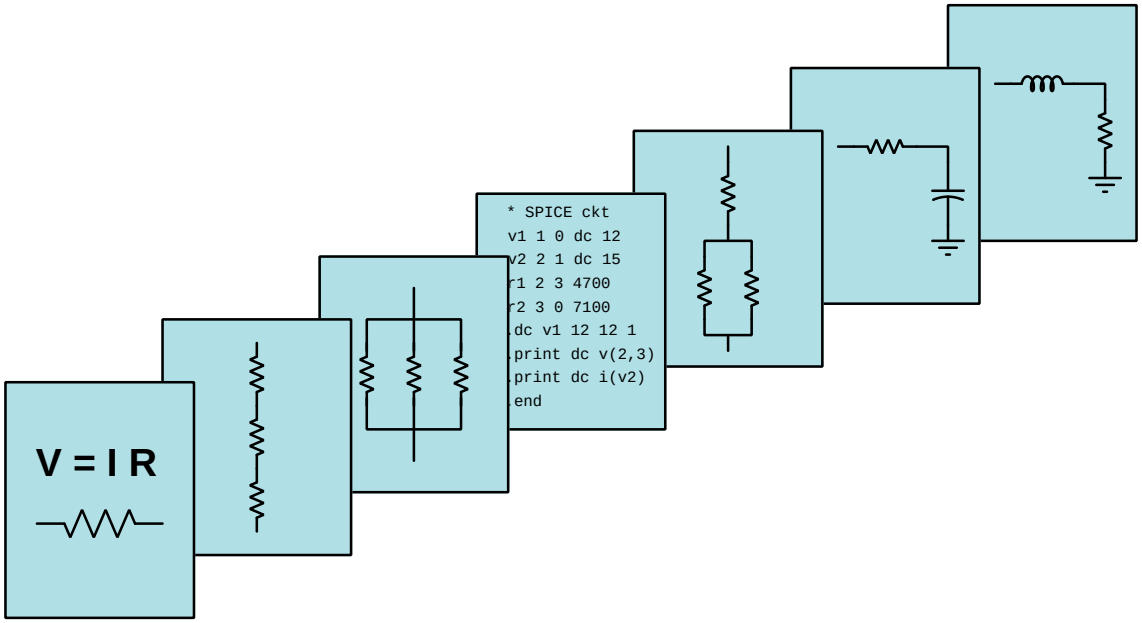
Modular Electronics Learning (ModEL) project



Thévenin's and Norton's Theorems

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Chapter 1

Introduction

1.1 Recommendations for students

A *network theorem* is a useful tool for simplifying the analysis of some system, provable on the basis of more fundamental principles and/or theorems. Anyone familiar with the proofs and theorems of geometry (e.g. the Isosceles Triangle Theorem[1]) has experienced the benefit of theorems, allowing rigorous analysis of geometric systems without having to constantly resort to low-level definitions and axioms of geometry. Similarly, network theorems are proven tools useful for quickly and easily analyzing complex electrical networks without being limited to fundamental rules such as Ohm's Law and Kirchhoff's Laws.

Thévenin's and Norton's Theorems are two such network theorems, and they find frequent application in electronic circuit analysis. The end-result of each is simple: any linear network, no matter how complex, may be reduced to an equivalent network consisting of one voltage source and one resistor (a Thévenin equivalent) or one current source and one resistor (a Norton equivalent). As with the Superposition Theorem, both Thévenin's and Norton's Theorems are limited to *linear* functions, which means components having stable resistance (R) over their operational ranges and directly-mathematical functions.

Important concepts related to these network theorems include **voltage sources**, **current sources**, **internal resistance**, the effects of **opens** versus **shorts**, **light** versus **heavy** load conditions, **Ohm's Law**, properties of **series** networks, properties of **parallel** networks, **bridge** networks, **linear** versus **non-linear** functions, **equivalent** electrical networks, and **substitution** of source resistances.

A problem-solving technique found throughout this text is the *thought experiment*, whereby we imagine altering some facet of an electrical network and analyze the effects of that alteration, for the purpose of better understanding the network. You may think of this as akin to a real experiment except that we apply known principles to a hypothetical condition rather than set up real components and take real measurements.

[1] According to the Isosceles Triangle Theorem, any triangle having two sides of equal length must exhibit equal angles opposite to those two sides.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Another useful reading strategy is to write your own summary page of important principles, especially when those principles mirror each other. For this module I would recommend writing your own summaries of network Thévenization and Nortonization: the steps involved to perform each type of analysis.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate Thévenin's Theorem? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate Norton's Theorem? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to test whether or not two different electrical sources were equivalent to one another? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- In what ways do voltage sources differ from current sources?
- What is the definition of a "series" network?
- What is the definition of a "parallel" network?
- How much internal resistance does an ideal voltage source possess?
- How much internal resistance does an ideal current source possess?
- What makes a load either "heavy" or "light" from the perspective of the source supplying energy to it?
- How would you describe the procedure for "Thévenizing" a network, in your own words?
- How would you describe the procedure for "Nortonizing" a network, in your own words?
- How does either of these theorems help to simplify complex networks?
- What does it mean for two electrical networks to be considered "equivalent" to one another?
- What is a "linear" electrical component, and what are some practical examples of linear components?
- What is a "nonlinear" electrical component, and what are some practical examples of nonlinear components?

- Why is a five-resistor unbalanced bridge circuit impossible to analyze as a series-parallel network?
- Would the bridge circuit be easier to analyze, harder to analyze, or the same difficulty to analyze if it were balanced rather than unbalanced?
- Why is it permissible to "wire-bend" a circuit diagram to aid in its analysis?
- What do we replace a voltage source with when "disabling" it during Thévenin or Norton analysis, and why?
- What do we replace a current source with when "disabling" it during Thévenin or Norton analysis, and why?

1.2 Challenging concepts related to Thévenin's and Norton's Theorems

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Equivalent electrical networks** – the possibility that completely different networks of components may nevertheless *behave* identically from the perspective of two or more connection terminals is non-intuitive for many students new to electric circuits. However, there are many analogues in life that may be used to help explain. For example, a box containing an unknown assortment of items may have precisely the same weight as a different box containing something completely different from the first: i.e. from the perspective of weight sense at the handle when lifting each box, the boxes may behave the same even though their contents may be wholly different.
- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts! Both Thévenin's and Norton's Theorems embody this principle by reducing any collection of linear and bilateral electrical components to a single source and a single resistance behaving precisely the same as the original network from the perspective of two connection terminals.
- **Creative application of foundational concepts** – many students enter college-level study of electronics with an educational background stressing rote memorization at the expense of logical reasoning from trusted principles, and as such tend to find circuit analysis daunting where there is no single procedure or single formula always yielding the correct answer(s). The educational solution to this is for the instructor to *constantly* tie all new circuit concepts back to previously-learned principles and show the logical relationships between them, and then with every new circuit-analysis example challenge students to reason from those concepts on their own. When students construct a mental "toolbox" they may apply in a plurality of ways to any new challenge they encounter, they will be as free as any skilled mechanic who knows how and why each of their hand tools functions to freely select and use whichever of those tools is most appropriate at any given time. A word of caution for new instructors: *be very sparing in your direction on how to use those tools, instead engaging with students Socratically when they are "stuck" and cannot see a path to a solution!*

Thévenin's and Norton's Theorems are particularly challenging because they involve *spatial reasoning* more so than other electrical analysis techniques. This means it is particularly important to determine exactly *where* the equivalent network "fits" within the original network as shown. A similar spatial-relations challenge is in identifying series and parallel connections while sources are disabled, and/or when load terminals are shorted together (e.g. finding the Norton current).

A helpful resource for the confused is the *Animations* chapter section showing Thévenization performed step-by-step on a series-parallel electrical network. As usual, the *Case Tutorial* chapter also contains completely-worked examples of networks being reduced to Thévenin and Norton equivalents.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples. *This is especially important with Thévenin's and Norton's Theorems. Do not be surprised when students plead to have you verbally explain everything to them rather than closely inspect the approaches described in the text. Remember that their intelligence is up to the task – always treat them as capable thinkers!*

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of Thévenin's Theorem to the simplification of electrical networks

  Assessment – Reduce a given electrical network to a Thévenin equivalent; e.g. pose problems in the form of the "Thévenin and Norton equivalents of a single-source network" Quantitative Reasoning question.

  Assessment – Reduce a given electrical network to a Thévenin equivalent; e.g. pose problems in the form of the "Thévenin equivalent of an AC/DC power supply" Quantitative Reasoning question.

- **Outcome** – Apply the concept of Norton's Theorem to the simplification of electrical networks

  Assessment – Reduce a given electrical network to a Norton equivalent; e.g. pose problems in the form of the "Thévenin and Norton equivalents of a single-source network" Quantitative Reasoning question.

  Assessment – Reduce a given electrical network to a Norton equivalent; e.g. pose problems in the form of the "Norton equivalent of an electric arc welder" Quantitative Reasoning question.

Chapter 2

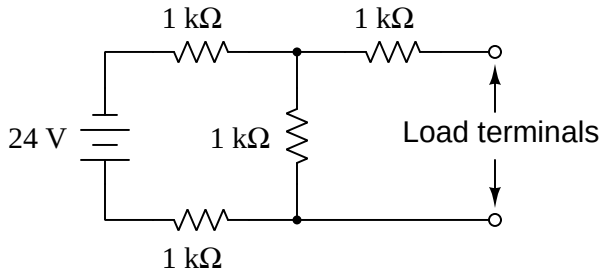
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

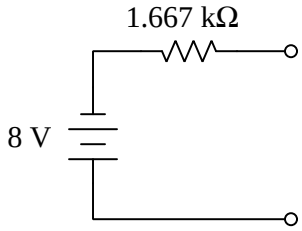
These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: Thévenizing series-parallel networks

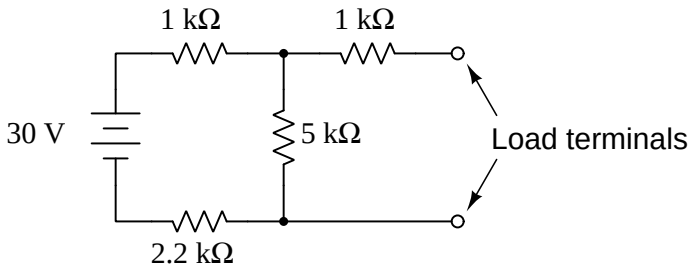
Original network #1:



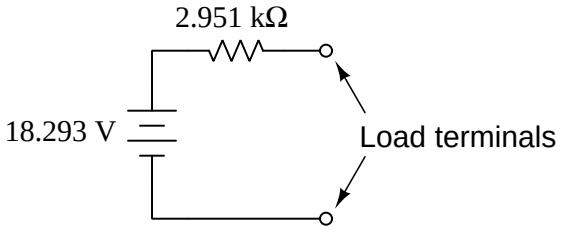
Thévenin equivalent network #1:



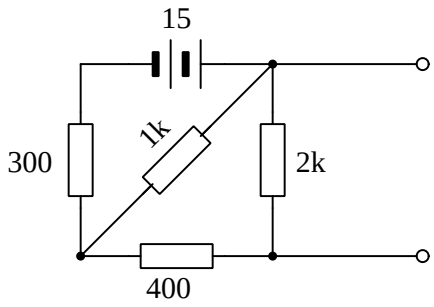
Original network #2:



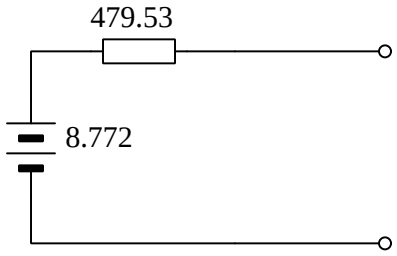
Thévenin equivalent network #2:



Original network #3:

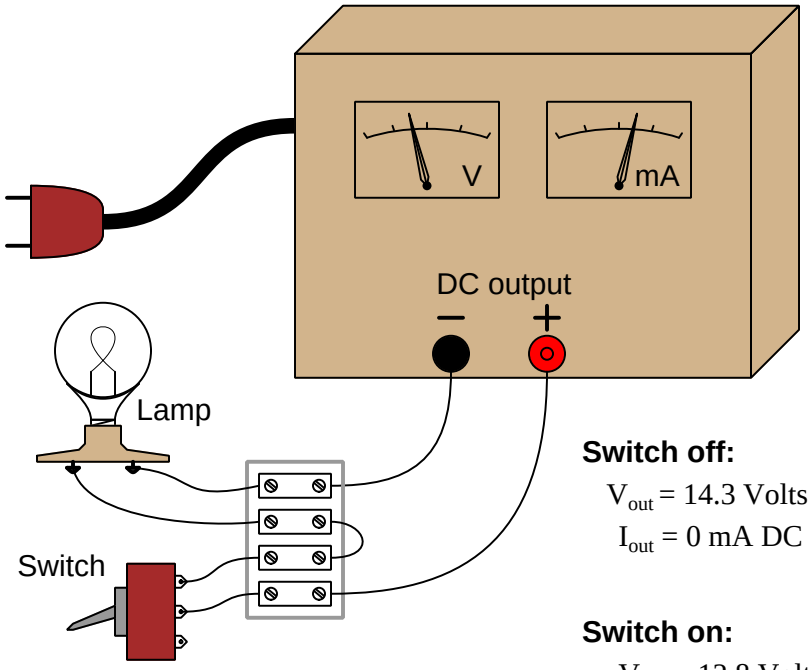


Thévenin equivalent network #3:



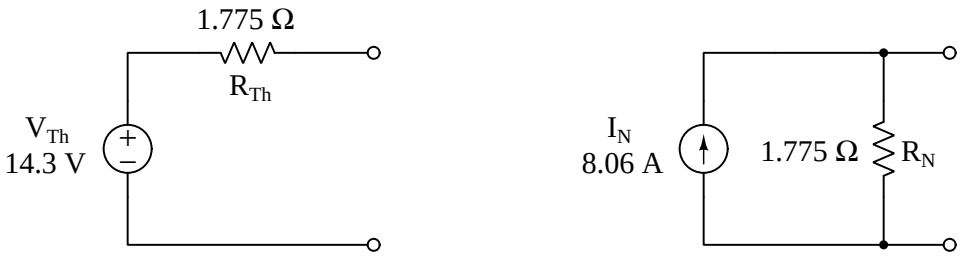
2.2 Example: Thévenizing and Nortonizing a power supply

Based on the following open-circuit and loaded tests of an AC-to-DC power supply (voltage being measured at the power supply's terminals), we may derive a Thévenin equivalent network:



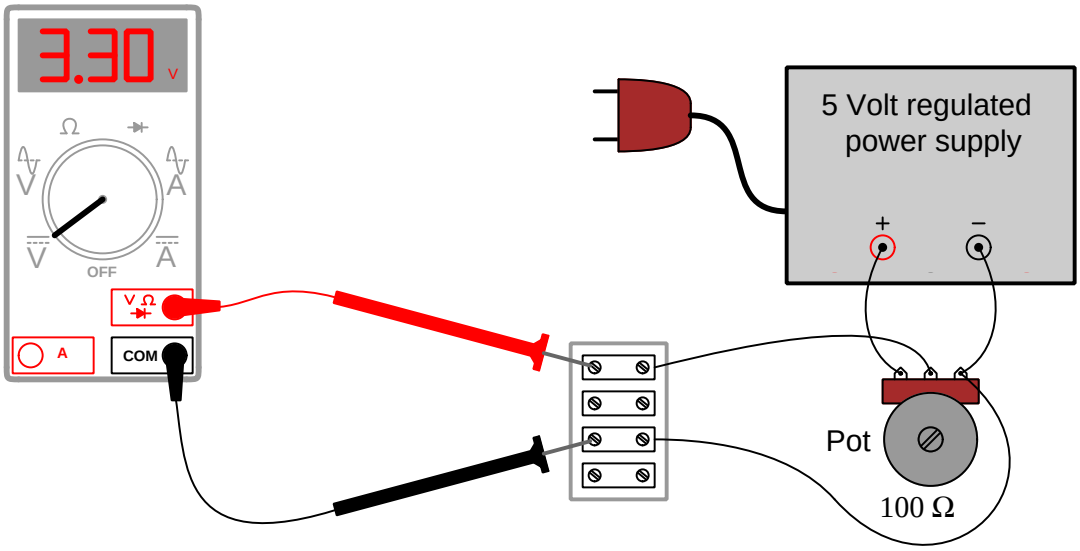
Thévenin voltage is simply the power supply's open-circuit voltage value of 14.3 Volts. By calculating how much that terminal voltage "sagged" under the 845 mA load (14.3 Volts − 12.8 Volts = 1.5 Volts of "sag"), we know how much voltage must be dropped across the Thévenin equivalent resistance at that amount of current. Ohm's Law then provides us with that R_{Th} value.

Thévenin and Norton equivalent networks:

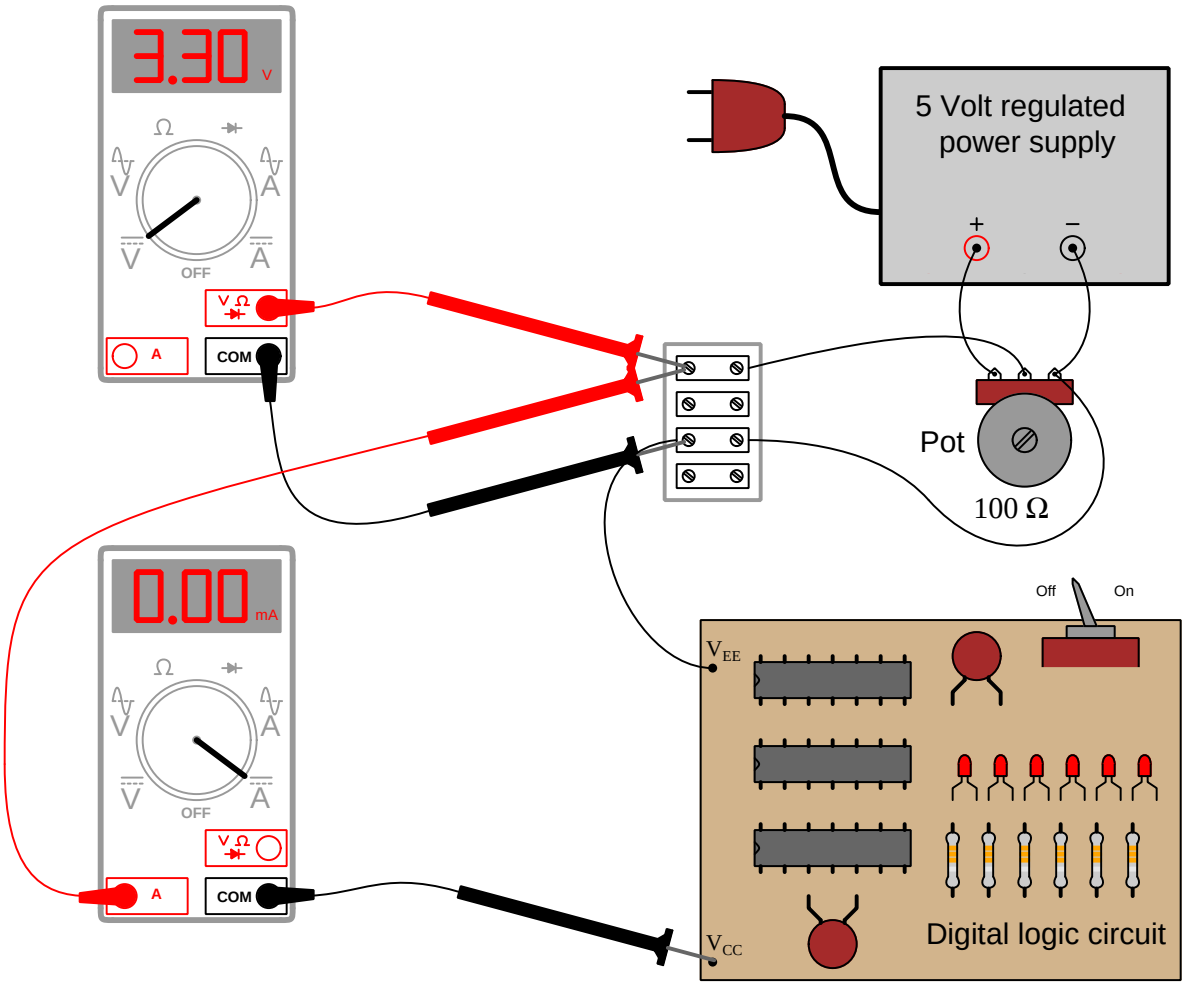


2.3 Example: improvised 3.3 Volt power supply

Suppose that a student builds a digital logic circuit designed to operate with a DC source voltage of 3.3 Volts, but they only have a 5.0 Volt DC power supply on hand. Improvising, the student connects a *potentiometer* to the output of the 5.0 Volt DC power supply terminals and carefully adjusts the knob to the 66% position where she measures 3.3 Volts between the two wires attached to the terminal block:



The student's plan is to connect the V_{CC} (+) and V_{EE} (−) DC terminals of her digital logic circuit to the same two terminal block terminals where the voltmeter connects. An ammeter installed in series between the digital circuit and the terminal block measures load current:



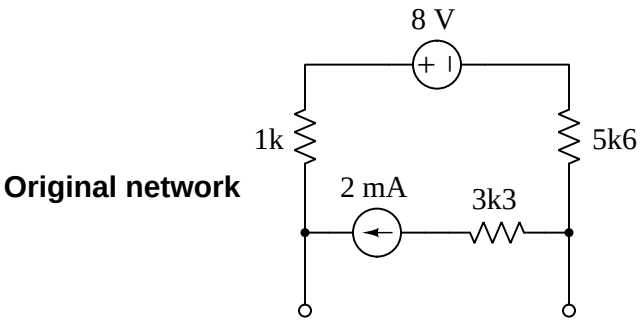
With both meters in place, the student will be able to monitor the stability of the DC power sent to the digital logic circuit as its current varies. The regulated 5.0 Volt power supply is designed to compensate for variations in load current, maintaining a stable 5 Volts for all loads within its rated capability. However, the resistance of the potentiometer will cause some voltage "sag" to occur at the digital circuit's power terminals as its current varies. This is an inherent limitation of the improvised power supply circuit, achieving a reduced output voltage at the expense of less stability.

This table shows the results of the student's testing:

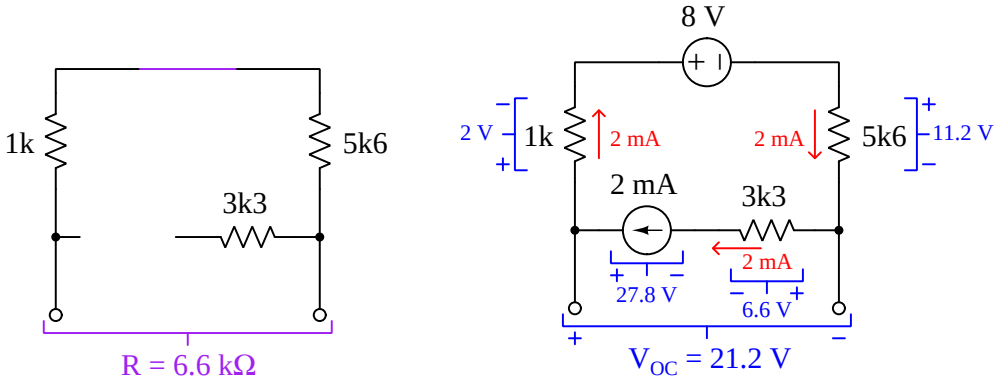
Switch	**Current**	**Voltage**
Off	0.00 mA	3.30 V
On	1.00 mA	3.28 V
On	2.00 mA	3.26 V
On	5.00 mA	3.19 V
On	7.40 mA	3.13 V
On	9.25 mA	3.09 V

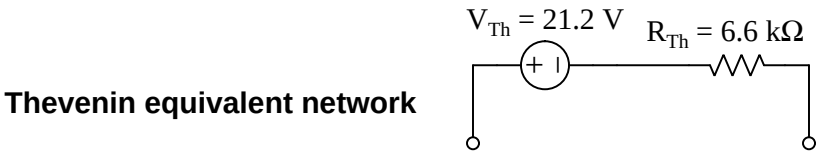
Using a 100 Ω potentiometer to reduce 5.0 Volts down to 3.3 Volts is equivalent to an ideal 3.3 Volt source with a series resistance value of 22.44 Ω.

2.4 Example: Thévenizing a multi-source network



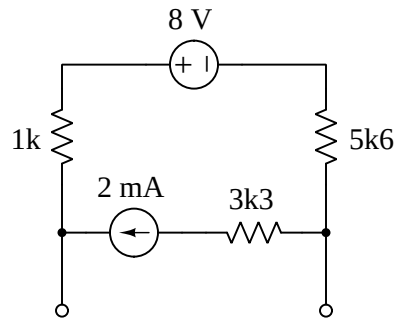
Calculating terminal resistance (left) and open-circuit terminal voltage (right):



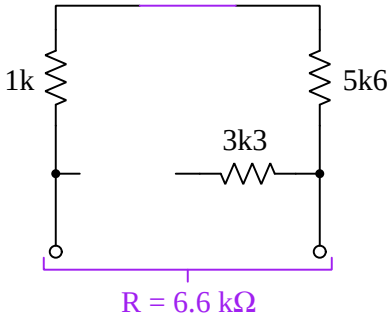


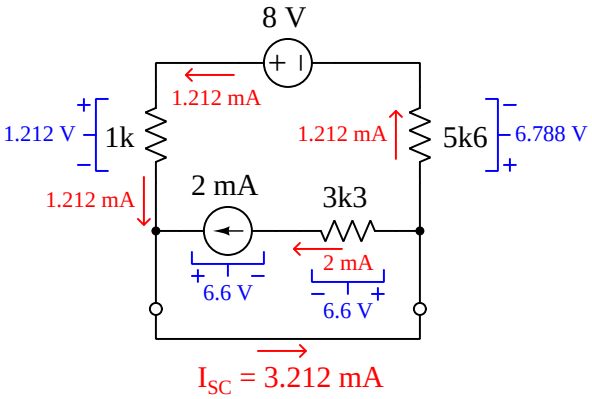
2.5 Example: Nortonizing a multi-source network

Original network

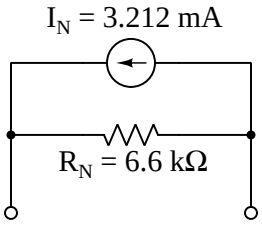


Calculating terminal resistance (left) and short-circuit terminal current (right):



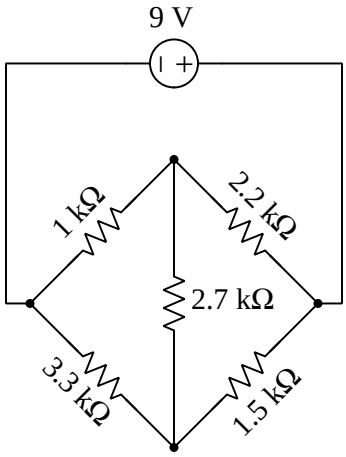


Norton equivalent network

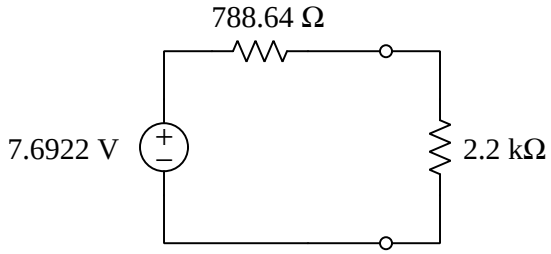


2.6 Example: Thévenizing an unbalanced bridge

Original network:



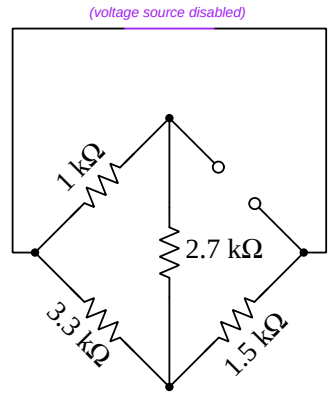
Thévenin equivalent network (from the perspective of the 2.2 kΩ resistor as the load:



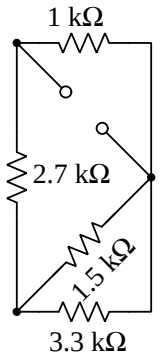
In this Thévenin equivalent network we may easily calculate the voltage across that 2.2 kΩ load resistor as 5.6624 Volts, because the equivalent network is a simple series circuit. The bridge circuit, however cannot be reduced to a single resistance as we can do with standard series-parallel circuits. In this scenario we *must* use a network theorem (or some other advanced circuit-analysis technique) to determine any of the voltage or currents.

The most difficult part of this analysis is determining the Thévenin equivalent resistance (R_{Th}). Here is a step-by-step reduction of the bridge into a single R_{Th} value from the perspective of the terminals the 2.2 kΩ resistor used to connect to:

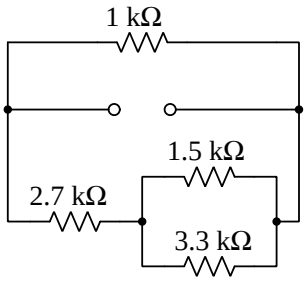
Step 1: (disable the voltage source)



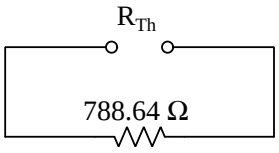
Step 2: ("bend" the wires)



Step 3: (re-arrange perpendicularly)



Step 4: (solve for R_{Th})

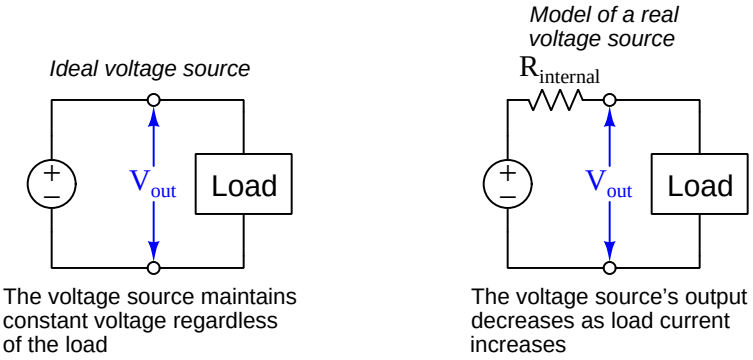


Chapter 3

Tutorial

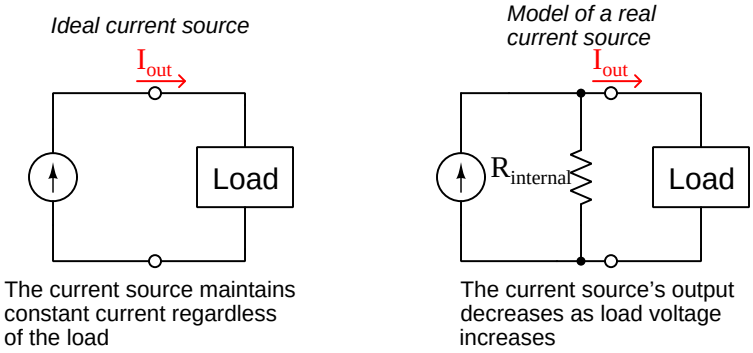
3.1 Ideal versus real sources

A perfect voltage source outputs constant voltage regardless of how much or little current passes through it: no matter the rate of electrical charge carriers flowing through a perfect voltage source, each charge carrier gains the exact same amount of energy from the source. Real voltage sources cannot maintain absolutely constant voltage over wide variations in current, and so we often model real voltage sources as ideal voltage sources connected in series with internal resistance:



The closer to ideal a voltage source is, the less its internal resistance. An ideal voltage source is considered to have zero internal resistance.

Similarly, a perfect current source outputs constant current regardless of how much or little voltage appears across its terminals: no matter how much energy is given to each electrical charge carrier as it passes through the source, the rate that charge carriers move through will remain constant. Real current sources cannot maintain absolutely constant current over wide variations in voltage, and so we often model real current sources as ideal current sources connected in parallel with internal resistance:



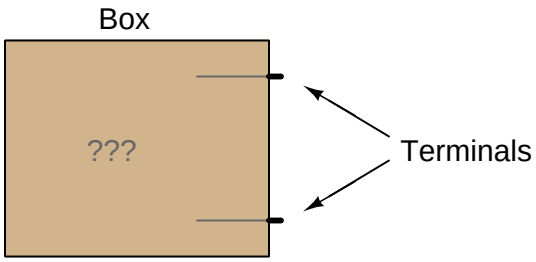
The closer to ideal a current source is, the more its internal resistance. An ideal current source is considered to have infinite internal resistance.

Real sources, both voltage and current, approach ideal behavior as less power is drawn from them, and deviate more from ideal behavior when more power is demanded. Common terms for describing the relative power demanded from a source are "heavy" for high-power loads and "light" for low-power loads, evocative of a person exerting energy to hoist a heavy object versus a light object. For a voltage source, a "light" load is one drawing little or no current, such as an open-circuit, because less current for any given voltage results in less power. For a current source, a "light" load is one dropping little to no voltage, such as a short-circuit, because less voltage for any given current also results in less power. If you examine the schematic models of real voltage and current sources previously shown, and imagine their respective minimum-power load conditions (i.e. open load for the voltage source and shorted load for the current source) you will see that in both cases the internal resistance becomes irrelevant. For the voltage source, a load drawing no current means that the source's $R_{internal}$ drops no voltage, leaving full source voltage at the output terminals. For the current source, a load dropping no voltage means that the source's $R_{internal}$ passes no current, leaving full source current at the output terminals.

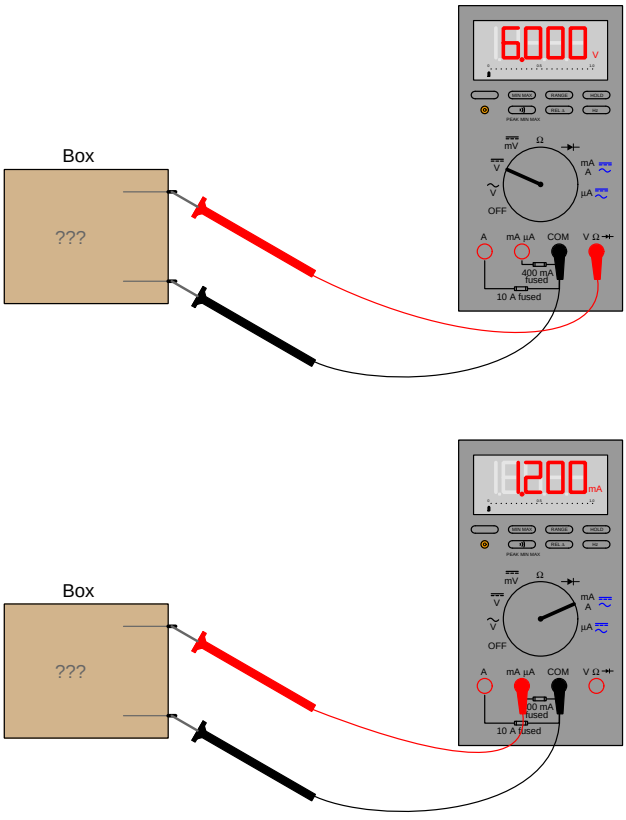
While the internal resistances of real voltage and current sources have many practical implications, the important point to recognize for this tutorial is that *voltage sources ideally have no internal resistance while current sources ideally have infinite internal resistance.* In particular, some electrical network theorems require this knowledge to apply, as one must consider the internal resistance of each source as a factor in the theorem.

3.2 Equivalent electrical networks

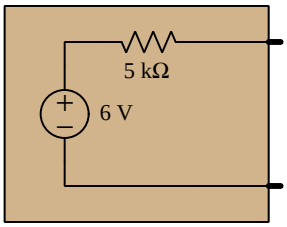
Suppose someone handed you a sealed wooden box with two metal terminals on its exterior. Inside this box, you are told, is a voltage source in series with a resistance connected to those two terminals:



Connecting a digital multimeter to those terminals, you decide to perform an open-circuit voltage test and a short-circuit current test to figure out what those internal component values are, measuring 6 Volts and 1.2 milliAmperes, respectively:

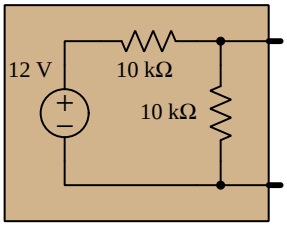


These two measurements provide enough information to identify the values of the voltage source and resistor we were told are both hidden inside the box:



A voltmeter connected to the two terminals will directly sense the 6 Volt source, since its negligible current draw ensures the box's series resistor will drop none of the source's voltage during the test. An ammeter connected to the same two terminals senses the 1.2 milliAmperes that the 6 Volt source is able to drive through the series 5 kiloOhm resistor. In other words, the open-circuit voltmeter test directly indicates the voltage source's value, while the short-circuit ammeter test indicates current which may be combined with the voltmeter measurement to yield the series resistance value by Ohm's Law ($R = \frac{V}{I}$).

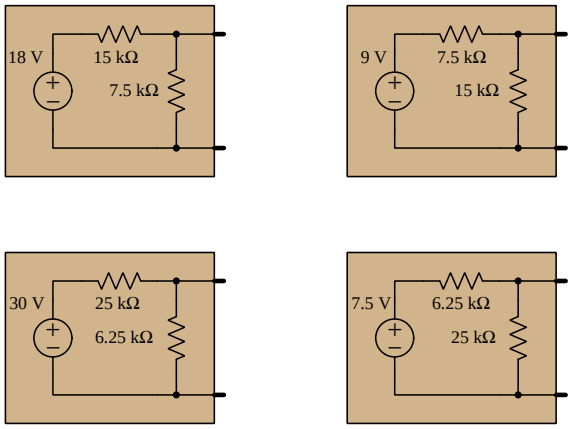
Later, though, when we open up this box and look inside, we discover the person who gave it to us did not tell us the whole truth about the circuit inside. Rather than containing a single 6 Volt source and a single 5 kiloOhm resistor, we discover a 12 Volt source and two 10 kiloOhm resistors:



A voltmeter connected to the box's external terminals will still register 6 Volts with this internal circuit, because the two 10 kiloOhm resistors divide the source's 12 Volts into two equal halves. An ammeter connected to the same terminals will still register 1.2 milliAmperes because the ammeter's negligible resistance shunts all current past the parallel 10 kiloOhm resistor, leaving just the 12 Volt source and a series 10 kiloOhm resistor to establish current ($\frac{12\ \text{V}}{10\ \text{k}\Omega} = 1.2\ \text{mA}$).

The fact that our hypothetical 6 Volt source and 5 kiloOhm series resistor would in fact behave identically to this more complex network comprised of a 12 Volt source and two 10 kiloOhm resistors means the two electrical networks are equivalent to one another. In other words, these two networks are *indistinguishable* from one another as measured from the perspective of their test terminals, and are guaranteed to behave identically to one another regardless of what we might connect between those terminals. Without peering into the box, there is absolutely no way for us to tell which of the equivalent networks is inside.

