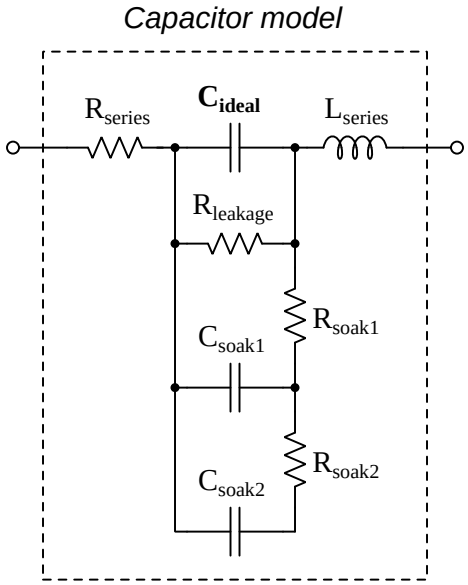
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Capacitor parasitics

4.1.1 Model of a real capacitor

An ideal capacitor exhibits only capacitance, with no inductance, resistance, or other characteristics to interfere. Real capacitors exhibit all these phenomena to varying degrees, and we collectively refer to these undesirable traits as *parasitic effects*. The following diagram models some of the parasitic effects observed in real capacitors:



In addition to the capacitance the capacitor is supposed to exhibit (C_{ideal}), the capacitor also has parasitic resistance (R_{series}, also known as *Equivalent Series Resistance*, or *ESR*), parasitic inductance (L_{series}), and additional energy storage in the form of *soakage* (also known as *dielectric absorption*) whereby the dielectric substance itself absorbs and releases energy after relatively long periods of time compared to the main (ideal) capacitance.

Some of these parasitic effects – such as leakage resistance and soakage – affect the capacitor's performance in DC applications. Most of the other parasitic effects cause problems in AC and pulsed applications. For example, the effective series capacitor-inductor combination formed by C_{ideal} and L_{series} will cause *resonance* to occur at a particular AC frequency, resulting in much less reactance at that frequency than what would be predicted by the capacitive reactance formula $X_C = \frac{1}{2\pi fC}$.

Next we will explore common mechanisms for each of these effects.

4.1.2 Parasitic resistance in capacitors

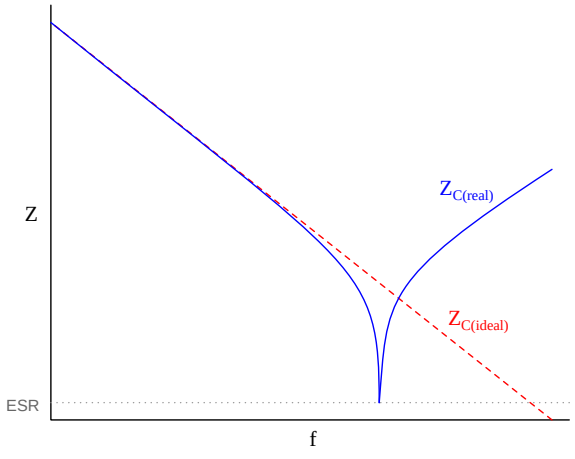
Wire resistance, of course, plays a part in this parasitic effect, but this is generally very small due to the short conductor lengths we typically see with capacitors. More significant is *dielectric losses* – energy dissipation caused by the stressing and relaxation of dipoles within the dielectric material – which act like resistance because energy ends up leaving the component (in the form of heat) and not returning to the circuit. Electrolytic capacitors have an additional source of parasitic resistance, in the form of the electrolytic gel substance used to make electrical contact from the metal-foil "plate" to the surface of the dielectric layer.

Another form of parasitic resistance within a capacitor behaves like a resistor connected in parallel with the ideal capacitance ($R_{leakage}$), resulting from the dielectric not being a perfectly insulating medium. This parasitic characteristic results in a small current passing through the capacitor even when the voltage across the capacitor is steady (i.e. $\frac{dV}{dt}$ is zero).

4.1.3 Parasitic inductance in capacitors

Any time a magnetic field forms around a current-carrying conductor, energy is stored in that magnetic field. We call this magnetic-based energy-storing capability *inductance*, and of course all capacitors must have some inductance due to the wire leads serving as connection points to the capacitor's metal plates. Much of a capacitor's parasitic inductance may be minimized by maintaining short lead lengths as it attaches to a printed-circuit board.

Parasitic inductance is a problem for capacitors in AC applications because inductive reactance (X_L) tends to cancel out capacitive reactance (X_C). If we plot the impedance of a capacitor as a function of frequency, we would expect an ideal capacitor to manifest a straight-line descent on a logarithmic plot. However, what we see is that at a certain frequency the series parasitic inductance resonates with the capacitance leaving only parasitic resistance (ESR), and then past that frequency the inductive effects overshadow the capacitance:



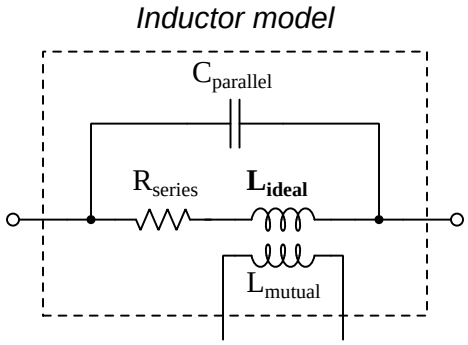
4.1.4 Other parasitic effects in capacitors

Soakage is an interesting effect resulting from dipole relaxation within the dielectric material itself, and may be modeled (as shown) by a series of resistor-capacitor subnetworks. This effect is especially prominent in aluminum electrolytic capacitors, and may be easily demonstrated by discharging a capacitor (by briefly connecting a shorting wire across the capacitor's terminals) and then monitoring the capacitor's DC voltage slowly "recover" with no connection to an external source.

4.2 Inductor parasitics

4.2.1 Model of a real inductor

An ideal inductor exhibits only inductance, with no capacitance, resistance, or other characteristics to interfere. Real inductors exhibit all these phenomena to varying degrees, and we collectively refer to these undesirable traits as *parasitic effects*. The following diagram models some of the parasitic effects observed in real inductors:



In addition to the inductance the inductor is supposed to exhibit (L_{ideal}), the inductor also has parasitic resistance (R_{series}, also known as *Equivalent Series Resistance*, or *ESR*), parasitic capacitance ($C_{parallel}$), and mutual inductance (L_{mutual}) with nearby wires and components.

Some of these parasitic effects – such as equivalent series resistance – affect the inductor's performance in DC applications. Most of the other parasitic effects cause problems in AC and pulsed applications. For example, the effective inductor-capacitor "tank circuit" formed by L_{ideal} and $C_{parallel}$ will cause *resonance* to occur at a particular AC frequency, resulting in much more reactance at that frequency than what would be predicted by the inductive reactance formula $X_L = 2\pi fL$.

Next we will explore common mechanisms for each of these effects.

4.2.2 Parasitic resistance in inductors

Wire resistance plays a dominant role in this parasitic effect due to the typically long lengths of wire necessary to wind the coil that forms most inductors. Wire resistance is not the only dissipative mechanism at work, though. Other losses include magnetic hysteresis of the iron core material as well as *eddy currents* induced in the iron core. An "eddy" current is a circulating electric current induced within the iron core of an inductor, made possible by the fact that iron is an electrically-conductive material as well as being ferromagnetic. These circulating currents do no useful work, and dissipate energy in the form of heating the iron. They may be minimized by forming the iron core from pieces of iron that are electrically insulated from one another, e.g. forming the iron core from *laminated sheets* or *powdered particles* of iron where each sheet or particle is electrically insulated from the next by a layer of non-conductive material on its outer surface.

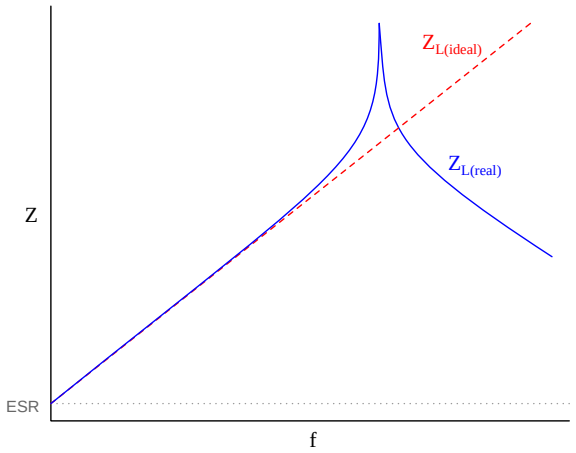
The series resistance of an inductor is always frequency-dependent. In DC conditions (i.e. frequency of zero Hertz) there will be the basic wire resistance of the coil at play. As frequency increases from zero, however, both the magnetic core losses from hysteresis and eddy currents also increase which add to the DC resistance to form a larger ESR. At extremely high frequencies the *skin effect*[1] further adds to the inductor's ESR.

[1] At high frequencies, electric current travels more toward the outer surface of a conductor rather than through the conductor's entire cross-section, effectively decreasing the conductor's cross-sectional area (gauge) as frequency rises.

4.2.3 Parasitic capacitance in inductors

Any time an electric field forms between two conductors, energy is stored in that electric field. We call this electric-based energy-storing capability *capacitance*, and of course all inductors must have some capacitance due to the insulating media between wire leads as well as between adjacent turns of wire within the coil (and between the wire turns and the iron core).

Parasitic capacitance is a problem for inductors in AC applications because capacitive reactance (X_C) tends to cancel out inductive reactance (X_L). If we plot the impedance of an inductor as a function of frequency, we would expect an ideal inductor to manifest a straight-line ascent on a logarithmic plot. However, what we see is that at a certain frequency the parallel parasitic capacitance resonates with the inductance to create a nearly-infinite impedance, and then past that frequency the capacitive effects overshadow the inductance:



Precious little may be done to eliminate parasitic capacitance within any inductor, whereas parasitic inductance is fairly easy to minimize within a capacitor. This explains why when faced with an equivalent choice between a circuit design using capacitors and a circuit design using inductors, capacitors nearly always win. Simply put, it is easier to make a nearly-ideal capacitor than it is to make a nearly-ideal inductor.

This also explains why the self-resonant frequency of most inductors is much lower than the self-resonant frequency of most capacitors: all other factors being equal, an inductor will have more parasitic capacitance in it than an equivalent capacitor will have parasitic inductance within it, making the LC product greater for the inductor than for the capacitor.

4.2.4 Other parasitic effects in inductors

Mutual inductance occurs whenever adjacent conductors' magnetic fields link with one another, which is difficult to avoid especially in physically dense circuit layouts. This parasitic effect may be minimized by proper placement of inductive components (e.g. keeping them spaced as far apart from each other as possible, orienting their axes perpendicular to each other rather than parallel) as well as by core designs with strong magnetic field containment (e.g. toroidal cores contain their magnetic fields better than rectangular cores).

4.2.5 Inductor testing

The following list shows typical tests for inductors at the time of manufacture:

- **DC winding resistance** – the simplest of these to measure, using a multimeter
- **Winding inductance/impedance** – performed by exciting the winding with an AC source and comparing voltage/current, typically using either an LCR meter or an impedance analyzer
- **Equivalent winding resistance** – based on the ratio of AC voltage to AC current (only the portions of each in-phase with each other) representing winding resistance as well as energy losses in the magnetic core
- **Core saturation** – performed by exciting the winding with a sinusoidal AC current and measuring self-induced winding voltage until a non-sinusoidal waveshape is detected, or more crudely by determining when the self-induced voltage no longer increases proportionately to increased excitation current
- **Insulation resistance** – performed with a "Hi-Pot" (high-potential) insulation tester, generally connecting one terminal to either or both ends of the winding and the other tester terminal to the conductive core material, this infers insulation resistance by applying (typically) several thousand Volts DC while measuring the small current passing through the stressed insulation

A more thorough battery of tests used during transformer design development might include the following:

- **BH loop** – performed by exciting the winding with an AC voltage and measuring the self-induced voltage across it, the phasor portion of that induced voltage shifted by 90 degrees from the current waveform representing the rate-of-change of magnetic flux which may be mathematically integrated to infer the amount of magnetic flux, those two variables being plotted using an oscilloscope's X-Y mode much like "curve tracing" for a semiconductor component ; often performed at different temperatures to fully characterize the magnetic properties of the core material
- **High-voltage impulse** – similar to an insulation resistance test, this uses a high-voltage DC pulse applied across the winding insulation to stress it
- **Volt-second** – performed by applying a DC voltage pulse to the winding while measuring current through it, the ideal response being a linear-ramping current whose magnitude is proportional to the Volt-second product of the applied pulse
- **Overload** – subjecting a winding to dramatic overcurrent to thermally and mechanically stress the inductor as it might be in an overloaded power system scenario
- **External magnetic field rejection** – subjecting the entire inductor to an external (alternating) magnetic field and measuring any voltage induced across any winding, usually tested with the magnetic field applied across different physical axes
- **Vibration** – not an electrical test per se, but a useful stressor when followed by repeated electrical tests to ensure electrical/magnetic properties have not substantially changed following the vibration

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Simplification as a problem-solving strategy