What is even more interesting is that other equivalent networks are possible[1]. Consider the following boxes, each one containing a network behaving exactly the same as the single 6 Volt source and series 5 kiloOhm resistor from the perspective of its external terminals:

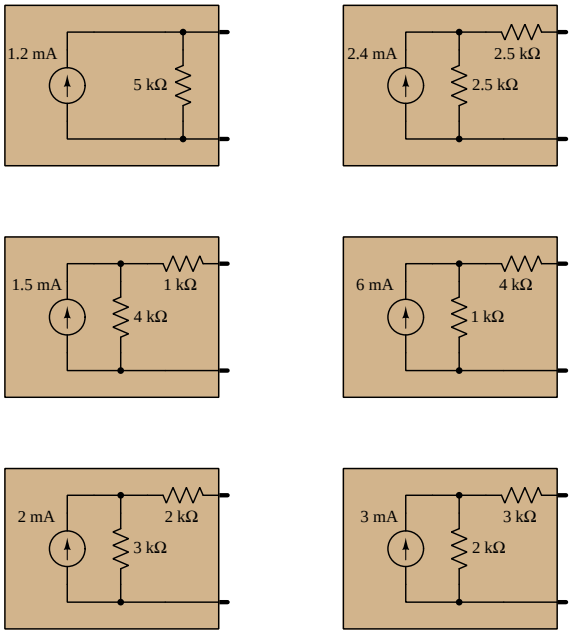


It is highly recommended as an *active reading* exercise to analyze each of these equivalent networks by calculating the amount of voltage between each network's open terminals as well as the amount of current that would flow if each network's terminals were directly "shorted" together with wire. In each case you should find 6 Volts (open-circuited) and 1.2 milliAmperes (short-circuited).

Each of these equivalent voltage-sourced networks behaves identically to the original "sealed box" circuit and to our simple single-source-single-resistor network. They are *all* equivalent from the perspective of their two test terminals, in that every one of them manifests a 6 Volt open-circuit voltage and a 1.2 milliAmpere short-circuit current at their terminals. If some arbitrary load resistance value were connected between their terminals, they would all power that load at identical voltage and current levels. This is another application for *active reading*: try adding an external resistance of any value to the original one-source-one resistor boxed circuit and then calculate voltage and current for that external resistance. After that, apply the same external resistance to any of the equivalent networks shown here and see for yourself how that external resistance will experience the same voltage and current as it forms a series-parallel combination network with the components inside the box. This demonstrates electrical equivalence of each box's internal network.

[1] These examples represent just a few possible with (fairly) round-number component values. As it so happens, there exist an *infinite* number of possible voltage-source-and-two-resistor networks equivalent to our original 6 Volt and 5 kΩ network!

We may even construct equivalent networks using *current* sources rather than voltage sources:



As with the voltage-sourced equivalent networks, it is highly recommended as an *active reading* exercise to analyze each of these equivalent networks by calculating the amount of voltage between each network's open terminals as well as the amount of current that would flow if each network's terminals were directly "shorted" together with wire. In each case you should find 6 Volts (open-circuited) and 1.2 milliAmperes (short-circuited).

Just like the set of equivalent voltage-sourced networks shown previously, this set of current-sourced networks also behaves identically to the original "sealed box" circuit and to our simple single-source-single-resistor network. They are *all* equivalent from the perspective of their two test terminals, in that every one of them manifests 6 Volts when open-circuited and 1.2 milliAmpere when short-circuited, and when connected to some load resistance would all power that load at identical voltage and current levels.

The existence of electrically-equivalent networks means it is possible, at least in theory, to take most any complex electrical network and reduce it down to a much simpler network behaving the same. In the following sections of this Tutorial we will explore two such theorems based on this principle of electrical equivalence, *Thévenin's Theorem* and *Norton's Theorem*, the only difference between the two theorems being that the former yields an equivalent network using a single *voltage source* while the latter gives an equivalent network using a single *current source*.

3.3 Thévenin's and Norton's theorems

Two network theorems particularly useful for analyzing complex circuits are *Thévenin's and Norton's Theorems*. Each of these may be stated quite simply:

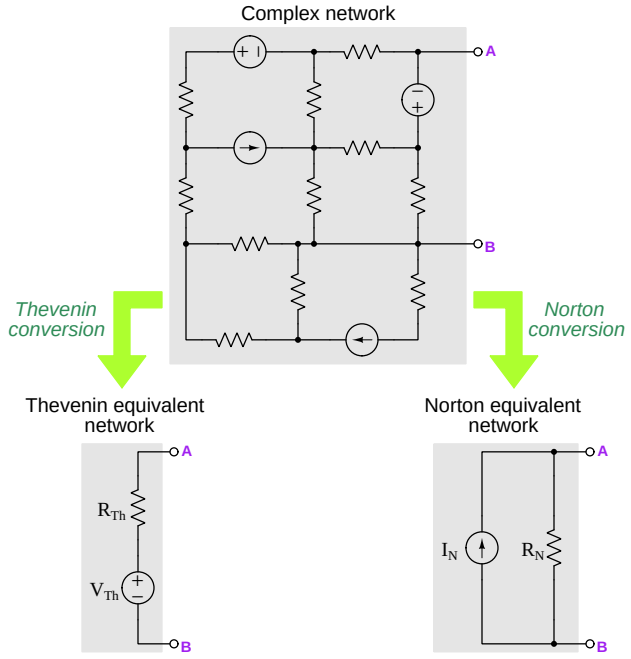
Thévenin's Theorem

> **Any linear network connected between two terminals may be modeled as an ideal voltage source in series with a single equivalent resistance.**

Norton's Theorem

> **Any linear network connected between two terminals may be modeled as an ideal current source in parallel with a single equivalent resistance.**

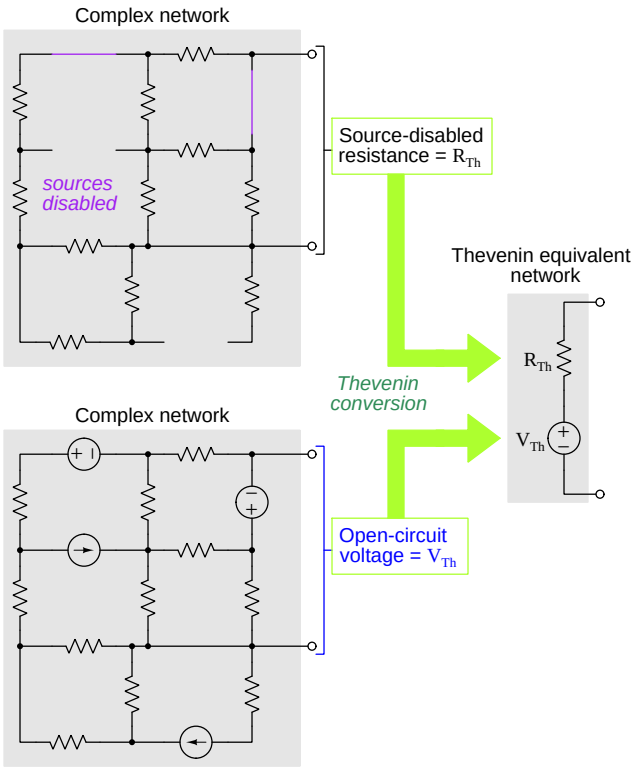
So long as all the components behave linearly[2], both Thévenin's and Norton's Theorems apply equally well to *any* network no matter its complexity:



[2]For the purpose of this discussion, a "nonlinear" component is one where voltage and current are *not* directly proportional to each other at any portion of the component's range. Another way to consider this characteristic would be to think of such components as having unstable resistance values. Not only are semiconductor PN junctions quite nonlinear, but so are some non-semiconductor devices such as incandescent lamps (where filament resistance changes with temperature, which in turn is affected by voltage and current). Some texts also add the condition of *bilateral symmetry* to the use of the Superposition Theorem, meaning that components must behave the same when energized in both polarities. However, the concept of linearity applied across a range of values both positive and negative covers this criterion. If a component does not exhibit bilateral symmetry, then it cannot be linear across its *entire* range of permitted values.

Converting a complex network into a Thévenin equivalent network consists of two steps. First, you will need to determine the complex network's open-circuit[3] voltage between its two output terminals. This open-circuit output voltage will be the value of V_{Th} in the Thévenin equivalent network. Next, you will need to determine the complex network's resistance as measured from its output terminals and with all of its internal sources disabled and substituted with their equivalent resistances (i.e. voltage sources replaced by shorts and current sources replaced by opens). This resistance will be the value of R_{Th} in the Thévenin equivalent network.

Illustrating the "Thévenization" process:

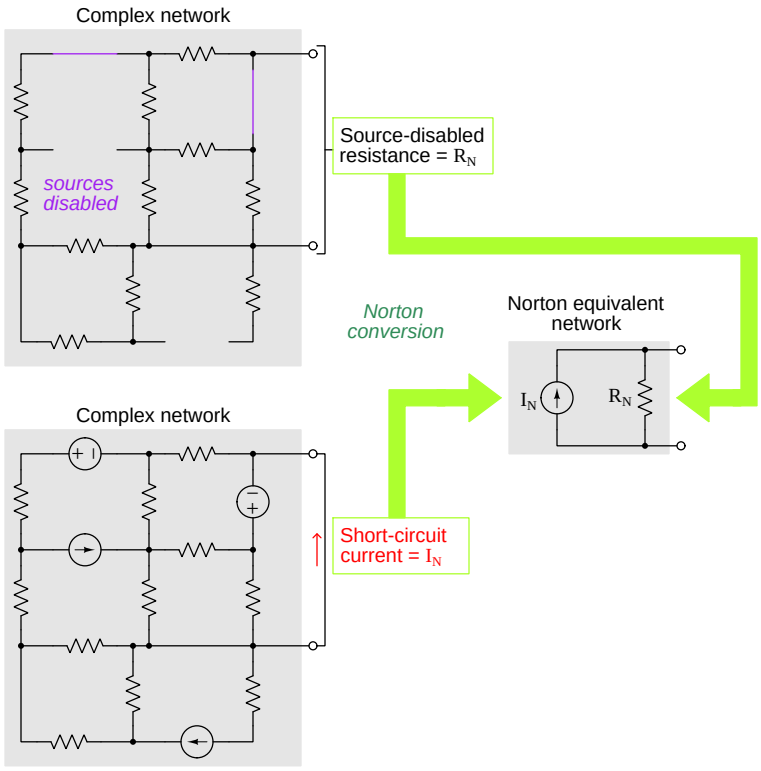


The rationale for these steps is easiest to understand by examining the Thévenin equivalent network. Once we accept the fact that there is such a thing as a Thévenin equivalent network for any complex linear-component network, whose behavior as measured from its output terminals is indistinguishable from the complex network's, it becomes easy to see why V_{Th} must be the open-circuit voltage and R_{Th} must be the resistance with V_{Th} replaced by a short. Open-circuiting the Thévenin equivalent network *must* result in V_{Th} being present between the output terminals because R_{Th} would drop zero voltage in an open-circuit (i.e. zero-current) state. Resistance between the output terminals likewise *must* be equal to R_{Th} if V_{Th} is disabled by a short. If the Thévenin equivalent behaves like this, then so must the complex network it is equivalent to.

[3] "Open-circuit" in this context means nothing else connects to the network's terminals but the network itself. For example, if the network happened to be powering a load from those two terminals, you would omit that load before determining voltage between those terminals.

Converting a complex network into a Norton equivalent network likewise consists of two steps. First, you will need to determine the complex network's short-circuit current between its two output terminals. This short-circuit output current will be the value of I_N in the Norton equivalent network. Next, you will need to determine the complex network's resistance as measured from its output terminals and with all of its internal sources disabled and substituted with their equivalent resistances (i.e. voltage sources replaced by shorts and current sources replaced by opens). This will be the Norton resistance value (R_N). You will note that this second step is identical to the step involved for calculating R_{Th} in a Thévenin equivalent network.

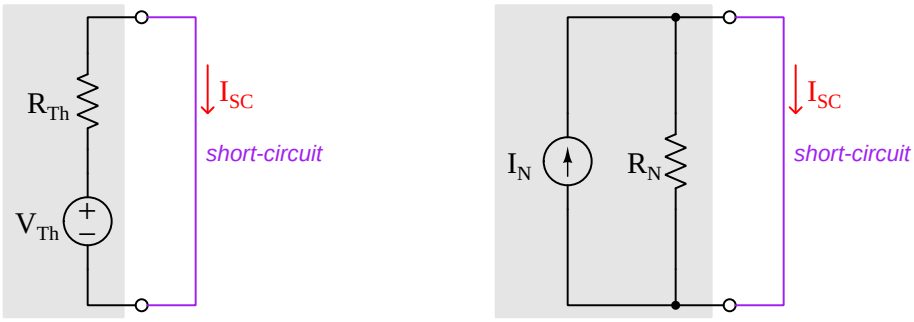
Illustrating the "Nortonization" process:



As with Thévenization, the rationale for the steps required to "Nortonize" a network is easiest to understand by examining the Norton equivalent network. Once we accept the fact that there is such a thing as a Norton equivalent network for any complex linear-component network, whose behavior as measured from its output terminals is indistinguishable from the complex network's, it becomes easy to see why I_N must be the short-circuit current and R_N must be the resistance with I_N replaced by an open. Short-circuiting the Norton equivalent network *must* result in I_N being present at the output terminals because the short would "shunt" the entirety of the source's current past R_N. Resistance between the output terminals likewise *must* be equal to R_N if I_N is disabled by an open. If the Norton equivalent behaves like this, then so must the complex network it is equivalent to.

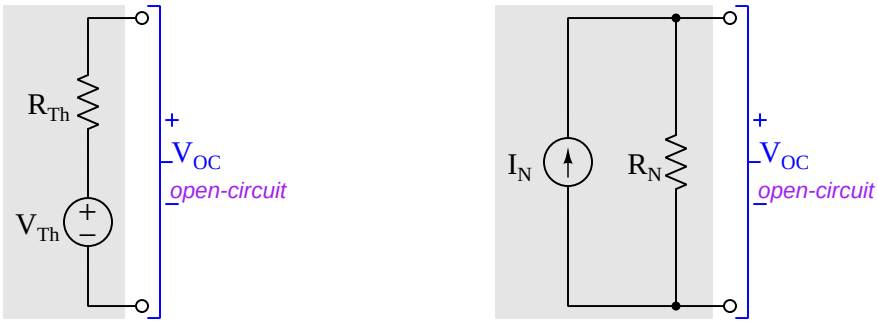
Not surprisingly, it is fairly easy to convert from any circuit's Thévenin equivalent to its corresponding Norton equivalent network, and vice-versa. A good way to understand this process is to apply the same "thought experiment" of shorting and opening the output terminals of each network that we performed in obtaining either a Thévenin or Norton equivalent from the original circuit. If a Thévenin and Norton network pair are to be equivalent to the same circuit (i.e. and therefore equivalent to each other), they must behave identically when short-circuited and when open-circuited.

First, let's imagine a Thévenin and Norton pair under short-circuit conditions:



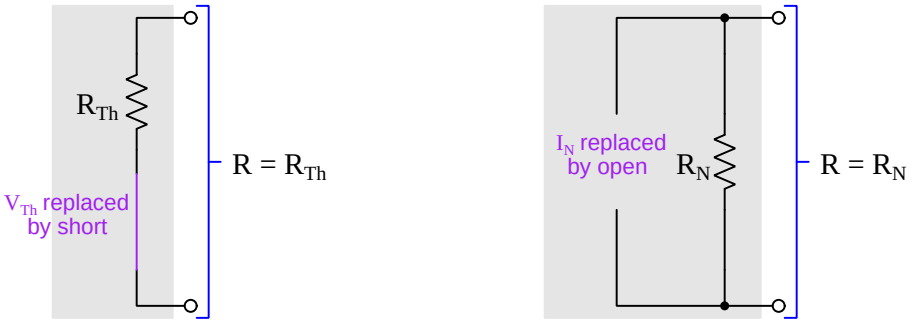
Calculating the short-circuit current (I_{SC}) in the Thévenin circuit is as simple as applying Ohm's Law ($I_{SC} = \frac{V_{Th}}{R_{Th}}$). In the Norton circuit, I_{SC} is simply equal to I_N because the short-circuit "jumper" wire bypasses all current around R_N. Therefore, the equivalent Norton current value must be equal to the Thévenin voltage divided by the Thévenin resistance ($I_N = \frac{V_{Th}}{R_{Th}}$).

Next, let's imagine a Thévenin and Norton pair under open-circuit conditions:



Calculating the open-circuit voltage (V_{OC}) in the Norton circuit is as simple as applying Ohm's Law ($V_{OC} = I_N R_N$). In the Thévenin circuit, V_{OC} is simply equal to V_{Th} because R_{Th} carries no current and therefore drops none of V_{Th}'s voltage, allowing all of V_{Th} to appear between the networks' terminals. Therefore, the equivalent Thévenin voltage value must be equal to the Norton current times the Norton resistance ($V_{Th} = I_N R_N$).

Converting between Thévenin and Norton internal resistance values is even simpler: *they are identical*. The equality between R_{Th} and R_N is evident if you disable each source (replacing it with its ideal resistance[4]) and imagine measuring resistance between the network terminals:



Both Thévenin and Norton equivalents are useful *tools* for analyzing complex electrical networks, and the choice between the two is arbitrary. If you prefer to think in terms of voltage sources, you are free to use Thévenin equivalents. If current sources are preferred, you are free to use Norton equivalents. And, if you end up wishing to switch from one type to the other, the conversion (as we have just seen) is very simple: Thévenin and Norton resistance values for equivalent networks are identical, and their respective voltage and current source values may be computed using Ohm's Law ($V_{Th} = I_N R_N$ and $I_N = \frac{V_{Th}}{R_{Th}}$).

As interesting as Thévenin's and Norton's Theorems may be, their utility may not be obvious. The practical value of any network theorem is to somehow *reduce the complexity of a given problem* for the purpose of making it easier to solve. In the next section we will explore this concept in more detail. In the sections following that, we will explore some specific example problems illustrating the power of Thévenin's and Norton's Theorems.

[4] Another way to conceptualize the disabling of a source is to consider what we would have to replace it with in order to ensure a "zero" condition. For example, in order to ensure a zero-voltage condition when disabling a voltage source, it makes sense that we would substitute a short for the voltage source since the guaranteed effect of a short is a condition of zero voltage. Similarly, in order to ensure a zero-current condition with disabling a current source, it makes sense we would replace that current source with an open since we know the guaranteed effect of an open is to halt current.

3.4 Uses for Thévenin's and Norton's Theorems

The real-world practicality of any theorem converting a complex network of electrical components into a simpler (equivalent) network may not be apparent at first. Sure, this might be useful for the kinds of convoluted resistor networks you see in a textbook, but how does this help us analyze *real* circuits? Here we will briefly discuss two important applications:

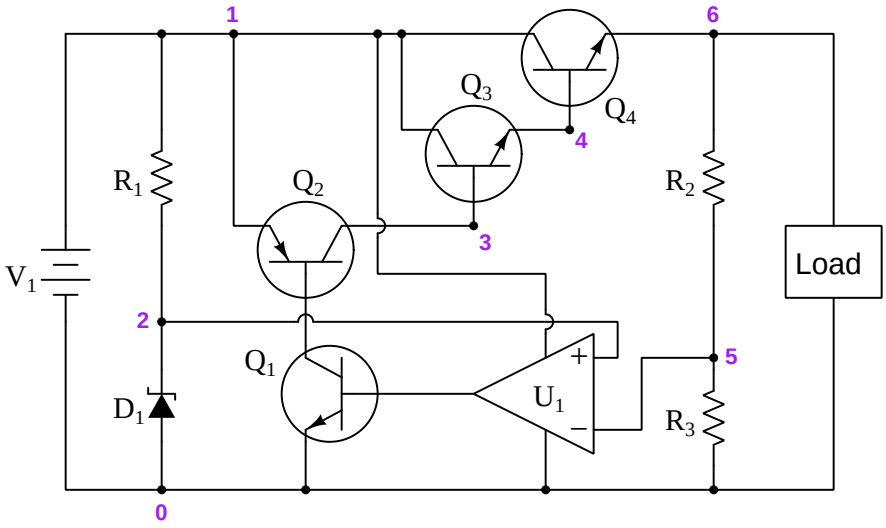
- Load-testing power supply designs
- Selecting components to achieve required input or output impedance[5]

[5] For any reader unfamiliar with the term *impedance*, for the sake of this introduction to Thévenin's and Norton's Theorems we may consider impedance to be the same thing as resistance. We typically use the word "impedance" to describe the ratio of voltage to current when the voltage and current in question change value over time – such as in alternating-current (AC) circuits – and resistance to describe the ratio of a constant (DC) voltage to a constant (DC) current. There is more to the concept of impedance than this simple distinction, but to explore the concept in more detail would detract from the purpose of this Tutorial.

3.4.1 Power supply design testing

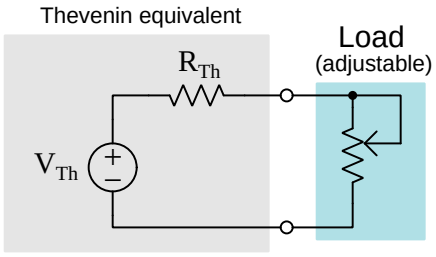
A *power supply* is any circuit designed to condition electrical power between a given source and a specific load, often performing such tasks as transforming power between different voltage and current levels and/or regulating voltage or current to prescribed values. Such circuits can be quite complex, containing many individual components, but at least for certain ranges of operation we may apply either Thévenin's Theorem or Norton's Theorem to reduce such a complex circuit to a single equivalent network consisting of a perfect voltage or current source and an internal resistance. If we apply one of these network theorems and thereby derive an equivalent network, we may then very quickly and easily run a series of calculations predicting how stable that power supply's output voltage and/or current will be for a range of load conditions.

The following schematic diagram shows a standard "pass"-style voltage regulator circuit, designed to take electrical power from any imperfect voltage source and deliver it at nearly constant voltage to a load whose resistance may vary substantially:



If we are careful to define the limits of operation for this circuit, it is possible to apply either Thévenin's or Norton's Theorem to it and arrive at an elementary network behaving very nearly the same. An ideal voltage source plus low-valued internal resistance makes sense to model a power supply circuit designed to maintain constant load voltage, making Thévenin's Theorem a natural fit for this problem.

Once we have determined[6] the values of V_{Th} and R_{Th} in the equivalent network, it then becomes a trivial matter to explore how stable the circuit's output terminal voltage will be for varying load conditions because the power supply and attached load resistance is nothing more than a simple series circuit:



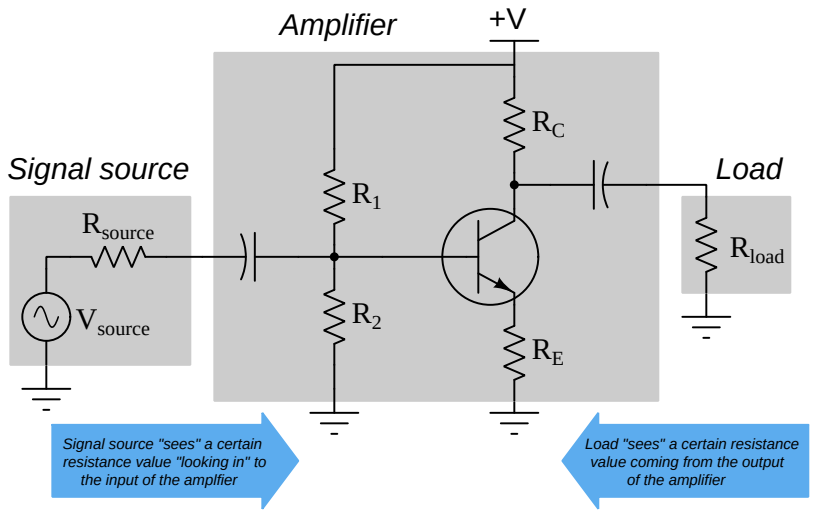
Rather than have to perform a long series of calculations over and over again to determine load voltage and current for different load resistance values applied to the original circuit, we may perform a long series of calculations *once* to derive the Thévenin equivalent network and then after that perform *very simple* calculations over and over again for each proposed load resistance value. Not only will this be less tedious for us, but it may also allow us to gain insights we might otherwise miss if we are cognitively burdened by the complexity of the original power supply circuit.

[6] Details of how to perform this determination on the voltage-regulator circuit shown will not be given here. For now we are merely exploring *why* these theorems are useful, not precisely *how* they apply to all scenarios. In order to explain how to apply Thévenin's or Norton's Theorem to this voltage regulator circuit, we would first have to understand how transistors, Zener diodes, and operational amplifiers function which is far beyond the scope of this Tutorial.

3.4.2 Input/output impedances

An important parameter for many electronic circuits is something called *impedance*, which if you have not yet studied this term is equivalent to *resistance* for the sake of this discussion. Any circuit receiving power or signal from an external source will present a load to that source as it extracts energy from it, and we may model that loading effect as though it were a single resistance (called the *input impedance* of that circuit). Similarly, any circuit delivering power to an external load will itself manifest some amount of internal resistance, which is the Thévenin or Norton equivalent *output impedance* of that circuit.

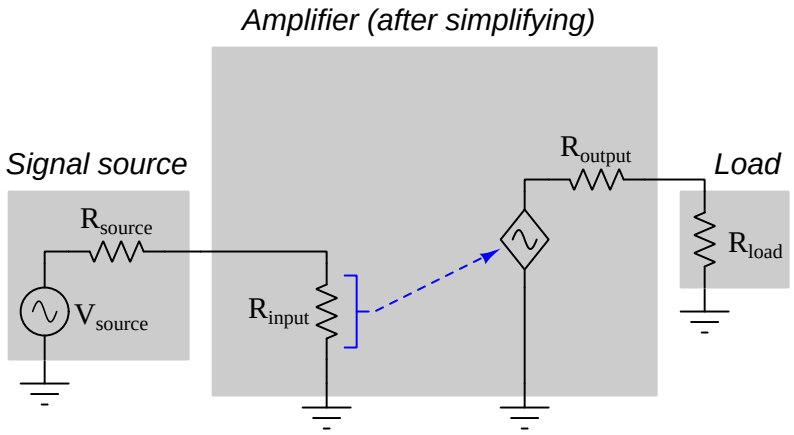
Below we see a schematic diagram of a single-transistor amplifier receiving an AC signal from a signal source and delivering an amplified version of that signal to a resistive load:



In order to operate at peak efficiency, the input impedance (resistance) of the amplifier must match the signal source's internal resistance to satisfy the Maximum Power Transfer Theorem. If we know the source's internal resistance value, it is possible to choose component values within the amplifier circuit to make its input impedance match. In other words, we treat the AC signal source as a Thévenin equivalent network and apply Thévenin/Norton reduction techniques to the amplifier to reduce its input circuitry to a single equivalent resistance, then determine what the amplifier's component values must be to yield that matching amount of input resistance.

Similarly, the amount of internal impedance (resistance) exhibited by the amplifier at its output is a function of its component values, and by applying Thévenin/Norton reduction techniques we may find out exactly what dictates this internal value too.

Here we see the simplified[7] equivalent of the amplifier circuit, its input presenting a simple resistance "load" to the signal source, and its output transferring power to the load through an internal output resistance:

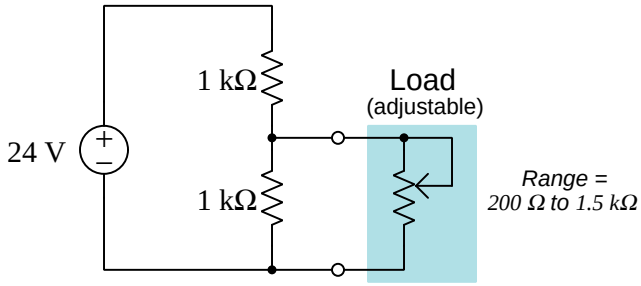


In summary, we find Thévenin's and Norton's Theorems to be useful tools to *model* complex networks as much simpler networks. This in turn allows us to assess in a much easier and intuitive fashion how the original (complex) network will perform when interfaced with other sources and loads.

[7] Again, details of how to perform this simplification from the original amplifier circuit to this equivalent will not be given here. For now we are merely exploring *why* these theorems are useful, not precisely *how* they apply to all scenarios. In order to explain how to apply either Thévenin's or Norton's Theorem to this amplifier circuit, we would have to also explain what transistors are and how exactly they function.

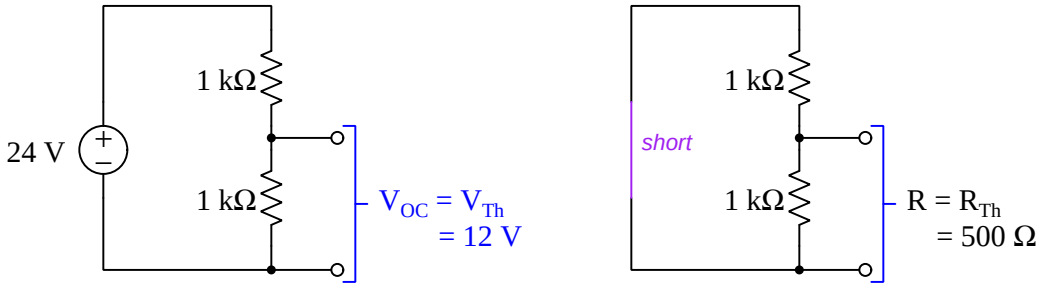
3.5 Example: Voltage dropped by a varying load resistance

For our first example we will consider a voltage divider circuit providing power to a load of varying resistance, the variable load represented by a potentiometer wired as a *rheostat*[8]:



In an unloaded condition, this voltage divider consisting of identical resistors and a 24 Volt source would output half of its internal source voltage (i.e. 12 Volts). However, we know that under loaded conditions the divider's output voltage will "sag" below the ideal value of 12 Volts. Our task is to determine how much that voltage sags as the load's resistance varies between 200 Ω and 1.5 kΩ.

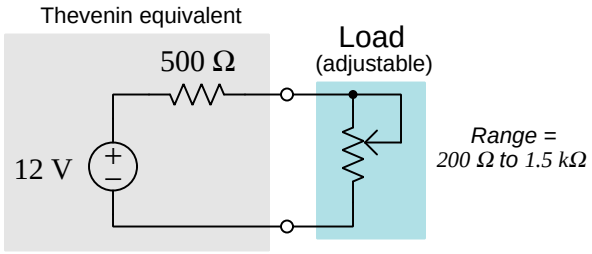
The most direct method for calculating the divider's output voltage under varying load conditions would be to analyze this as a series-parallel circuit for different values of R_{load}. However, Thévenin's Theorem provides a faster solution. Our first task is to "Thévenize" the voltage divider network to obtain an equivalent network that will behave the same as viewed from its output terminals:



From the unloaded (open-circuit) and disabled-source tests, we see the Thévenin equivalent network must consist of a 12 Volt source in series with a 500 Ohm resistance.

[8] A "rheostat" is simply a variable resistance. By connecting the wiper of the potentiometer to one of its outside terminals, it simply functions as a variable resistance.

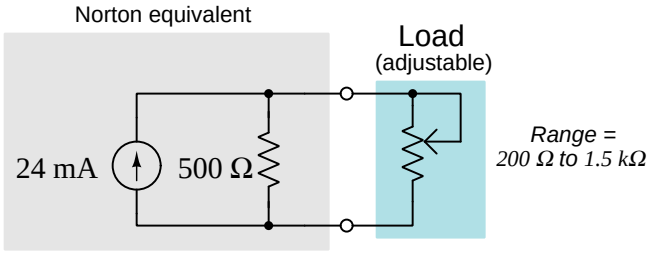
With this Thévenin equivalent in hand, our loaded voltage divider circuit becomes a *simple-series circuit* rather than a series-parallel circuit, thereby simplifying our task of computing load voltage at different load resistances:



Now, instead of having to perform series-parallel analysis for each and every load resistance value, we may simply use the voltage divider formula:

- V_{Load} @ 200 Ohms = (12 V) $\left(\frac{200\ \Omega}{500\ \Omega + 200\ \Omega}\right)$ = 3.429 Volts
- V_{Load} @ 1000 Ohms = (12 V) $\left(\frac{1000\ \Omega}{500\ \Omega + 1000\ \Omega}\right)$ = 8.000 Volts
- V_{Load} @ 1500 Ohms = (12 V) $\left(\frac{1500\ \Omega}{500\ \Omega + 1500\ \Omega}\right)$ = 9.000 Volts

We could have just as easily reduced the original voltage divider circuit to a *Norton* equivalent instead[9], and we would still have a circuit easier to analyze for multiple load resistance values than the original series-parallel network:



- V_{Load} @ 200 Ohms = (24 mA) $\left(\frac{1}{\frac{1}{500\ \Omega} + \frac{1}{200\ \Omega}}\right)$ = 3.429 Volts
- V_{Load} @ 1000 Ohms = (24 mA) $\left(\frac{1}{\frac{1}{500\ \Omega} + \frac{1}{1000\ \Omega}}\right)$ = 8.000 Volts
- V_{Load} @ 1500 Ohms = (24 mA) $\left(\frac{1}{\frac{1}{500\ \Omega} + \frac{1}{1500\ \Omega}}\right)$ = 9.000 Volts

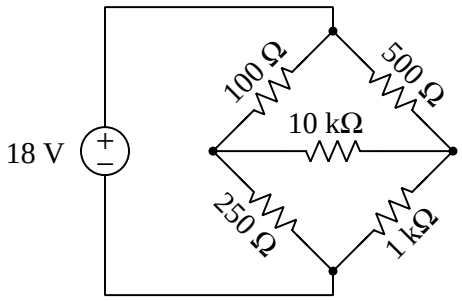
Bear in mind that Thévenin and Norton equivalents make *any* circuit with a varying load as easy to analyze as simple-series and simple-parallel networks, respectively.

[9] Since we already know V_{Th} and R_{Th}, we may quickly calculate the Norton equivalent values $I_N = \frac{V_{Th}}{R_{Th}} = 24$ mA and $R_N = R_{Th} = 500\ \Omega$.

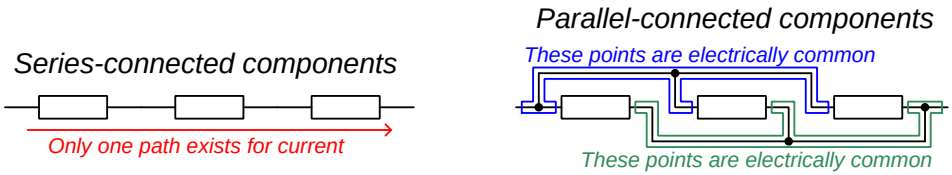
3.6 Example: Unbalanced bridge circuit

In the previous example we saw how either Thévenin's or Norton's Theorem could be used to simplify a series-parallel circuit into a simple-series or simple-parallel circuit (respectively) to facilitate modeling a variable load resistance. The resulting Thévenin or Norton equivalent network saved us the trouble of performing longer calculations, and therefore was a useful problem-solving strategy. In this example, however, we will see a more powerful application of Thévenin's and Norton's Theorems – one where the original circuit is *impossible* to analyze as a series-parallel network.

Consider the following *bridge* circuit, where each of the five resistance values are unique:



Not only are all resistors unequal in value, but their ratios are unequal as well. This fact makes the bridge circuit *unbalanced*[10] and therefore challenging to analyze. As tempting as it may be to try to condense these five resistors into a single equivalent resistance representing a series-parallel network, we will find this task impossible because *no two resistors are in series or in parallel with each other*. Recall the respective definitions of series and of parallel, shown in the following illustrations:



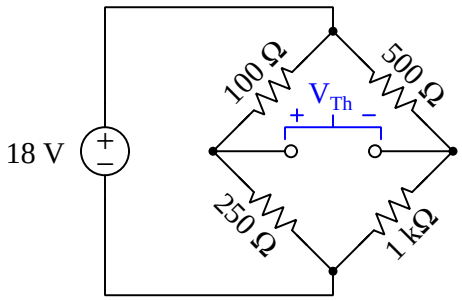
Series is defined as having only one path for current. *Parallel* is defined as the components all connected across the same two sets of electrically-common points. You will search in vain for any two resistors in the bridge circuit sharing just one path for current between them, or sharing the same two sets of electrically-common points. Bridge circuits are *not* series-parallel networks, and so we cannot use the techniques of series-parallel analysis to reduce any bridge circuit to a single equivalent resistance!

[10] Recall that a *balanced* bridge circuit has equal ratios between pairs of resistors on opposite sides of the bridge, resulting in zero voltage across the middle resistor (in this case, the 10 kΩ resistor). Balanced bridge circuits are easy to analyze because the lack of voltage across the middle resistance means there is no current flowing through that resistor, and therefore we may effectively regard that resistor as not being there at all (i.e. equivalent to an open). This is why balanced bridge circuits may be considered nothing more than a pair of two-resistor voltage dividers.

How does either Thévenin's or Norton's Theorem help us in this case? We may apply either theorem by arbitrarily selecting one of the resistors in the bridge circuit and call it our *load*, and then reducing the rest of the bridge circuit to a Thévenin or Norton equivalent. The reduction of the single source and four resistors to a single source and single resistor will greatly simplify our determination of voltage and/or current for that arbitrarily-selected resistor. This will become clear by example.

Suppose we wish to know the voltage and current for the 10 kΩ resistor in this bridge circuit. We may declare this resistor to be the load, then Thévenize or Nortonize the *rest* of the bridge circuit to arrive at an equivalent network of just one source and one resistance, and finally re-connect our 10 kΩ load to this equivalent network to solve for its voltage and current. The temporary removal of the 10 kΩ resistor from the original bridge circuit is the key to success: eliminating this resistor simplifies the bridge into a series-parallel network, for which we have applicable analytical tools.

First, removing the 10 kΩ "load" resistor and solving for the Thévenin voltage (V_{Th}):



Kirchhoff's Voltage Law informs us that the voltage between the open terminals (V_{Th}) must be equal to the difference in voltage dropped across the two lower[11] resistors.

$$V_{Th} = V_{250\Omega} - V_{1000\Omega}$$

$$V_{Th} = (18\text{ V})\left(\frac{250\,\Omega}{100\,\Omega + 250\,\Omega}\right) - (18\text{ V})\left(\frac{1000\,\Omega}{500\,\Omega + 1000\,\Omega}\right)$$

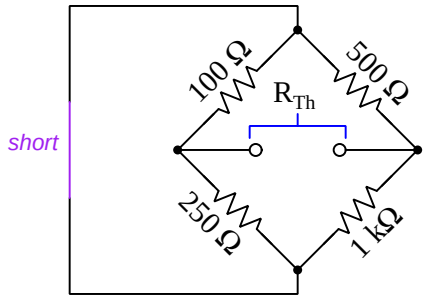
$$V_{Th} = 12.857\text{ V} - 12\text{ V}$$

$$V_{Th} = 0.857\text{ V (+ on left)}$$

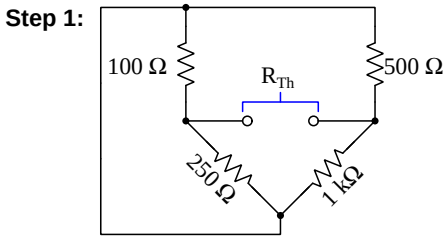
When we re-draw the circuit as a Thévenin equivalent connected to the 10 kΩ "load" resistor, the Thévenin equivalent will have a V_{Th} voltage value of 0.857 Volts.