Thought experiments as a problem-solving strategy

Limiting cases as a problem-solving strategy

Annotating diagrams as a problem-solving strategy

Interpreting intermediate results as a problem-solving strategy

Graphing as a problem-solving strategy

Converting a qualitative problem into a quantitative problem

Converting a quantitative problem into a qualitative problem

Working "backwards" to validate calculated results

Reductio ad absurdum

Re-drawing schematics as a problem-solving strategy

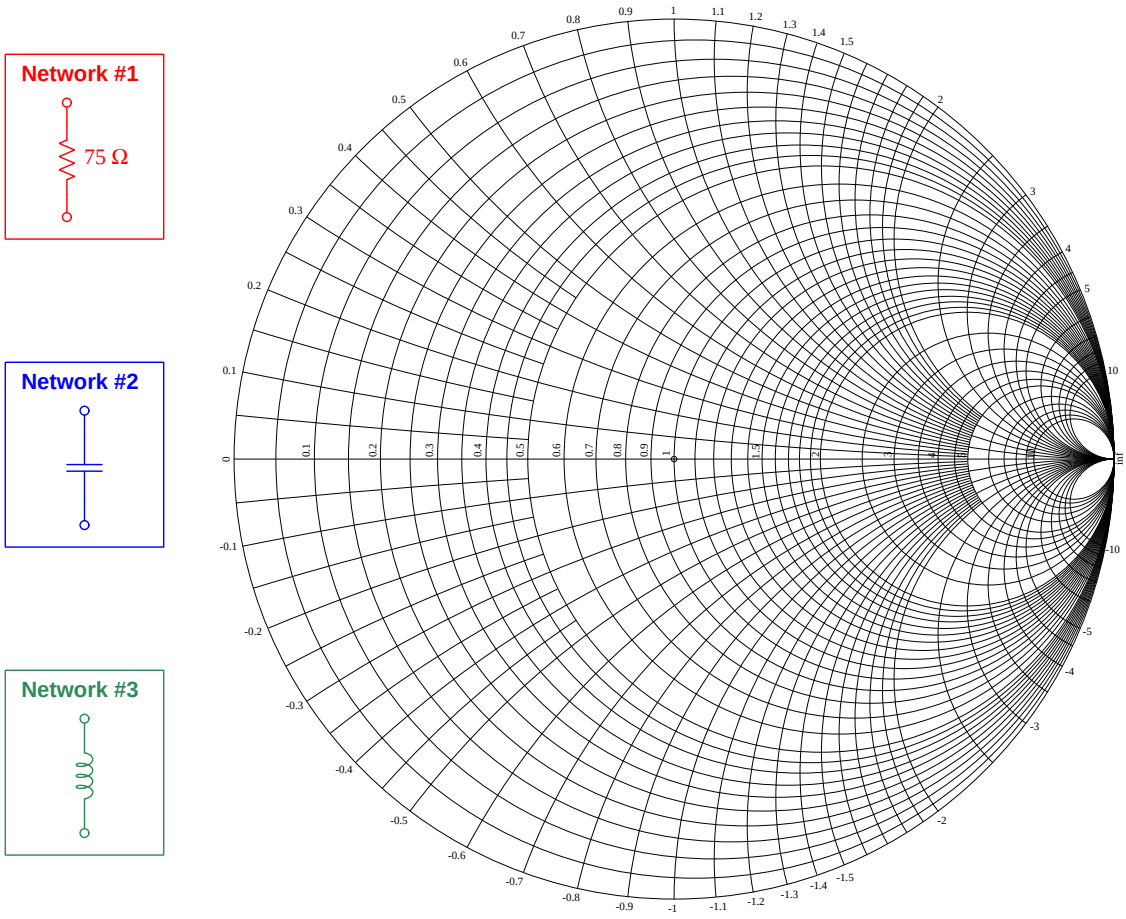
Cut-and-try problem-solving strategy

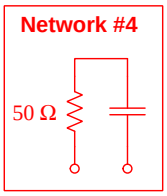
Algebraic substitution

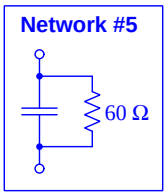
???

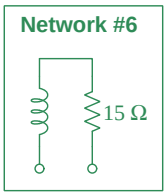
5.1.3 Network frequency-sweep tests

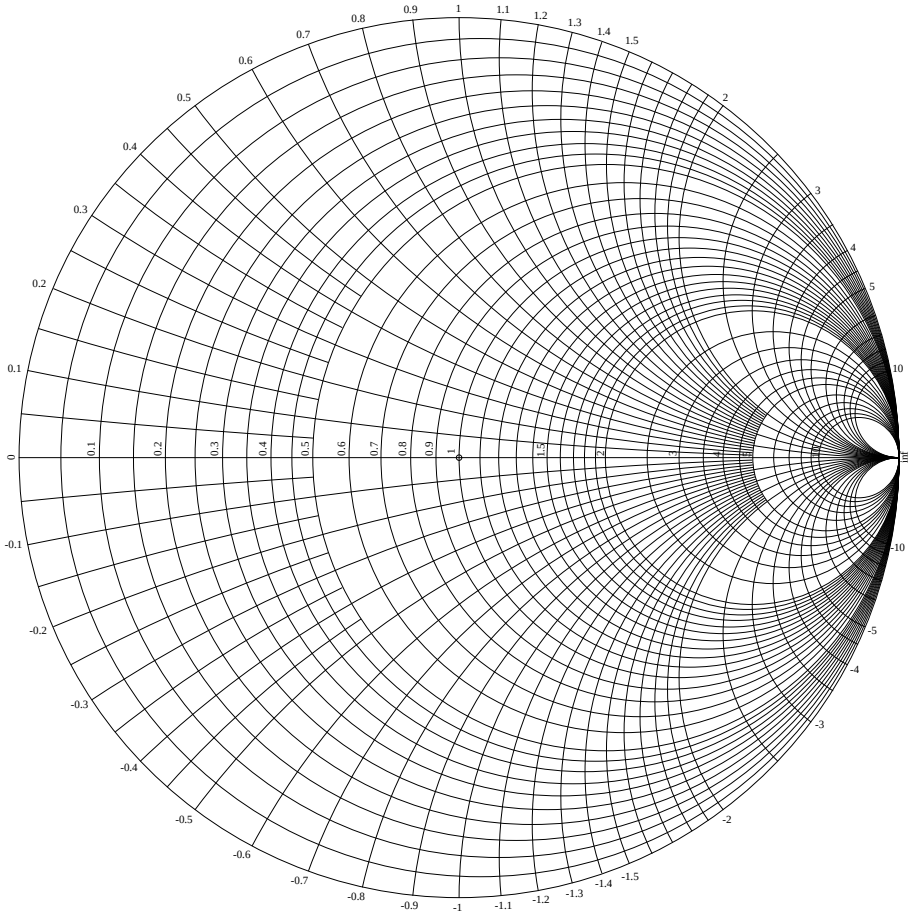
Qualitatively sketch the results of frequency-sweep tests for the following networks, showing their impedance values on the Smith Chart as the test frequency is swept from zero Hz (DC) to some arbitrarily high frequency. Sketch a circular marker symbol at the low-frequency (DC) end of the trace and a triangular marker symbol at the high-frequency end, assuming the Smith Chart is normalized to 50 Ohms:









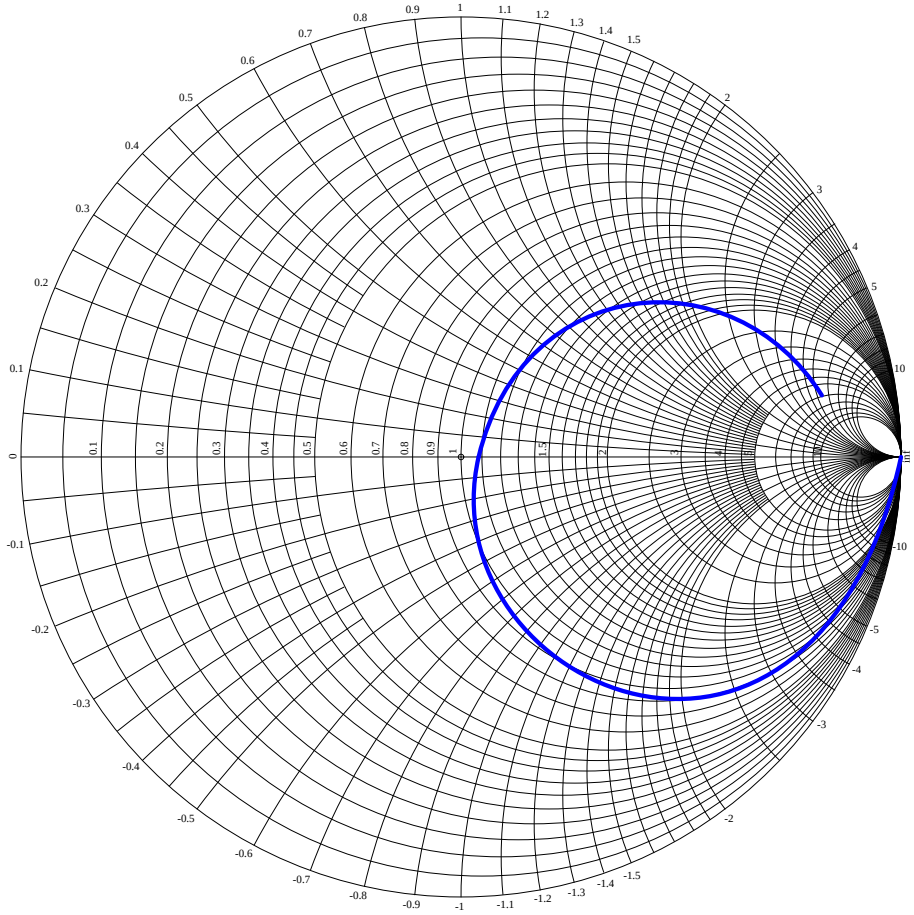


Challenges

- Explain how the problem-solving strategy of *limiting cases* applies to these sweeps.

5.1.4 Smith Chart test of a dipole antenna

Below we see the results of frequency-sweep test for a dipole antenna:



Identify the point on this curve representing a test frequency of 0 Hz (DC), and also the point representing the highest test frequency.

Where on this frequency sweep does the antenna *resonate* as it should for maximum effectiveness?

Identify the potential for this antenna to exhibit a *second* resonant frequency, and explain how this is possible.

Challenges

- Explain how the problem-solving strategy of *limiting cases* applies to these sweeps.

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = 1.25663706212(19) $\times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = 8.8541878128(13) $\times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = 6.67430(15) $\times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 Practice: complex number calculations

These complex-number arithmetic problems are presented to you, complete with answers (shown in **bold**), for the purpose of practice, since nearly all AC circuit calculations will need to be performed using complex numbers. Use these practice calculations to check your ability either to perform these calculations "by hand" (using trigonometric functions) or your ability to use your calculator's complex-number functionality.

Note: electronic hand calculators and computer-based calculation programs use the proper mathematical notation i to represent imaginary numbers rather than j. The letter j is used in electrical engineering work in order to avoid confusion with i being misinterpreted as "current". Also note that calculators and software programs usually default to *radians* for angle measurement rather than *degrees*, and will have to be configured (or converted) for degrees in order to handle the polar-form complex quantities shown here. Check the "mode" options of your hand calculator to ensure angles are in the correct unit and also that it will display in either rectangular or polar.

Addition and subtraction:

$(5 + j6) + (2 - j1) = \mathbf{7 + j5}$
$(10 - j8) + (4 - j3) = \mathbf{14 - j11}$
$(-3 + j0) + (9 - j12) = \mathbf{6 - j12}$

$(3 + j5) - (0 - j9) = \mathbf{3 + j14}$
$(25 - j84) - (4 - j3) = \mathbf{21 - j81}$
$(-1500 + j40) + (299 - j128) = \mathbf{-1201 - j88}$

$(25\angle 15^o) + (10\angle 74^o) = \mathbf{31.35\angle 30.87^o}$
$(1000\angle 43^o) + (1200\angle -20^o) = \mathbf{1878.7\angle 8.311^o}$
$(522\angle 71^o) - (85\angle 30^o) = \mathbf{461.23\angle 77.94^o}$

Multiplication and division:

$(25\angle 15^o) \times (12\angle 10^o) = \mathbf{300\angle 25^o}$
$(1\angle 25^o) \times (500\angle -30^o) = \mathbf{500\angle -5^o}$
$(522\angle 71^o) \times (33\angle 9^o) = \mathbf{17226\angle 80^o}$

$\frac{10\angle -80^o}{1\angle 0^o} = \mathbf{10\angle -80^o}$

$\frac{25\angle 120^o}{3.5\angle -55^o} = \mathbf{7.142\angle 175^o}$

$\frac{-66\angle 67^o}{8\angle -42^o} = \mathbf{8.25\angle -71^o}$

$(3 + j5) \times (2 - j1) = \mathbf{11 + j7}$
$(10 - j8) \times (4 - j3) = \mathbf{16 - j62}$

$\frac{(3+j4)}{(12-j2)} = \mathbf{0.1892 + j0.3649}$

Reciprocation:

$$\frac{1}{(15\angle 60^o)} = \mathbf{0.0667\angle -60^o}$$

$$\frac{1}{(750\angle -38^o)} = \mathbf{0.00133\angle 38^o}$$

$$\frac{1}{(10+j3)} = \mathbf{0.0917 - j0.0275}$$

$$\frac{1}{\frac{1}{15\angle 45^o}+\frac{1}{92\angle -25^o}} = \mathbf{14.06\angle 36.74^o}$$

$$\frac{1}{\frac{1}{1200\angle 73^o}+\frac{1}{574\angle 21^o}} = \mathbf{425.7\angle 37.23^o}$$

$$\frac{1}{\frac{1}{23k\angle -67^o}+\frac{1}{10k\angle -81^o}} = \mathbf{7.013k\angle -76.77^o}$$

$$\frac{1}{\frac{1}{110\angle -34^o}+\frac{1}{80\angle 19^o}+\frac{1}{70\angle 10}} = \mathbf{29.89\angle 2.513^o}$$

$$\frac{1}{\frac{1}{89k\angle -5^o}+\frac{1}{15k\angle 33^o}+\frac{1}{9.35k\angle 45}} = \mathbf{5.531k\angle 37.86^o}$$

$$\frac{1}{\frac{1}{512\angle 34^o}+\frac{1}{1k\angle -25^o}+\frac{1}{942\angle -20}+\frac{1}{2.2k\angle 44^o}} = \mathbf{256.4\angle 9.181^o}$$

Sign reversal:

$-(45\angle 70^o) = \mathbf{45}\angle -\mathbf{110^o}$

$-(90\angle -20^o) = \mathbf{90}\angle\mathbf{160^o}$

$-(5 + j8) = -\mathbf{5} - \mathbf{j8}$

$-(-3 + j9) = \mathbf{3} - \mathbf{j9}$

$-(10 - j15) = -\mathbf{10} + \mathbf{j15}$

Practical suggestions for using your calculator to perform these operations:

- Surround each complex-number quantity with parentheses when setting up arithmetic operations; e.g., `(3 + j5) * (4 - j2)` instead of `3 + j5 * 4 - j2`. This habit will guarantee your calculator executes the desired order of operations rather than assert its own. For instance, in the example given here the calculator may choose to multiply $j5$ by 4 and then add on 3 and $-j2$ since multiplication typically precedes addition, if the two complex numbers are not encapsulated in their own sets of parentheses.

- Store all calculated results in memory and then recall from memory when re-using those values, rather than re-entering previously-calculated values by hand or sampling previously-calculated values from the multi-line display. Manually re-entering values invites rounding errors and keystroke errors in all cases, and I've found certain calculators (I'm looking at you, *TI*!) fail to properly enter complex-number values when sampled from their multi-line displays. Getting in the habit of using your calculator's memory locations is an all-around good habit that will serve you very well!

Some Texas Instruments brand calculators such as the TI-84 offer an exponential key and imaginary (i) key which allows you to enter numbers in complex exponential form (i.e. $e^{i\theta}$). With the TI-84, for example, the complex number $10 - j8$ may be entered in either of the two following forms:

`(10 - i8)` or `(10 - 8i)`

The result may be displayed in either rectangular or polar forms according to the complex-number display *mode* the TI-84 calculator has been set to. In rectangular mode the displayed result for $10 - j8$ will be `10 - 8i`, whereas in polar mode the displayed result will be `12.806` $\mathtt{e}^{-38.66i}$. Note how the TI-84 uses exponential notation for polar display, where the angle (-38.66 degrees, in this example) is an imaginary power of e.

If you wish to enter a complex number in polar form on a TI-84, you must unfortunately express the angle in units of *radians* (even though the calculator is able to display the result in *degrees*). For example, to enter the number $25\angle 15^o$ into a TI-84 calculator, you must type:

`25` $\mathtt{e}^{i15\pi/180}$

The fraction $\pi/180$ is the conversion factor from degrees to radians, since there are 2π radians to a full circle, or π radians to every 180 degrees. Thus, writing $15\pi/180$ multiplies the desired angle (15 degrees) by the conversion factor $\pi/180$ to yield a power in radians. The obligatory i simply makes this power an imaginary quantity, which is mathematically necessary with exponential notation for describing a complex number. It should be noted that the order of entry for the power matters little. $i15\pi/180$ works just as well as $15i\pi/180$ or $15\pi/180i$.

A time-saving step some students find useful is to save the imaginary quantity $i\pi/180$ to a memory location in the TI-84 such as Z. That way, they can recall that imaginary factor from memory instead of typing the whole thing by hand every time they wish to enter a polar-form complex number. Supposing the memory location Z contains $i\pi/180$, entering the number $25\angle 15^o$ becomes as simple as:

25 e^{15Z} or 25 e^{Z15}

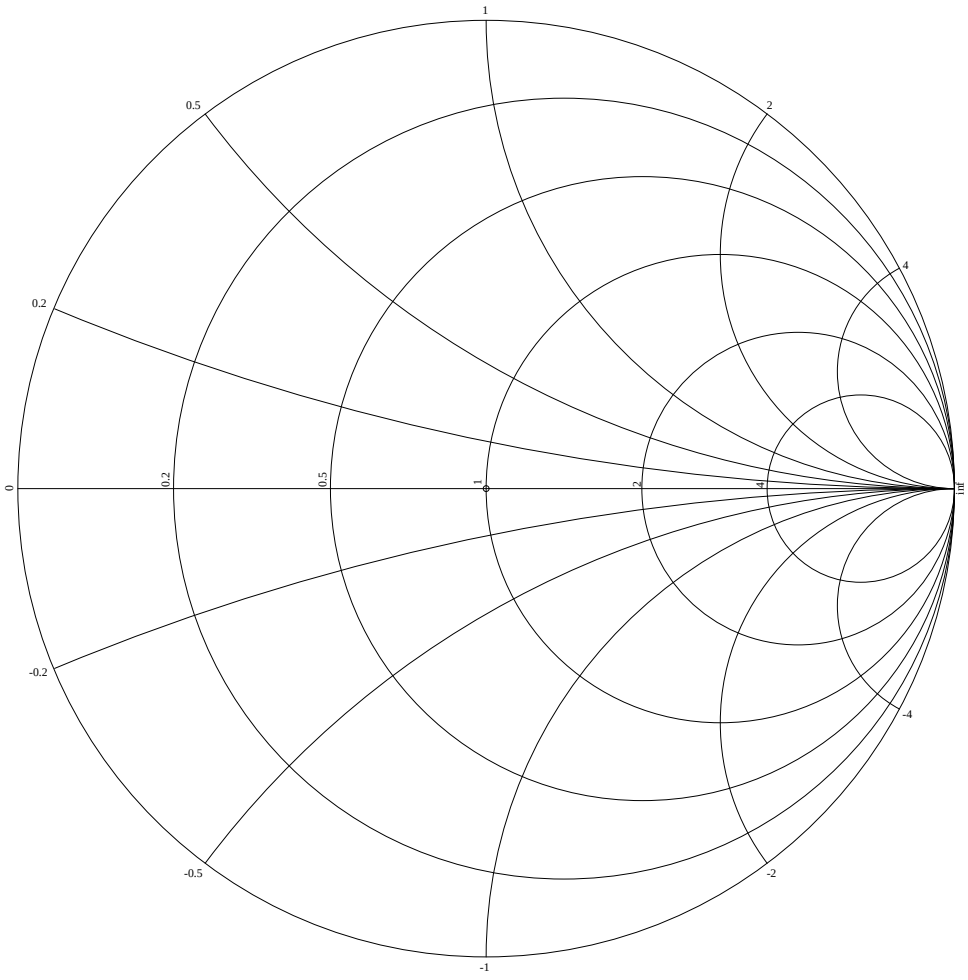
It should be understood that *any* memory location in your calculator is suitable for storing $i\pi/180$, not just Z. The TI-84 calculator even provides a Θ memory location (<Alpha>-<3>) that you may use and find easy to remember because of its common association with angles. It should also be understood that this imaginary quantity is not the same as i or j, which the calculator already provides a dedicated function for. The imaginary quantity we're storing in memory for the purpose of entering polar-notation angles contains not only i but also the $\pi/180$ conversion factor necessary for translating your *degree* entry into *radians*.