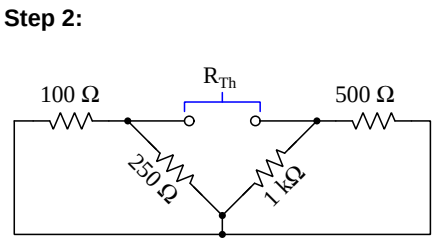
[11] Also, equal to the difference in voltage dropped across the two upper resistors. The choice of which resistor-pair to use is arbitrary.

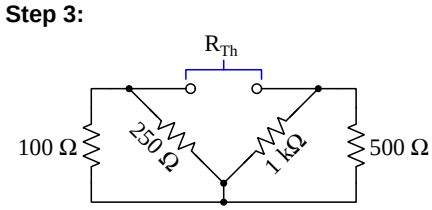
Next, we determine the Thévenin equivalent resistance of the four remaining resistors in the bridge circuit. To do this, of course, we must replace the 18 Volt source by a shorting wire:

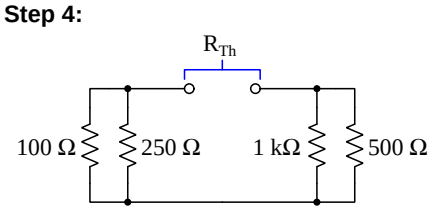


The result is a terminal-to-terminal resistance equal to the 100 Ω and 250 Ω resistors in parallel with each other, in series with the 500 Ω and 1 kΩ resistors in parallel with each other. This is perhaps the most confusing aspect of the Thévenization/Nortonization process for this bridge circuit, and it may be more easily grasped by viewing a sequence of "wire-bending" steps:





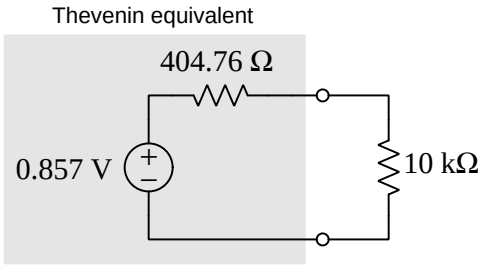




$$R_{Th} = \frac{1}{\frac{1}{100\ \Omega} + \frac{1}{250\ \Omega}} + \frac{1}{\frac{1}{1000\ \Omega} + \frac{1}{500\ \Omega}} = 404.76\ \Omega$$

When we re-draw the circuit as a Thévenin equivalent connected to the 10 kΩ "load" resistor, the Thévenin equivalent will have an internal R_{Th} resistance of 404.76 Ohms.

Re-drawing the circuit as a Thévenin equivalent plus the 10 kΩ "load" resistor:

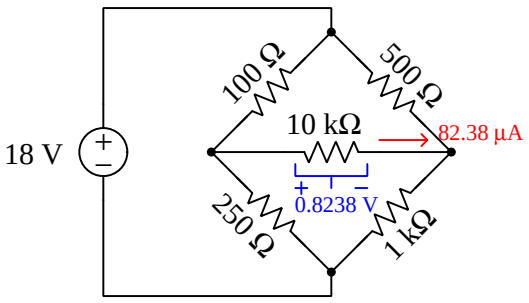


Now, it is a very simple matter indeed to calculate the 10 kΩ resistor's voltage and current using the properties of series circuits and Ohm's Law:

$$I = \frac{V_{Th}}{R_{total}} = \frac{0.857 \text{ V}}{404.76\ \Omega + 10000\ \Omega} = 82.380\ \mu\text{A}$$

$$V_{load} = IR_{load} = (82.380\ \mu\text{A})(10000\ \Omega) = 0.8238 \text{ V}$$

Therefore, the 10 kΩ resistor in the original bridge circuit must drop 0.8238 Volts and pass 82.380 μA of current:



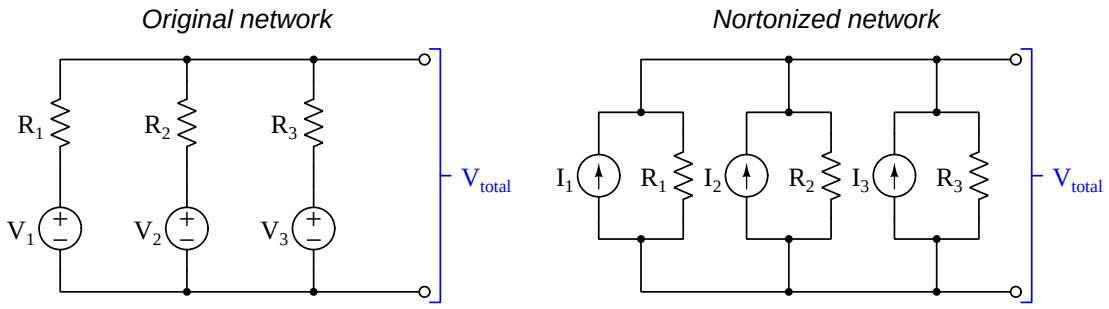
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Derivation of Millman's Theorem

Whenever we encounter a network of multiple voltage sources connected together in parallel, each with its own associated series-connected resistance, there is an easy way to calculate that parallel network's voltage value. This method treats each source-resistor pair as a Thévenin voltage source and then converts each into a corresponding Norton current source (current source and parallel-connected resistance). In the converted network all sources and resistors are in parallel with each other, which means all the current source currents simply add together and pass through a parallel-equivalent resistance value determined by the individual resistors:



Voltage across the Nortonized network is fairly simple to calculate, multiplying total current by the total (parallel-diminished) resistance in accordance with Ohm's Law ($V = \frac{I}{R}$):

$$V_{total} = \frac{I_1 + I_2 + I_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Expanding back to the original network with three voltage sources, we must re-write the numerator of the fraction to show each Norton-equivalent current as the quotient of a voltage source and its respective resistance:

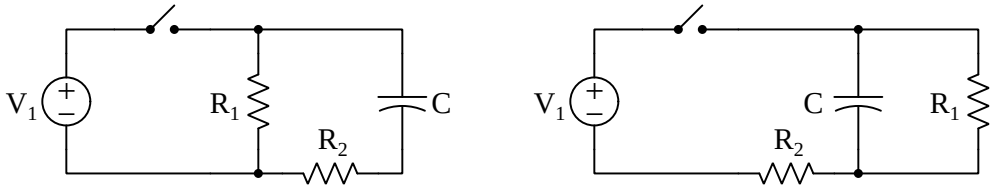
$$V_{total} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

That last equation is the mathematical expression of *Millman's Theorem*, useful for predicting voltage in any network comprised of parallel-connected voltage sources (with individual resistances).

4.2 Time delay circuits and Thévenin's/Norton's Theorems

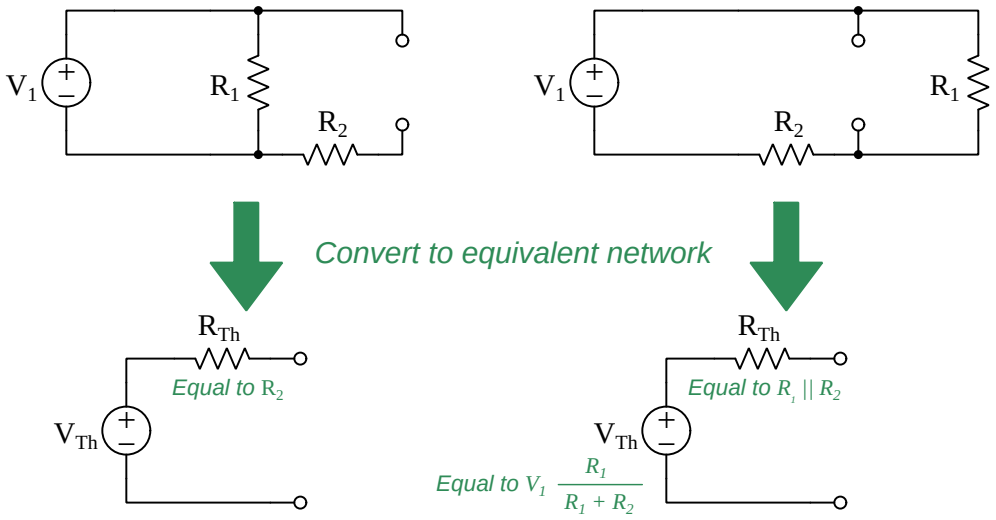
Circuits containing resistors along with capacitors or inductors typically result in voltage and current values that vary over time. Determining starting and final values for these parameters, as well as calculating the "time constant" ($\tau = RC = \frac{L}{R}$) can be challenging especially if the circuit in question consists of more than just a single resistor, single source, and single reactive (C or L).

Take for example the following two RC circuits, each one containing one capacitor but two resistors:



If the capacitor begins in a completely de-energized state with the switch in the open position, capacitor voltage for each circuit is guaranteed to begin at 0 Volts because energy stored in any capacitance always manifests as a voltage across that capacitance. When we close the switch, we would expect the capacitor in both circuits to begin energizing as they absorb energy from the voltage source V_1, and this means their voltages will rise over time. However, what is not as clear upon first inspection is what the final voltage will be for either capacitor after the switch has been left in its closed state for a long time, as well as how we might calculate the time constant $\tau = RC$ given the fact each circuit has *two* resistors and not just one.

Obviously, an RC circuit containing just one resistor and one capacitor instead of two resistors and one capacitor would be simpler to analyze, so why don't we just apply either Thévenin's Theorem or Norton's Theorem to each circuit in order to reduce it to just a single source and single resistance? Choosing the capacitor to be the load, we will convert the rest of each circuit into its Thévenin equivalent:

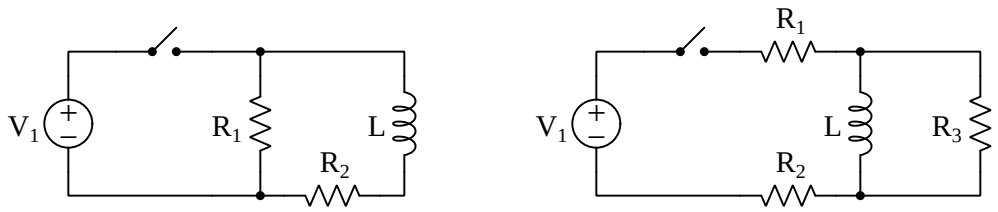


Recall that determining either the Thévenin or the Norton equivalent resistance of any complex network consists of disabling all sources in the original circuit and determining resistance as measured from load terminal to load terminal. In the left-hand circuit the disabled V_1 is replaced by a short which completely bypasses R_1 and leaves only R_2 remaining. In the right-hand circuit the disabled V_1 (again, replaced by a short) causes R_1 and R_2 to be in parallel with each other from the perspective of the two load terminals. Thus, the time constant for the left-hand circuit is simply $\tau = R_2C$ while the time constant for the right-hand circuit will be $\tau = \frac{C}{\frac{1}{R_1} + \frac{1}{R_2}}$.

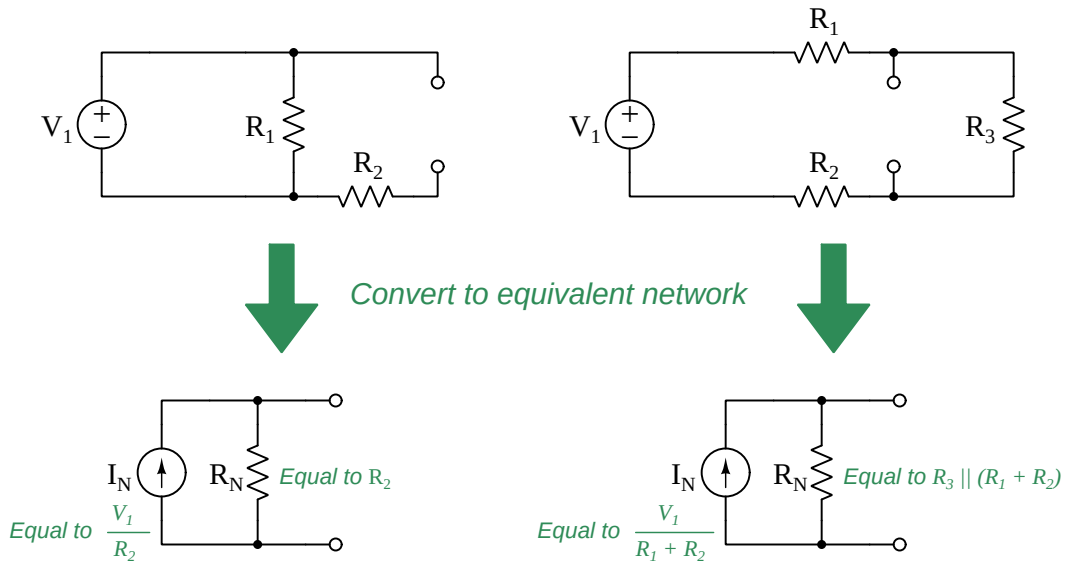
Recall that determining the Thévenin equivalent voltage of any complex network consists of calculating voltage from one load terminal to the other with all original sources active. Here we see that the left-hand circuit's open-circuit load terminal voltage must simply be V_1, while the right-hand circuit's open-circuit terminal voltage will be the result of V_1 powering a two-resistor voltage divider.

Re-attaching the capacitor C to the load terminals of the Thévenin equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = R_{Th}C$ and the final capacitor voltage after full energization is V_{Th}.

Norton's Theorem, of course, works just as well as Thévenin's Theorem for this purpose. To illustrate, we will apply Norton's Theorem to a pair of resistor-inductor circuits to determine how multiple resistors affect the inductors' energization:



Removing the inductor from each circuit and converting the remaining networks into Norton equivalents:



The process for determining Norton equivalent resistance is exactly the same as it is for Thévenin equivalent resistance: simply disable all original sources and determine what an ohmmeter would register if connected between the two load terminals. In the left-hand circuit we see once again that R_1 is of no effect to the Norton equivalent value of R_N, while in the right-hand circuit R_N is a series-parallel combination of the three original resistors.

Recall that determining the Norton equivalent current of any complex network consists of calculating current passing through a shorting wire placed between the two load terminals with all the original sources active. In the left-hand circuit we see that this short-circuit current is the limited only by R_2 and so $I_N = \frac{V_1}{R_2}$. In the right-hand circuit we see only R_1 and R_2 serving to limit current, and so $I_N = \frac{V_1}{R_1+R_2}$.

Re-attaching the inductor L to the load terminals of the Norton equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = \frac{L}{R_N}$ and the final inductor current after full energization is I_N.

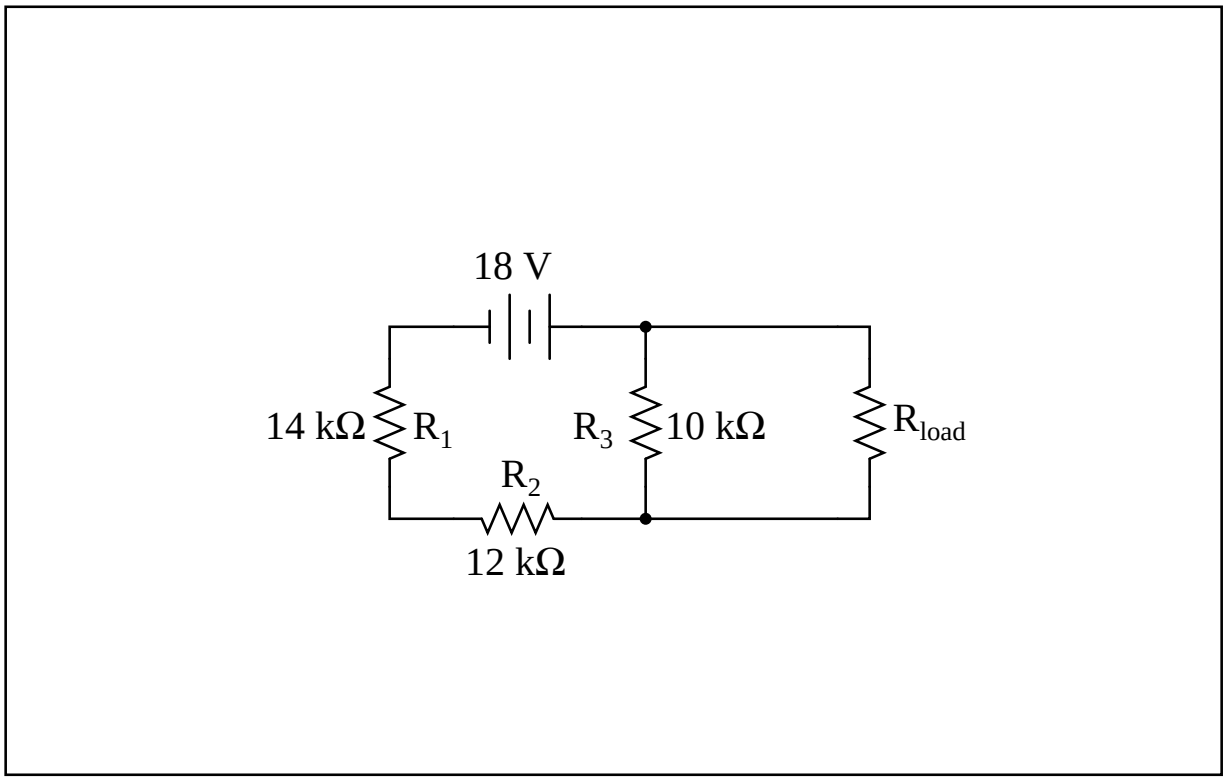
Chapter 5

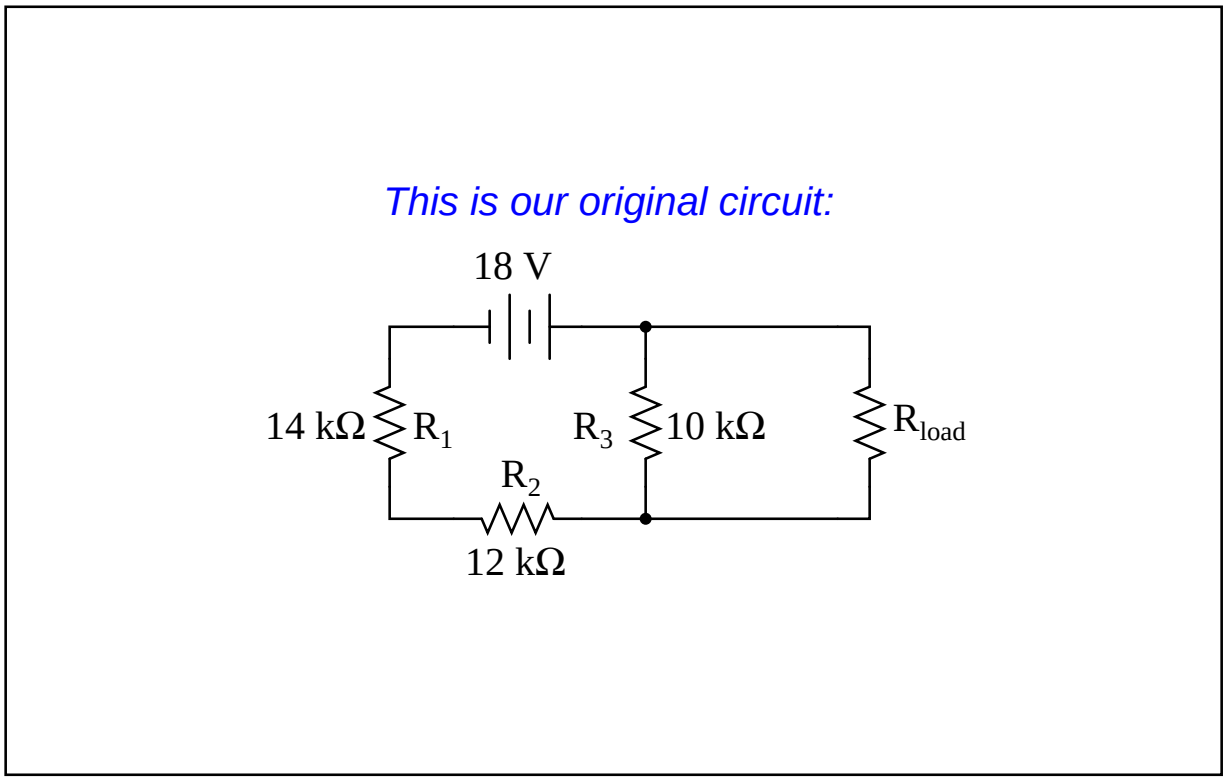
Animations

Some concepts are much easier to grasp when seen in *action*. A simple yet effective form of animation suitable to an electronic document such as this is a "flip-book" animation where a set of pages in the document show successive frames of a simple animation. Such "flip-book" animations are designed to be viewed by paging forward (and/or back) with the document-reading software application, watching it frame-by-frame. Unlike video which may be difficult to pause at certain moments, "flip-book" animations lend themselves very well to individual frame viewing.

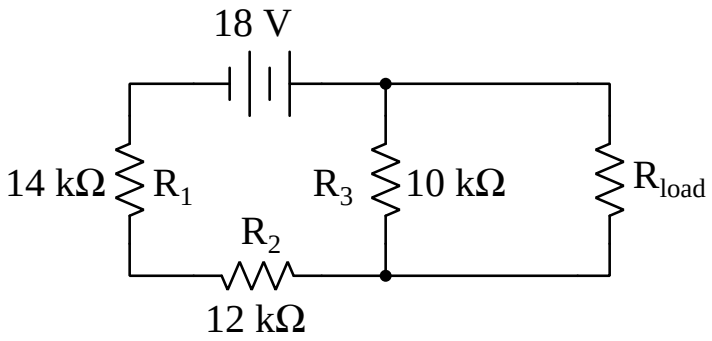
5.1 Animation of Thévenin's Theorem

The following animation shows the steps involved in "Thévenizing" a circuit.

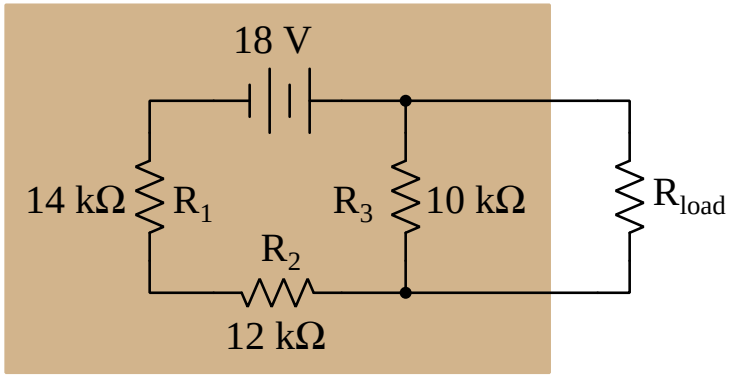

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ


This is our original circuit:
18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ

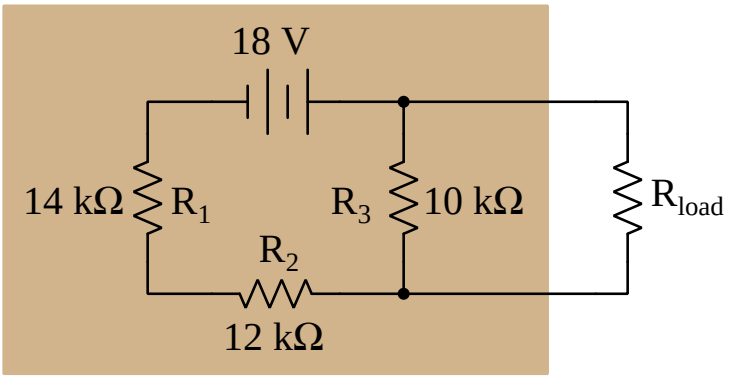
We may use Thevenin's Theorem
to simplify this portion of the circuit . . .

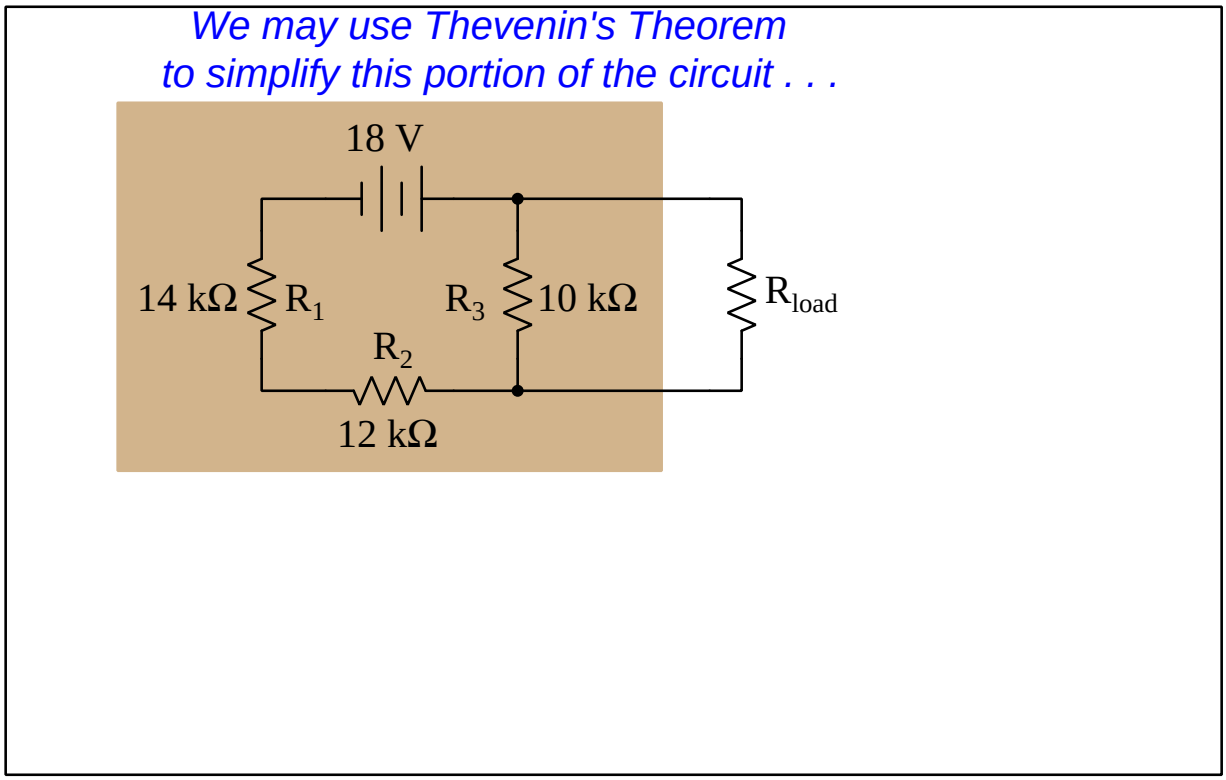


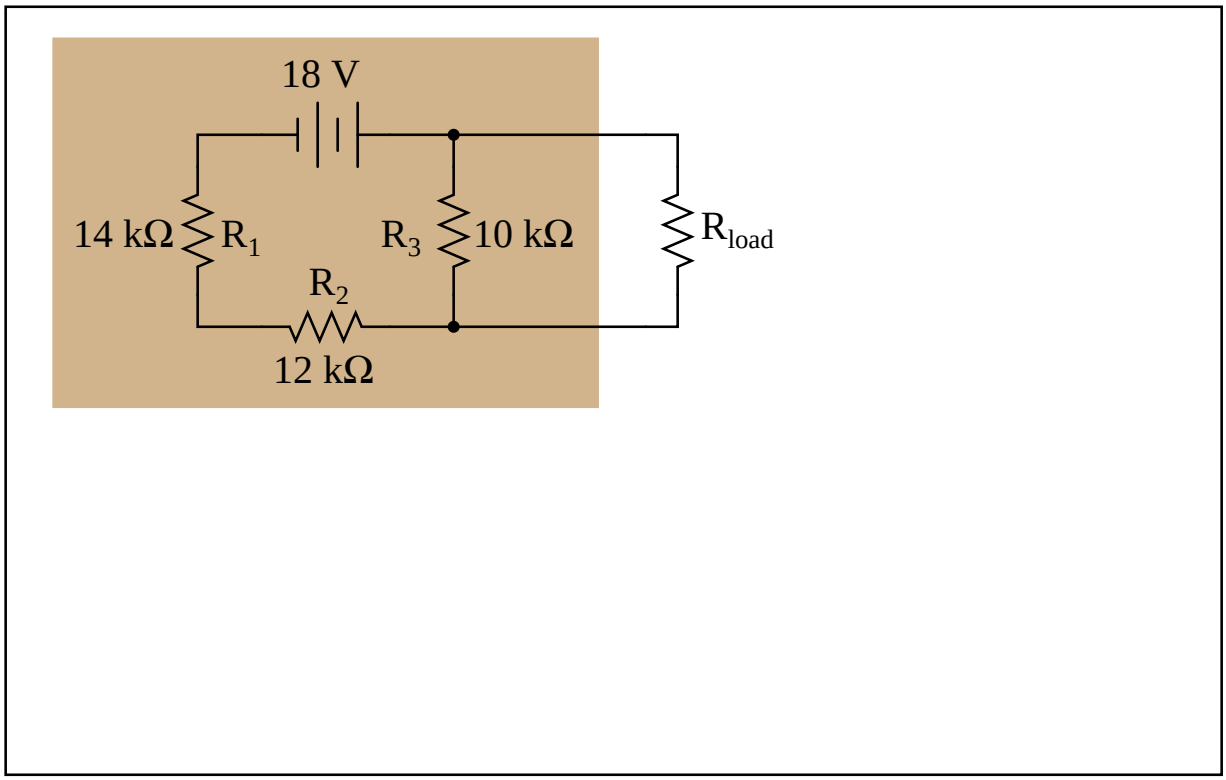
We may use Thevenin's Theorem
to simplify this portion of the circuit . . .

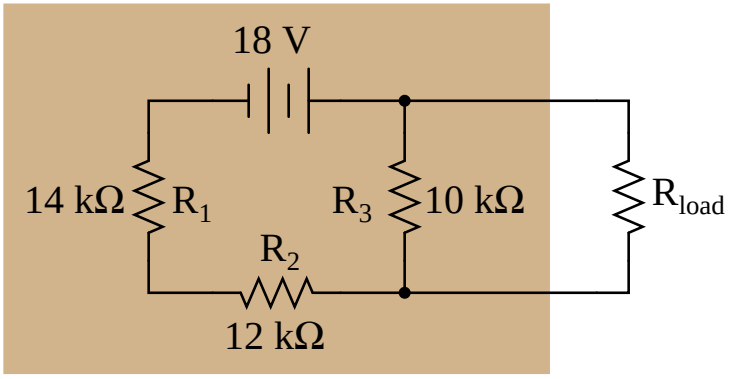


We may use Thevenin's Theorem
to simplify this portion of the circuit . . .

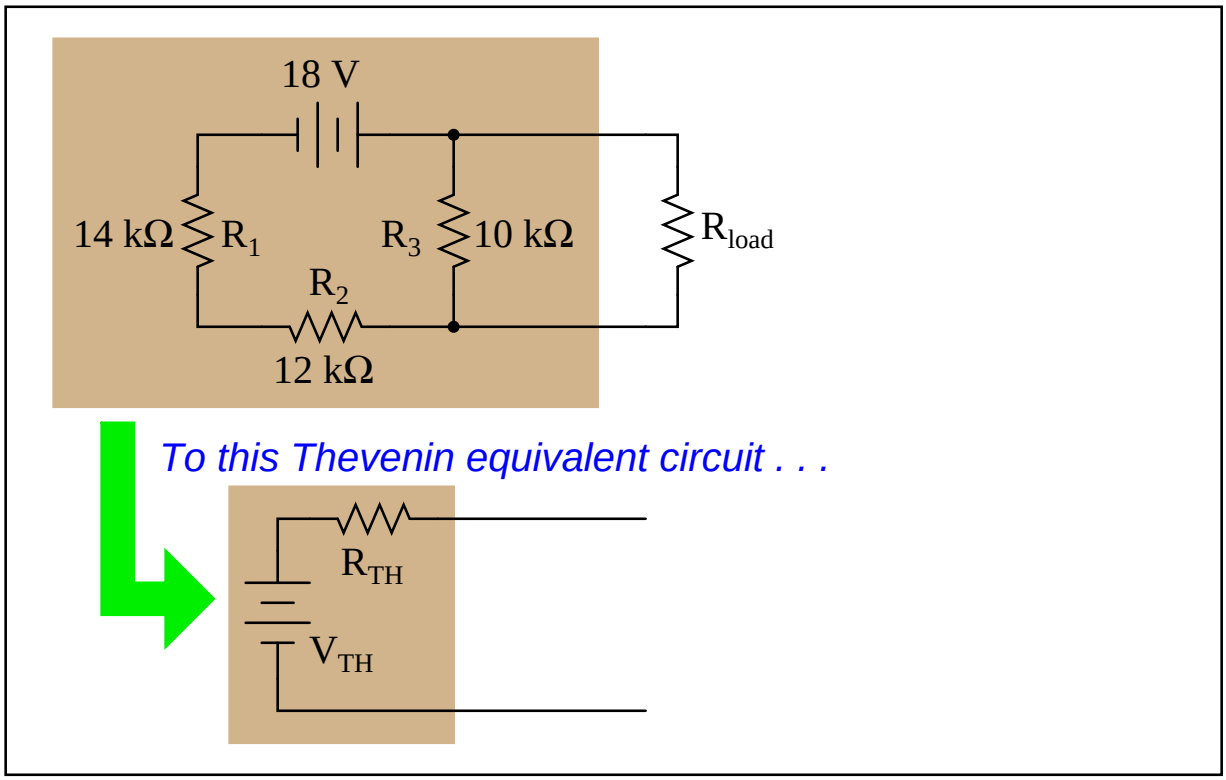


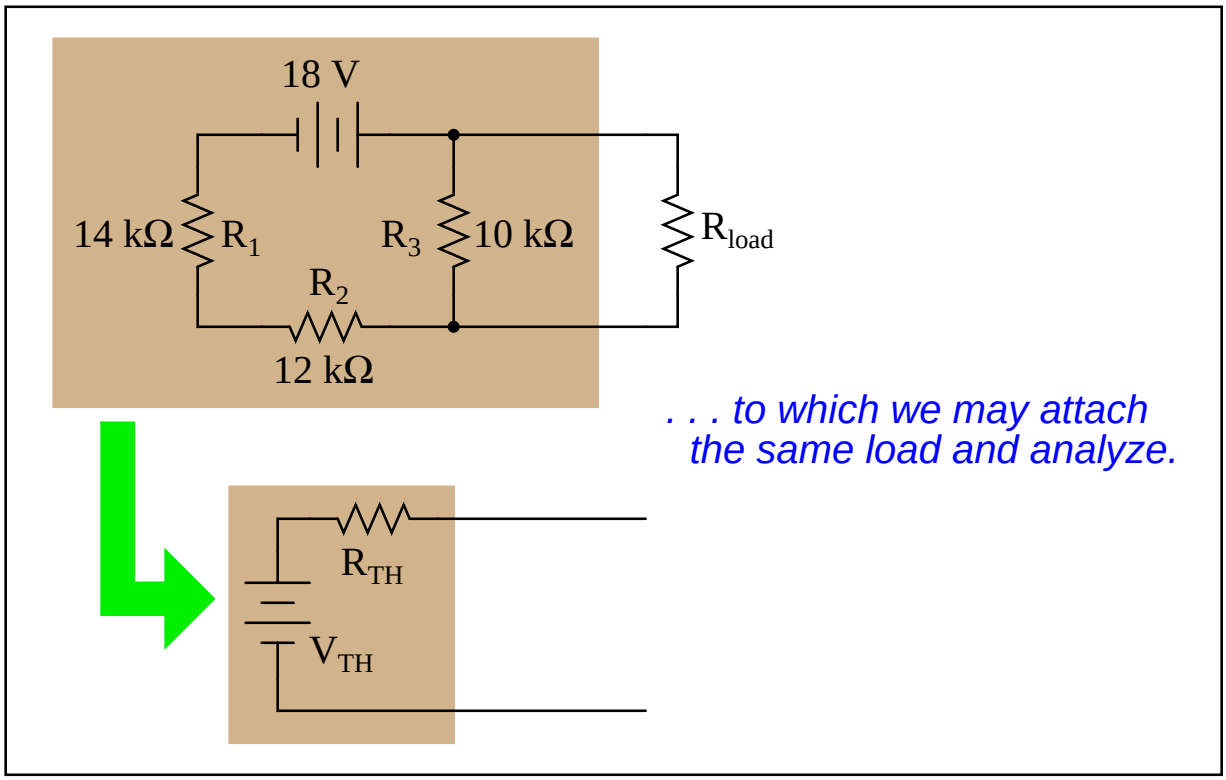

We may use Thevenin's Theorem
to simplify this portion of the circuit . . .
18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ

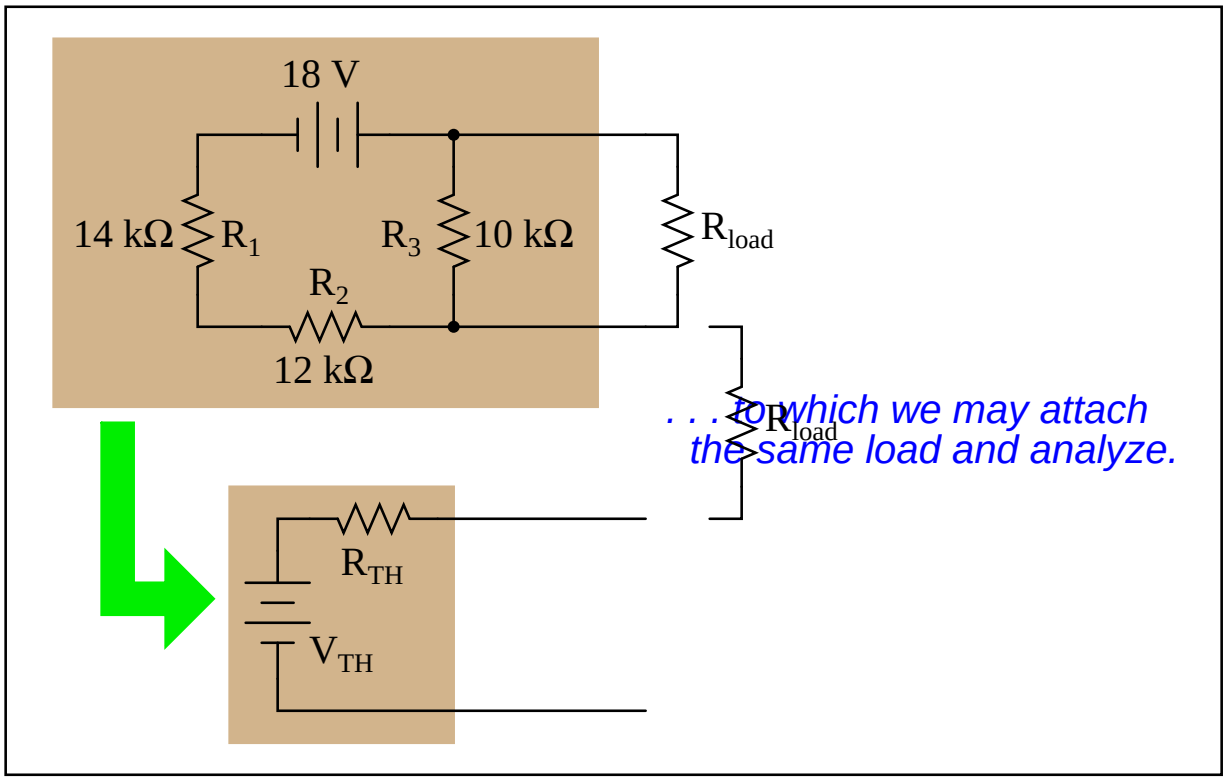

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ

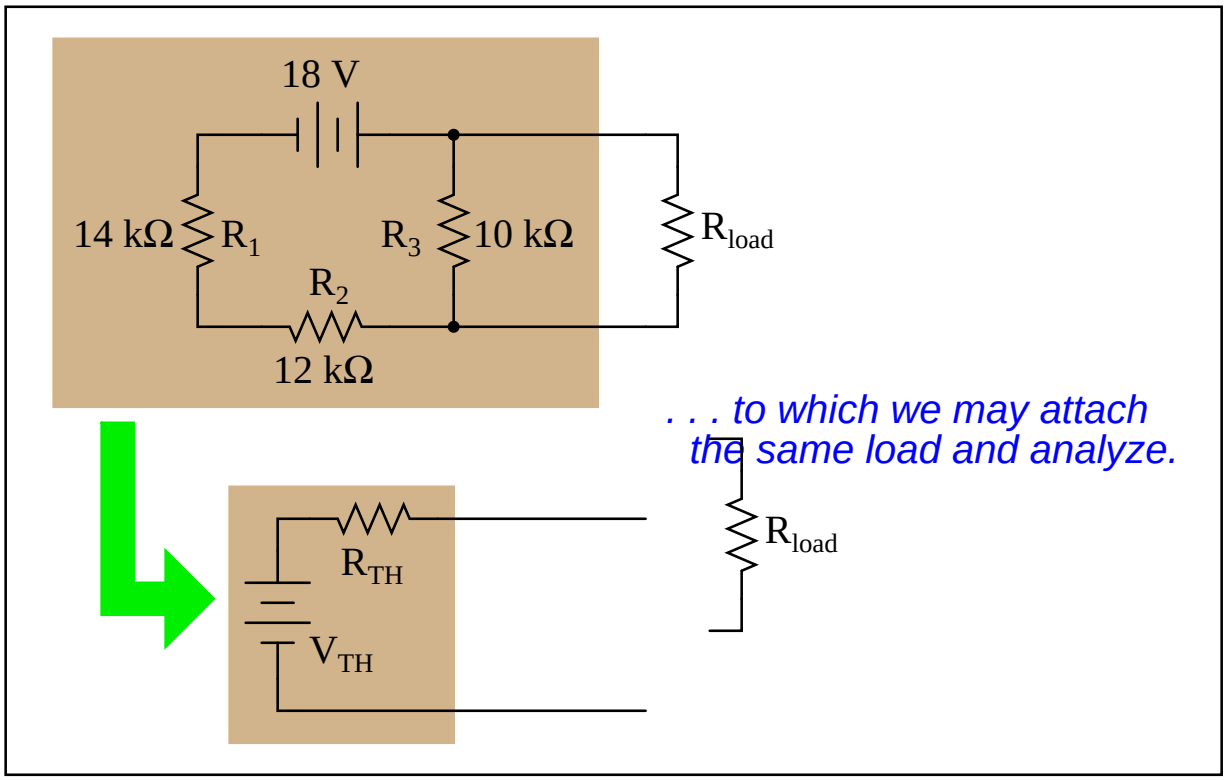


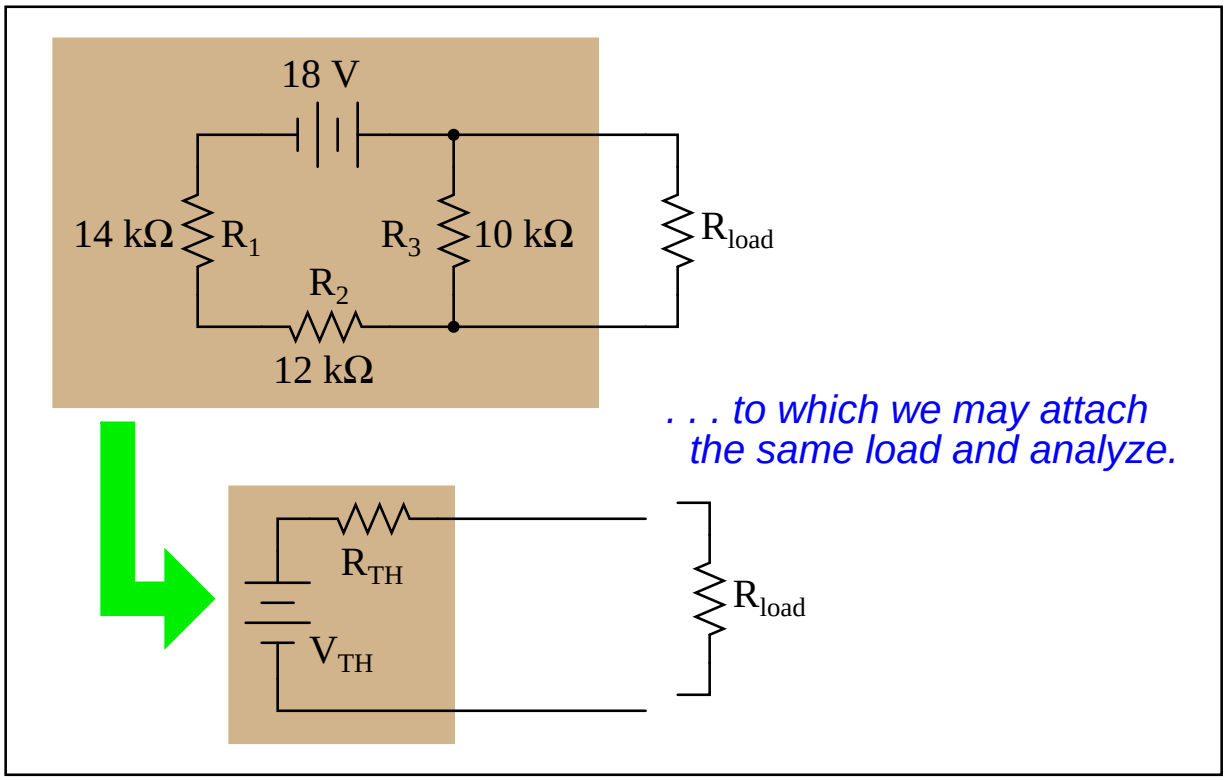
To this Thevenin equivalent circuit . . .

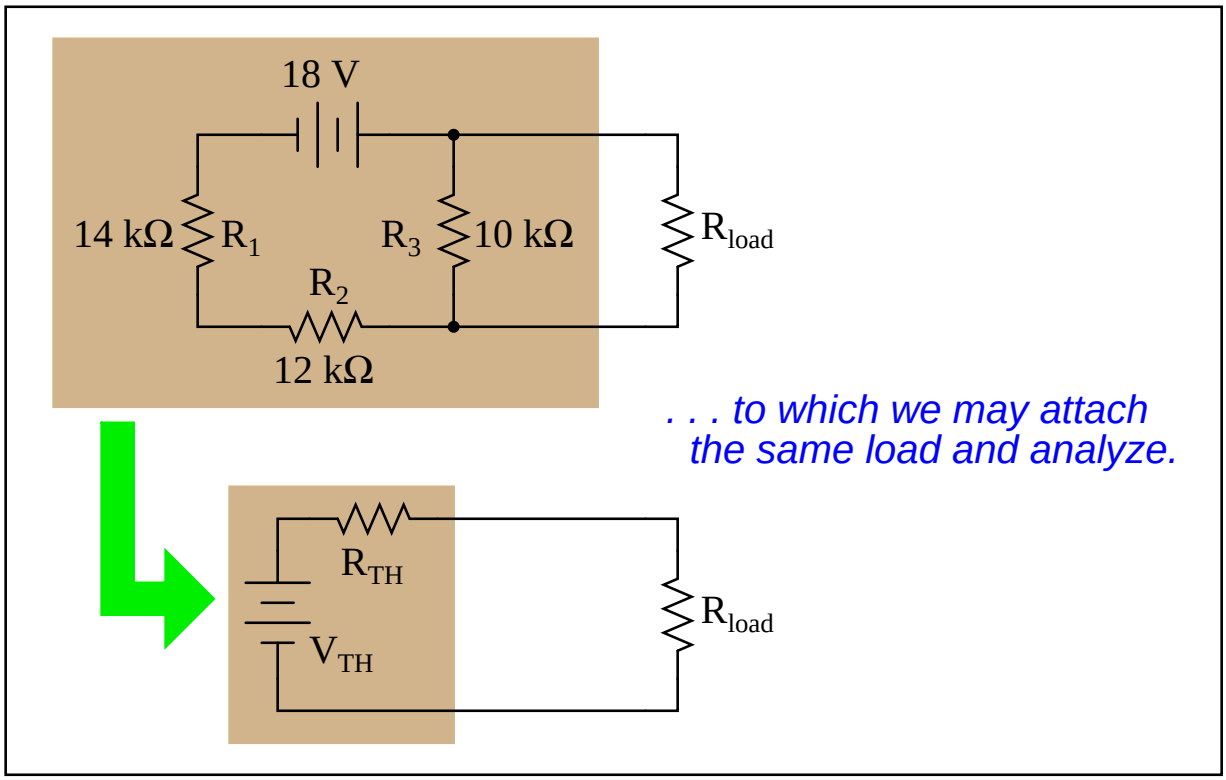

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
To this Thevenin equivalent circuit . . .
RTH
VTH

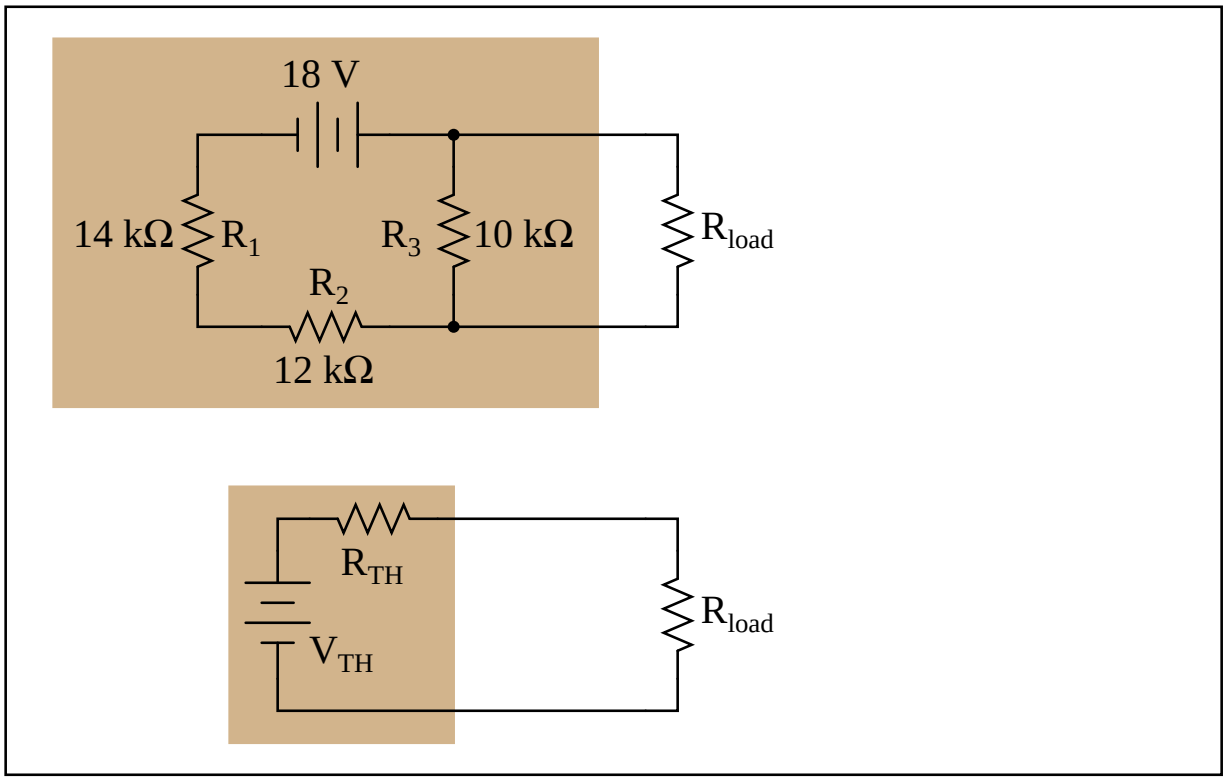

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
. . . to which we may attach
the same load and analyze.
RTH
VTH

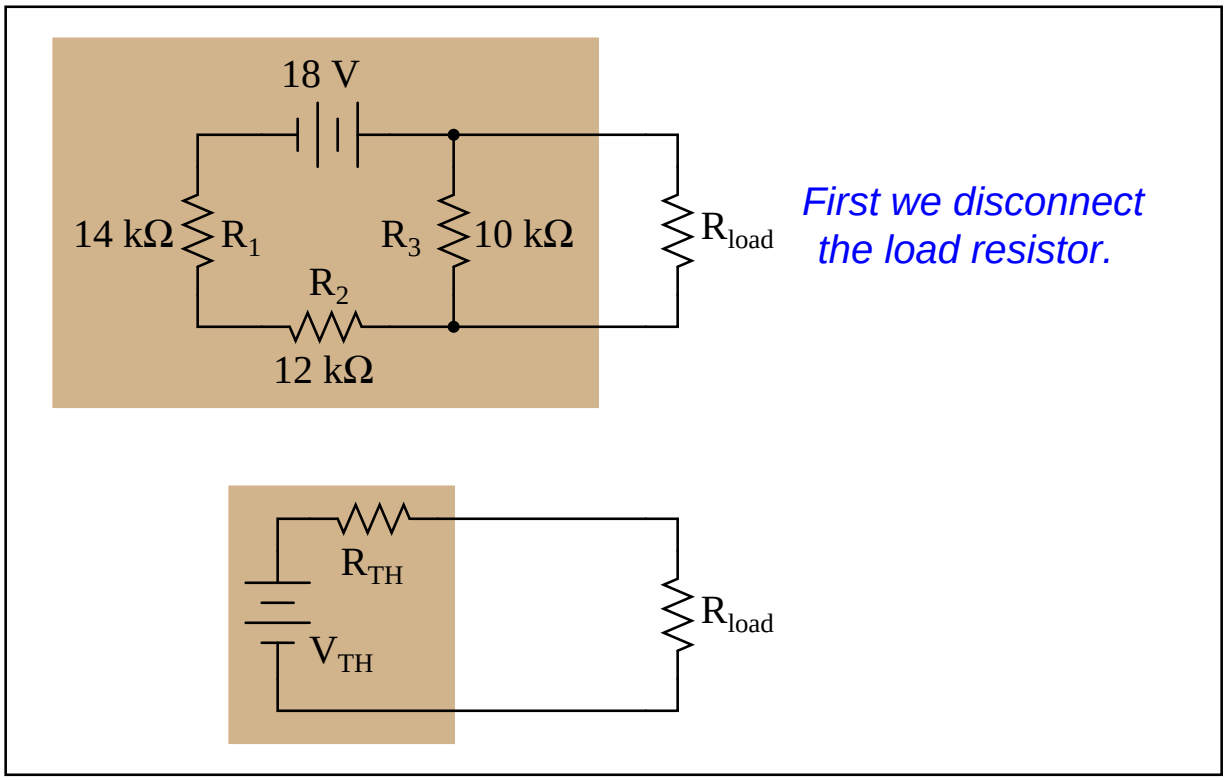

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
. . . to which we may attach
the same load and analyze.
Rload
RTH
VTH

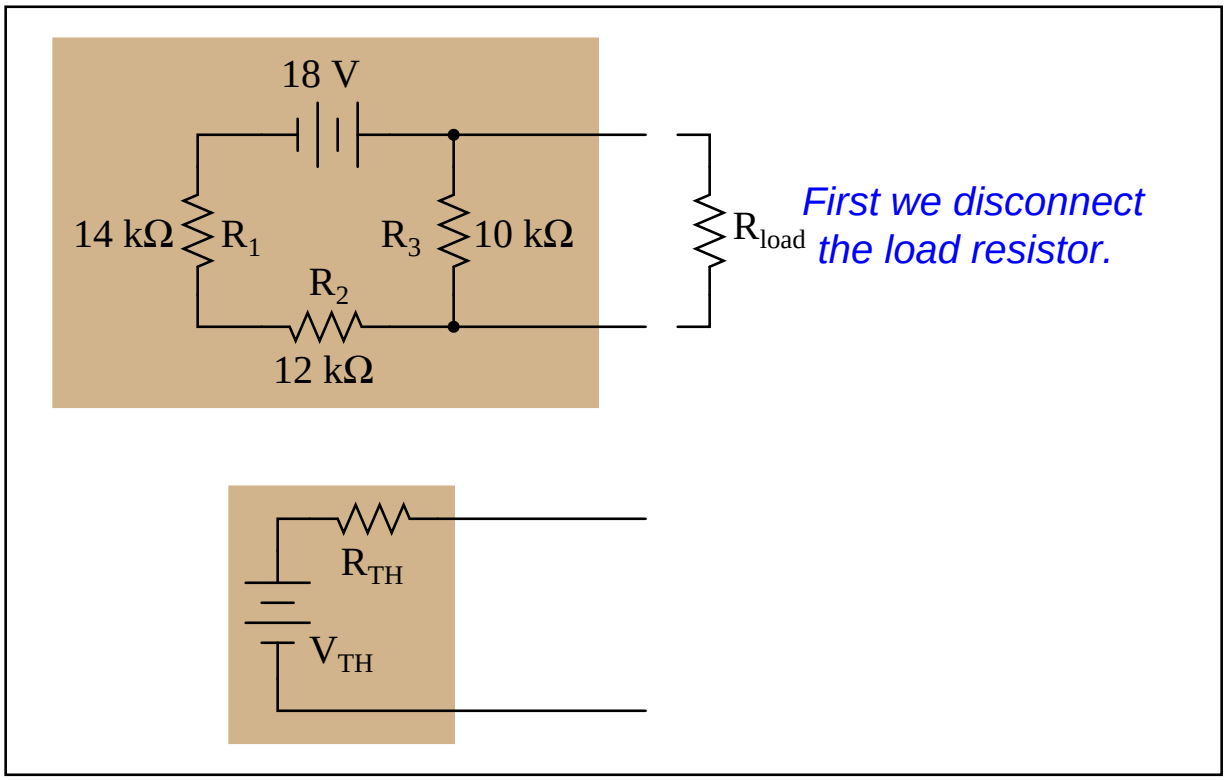

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
. . . to which we may attach
the same load and analyze.
RTH
VTH
Rload

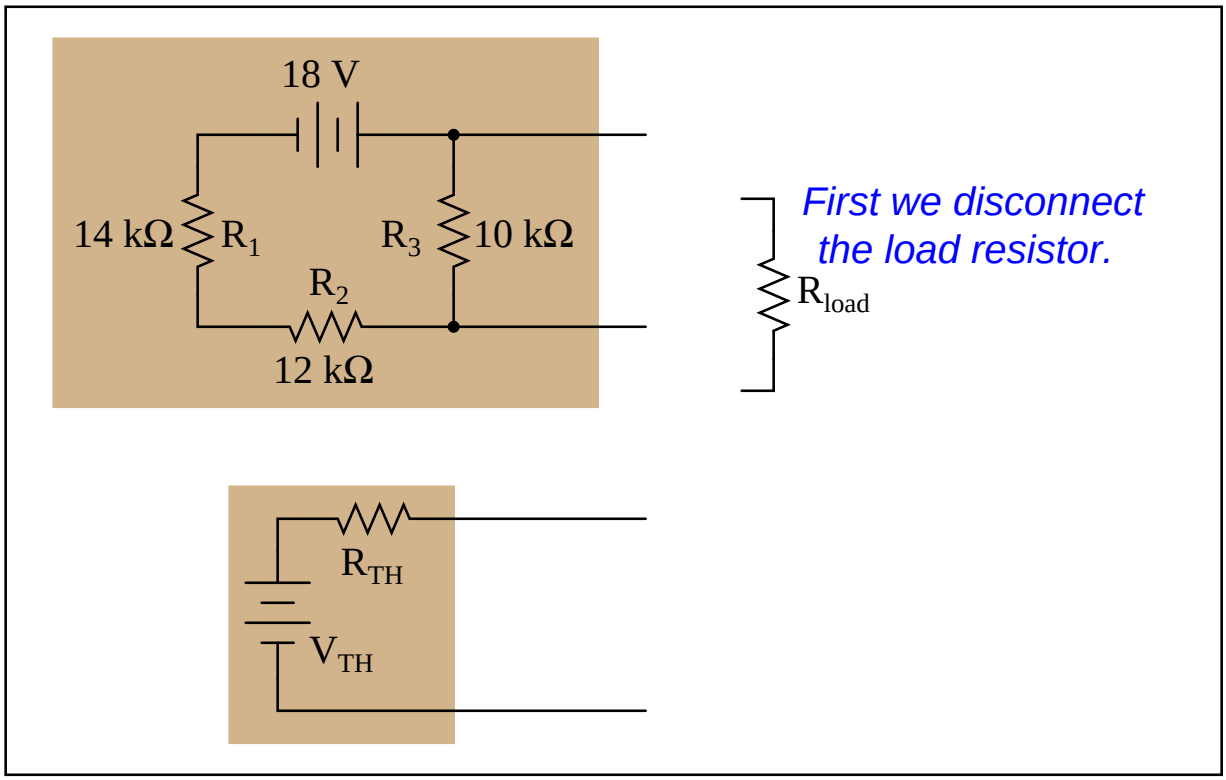

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
. . . to which we may attach
the same load and analyze.
RTH
VTH
Rload

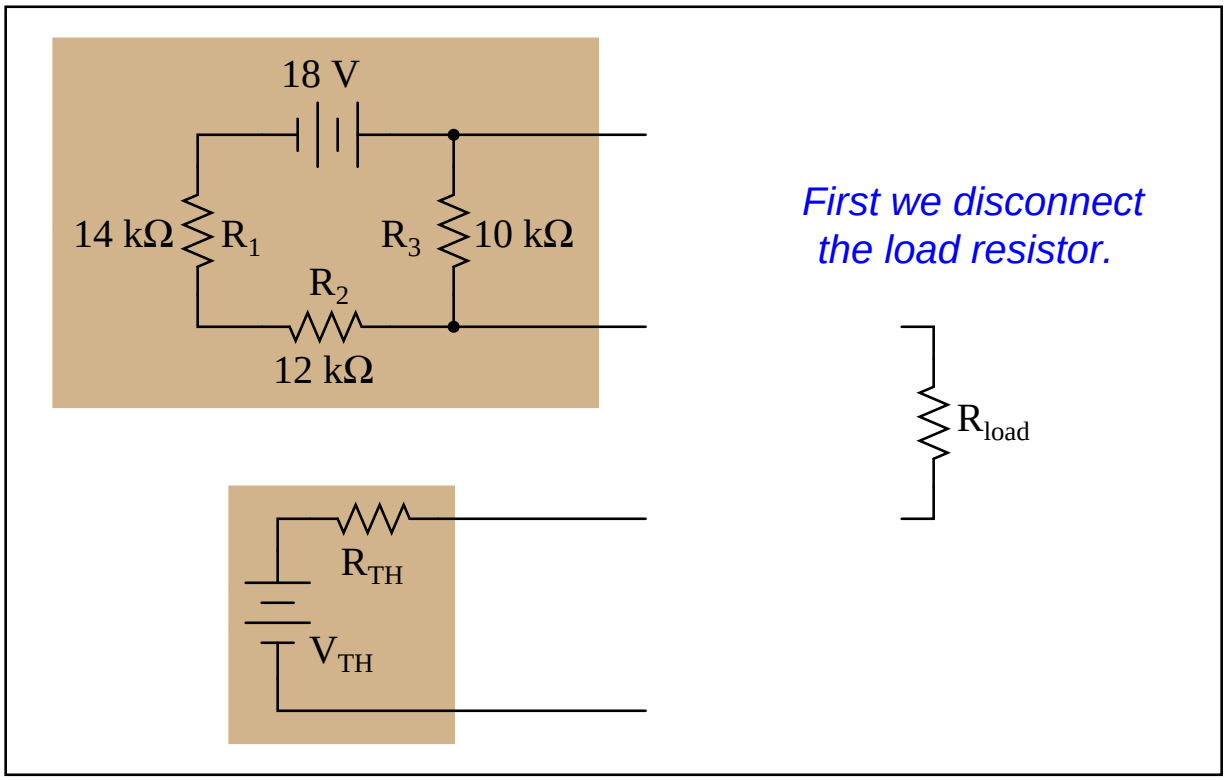

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
. . . to which we may attach
the same load and analyze.
RTH
VTH
Rload

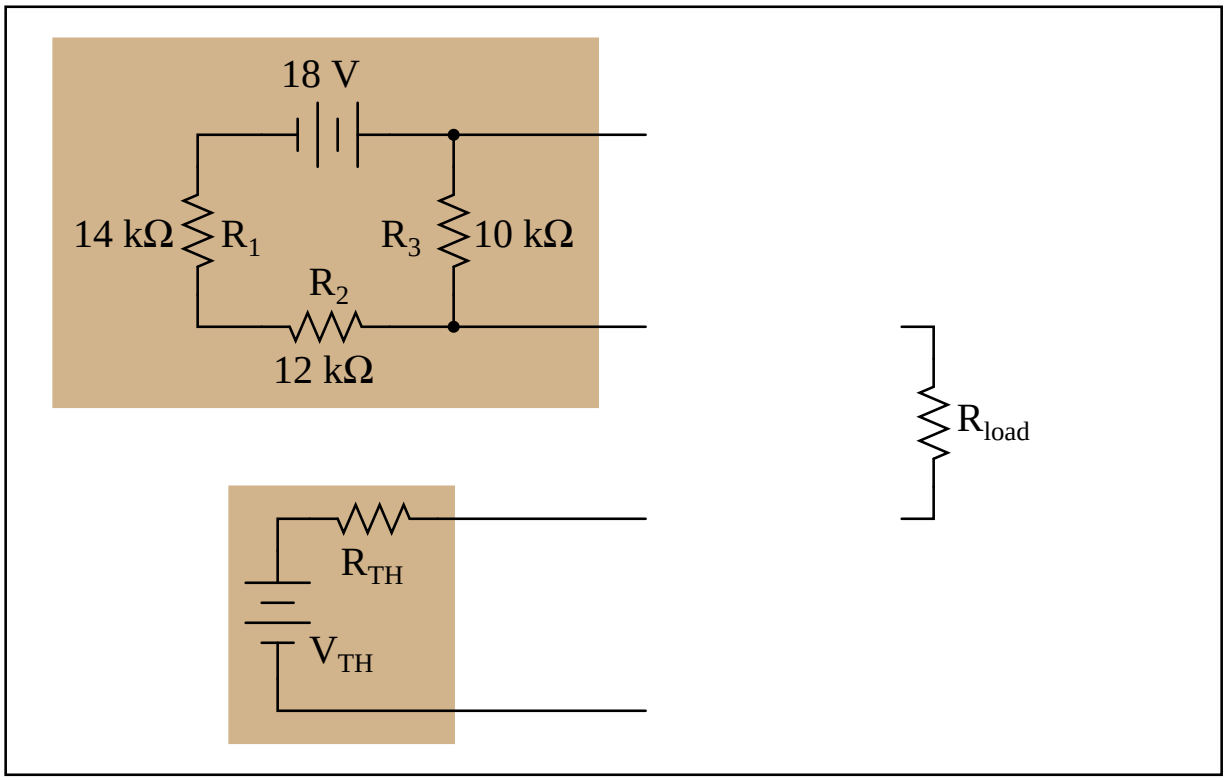

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
RTH
VTH
Rload

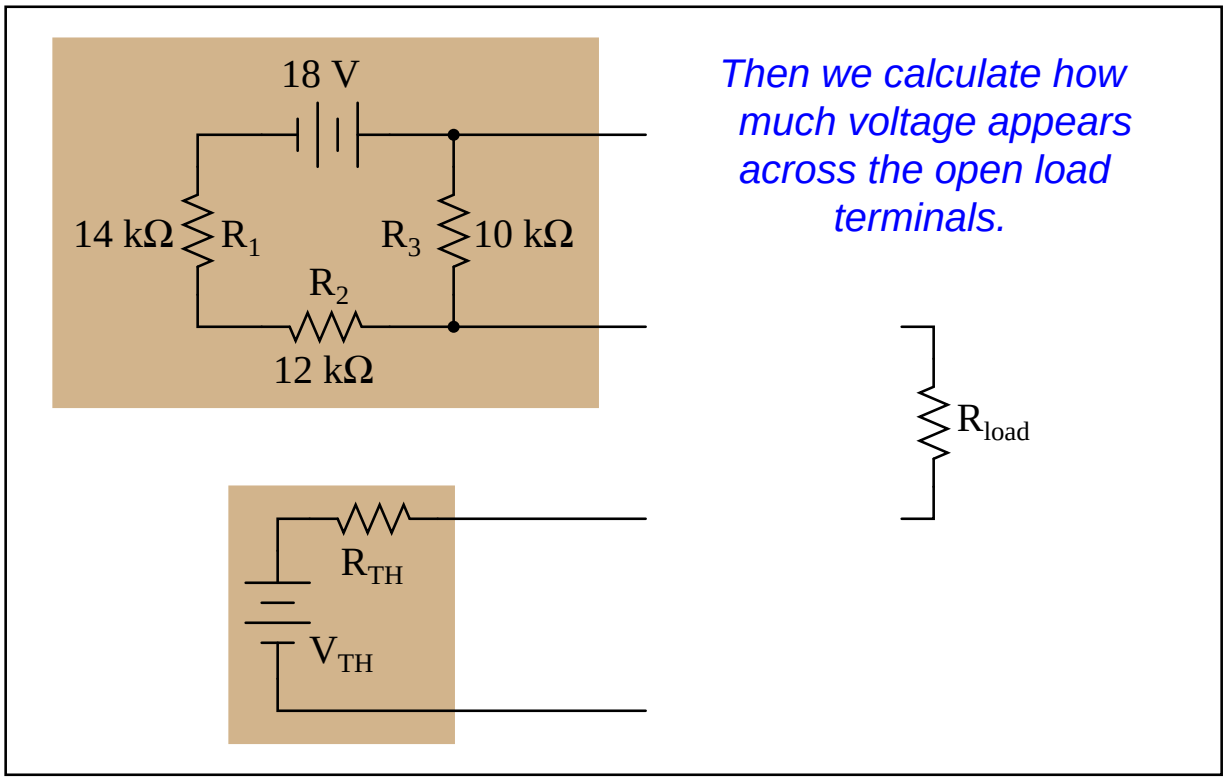

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
First we disconnect
the load resistor.
RTH
VTH
Rload

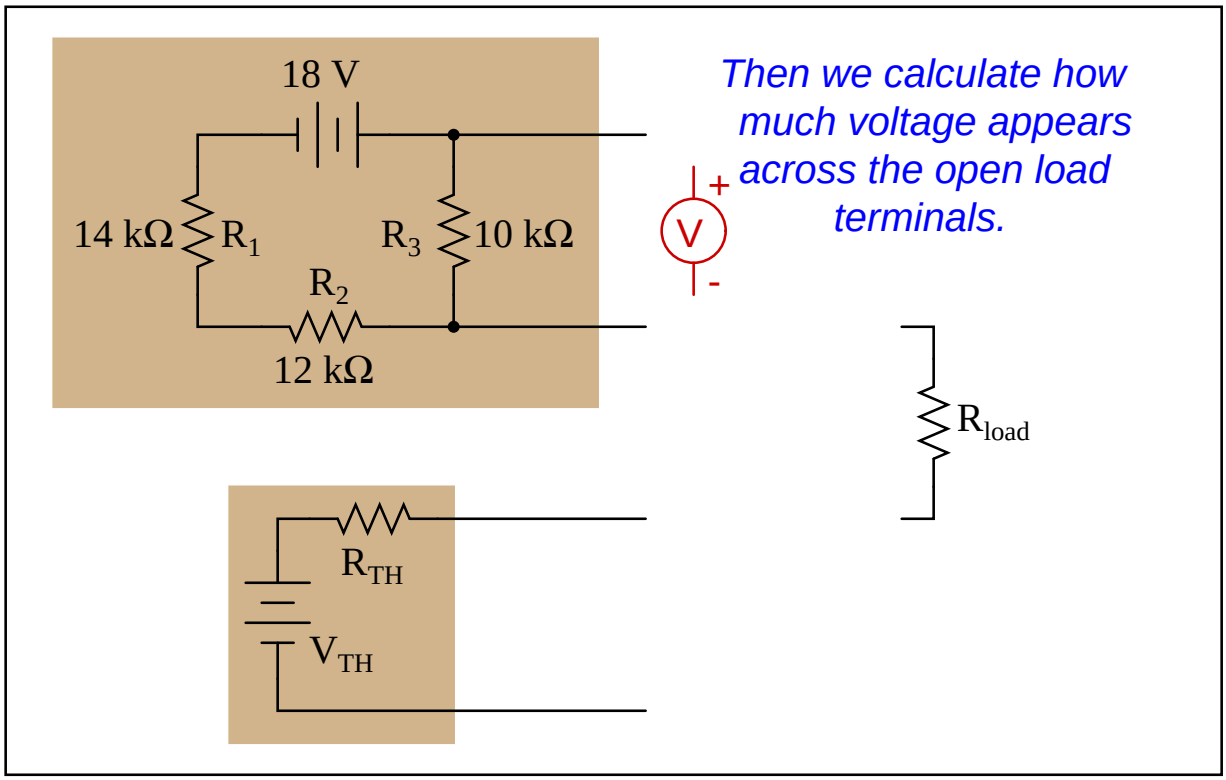

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
First we disconnect
the load resistor.
RTH
VTH

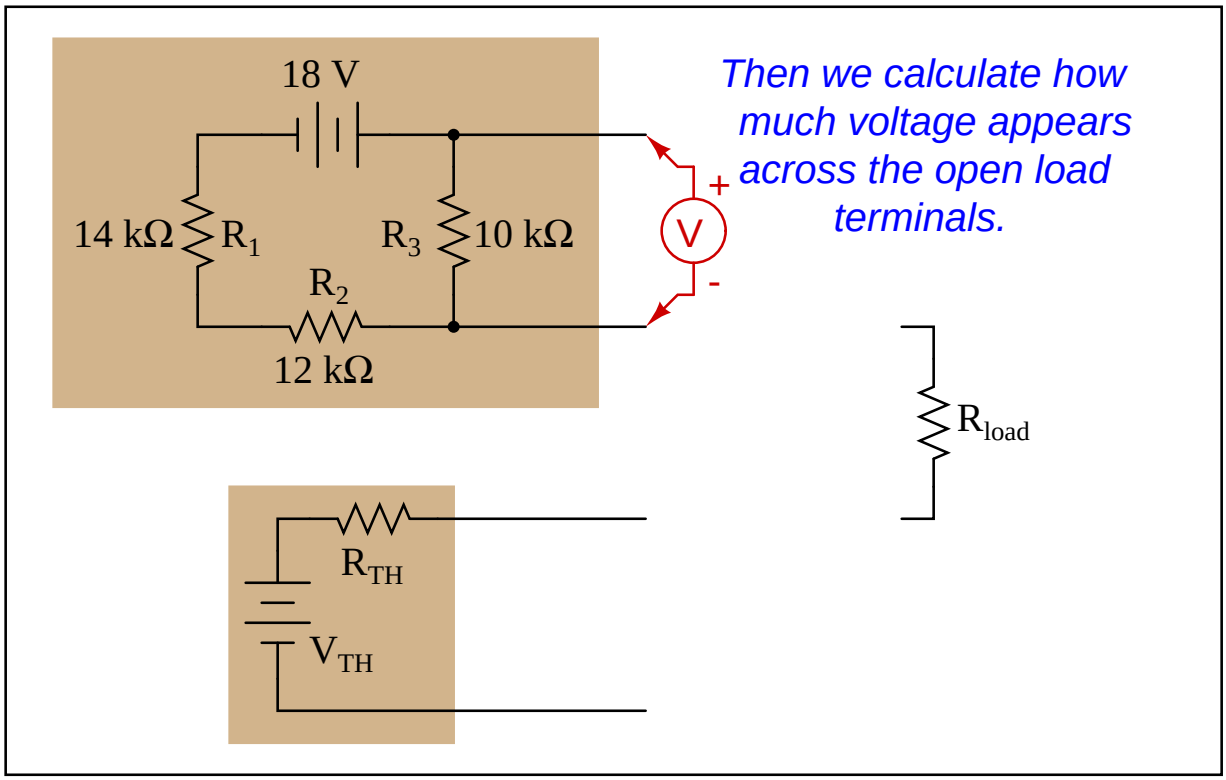

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
First we disconnect the load resistor.
Rload
RTH
VTH