Some calculators provide easier means of entering and displaying complex numbers in polar form. Both the Texas Instruments TI-36X Pro and TI-89 offer an angle symbol ($\angle$) for this purpose, the TI-36X Pro being a much less expensive and less complex calculator than the TI-89. Entering the number $25 \angle 15^o$ into one of these calculators is as easy as typing 25 $\angle$ 15, and likewise the result will be displayed in this same form when the calculator is set to "polar" complex mode.

5.2.4 Plotting component impedances on a Smith Chart

Plot points on a Smith Chart for the following components, assuming a 50 Ohm normalizing value for the Chart:

- 200 Ohm resistor at 55 kHz
- 270 μH inductor at 10 kHz
- 150 nF capacitor at 14 kHz



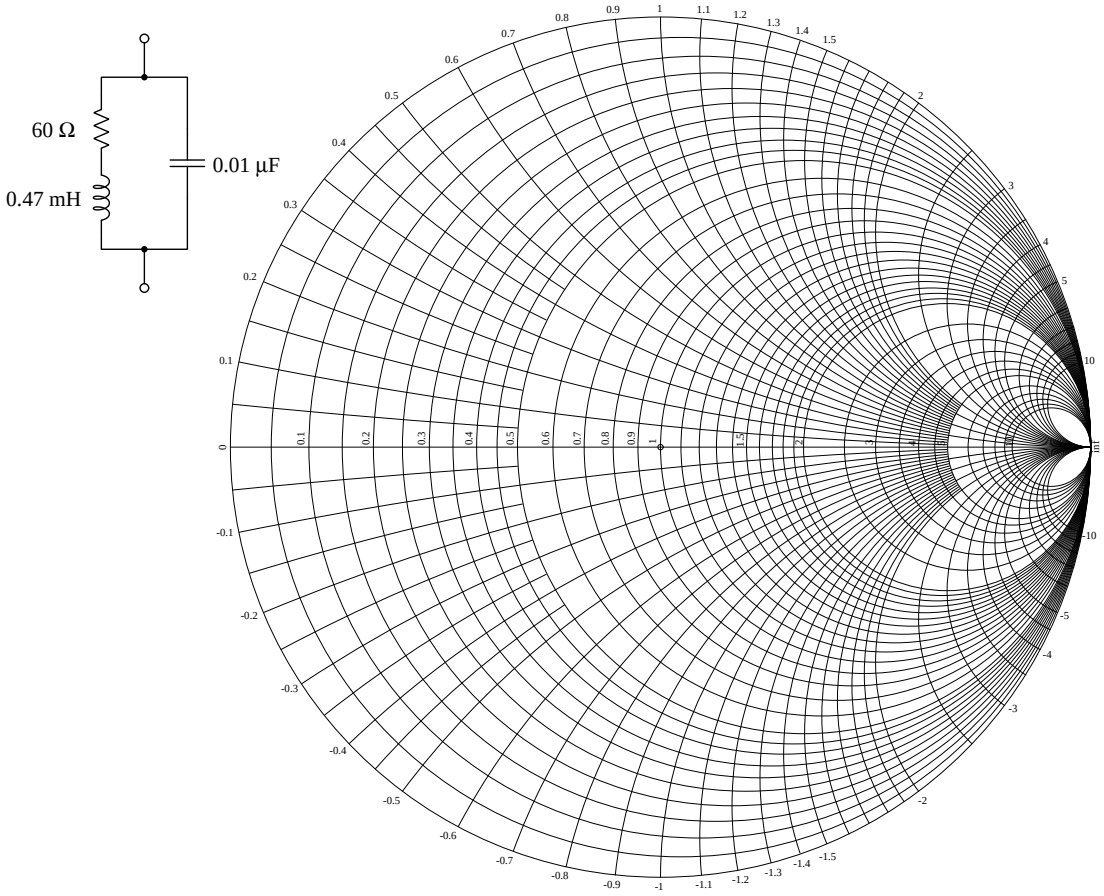
Challenges

- Identify how the real-life presence of *parasitic* properties would affect each of these components' placement on the Smith Chart.