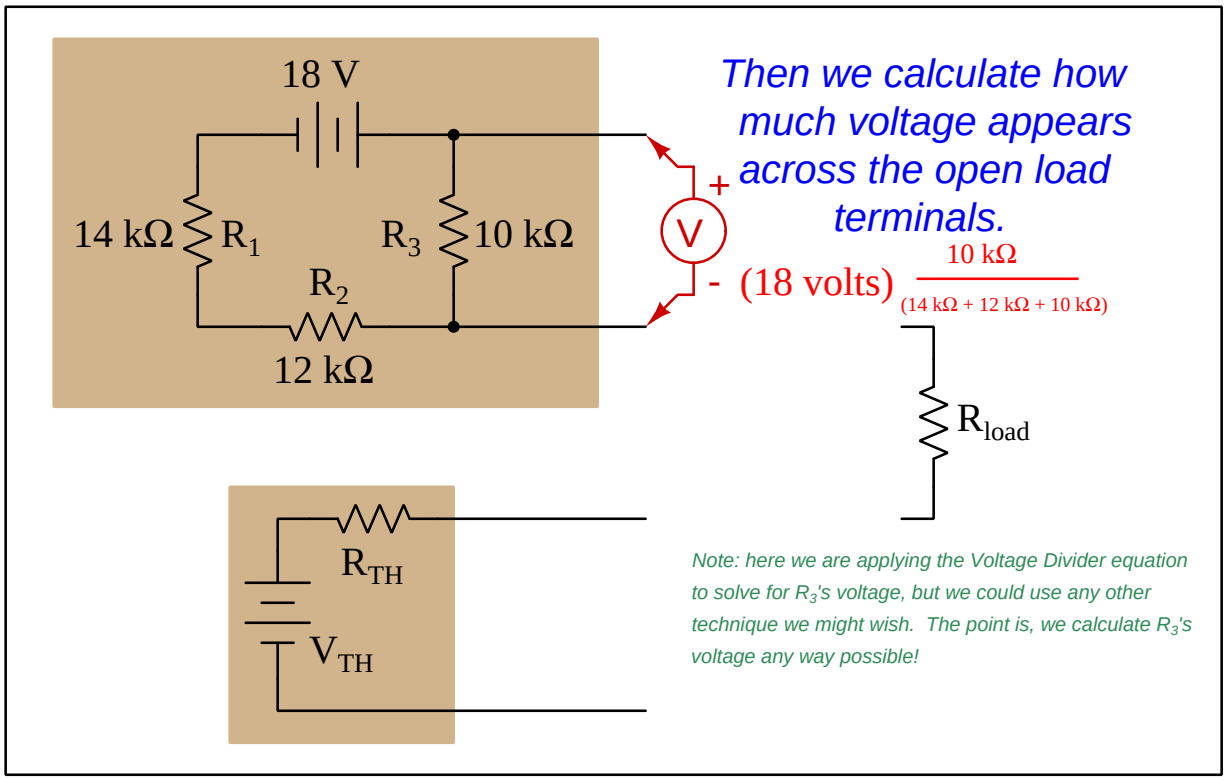

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
First we disconnect
the load resistor.
Rload
RTH
VTH

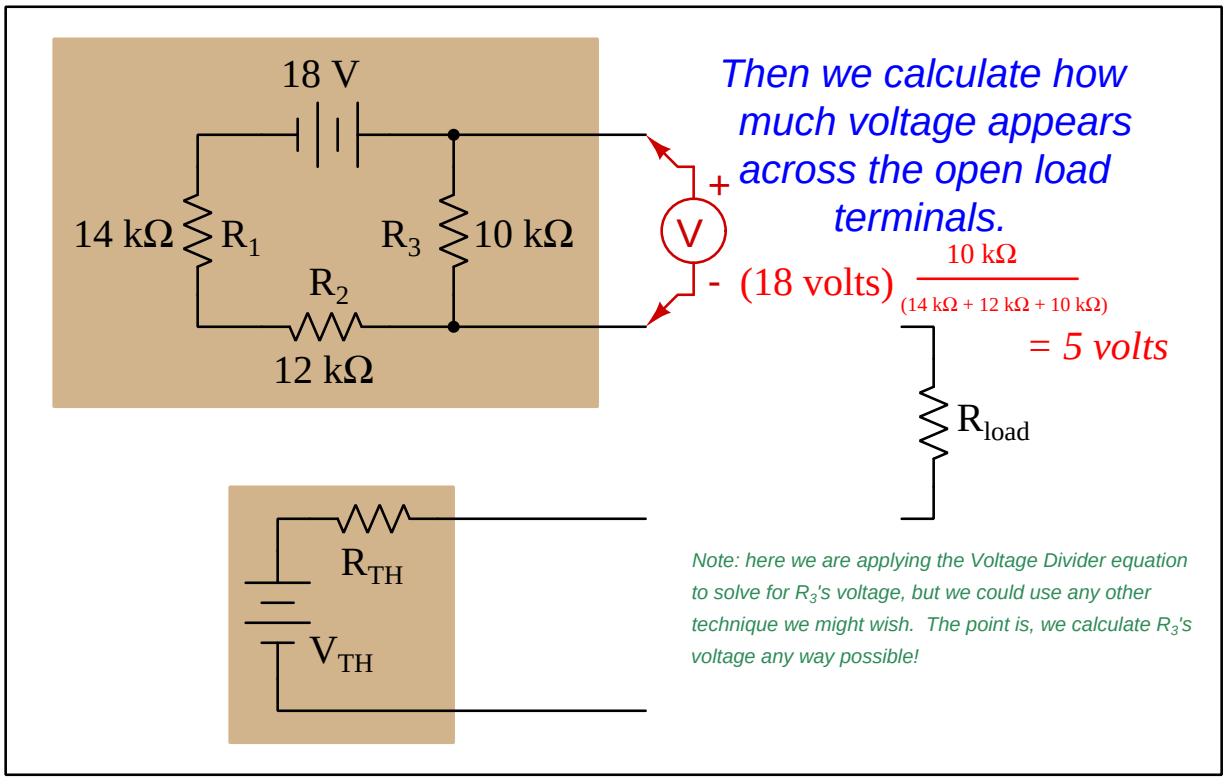

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
RTH
VTH

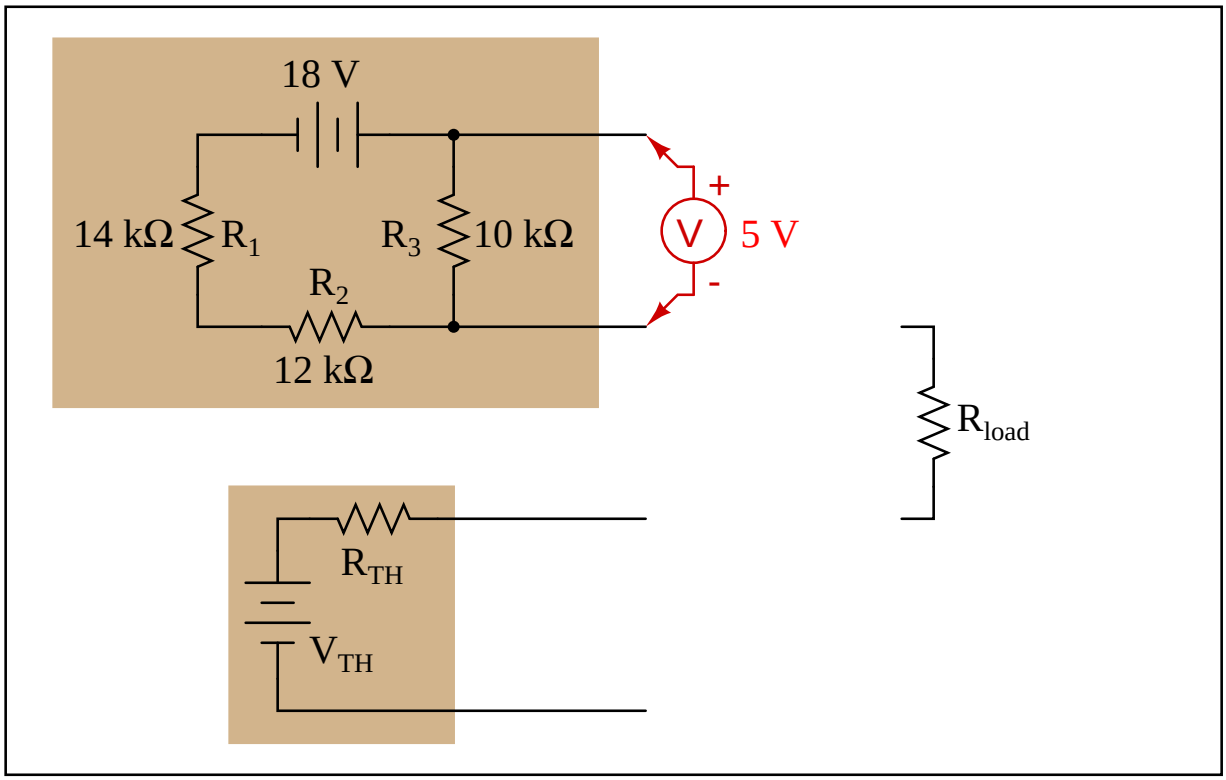

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Then we calculate how much voltage appears across the open load terminals.
Rload
RTH
VTH

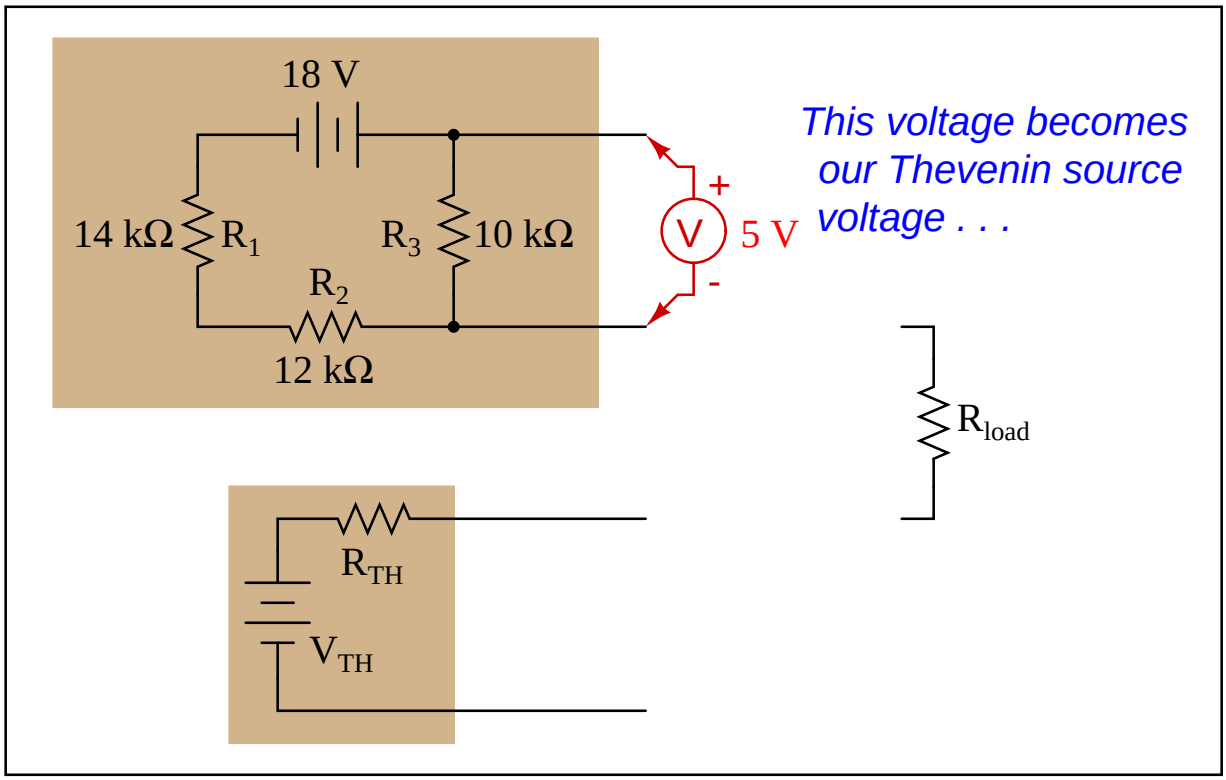

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
-
Then we calculate how much voltage appears across the open load terminals.
Rload
RTH
VTH

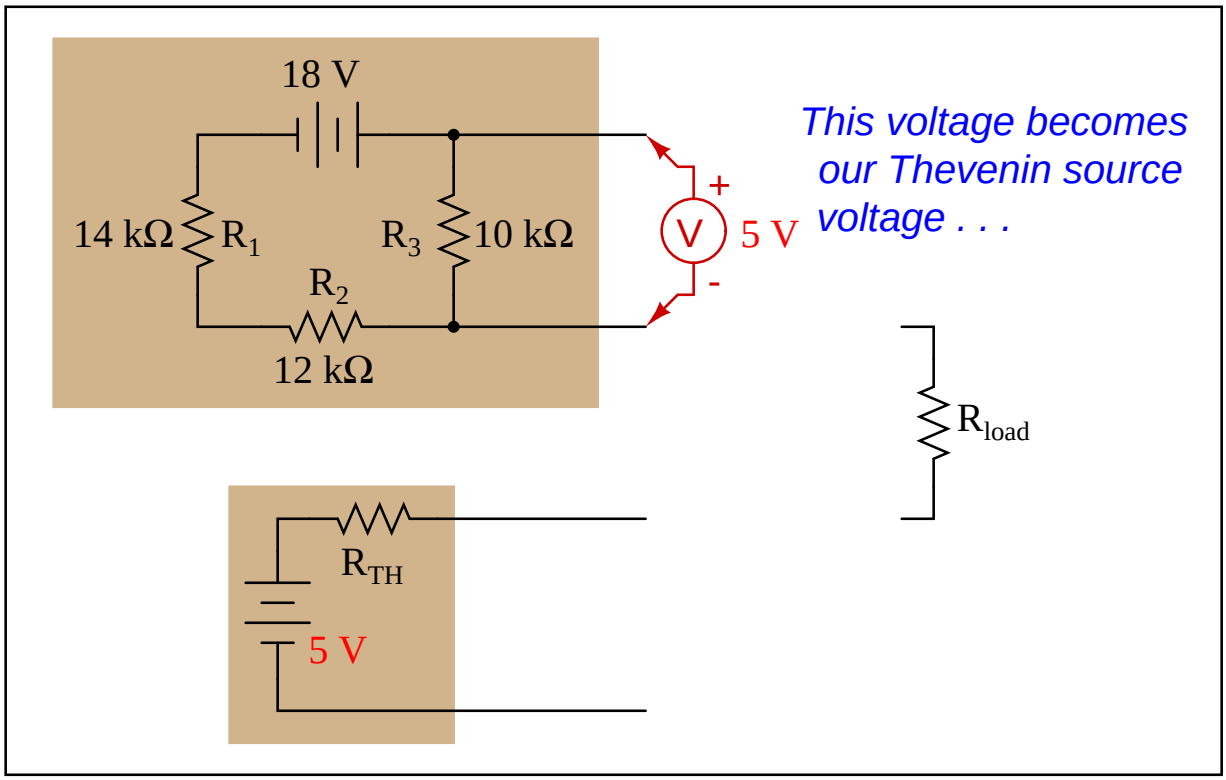

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
-
Then we calculate how much voltage appears across the open load terminals.
Rload
RTH
VTH

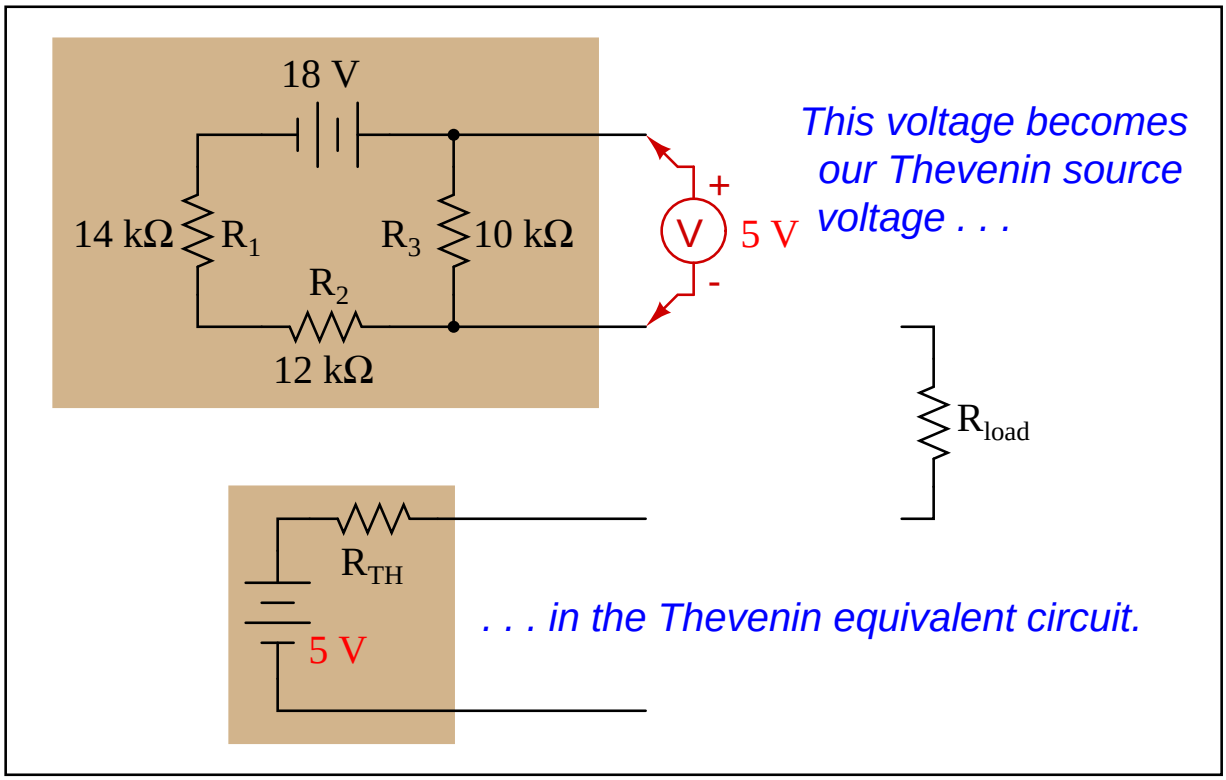

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
-
Then we calculate how much voltage appears across the open load terminals.
(18 volts) 10 kΩ / (14 kΩ + 12 kΩ + 10 kΩ)
Rload
RTH
VTH
Note: here we are applying the Voltage Divider equation to solve for R3's voltage, but we could use any other technique we might wish. The point is, we calculate R3's voltage any way possible!

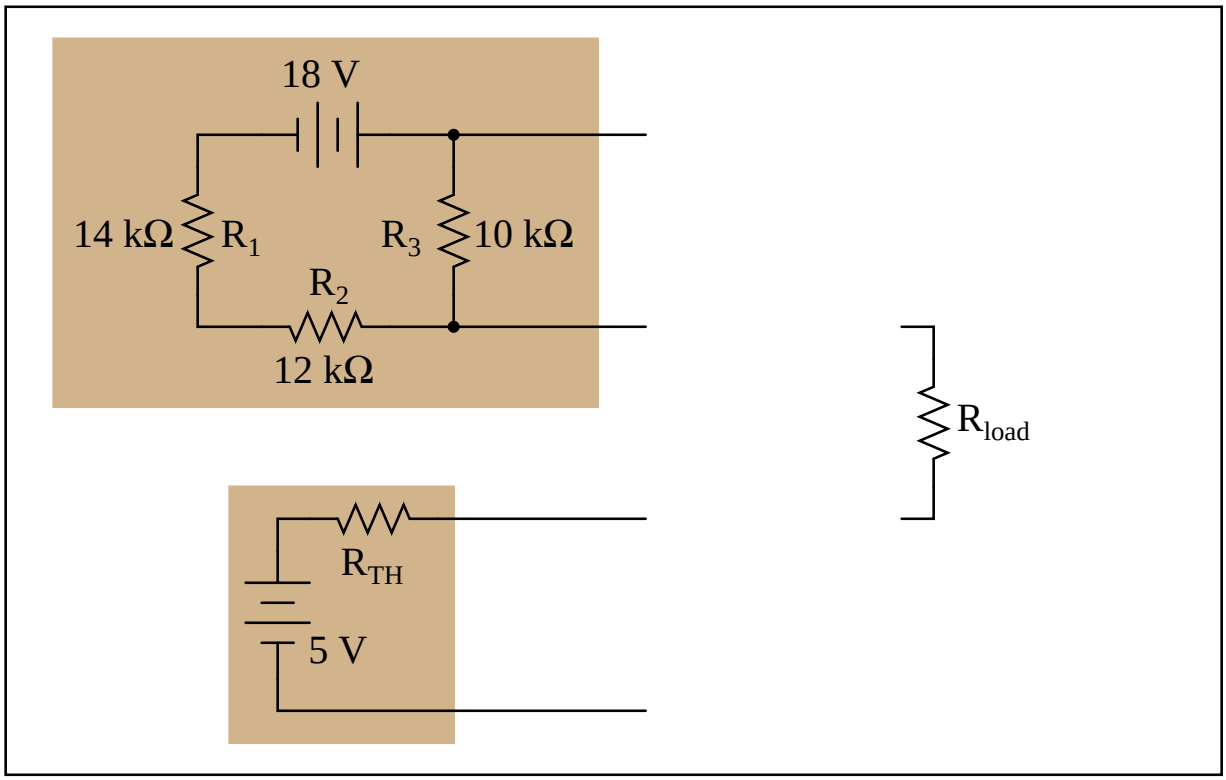

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
-
Then we calculate how much voltage appears across the open load terminals.
(18 volts) 10 kΩ / (14 kΩ + 12 kΩ + 10 kΩ)
= 5 volts
Rload
RTH
VTH
Note: here we are applying the Voltage Divider equation to solve for R3's voltage, but we could use any other technique we might wish. The point is, we calculate R3's voltage any way possible!

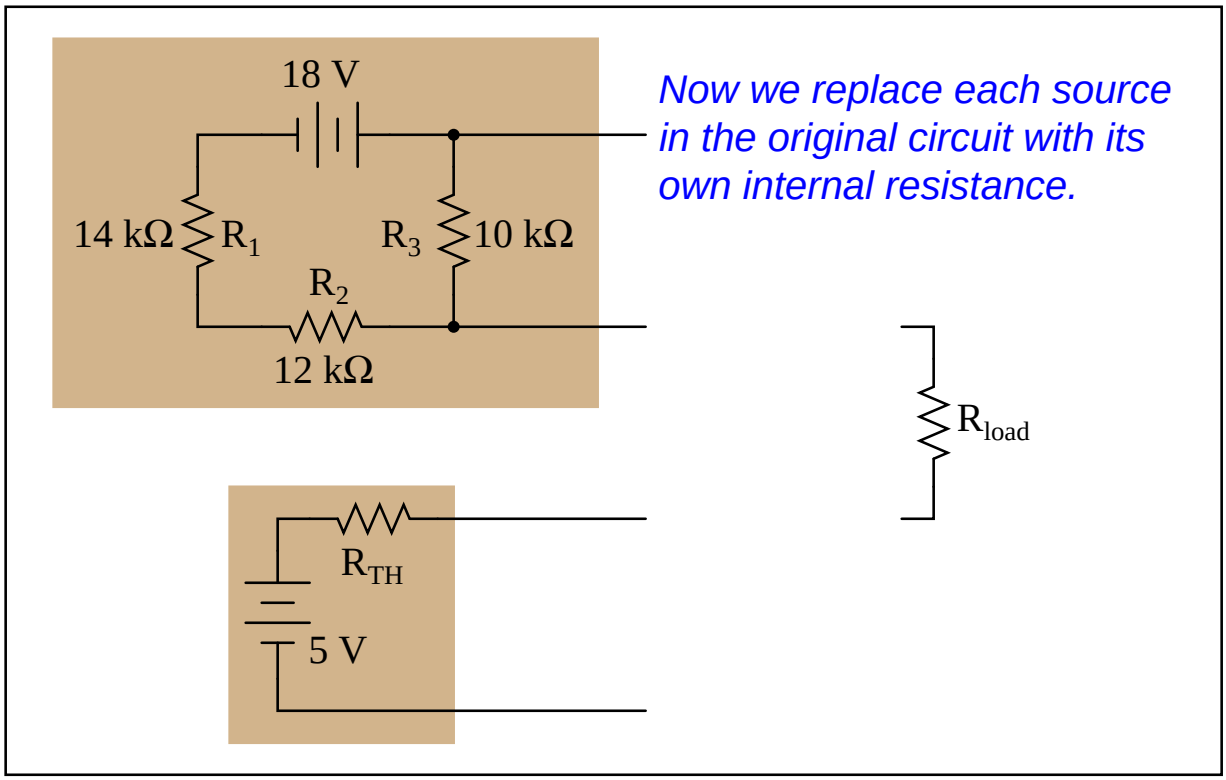

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
5 V
-
Rload
RTH
VTH

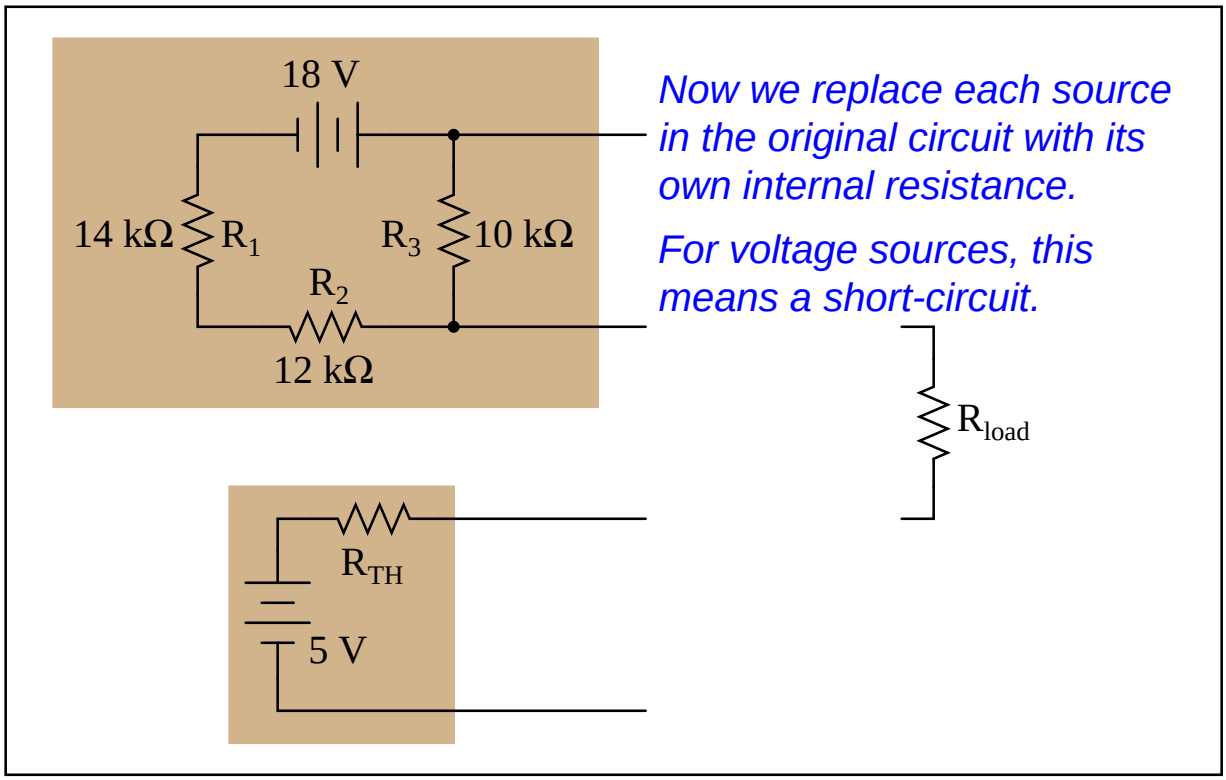

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
5 V
-
This voltage becomes our Thevenin source voltage . . .
Rload
RTH
VTH

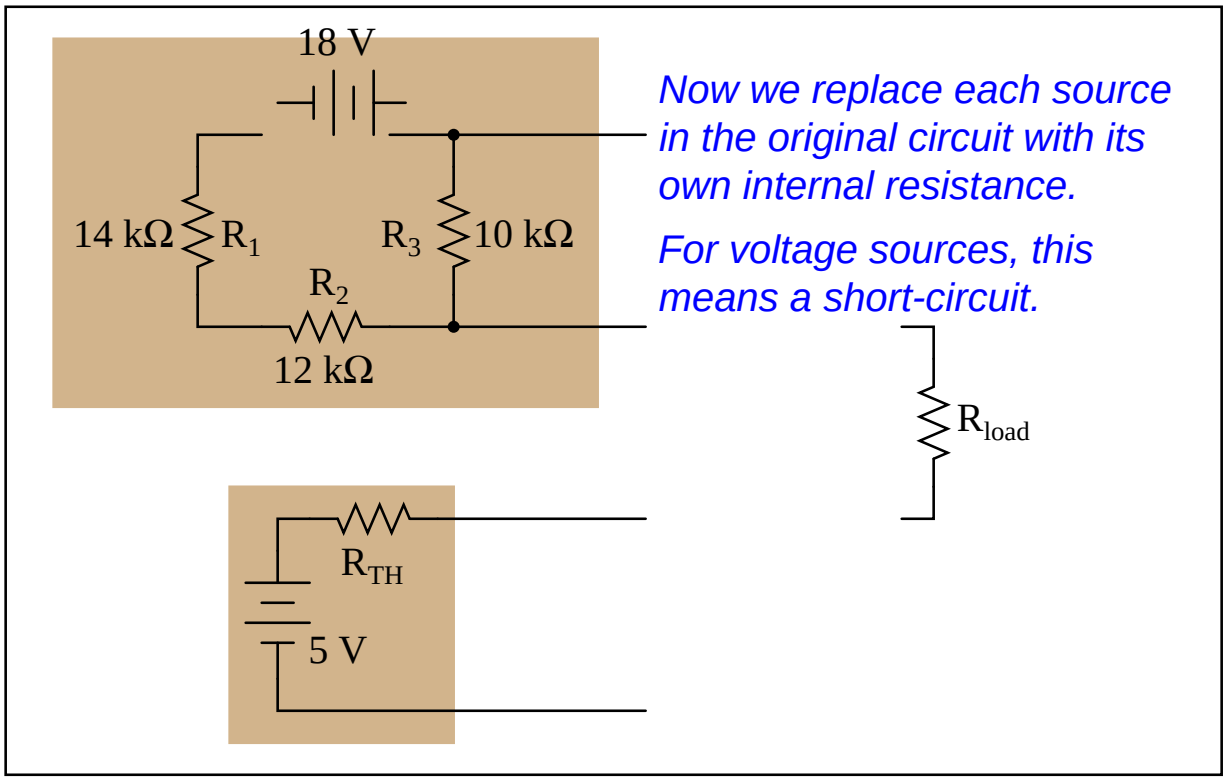

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
5 V
-
This voltage becomes our Thevenin source voltage . . .
Rload
RTH
5 V

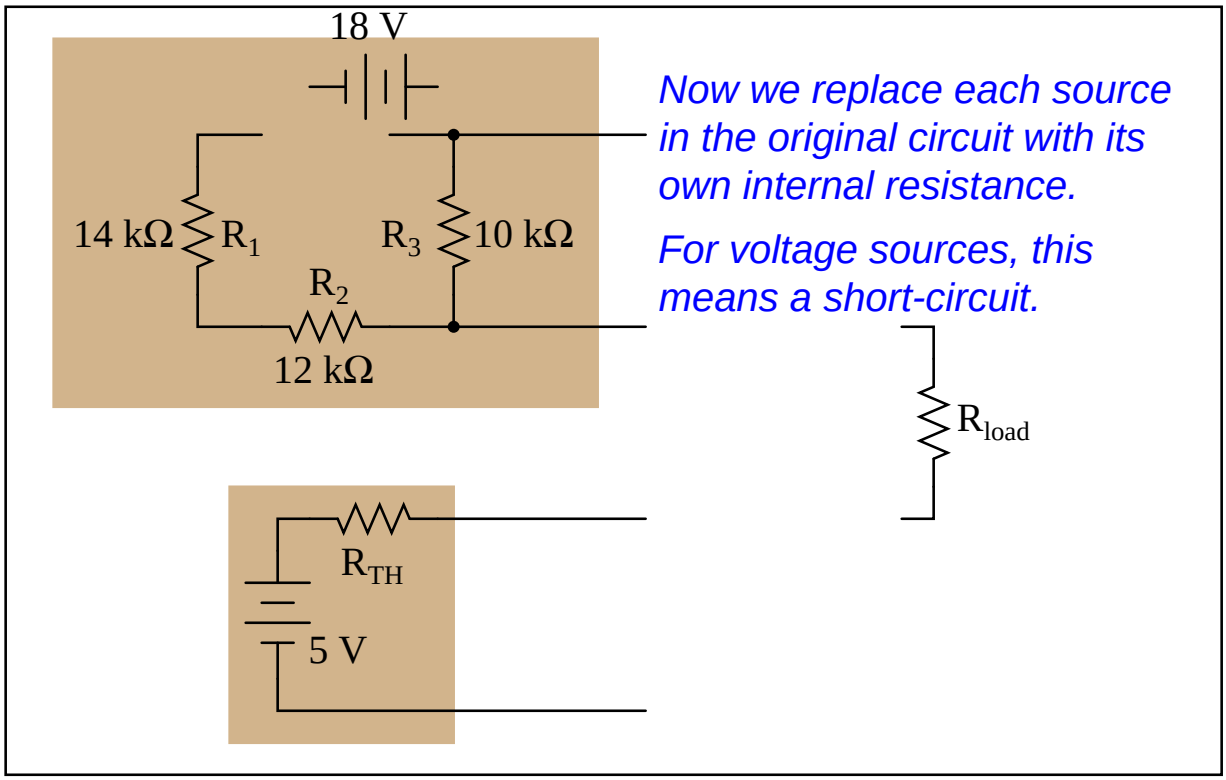

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
+
V
5 V
-
This voltage becomes our Thevenin source voltage . . .
Rload
RTH
5 V
. . . in the Thevenin equivalent circuit.