5.2.5 Frequency-sweep simulation program

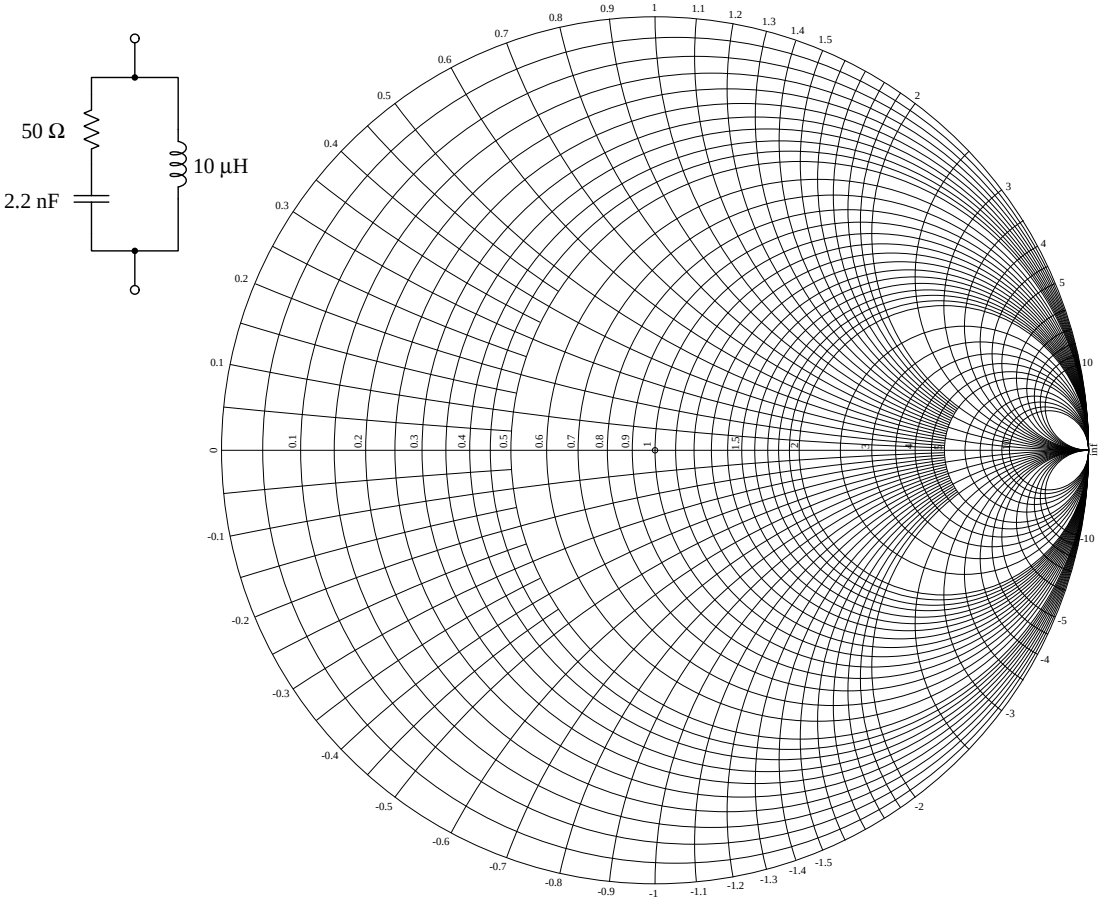
Write text-based computer programs (e.g. C, C++, Python) to calculate the complex impedances of the following RLC networks over a frequency range of DC to 100 kHz, and then trace the results of this sweep on the corresponding Smith Charts assuming they are all normalized to 50 Ohms:

Example #1



Example #2

Frequency sweep range of DC to 3 MHz, normalized to 50 Ohms:



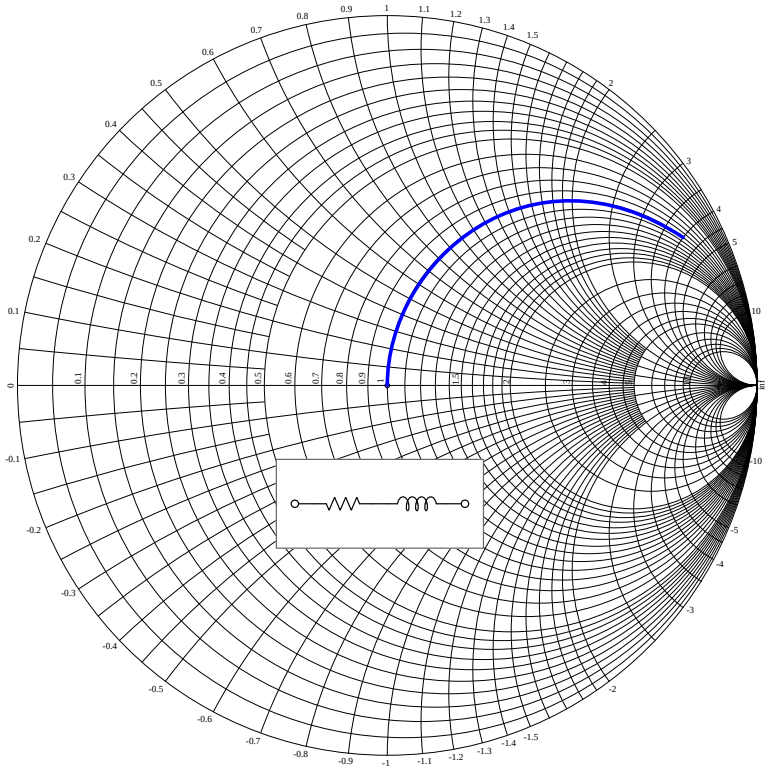
Challenges

- At what frequency does each of these networks' power factor values equal unity?.

5.2.6 Frequency-sweep test modifications

Example #1

First, calculate the R and L values for this network, assuming a Smith Chart normalized to 50 Ohms and frequency sweeping from 0 Hz to 25 kHz:



How would this frequency-sweep test curve change if the resistor were to fail open?

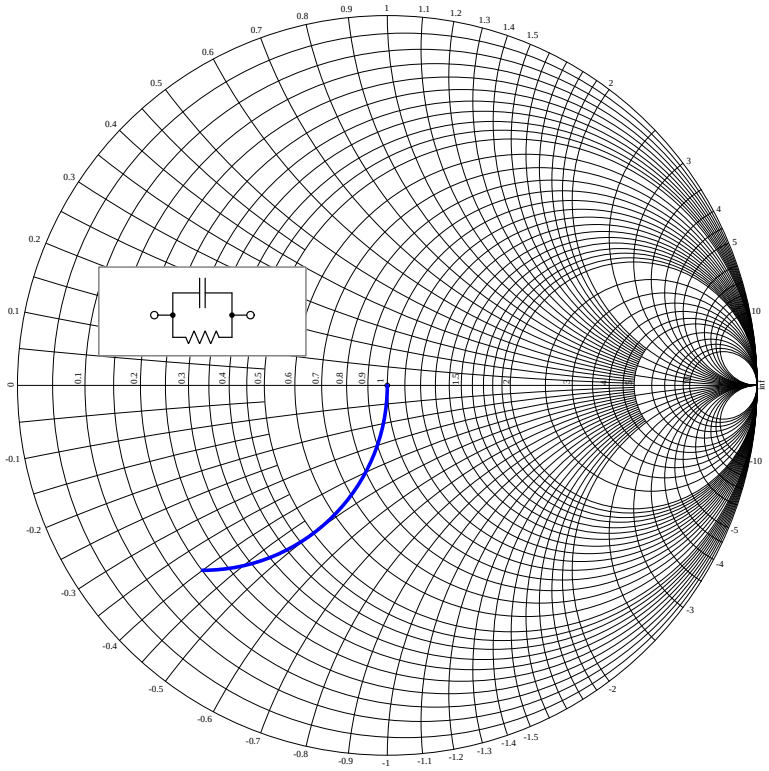
How would this frequency-sweep test curve change if the inductor were to fail shorted?