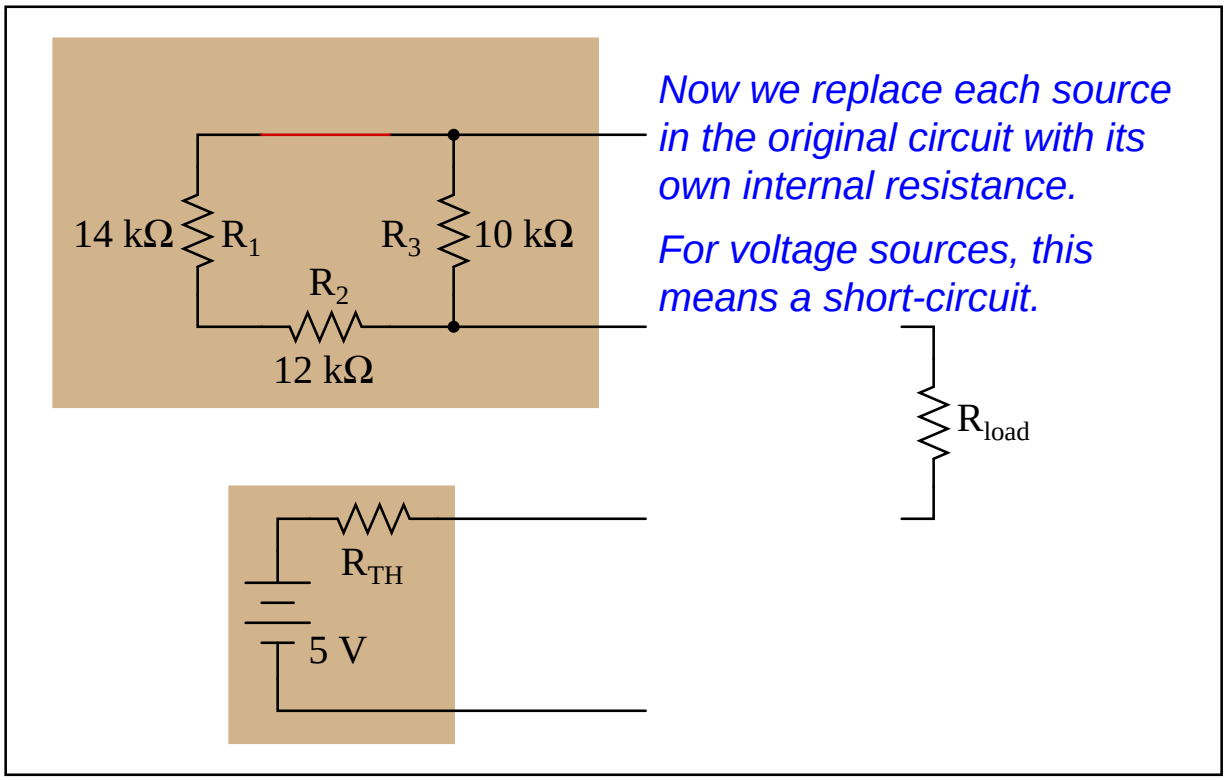

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
RTH
5 V

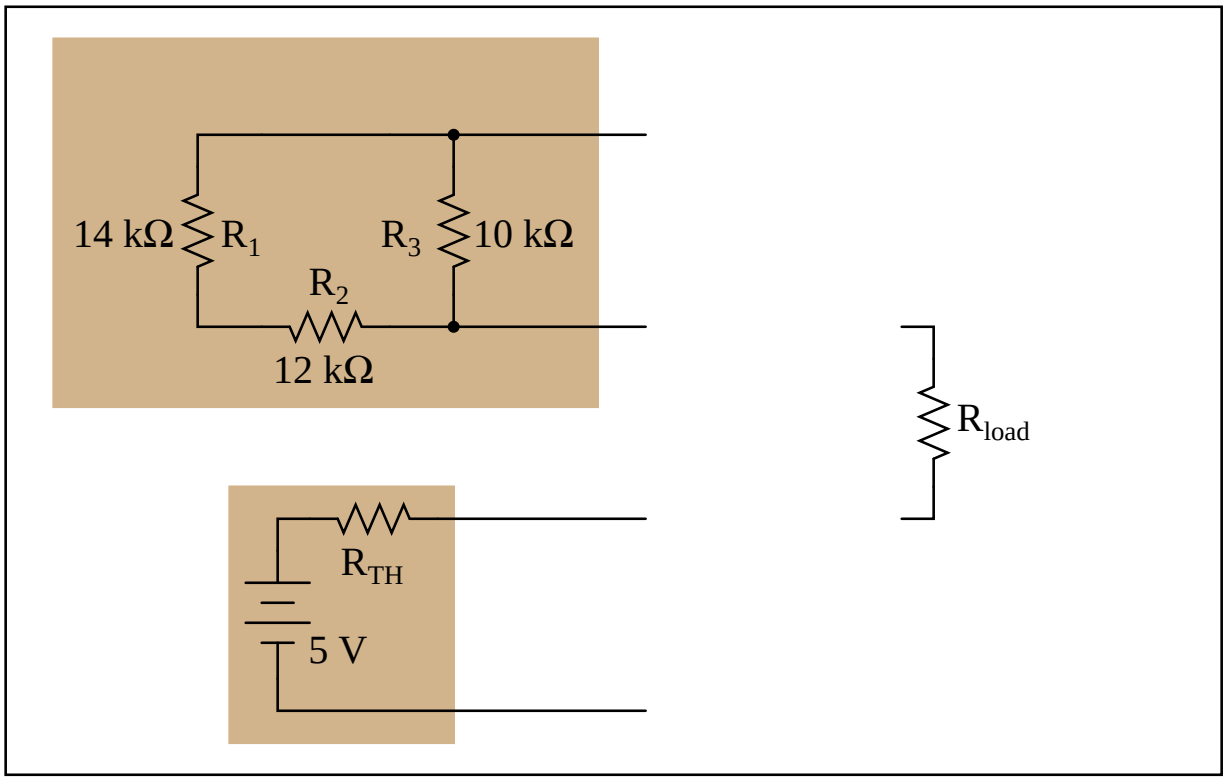

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now we replace each source in the original circuit with its own internal resistance.
Rload
RTH
5 V

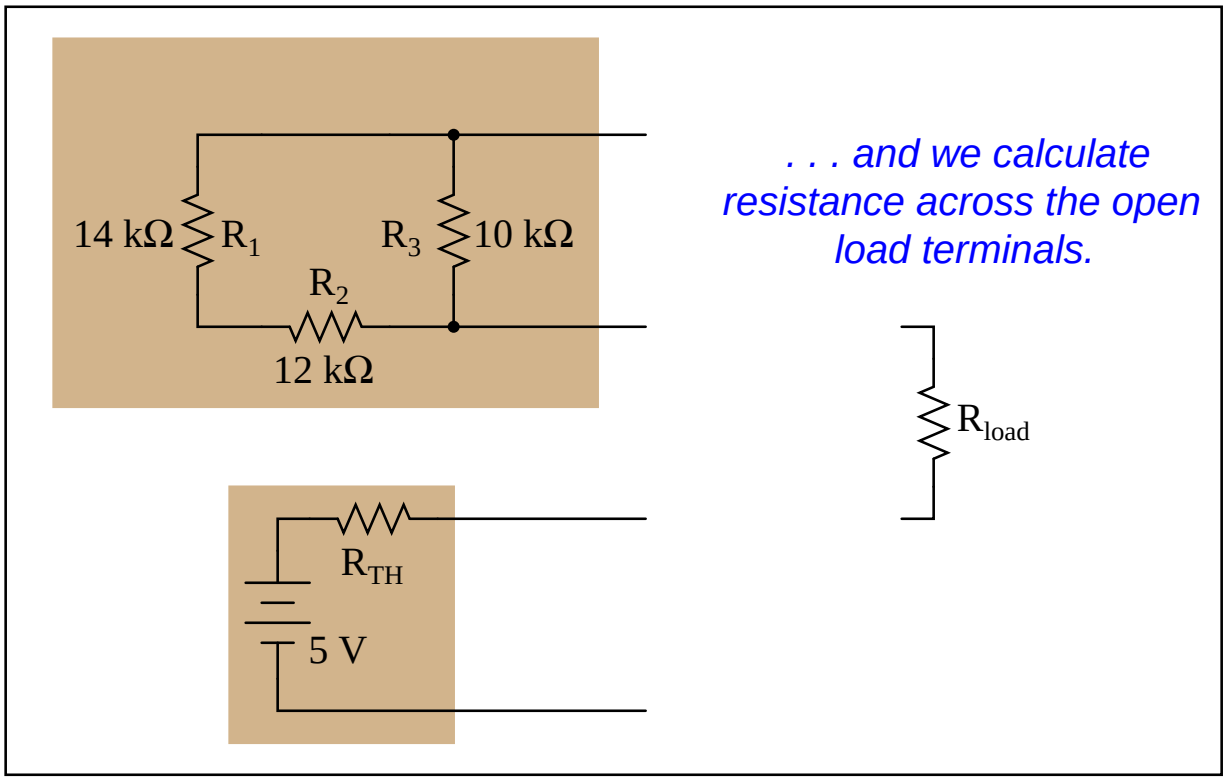

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now we replace each source in the original circuit with its own internal resistance.
For voltage sources, this means a short-circuit.
Rload
RTH
5 V

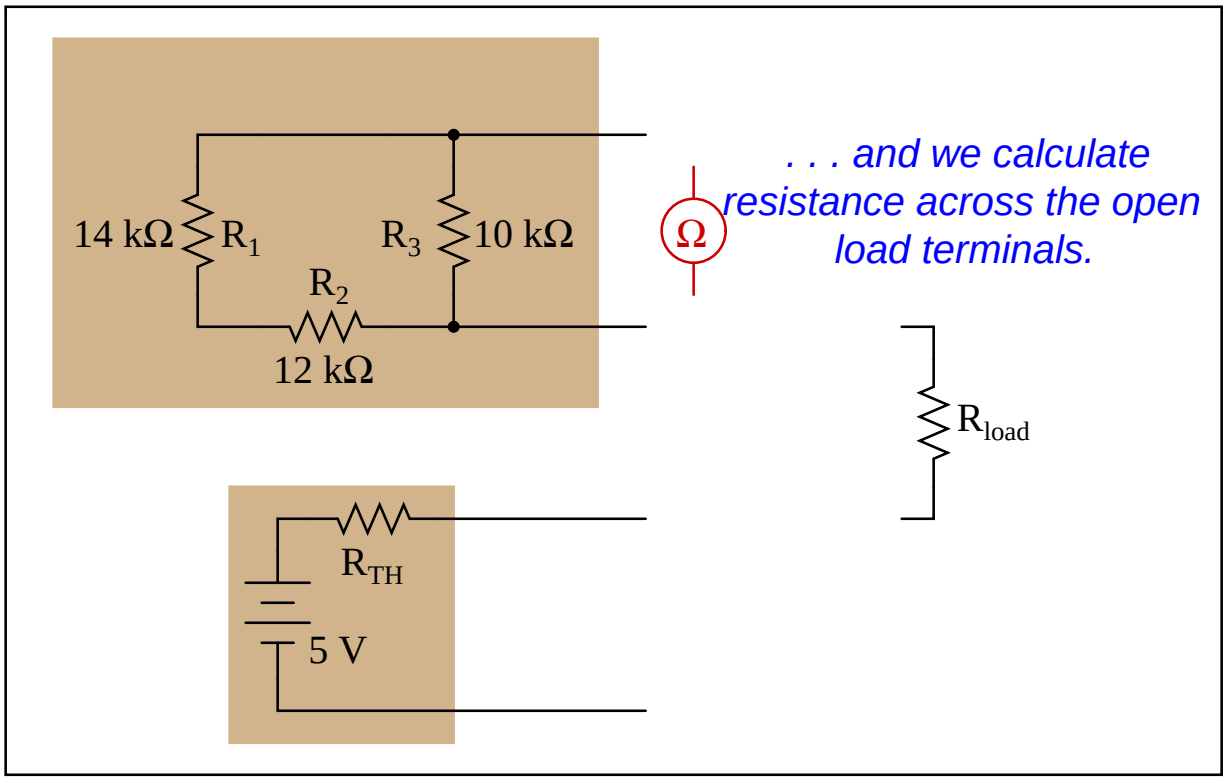

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now we replace each source in the original circuit with its own internal resistance.
For voltage sources, this means a short-circuit.
Rload
RTH
5 V

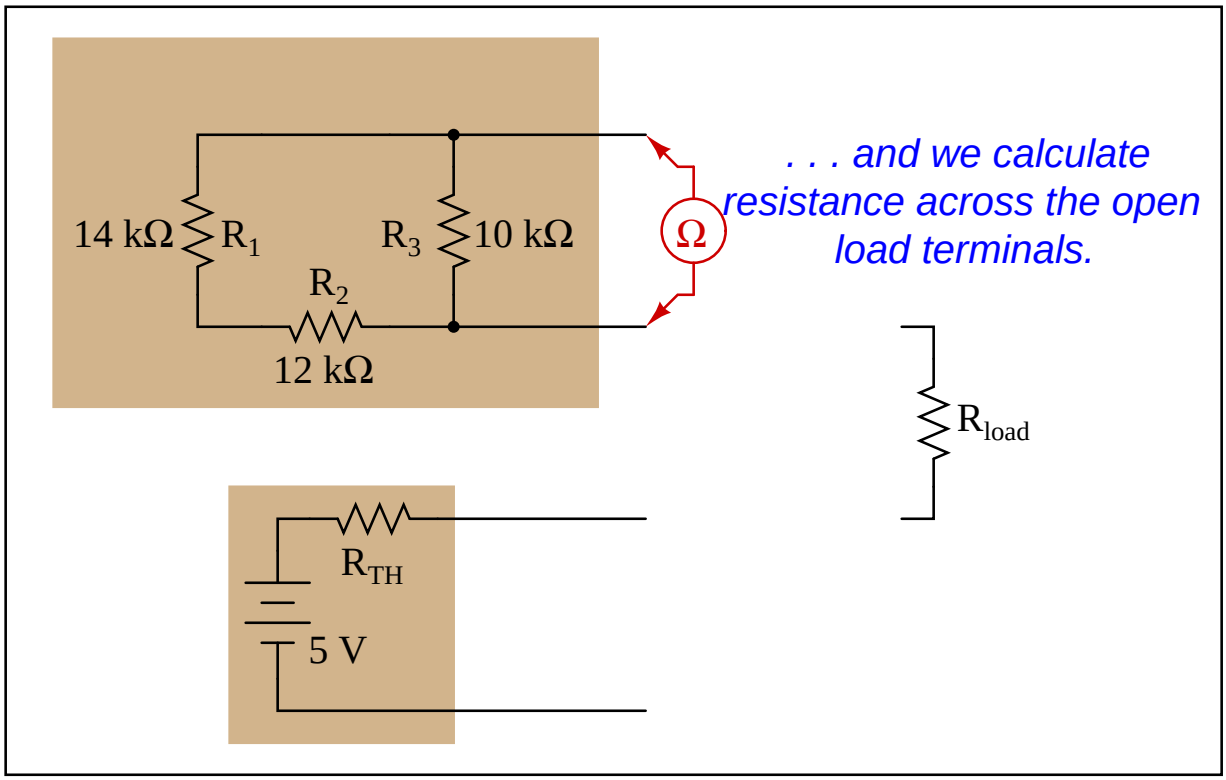

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now we replace each source in the original circuit with its own internal resistance.
For voltage sources, this means a short-circuit.
Rload
RTH
5 V

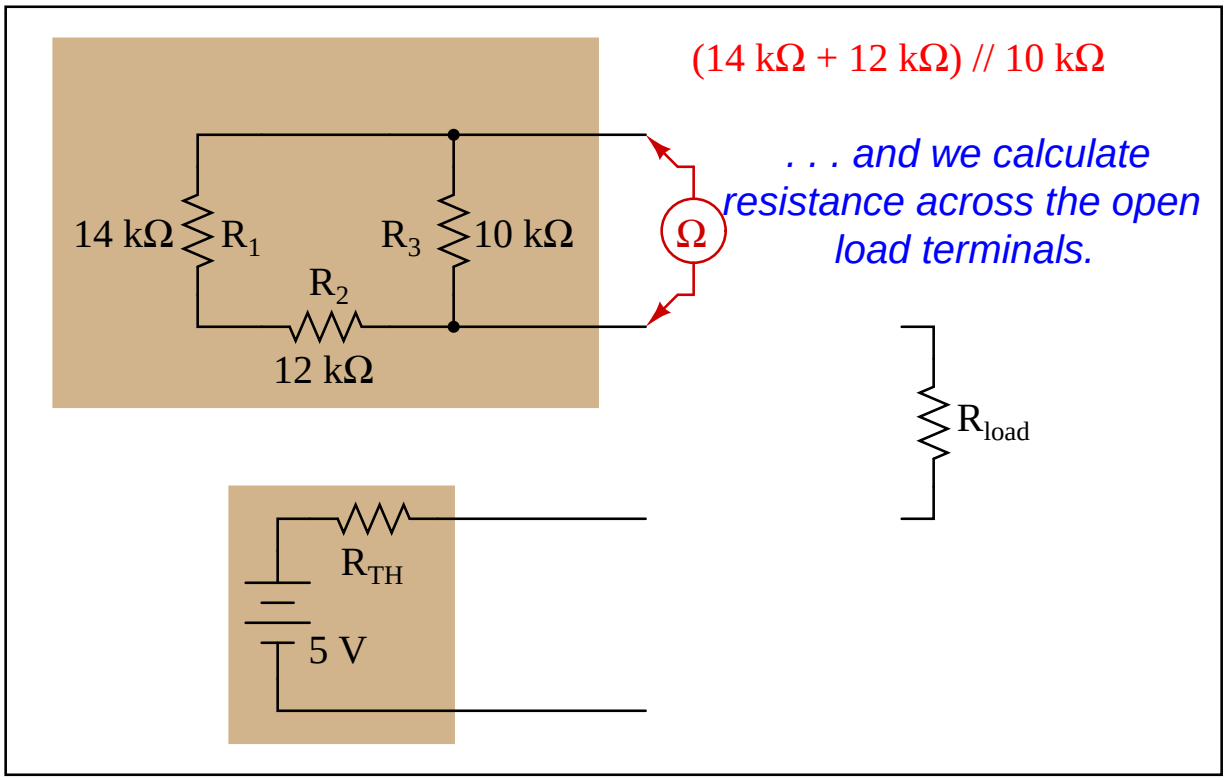

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now we replace each source in the original circuit with its own internal resistance.
For voltage sources, this means a short-circuit.
Rload
RTH
5 V

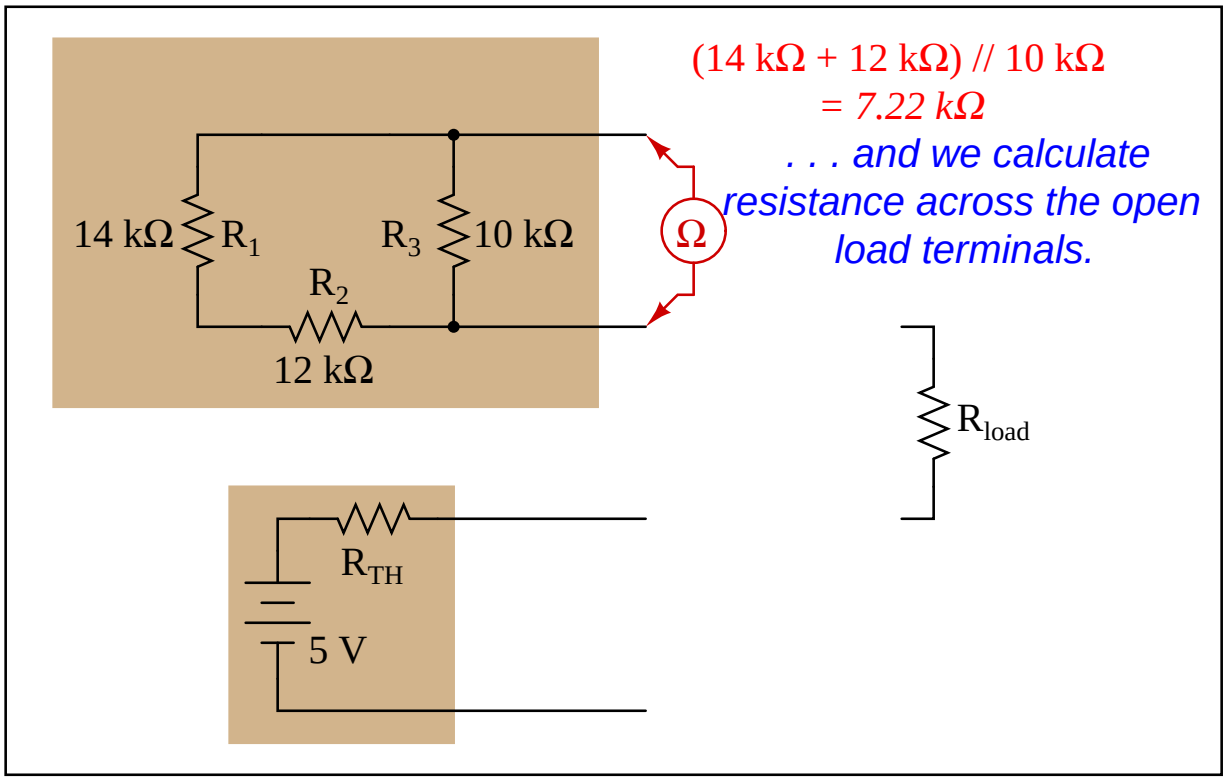

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
RTH
5 V

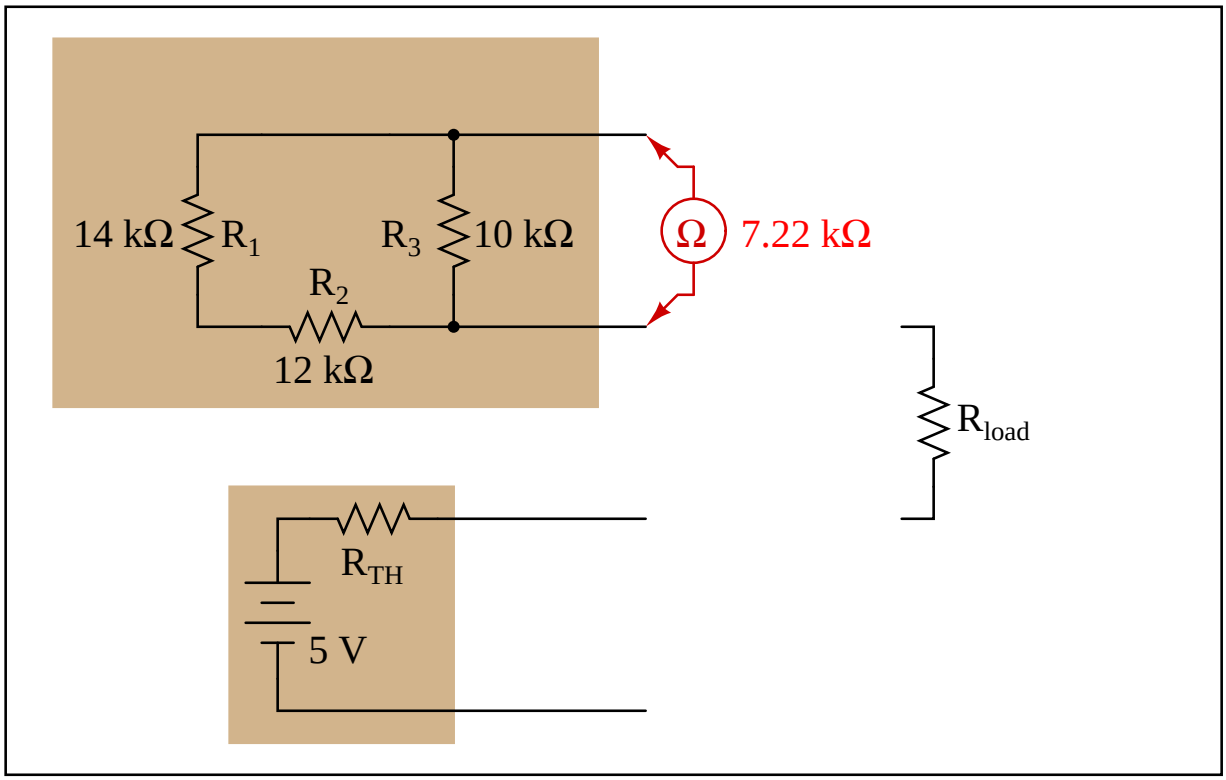

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
. . . and we calculate resistance across the open load terminals.
Rload
RTH
5 V

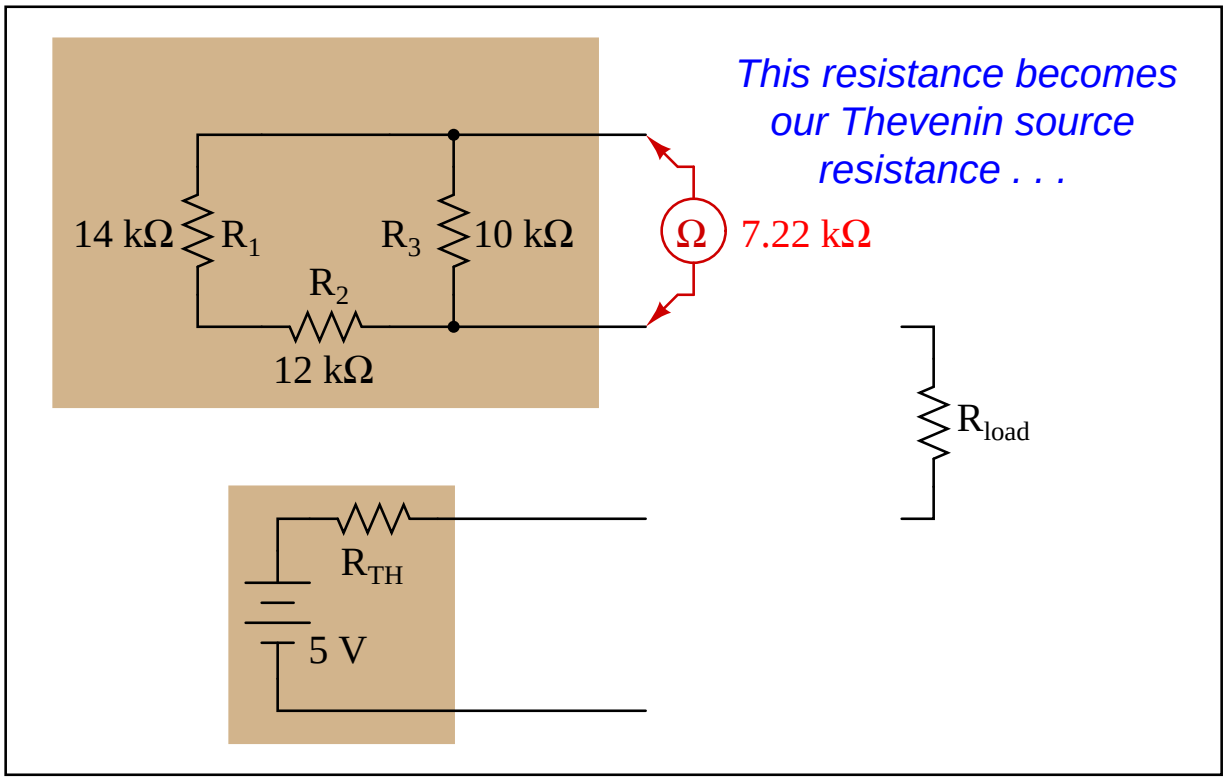

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
. . . and we calculate resistance across the open load terminals.
Rload
RTH
5 V

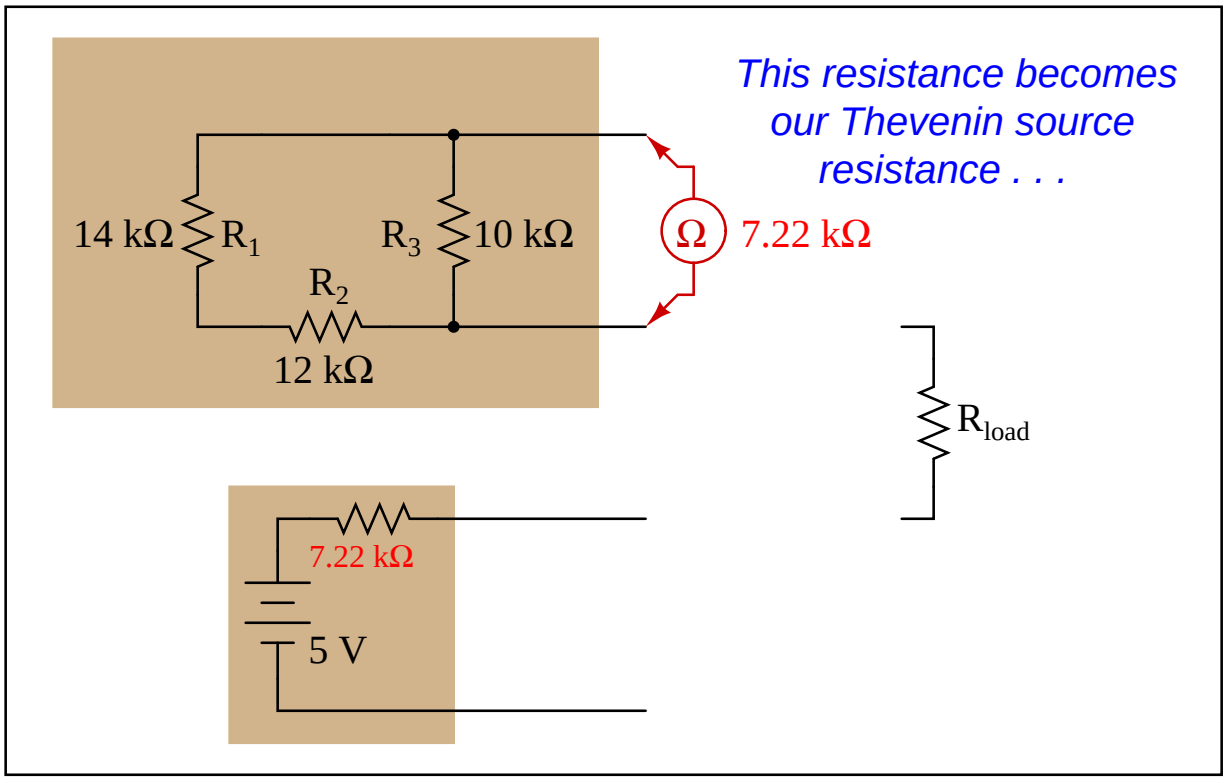

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
. . . and we calculate resistance across the open load terminals.
Rload
RTH
5 V

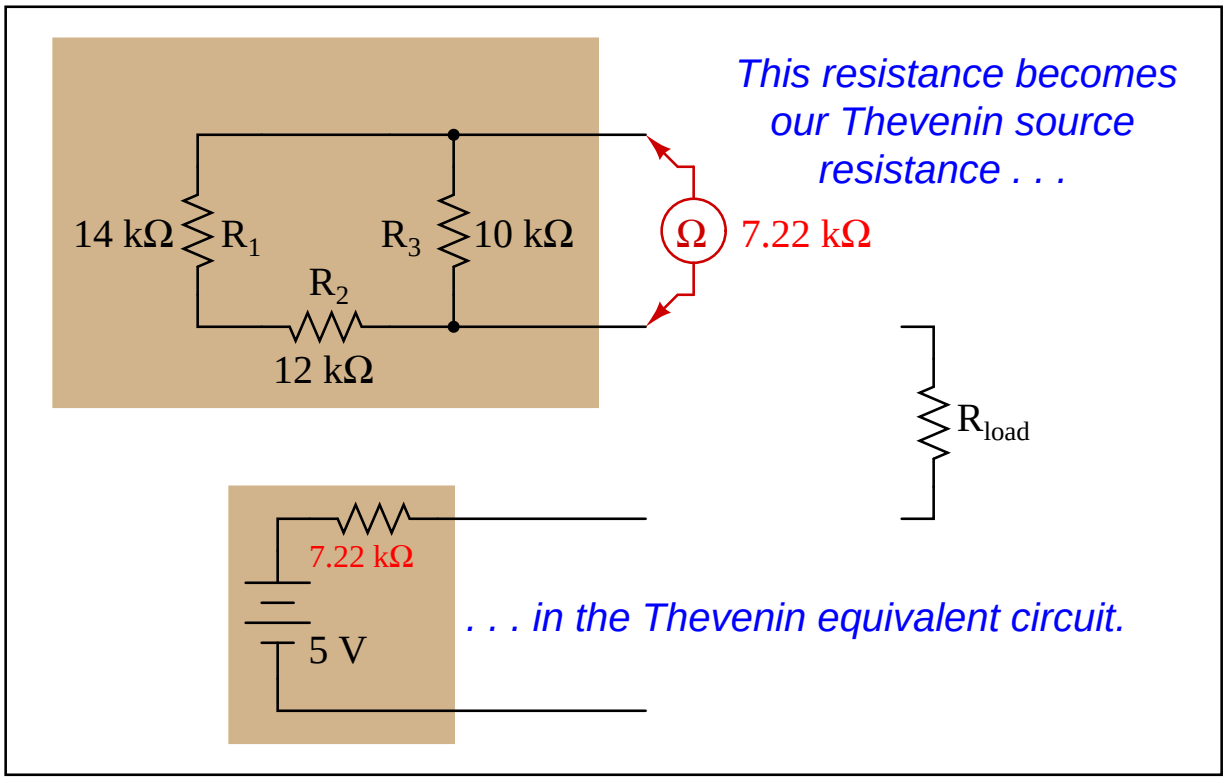

(14 kΩ + 12 kΩ) // 10 kΩ
. . . and we calculate resistance across the open load terminals.
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
Rload
RTH
5 V

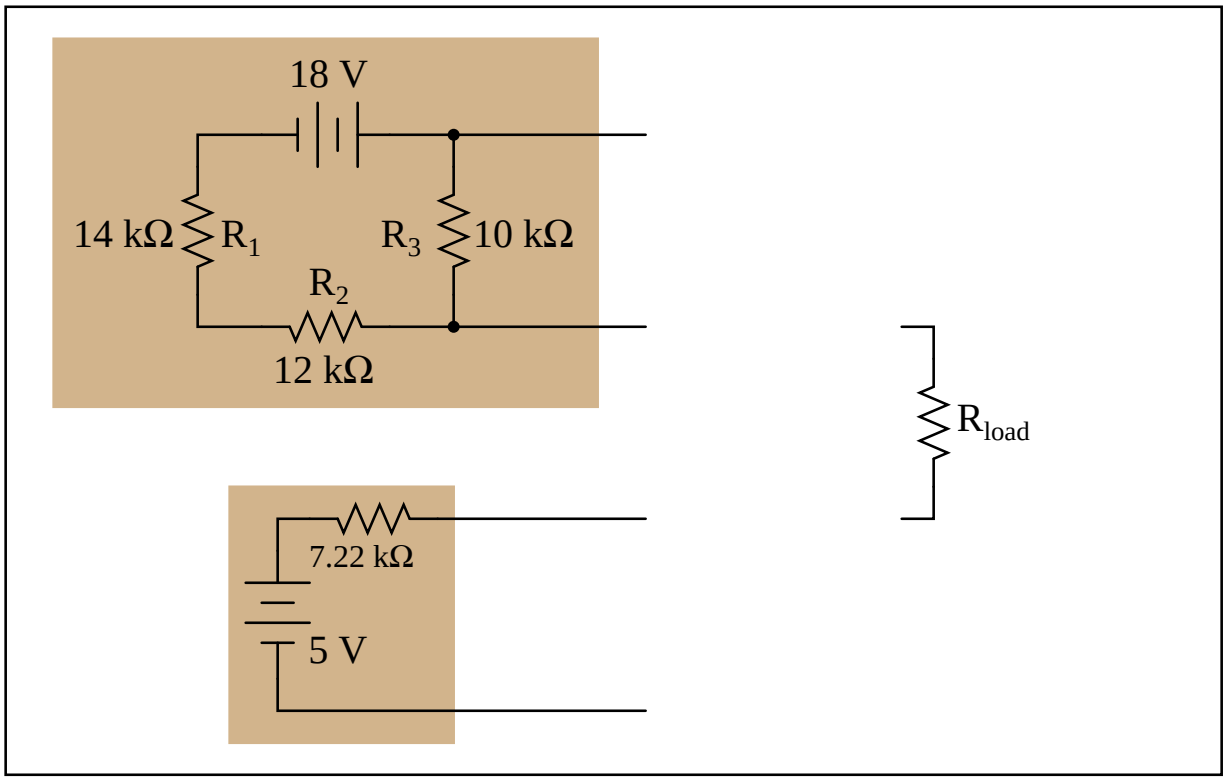

(14 kΩ + 12 kΩ) // 10 kΩ
= 7.22 kΩ
. . . and we calculate
resistance across the open
load terminals.
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
Rload
RTH
5 V

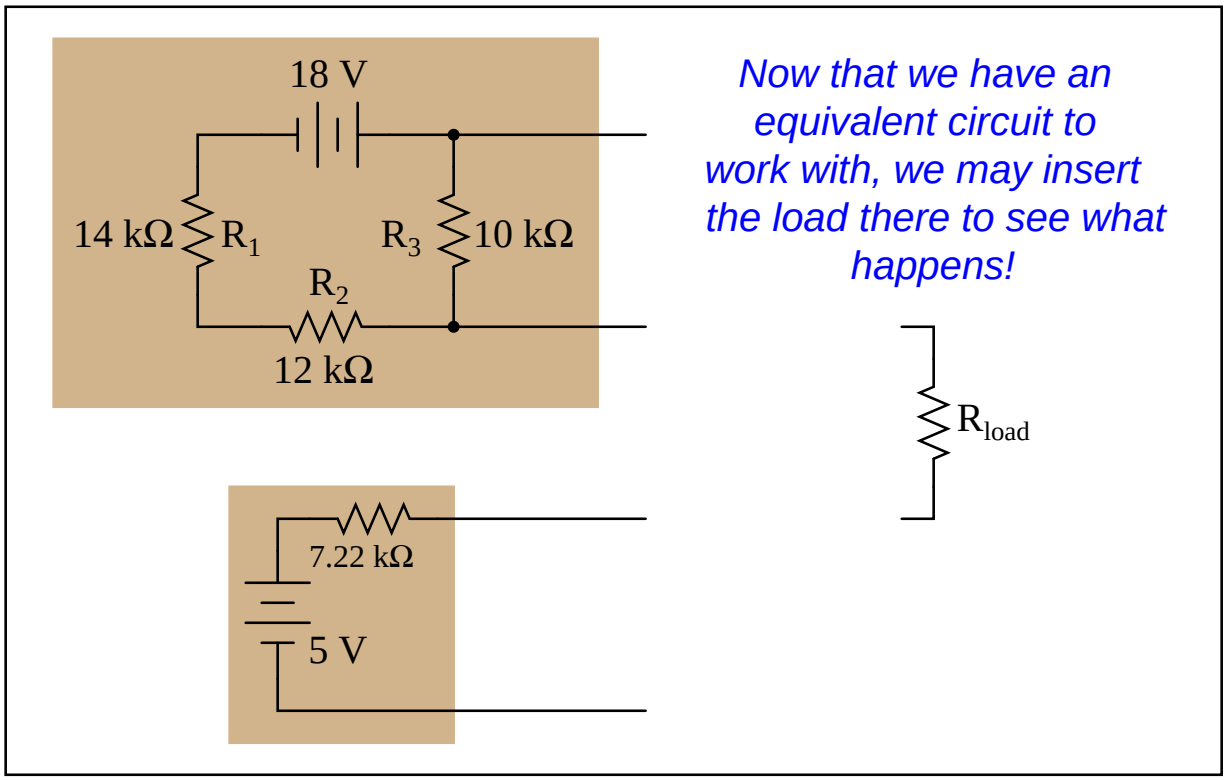

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
7.22 kΩ
Rload
RTH
5 V

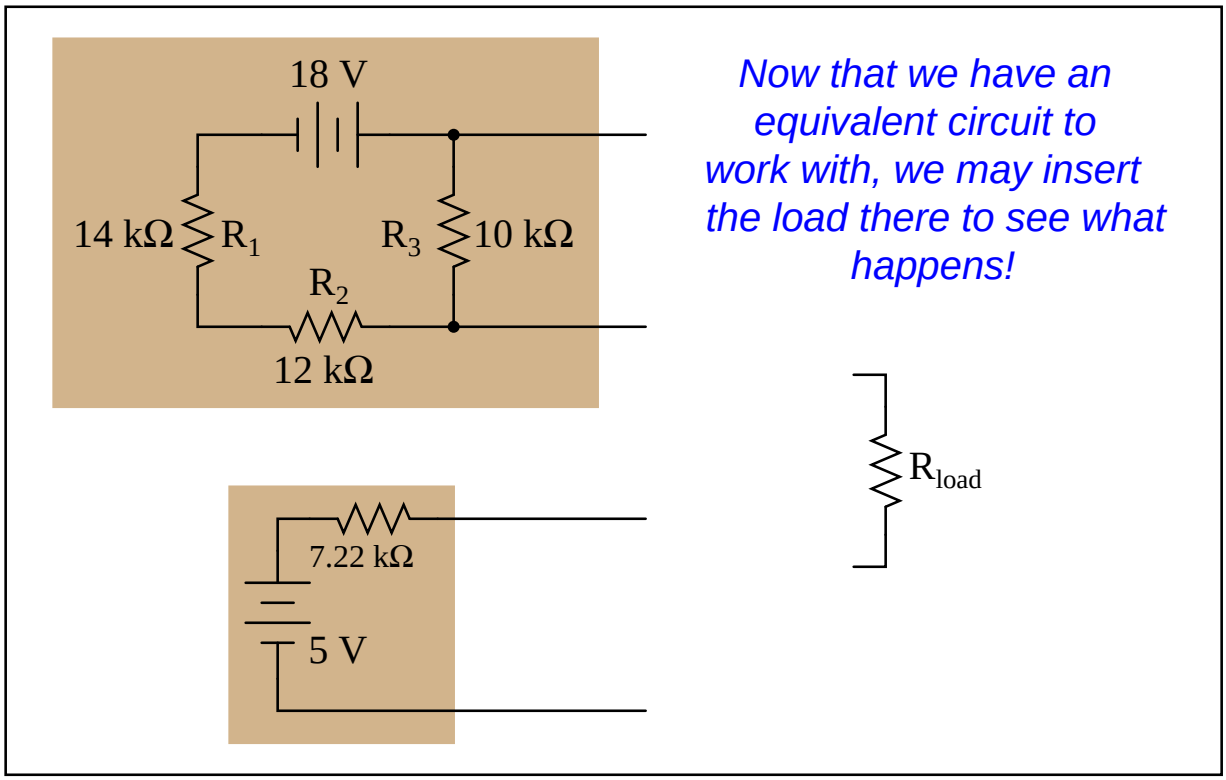

This resistance becomes our Thevenin source resistance . . .
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
7.22 kΩ
Rload
RTH
5 V

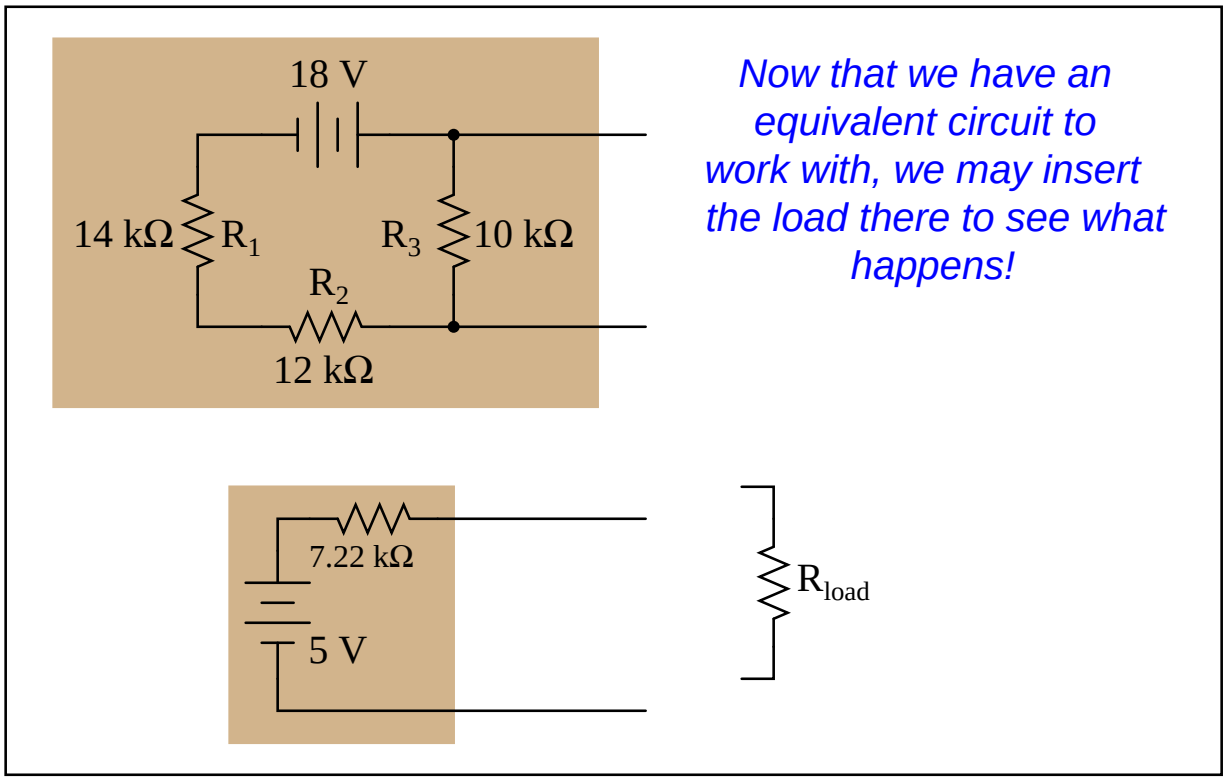

This resistance becomes our Thevenin source resistance . . .
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
7.22 kΩ
Rload
7.22 kΩ
5 V

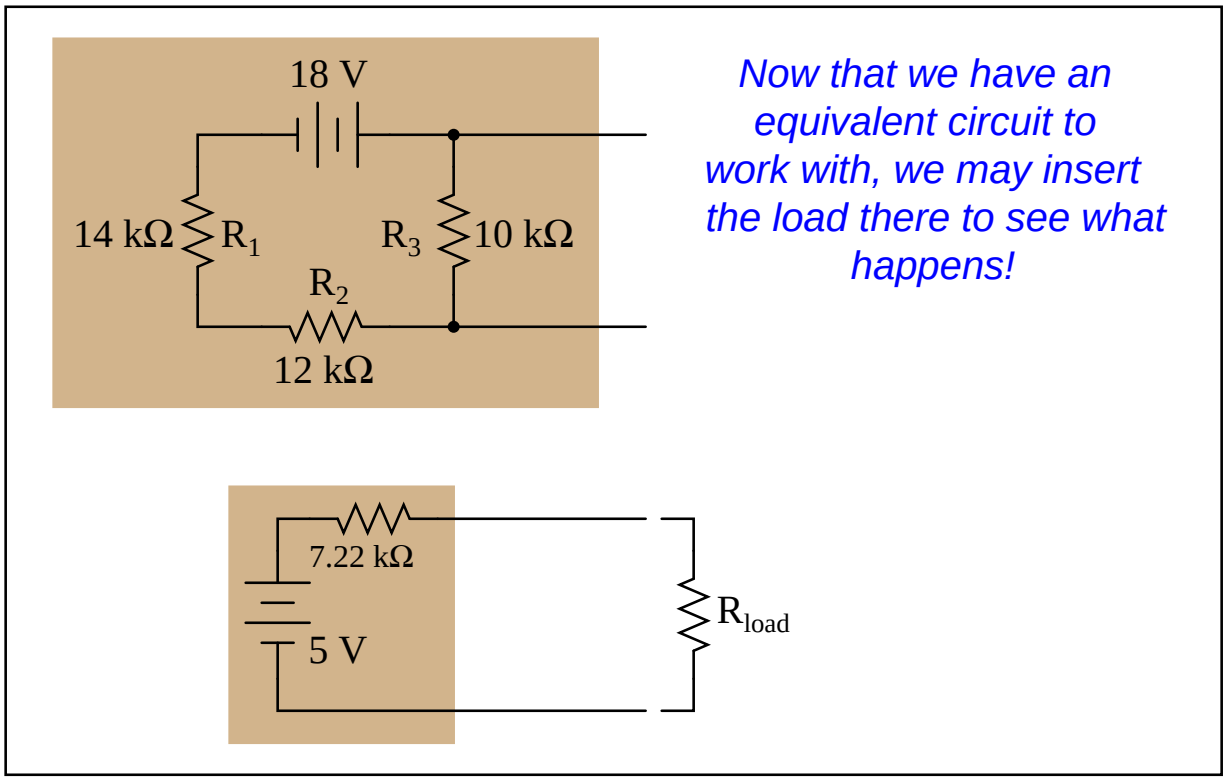

14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Ω
7.22 kΩ
This resistance becomes our Thevenin source resistance . . .
Rload
7.22 kΩ
5 V
. . . in the Thevenin equivalent circuit.