How would this frequency-sweep test curve change if the resistor were to fail shorted?

How would this frequency-sweep test curve change if the test were re-done to cover a range of 0 Hz to 10 kHz instead of the original range which was 0 Hz to 25 kHz?

Example #2

First, calculate the R and C values for this network, assuming a Smith Chart normalized to 50 Ohms and frequency sweeping from 0 Hz to 2 kHz:



How would this frequency-sweep test curve change if the resistor were to fail open?

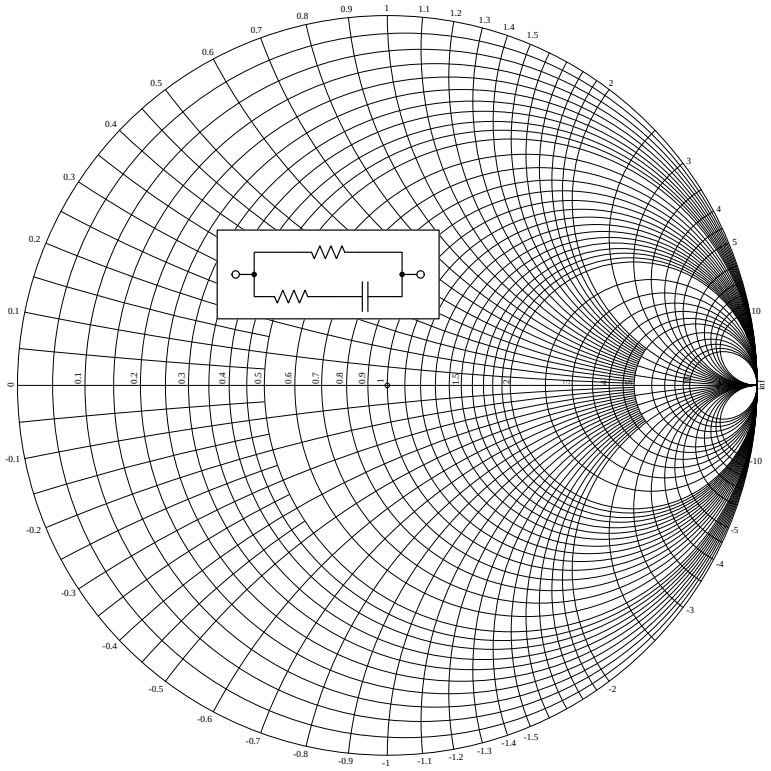
How would this frequency-sweep test curve change if the capacitor were to fail shorted?

How would this frequency-sweep test curve change if the test were re-done to cover a range of 0 Hz to 10 kHz instead of the original range which was 0 Hz to 2 kHz?

How would this frequency-sweep test curve change if the spacing between the capacitor's plates were reduced without altering any other parameters for the capacitor such as dielectric permittivity or plate area?

Example #3

Plot the frequency-sweep curve for the RC network shown, assuming a Smith Chart normalized to 50 Ohms, both resistors being 50 Ohms each, and frequency sweeping from DC to infinity:



How would this frequency-sweep test curve change if the capacitor were to fail shorted?

How would the frequency-sweep test curve change if the high-end of the frequency sweep stopped at some finite value instead of infinity?

How would the frequency-sweep test curve change if the capacitor were replaced by an inductor?

Challenges

- What common patterns do you see among all these examples?

5.2.7 Impedance-matching networks

Design an impedance-matching network to match a 250 Ohm resistive load to a 50 Ohm resistive source, at a frequency of 4 MHz.

Design an impedance-matching network to match a 20 Ohm resistive load to a 50 Ohm resistive source, at a frequency of 200 kHz.

Challenges

- For each of these, describe an alternative solution.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

9-11 March 2026 – edited an instructor note. Also edited image_???? to show normalized impedance and admittance values in addition to the de-normalized values.

7 October 2025 – corrected a coloring error in image_7109.

5 March 2025 – corrected a minor error in the Tutorial, where I said "standard Smith Chart" when I meant to say "Z-Y Smith Chart".

9 October 2024 – minor edits to the Tutorial, including a typographical error correction (the word "again" was missing the first letter and appearing as "gain").

26 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors. Also divided the Tutorial into sections.

7 March 2024 – elaborated on equivalent series and parallel network conversions in the "Impedance and admittance Smith Charts" section of the Tutorial. Also corrected a few minor typographical errors.

5 March 2024 – added more instructor notes.

1 March 2024 – made edits to the "Plotting impedance for an open" and "Plotting impedance for a short" subsections of "The Basic Smith Chart" section of the Tutorial to discuss the lack of energy storage in shorts and opens, and how this affects the complex-number impedance values of each. Also edited image_5885 to show the imaginary portion of the infinite impedance as negative in sign. Also corrected a spelling error.

29 February 2024 – minor edits to formatting on the "Frequency-sweep simulation program" Quantitative Reasoning question. Also added 50 Ohm normalized qualification to the "Network frequency-sweep tests" Conceptual Reasoning question. Also added a Technical References section with inductor and capacitor parasitics.

27 February 2024 – populated the Introduction chapter with questions.

11-12 February 2024 – added a new Quantitative Reasoning question challenging students to identify how various frequency-sweep test curves would be altered by changes made to their respective RC, RL, or RLC networks. Added a Qualitative Reasoning question about a Smith Chart's display of a dipole antenna's characteristic behavior.

7-8 January 2024 – added another network to the "Frequency-sweep simulation program" Quantitative Reasoning question. Also added a new Case Tutorial section showing frequency-sweep impedances on Smith charts for simple RLC networks using Elsie software, and corrected some spelling errors.

1 October 2023 – added a Quantitative Reasoning question.

29 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

18 May 2022 – placed the "Practice: complex number calculations" questions into its own file (case_complexpractice.latex) so it may be shared amongst multiple modules.

3-4 March 2022 – added more content to the Tutorial and edited some illustrations.

11-17 January 2022 – added more content to the Tutorial and also questions.

6 January 2022 – completed an Xcircuit template file for a Smith Chart (image_4874.eps). This required hand-editing of the Xcircuit PostScript output to get the circles and arcs placed precisely where they need to be, but the result is a compact (just over 15 kB) graphic file that may be opened by Xcircuit and annotated at will.

4 January 2022 – created sections for the Tutorial.

21 December 2021 – document first created.

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