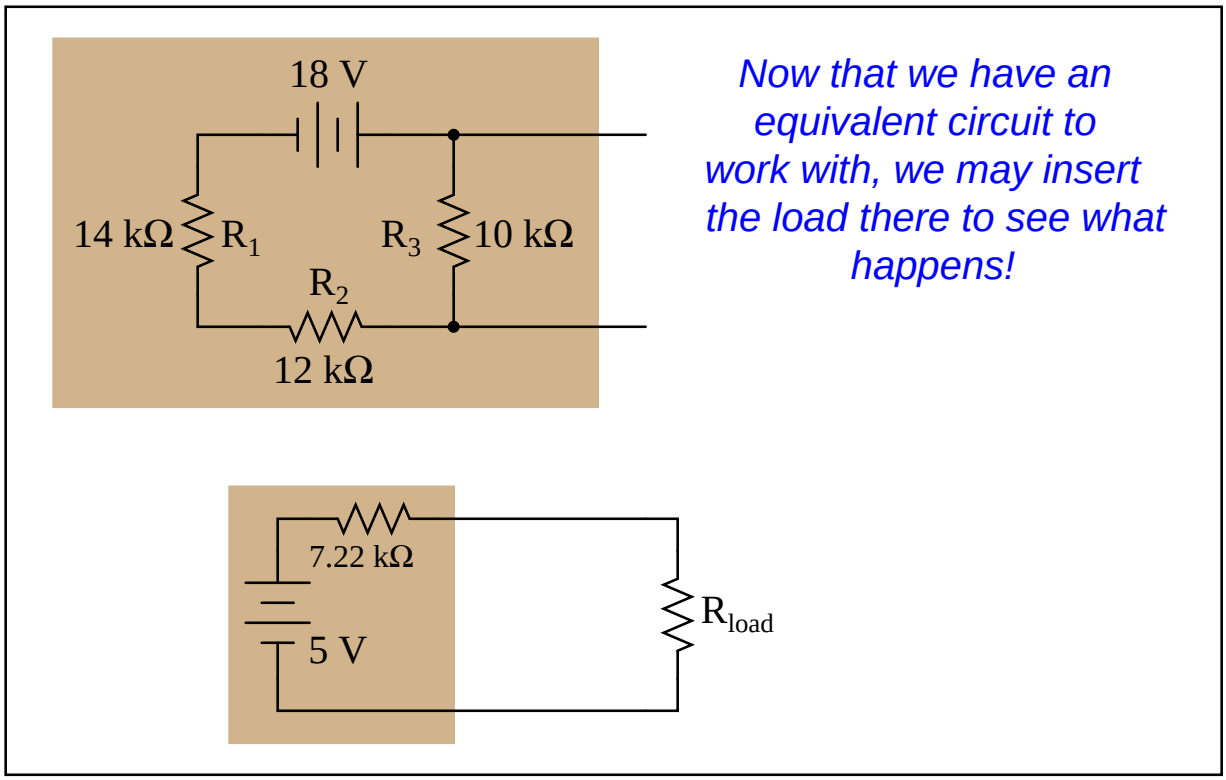

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
7.22 kΩ
5 V

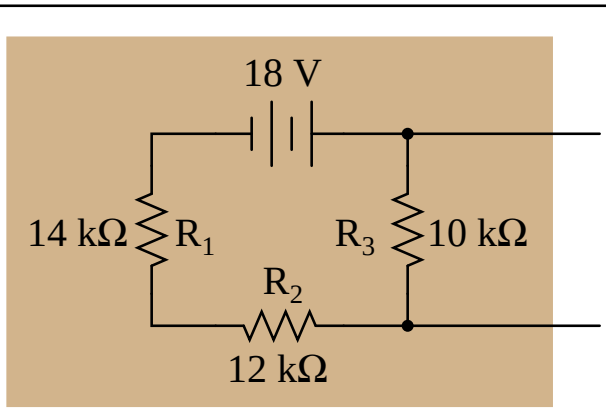

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
7.22 kΩ
5 V
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!
Rload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
7.22 kΩ
5 V
Rload
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!
7.22 kΩ
5 V
Rload

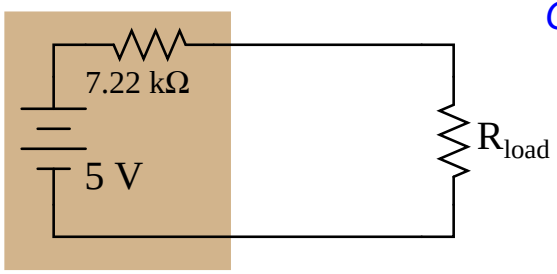

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!
7.22 kΩ
5 V
Rload

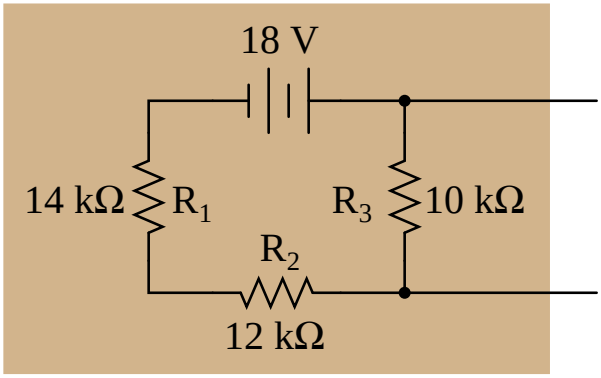

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!
7.22 kΩ
5 V
Rload



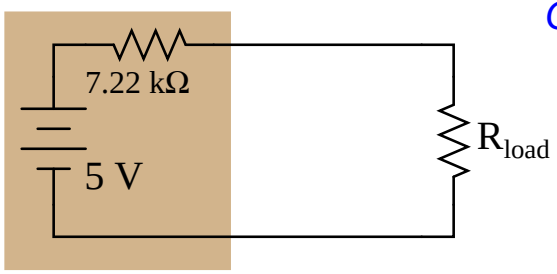
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!

Calculate:



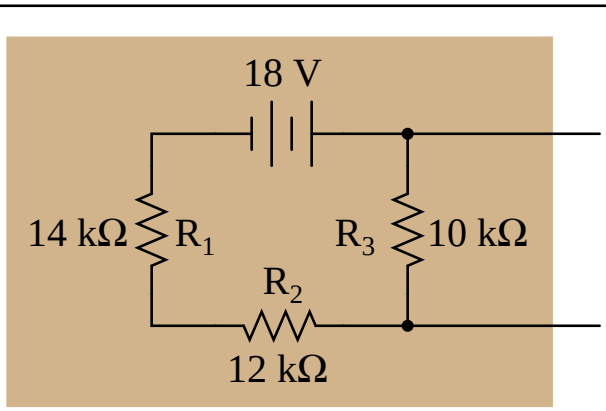


Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!

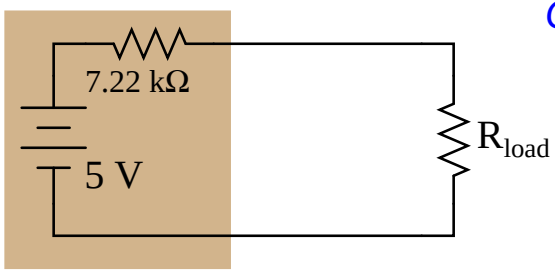


Calculate:

V_{load}



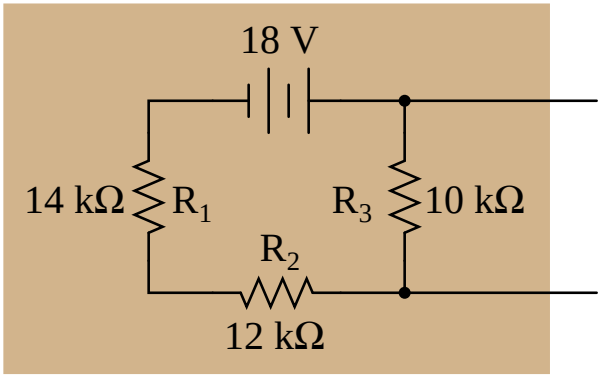
Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!



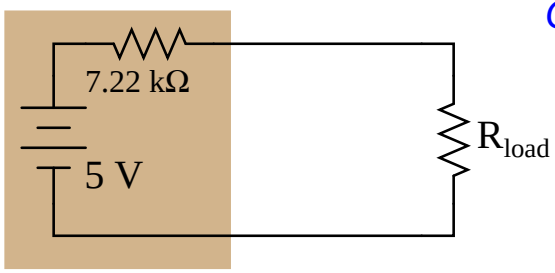
Calculate:

V_{load}

I_{load}



Now that we have an equivalent circuit to work with, we may insert the load there to see what happens!

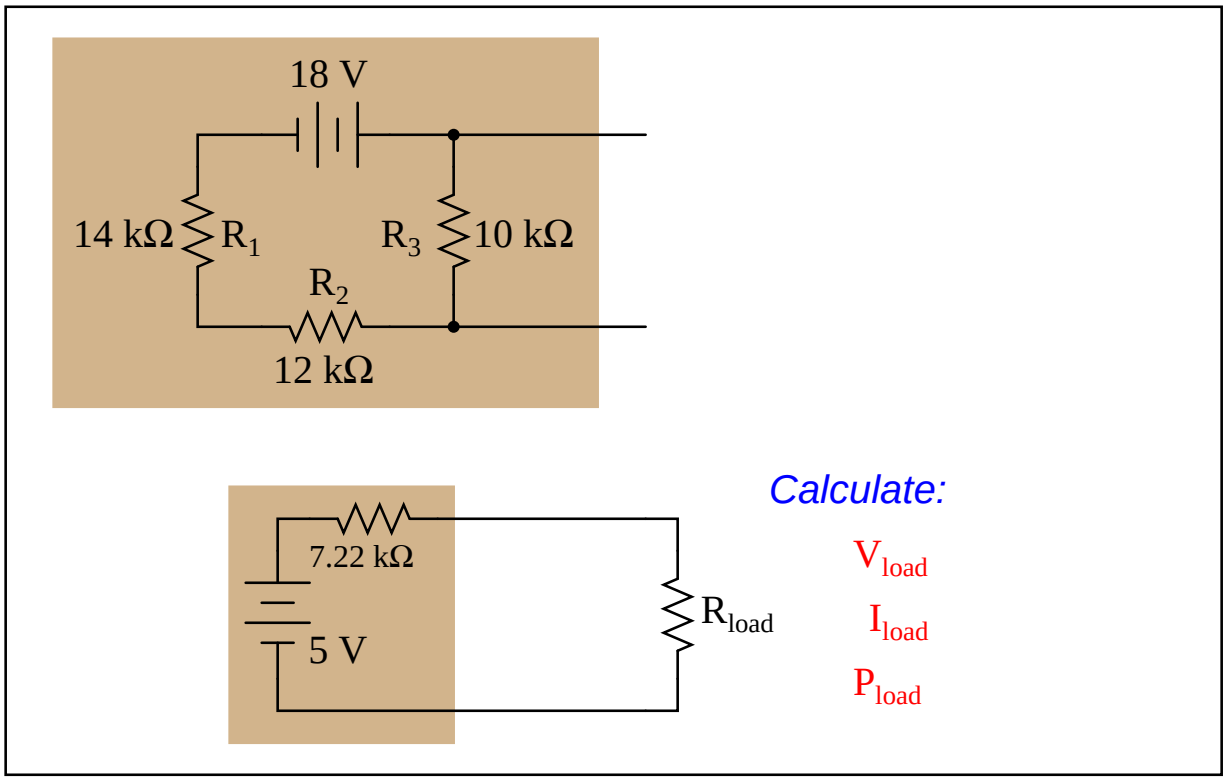


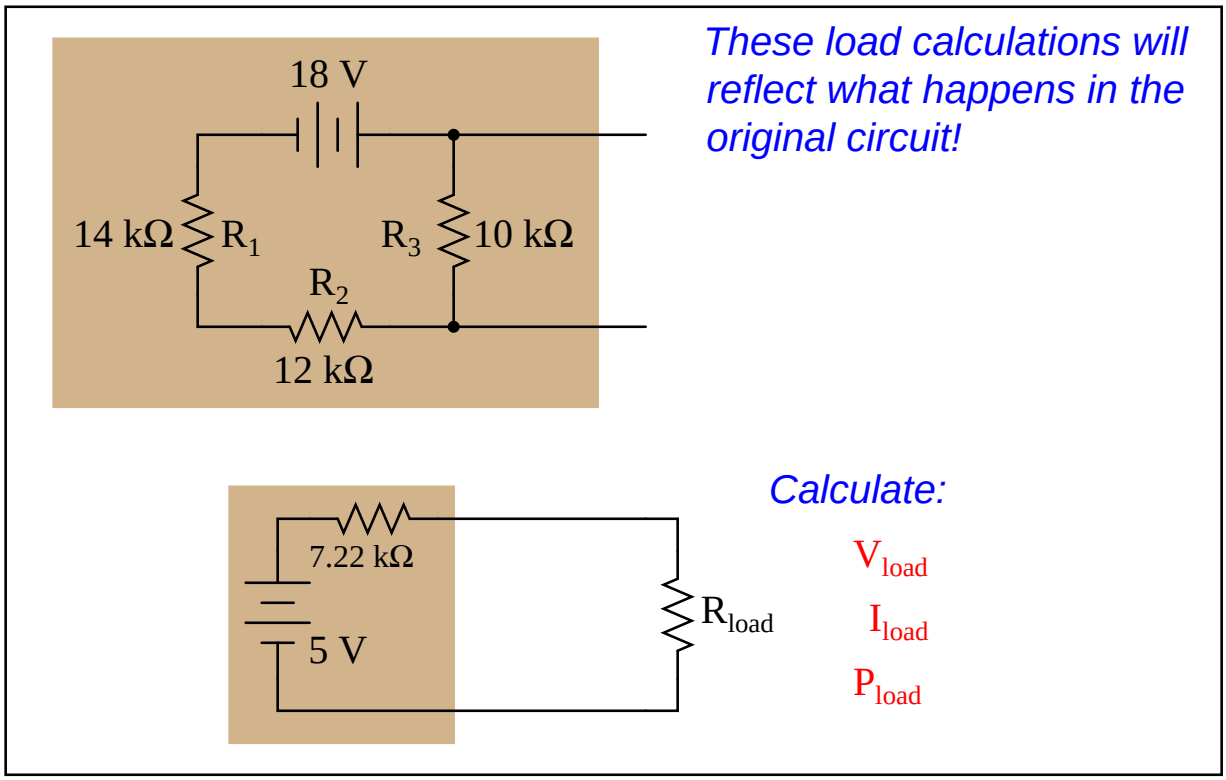
Calculate:

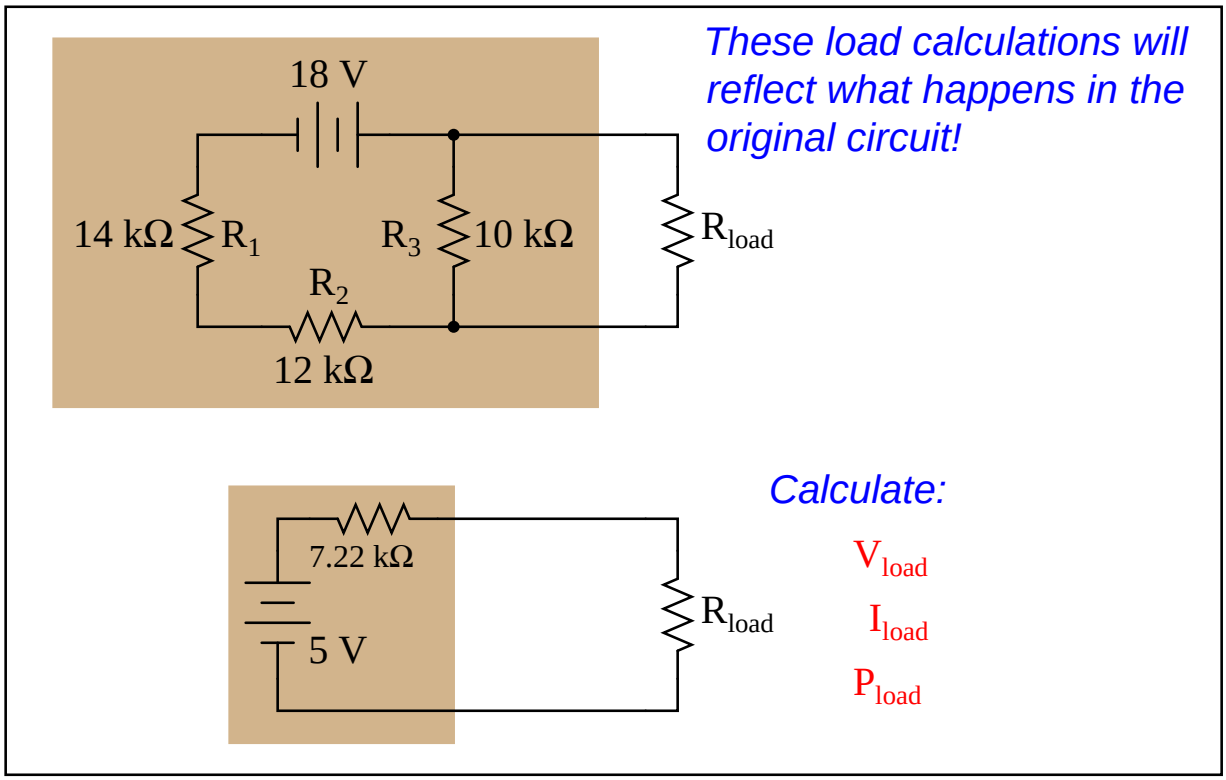
V_{load}

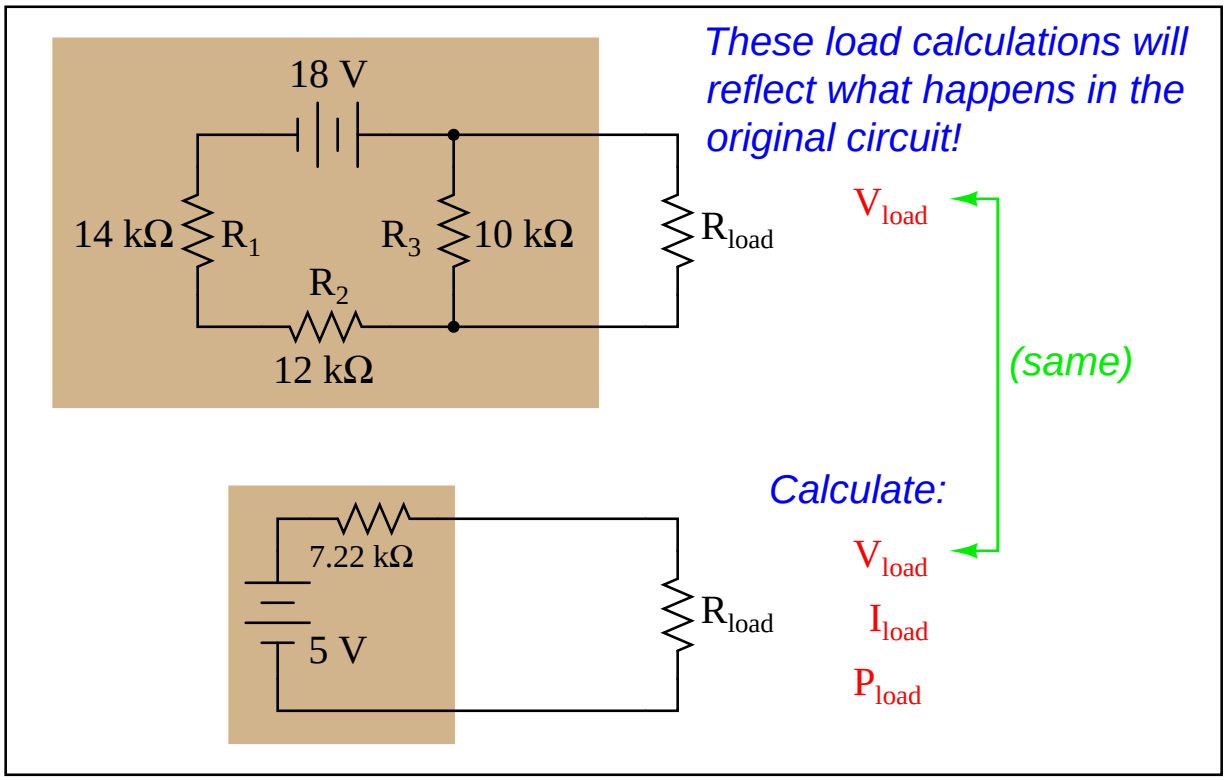
I_{load}

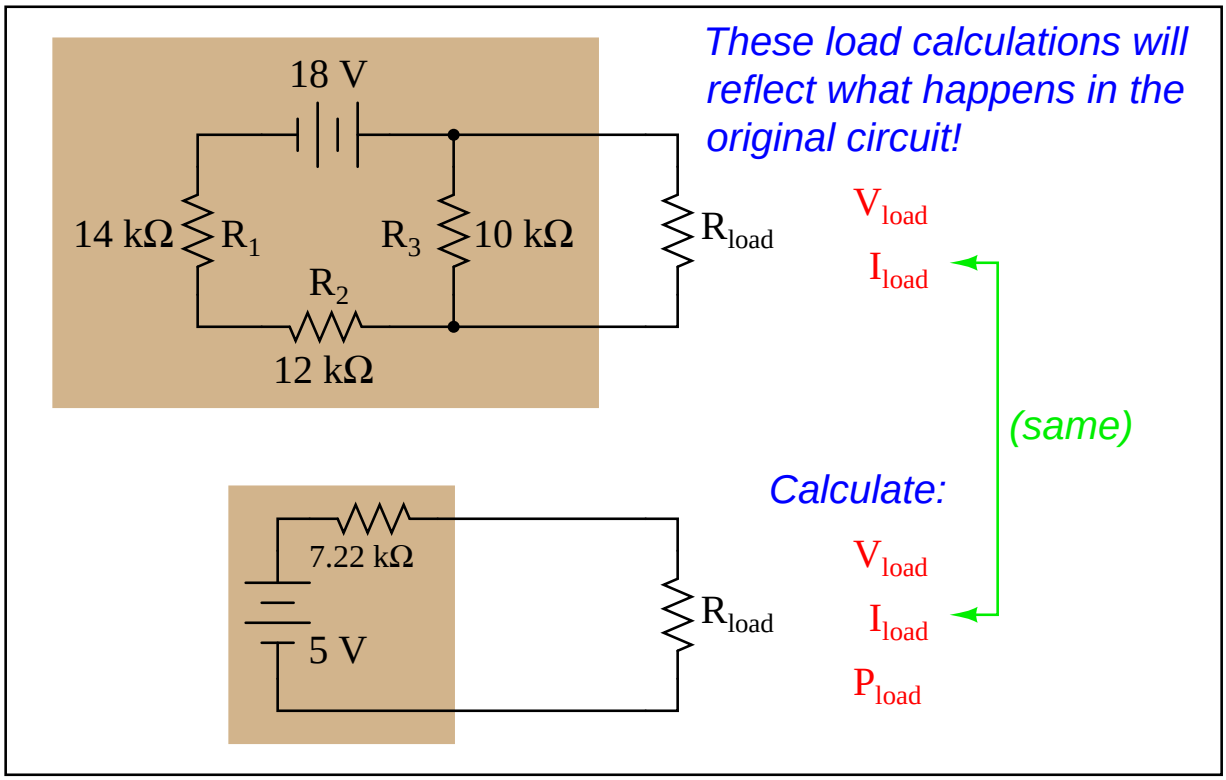
P_{load}

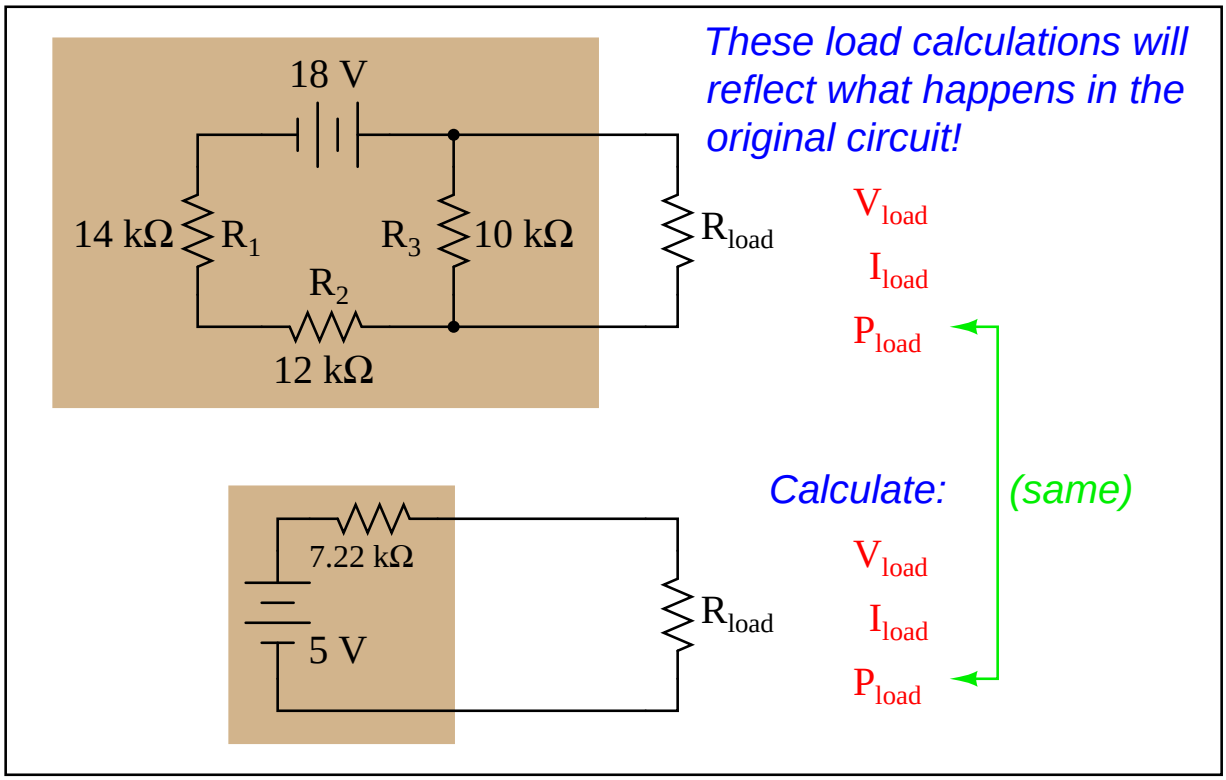

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
7.22 kΩ
5 V
Rload
Calculate:
Vload
Iload
Pload

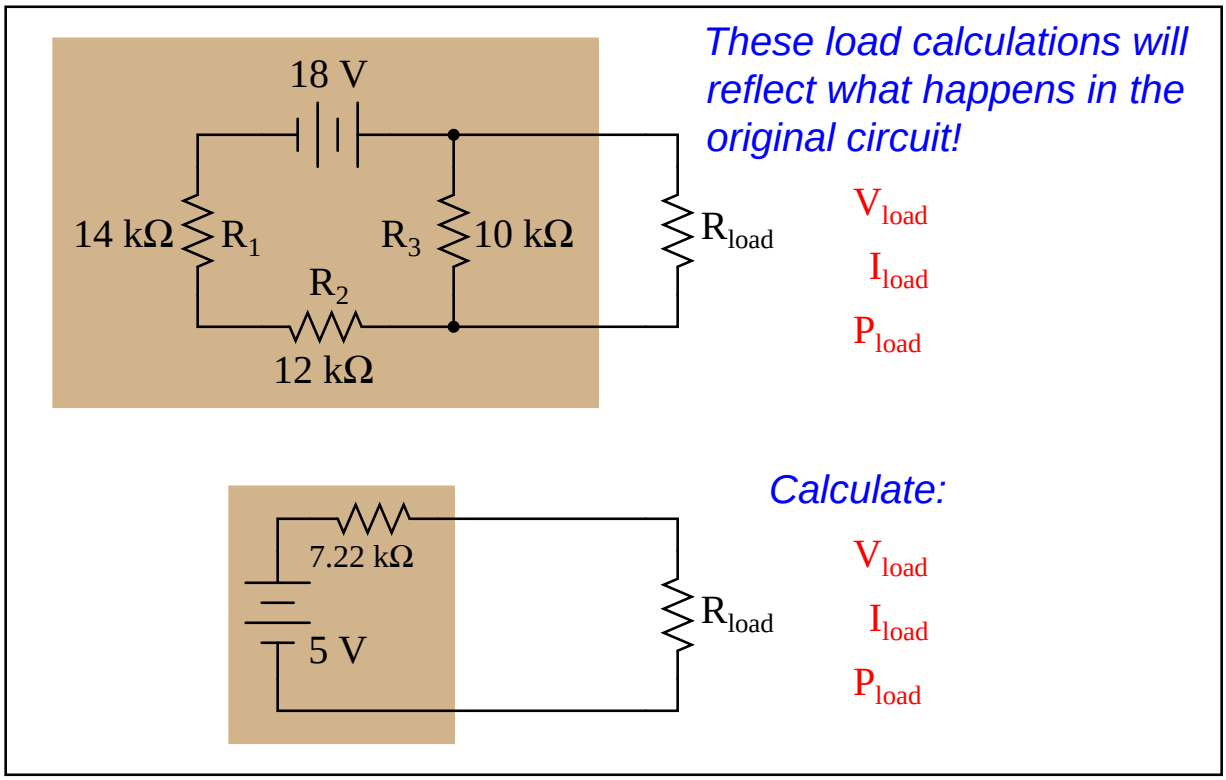

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
These load calculations will reflect what happens in the original circuit!
7.22 kΩ
5 V
Rload
Calculate:
Vload
Iload
Pload

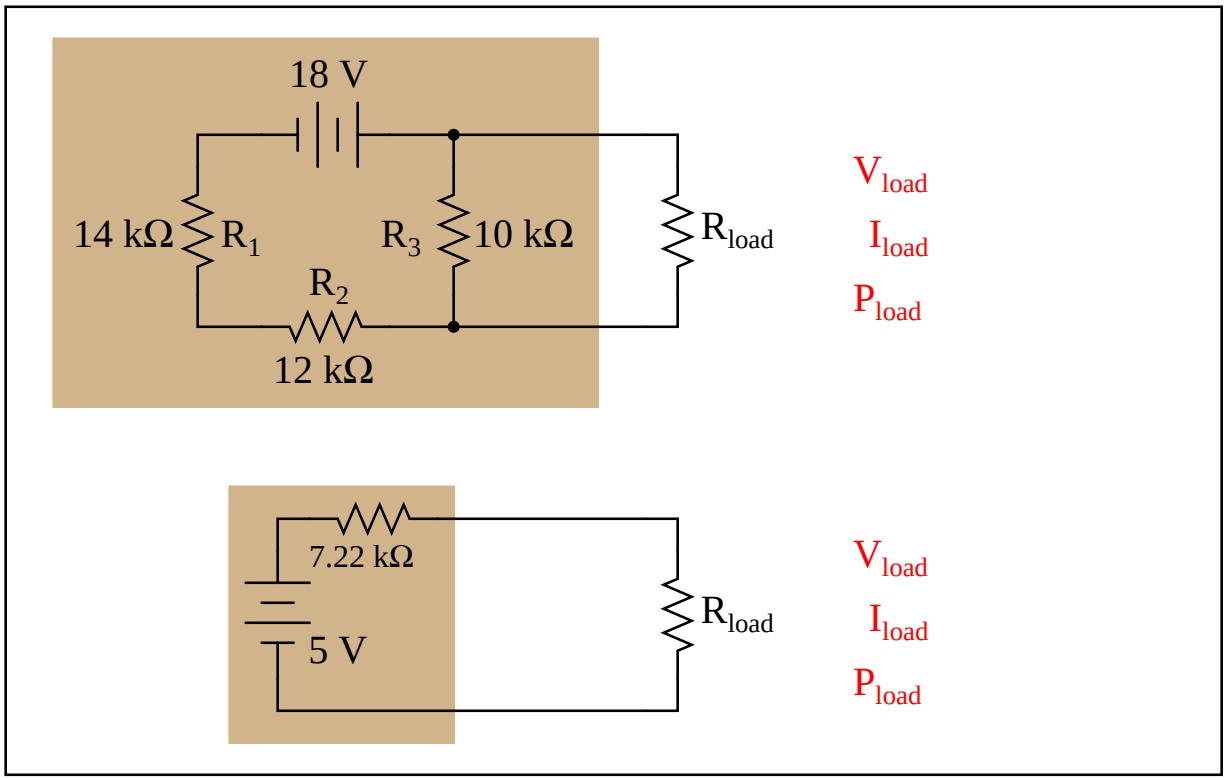

18 V
14 kΩ
R1
R3
10 kΩ
Rload
R2
12 kΩ
These load calculations will reflect what happens in the original circuit!
7.22 kΩ
5 V
Rload
Calculate:
Vload
Iload
Pload

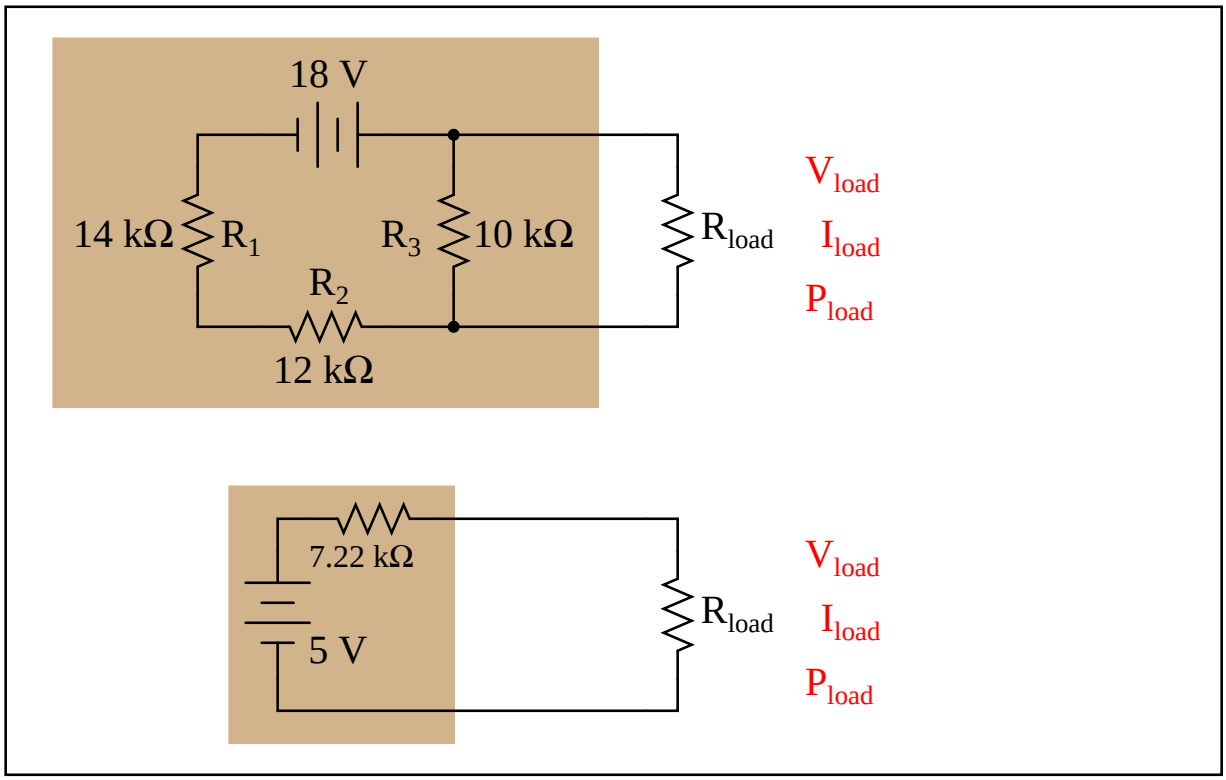

18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
These load calculations will reflect what happens in the original circuit!
Vload
(same)
Calculate:
Vload
Iload
Pload
7.22 kΩ
5 V
Rload

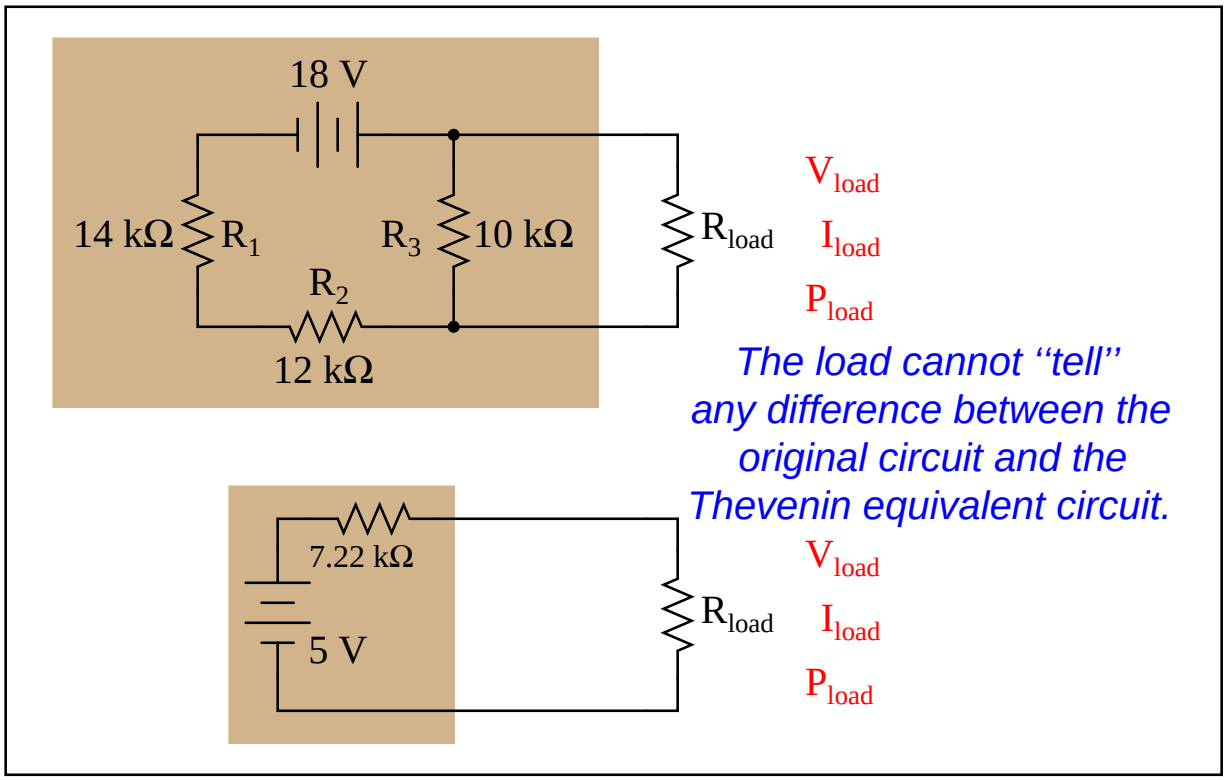

These load calculations will reflect what happens in the original circuit!
18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
Vload
Iload
(same)
Calculate:
7.22 kΩ
5 V
Rload
Vload
Iload
Pload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
These load calculations will reflect what happens in the original circuit!
Vload
Iload
Pload
Calculate:
(same)
7.22 kΩ
5 V
Rload
Vload
Iload
Pload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
These load calculations will reflect what happens in the original circuit!
Vload
Iload
Pload
7.22 kΩ
5 V
Rload
Calculate:
Vload
Iload
Pload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
Vload
Iload
Pload
7.22 kΩ
5 V
Rload
Vload
Iload
Pload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
Vload
Iload
Pload
7.22 kΩ
5 V
Rload
Vload
Iload
Pload


18 V
14 kΩ
R1
R3
10 kΩ
R2
12 kΩ
Rload
Vload
Iload
Pload
The load cannot "tell" any difference between the original circuit and the Thevenin equivalent circuit.
7.22 kΩ
5 V
Rload
Vload
Iload
Pload

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Thought experiments as a problem-solving strategy

Electrical source

Electrical load

Series connection

Equivalent networks

Current source

Voltage source

Open

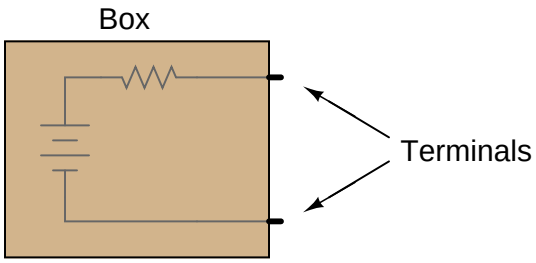
Short

Voltage divider

Re-drawing schematics as a problem-solving strategy

6.1.3 Testing a "black box" voltage source

Suppose you were handed a black box with two metal terminals on one side, for attaching electrical (wire) connections. Inside this box, you were told, was a voltage source (an ideal voltage source connected in series with a resistance):



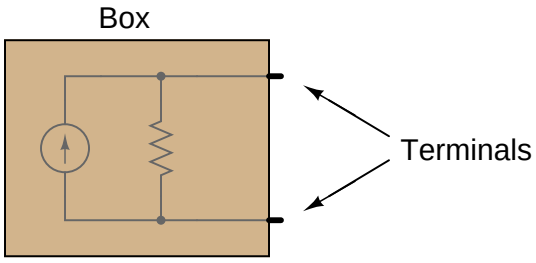
How would you empirically determine the voltage of the ideal voltage source inside this box, and how would you empirically determine the resistance of the series resistor? By "empirically" I mean determine voltage and resistance using actual test equipment rather than assuming certain component values (remember, this "black box" is sealed, so you cannot look inside!).

Challenges

- How does this empirical technique relate to the theoretical process of Thévenizing or Nortonizing a network?

6.1.4 Testing a "black box" current source

Suppose you were handed a black box with two metal terminals on one side, for attaching electrical (wire) connections. Inside this box, you were told, was a current source (an ideal current source connected in parallel with a resistance):



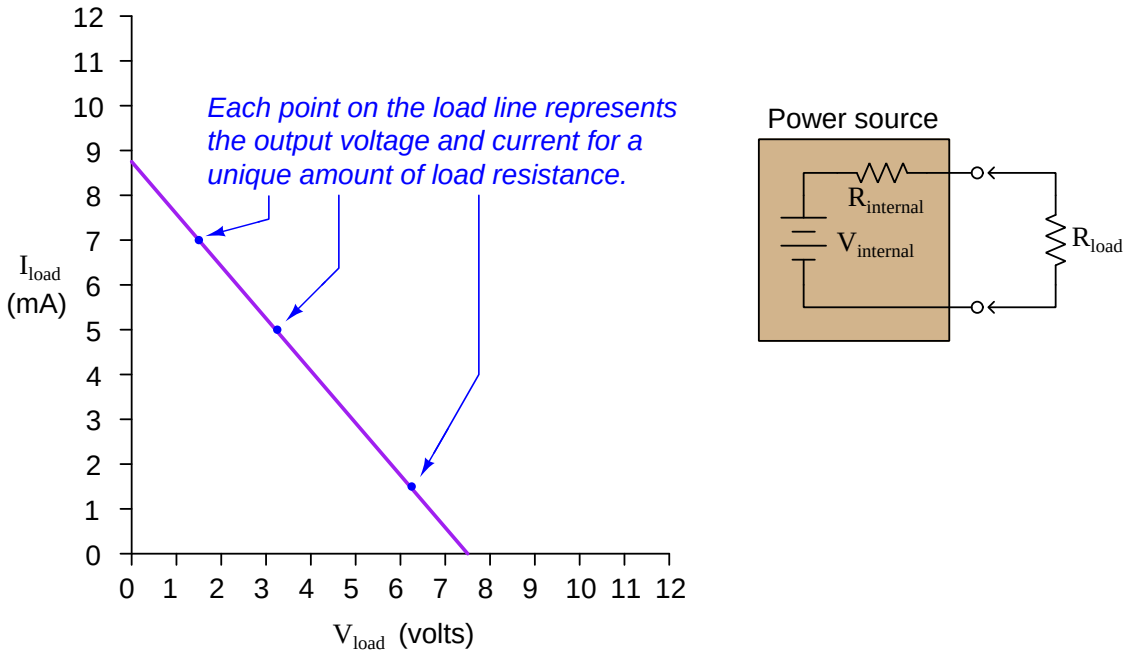
How would you empirically determine the current of the ideal current source inside this box, and how would you empirically determine the resistance of the parallel resistor? By "empirically" I mean determine current and resistance using actual test equipment rather than assuming certain component values (remember, this "black box" is sealed, so you cannot look inside!).

Challenges

- How does this empirical technique relate to the theoretical process of Thévenizing or Nortonizing a network?

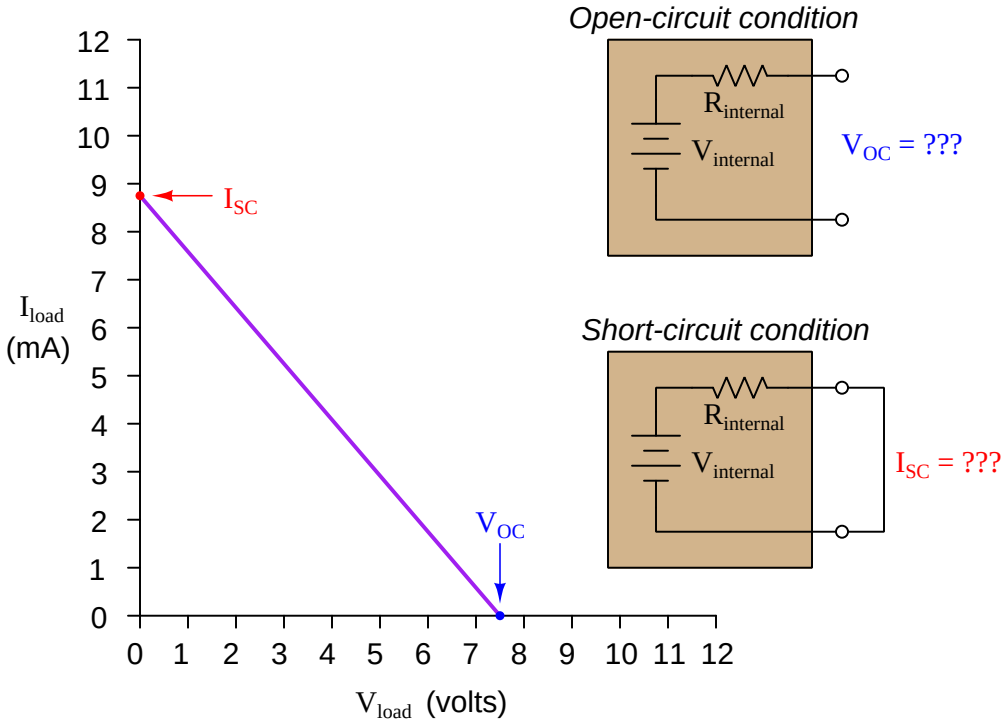
6.1.5 Load lines of equivalent sources

Load lines are special types of graphs used in electronics to characterize the output voltage and current behavior of different power sources:



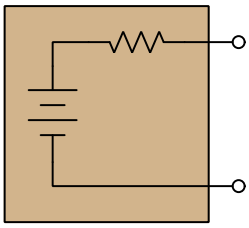
If we know that all the internal components of a power source are inherently *linear*, we know that the load line plot will indeed by a straight line. And, if we know the plot will be a straight line, all we need in order to plot a complete load line are *two* data points.

Usually, the easiest data points to gather for a circuit – whether it be a real circuit or an hypothetical circuit existing on paper only – is the *open-circuit* condition and the *short-circuit* condition. In other words, we see how much voltage the source will output with no load connected (I_{load} = 0 milliAmperes) and then we see how much current the source will output into a direct short (V_{load} = 0 Volts):



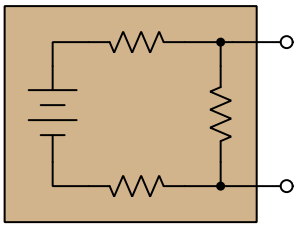
Suppose we have two differently-constructed power sources, yet both of these sources share the same open-circuit voltage (V_{OC}) and the same short-circuit current (I_{SC}). Assuming the internal components of both power sources are linear in nature, explain how we would know without doubt that the two power sources were electrically equivalent to one another. In other words, explain how we would know just from the limited data of V_{OC} and I_{SC} that these two power sources will behave exactly the same when connected to the same load resistance, whatever that load resistance may be.

How do we know these two power sources are completely equivalent to one another just from their equal open-circuit voltage and short-circuit current figures?



V_{OC} = 7.5 volts

I_{SC} = 8.75 mA



V_{OC} = 7.5 volts

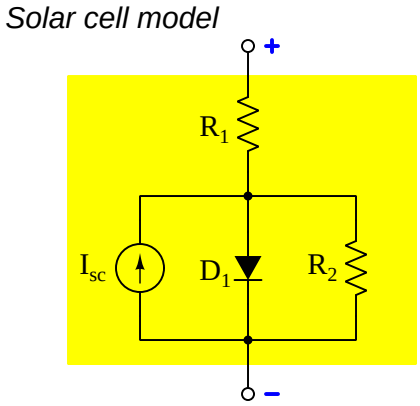
I_{SC} = 8.75 mA

Challenges

- A line may be mathematically defined with just two points of data. Explain why this is fact is significant to this problem.

6.1.6 Thévenizing a solar cell

A photovoltaic *solar cell* is a semiconductor component designed to act as an electrical source when struck by sunlight. Its behavior is approximated by the lumped-component model shown below:



The current output by source I_{SC} depends on the geometry of the solar cell and the amount of light striking it. The resistance values also depend on the geometry and chemical composition of the cell.

One of these modeling components – the *diode* labeled D_1 – has a non-linear characteristic. In other words, it does not exhibit a stable resistance value like the other modeling components. Although its real behavior is best described by the exponential *Shockley diode equation*, for the sake of our analysis here we may think of the diode as being either *on* or *off* depending on the voltage across its terminals: if the voltage is less than 0.7 Volts, this diode will be "off" (non-conducting); an attempt to raise its voltage above 0.7 Volts results in it turning "on" and maintaining a constant (regulated) voltage drop of 0.7 Volts as it conducts current. In other words, below 0.7 Volts it behaves as an open and at all other times it behaves as a constant 0.7 Volt device (e.g. a 0.7 Volt source).

Develop one Thévenin equivalent network for this solar cell assuming a condition where $V_{D1} <$ Volts, and another Thévenin equivalent network assuming $V_{D1} = 0.7$ Volts.

Challenges

- ???

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

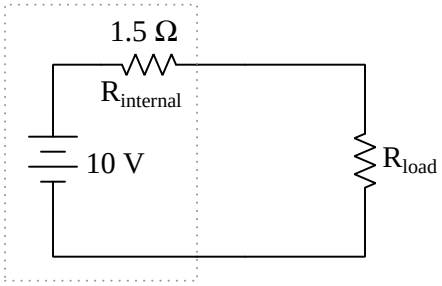
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

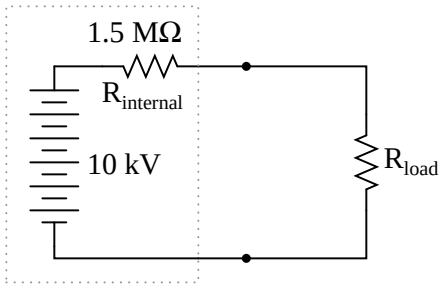
6.2.3 Testing sources under load

Calculate the voltage dropped across the load resistor, and the current through the load resistor, for the load resistance values of 1 kΩ, 2 kΩ, 5 kΩ, 8 kΩ, and 10 kΩ:



Do the "boxed" components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Now, do the same for this circuit:



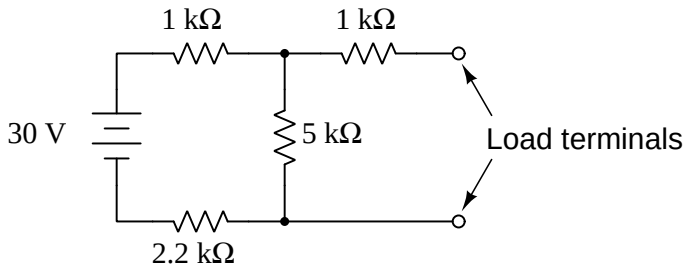
Do the "boxed" components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Challenges

- Would you say that voltage sources are typically characterized as having high internal resistances or low internal resistances? What about current sources? Explain your answers.

6.2.4 Thévenin and Norton equivalents of a single-source network

Reduce the following circuit to a Norton equivalent, and also to a Thévenin equivalent:

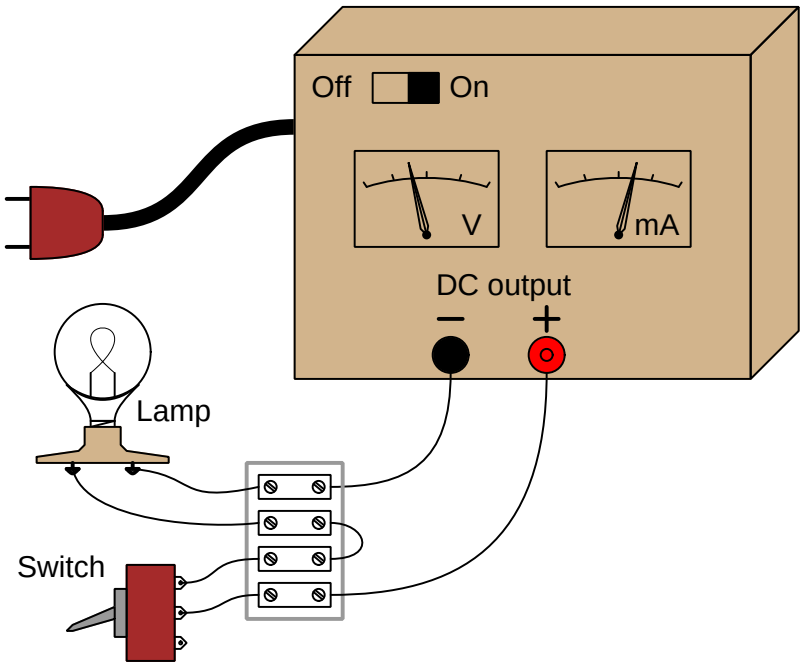


Challenges

- Explain why Thévenization and Nortonization are useful tools in circuit analysis.

6.2.5 Thévenin equivalent of an AC/DC power supply

Suppose you had an AC/DC power supply, which performed as follows (open-circuit and loaded test conditions):



Switch off:

V_{out} = 14.3 Volts DC

I_{out} = 0 mA DC

Switch on:

V_{out} = 12.8 Volts DC

I_{out} = 845 mA DC

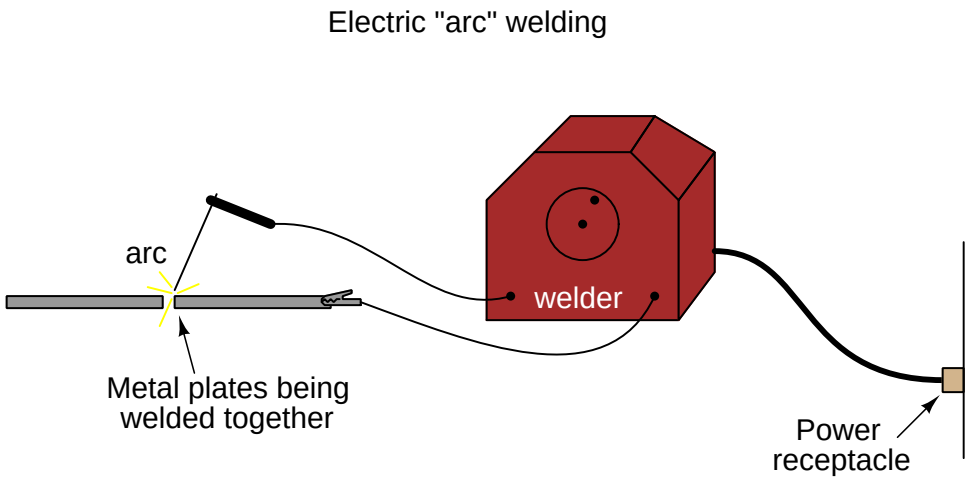
Draw a Thévenin equivalent network to model the behavior of this power supply.

Challenges

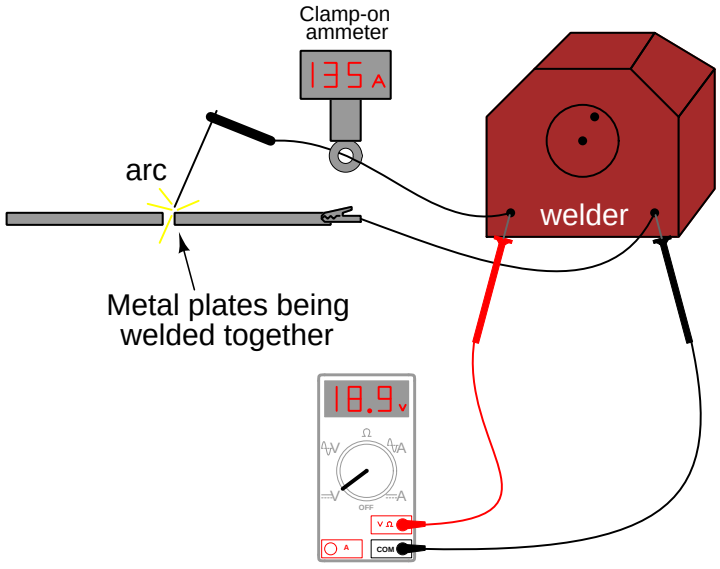
- Identify any assumptions we need to make about this power supply in order to apply Thévenin's Theorem to these measured values.

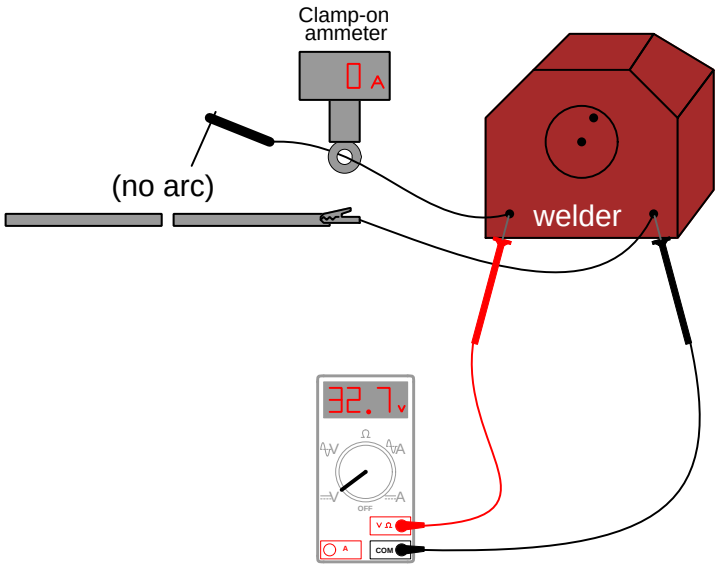
6.2.6 Norton equivalent of an electric arc welder

An *electric arc welder* is a low-voltage, high-current power source designed to supply enough electric current to sustain an arc capable of welding metal with its high temperature:



It is possible to derive a Norton equivalent network for an arc welder based on empirical measurements of voltage and current. Take for example these measurements, under *loaded* and *no-load* conditions:





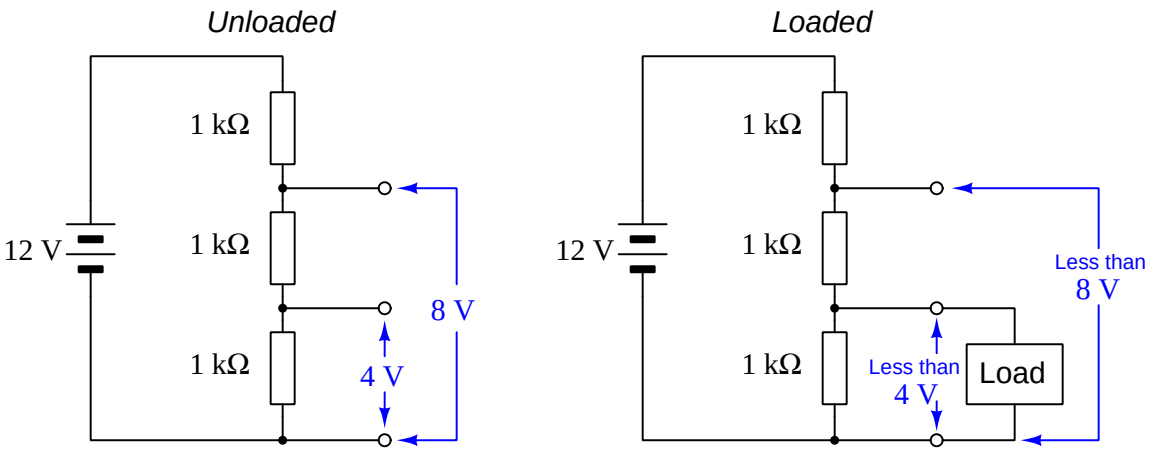
Based on these measurements, derive a Norton equivalent network for the arc welder.

Challenges

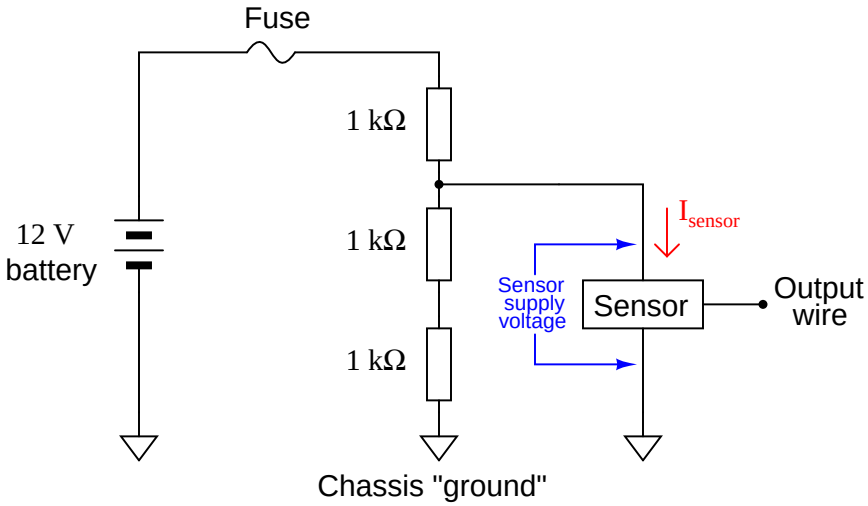
- Identify any assumptions we need to make about this arc welder in order to apply Thévenin's Theorem to these measured values.

6.2.7 Thévenizing a loaded voltage divider

Resistive voltage dividers are very useful and popular circuits. However, it should be realized that their output voltages "sag" under load:



Just how much a voltage divider's output will sag under a given load may be a very important question in some applications. Take for instance the following application where we are using a resistive voltage divider to supply an engine sensor with reduced voltage (8 Volts) from the 12 Volt battery potential in the automobile:



If the sensor draws no current (I_{sensor} = 0 mA), then the voltage across the sensor supply terminals will be 8 Volts. However, if we were asked to predict the voltage across the sensor supply terminals for a variety of different sensor current conditions, we would be faced with a much more complex problem:

- I_{sensor} = 0 mA ; Sensor terminal voltage = 8 Volts
- I_{sensor} = 1 mA ; Sensor terminal voltage =
- I_{sensor} = 2 mA ; Sensor terminal voltage =
- I_{sensor} = 3 mA ; Sensor terminal voltage =
- I_{sensor} = 4 mA ; Sensor terminal voltage =
- I_{sensor} = 5 mA ; Sensor terminal voltage =

One technique we could use to simplify this problem is to reduce the voltage divider resistor network into a Thévenin equivalent network. With the three-resistor divider reduced to a single resistor in series with an equivalent voltage source, the calculations for sensor supply voltage become much simpler.

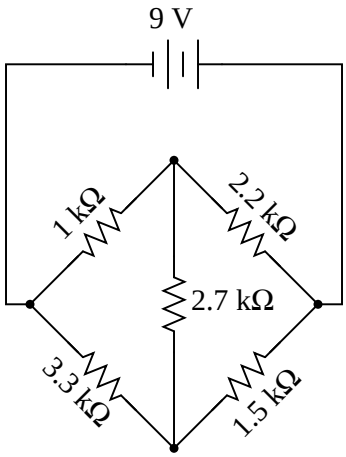
Show how this could be done, then complete the list of sensor supply voltages shown above.

Challenges

- If we are not supposed to let the sensor supply voltage to fall below 6.5 Volts, what is the maximum amount of current it may draw from this voltage divider circuit.
- Try solving for the sensor's terminal voltage without using Thévenin's Theorem, and describe the difficulty in doing so.

6.2.8 Unbalanced bridge circuit

Inspect the following circuit:



Note that it is not reducible to a single resistance and power source. In other words, it is *not* a series-parallel combination circuit. In fact, *no two resistors in this network are directly in series with each other or directly in parallel with each other!* And, while it is a bridge circuit, you are not able to simply analyze the resistor ratios because it is obviously not in a state of balance! If you were asked to calculate voltage or current for any component in this circuit, it would be a difficult task . . . unless you know either Thévenin's or Norton's Theorems, that is!

Apply either one of these theorems to the determination of voltage across the 2.2 kΩ resistor (the resistor in the upper-right corner of the bridge). Hint: consider the 2.2 kΩ resistor as the *load* in a Thévenin or Norton equivalent network.

Challenges

- What is the definition of a series network?
- What is the definition of a parallel network?

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

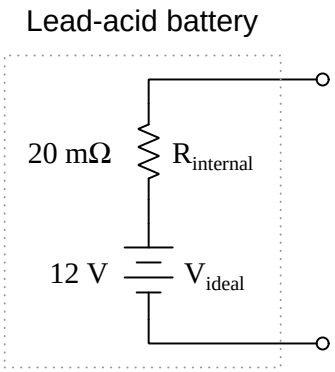
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

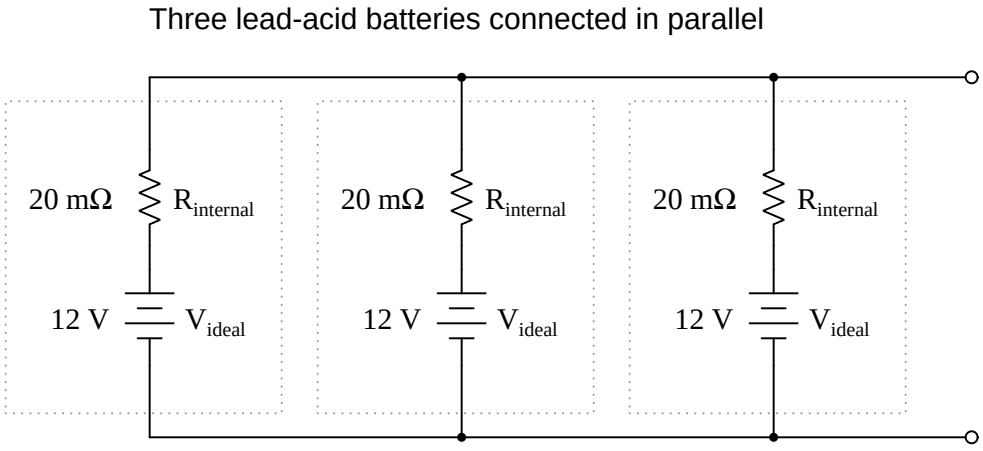
6.3.1 Fault current of a battery bank

Suppose a 12 Volt lead-acid battery has an internal resistance of 20 milli-Ohms (20 mΩ):



If a short-circuit were placed across the terminals of this large battery, the fault current would be quite large: 600 Amperes!

Now suppose three of these batteries were connected directly in parallel with one another:



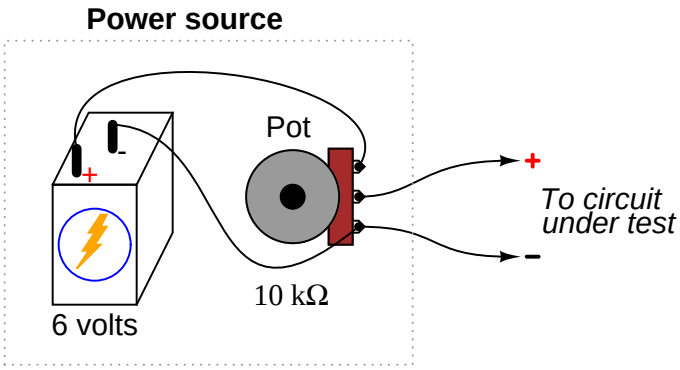
Re-calculate the fault current available at the terminals of the three-battery "bank" in the event of a direct short-circuit.

Challenges

- Explain what practical importance this question has for parallel-connected batteries, and how either Thévenin's or Norton's Theorems makes the concept easier to explain to someone else. What safety issues might be raised by the parallel connection of large batteries such as these?

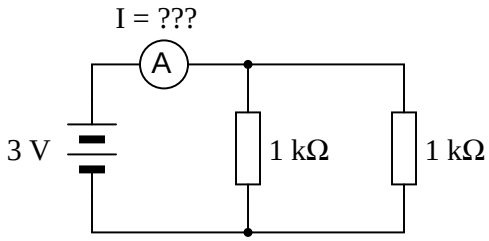
6.3.2 Variable-voltage power source

One day an electronics student decides to build her own variable-voltage power source using a 6 Volt battery and a 10 kΩ potentiometer:



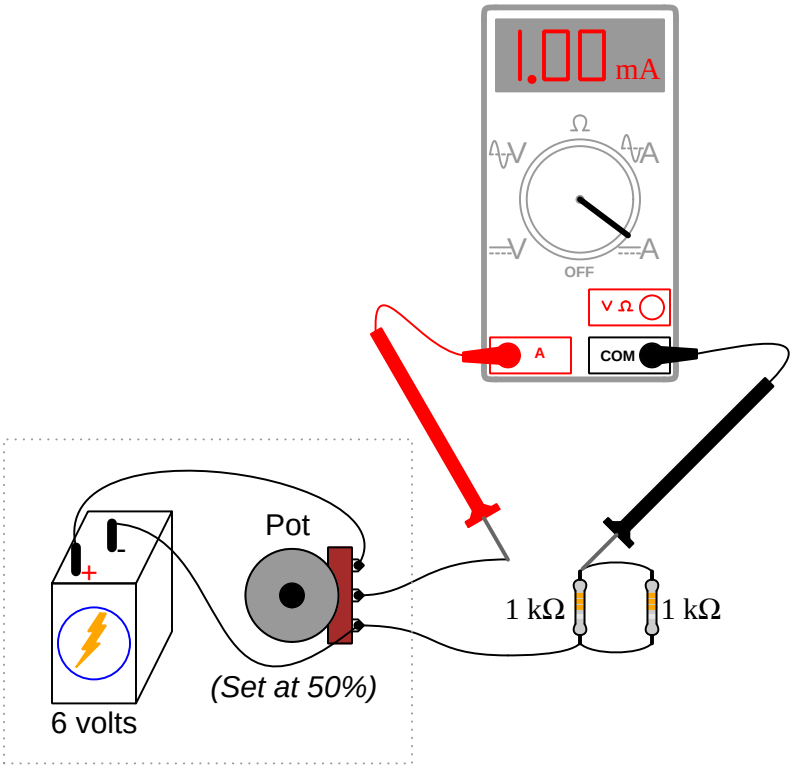
She tests her circuit by connecting a voltmeter to the output terminals and verifying that the voltage does indeed increase and decrease as the potentiometer knob is turned.

Later that day, her instructor assigns a quick lab exercise: measure the current through a parallel resistor circuit with an applied voltage of 3 Volts, as shown in the following schematic diagram.



Calculating current in this circuit is a trivial exercise, she thinks to herself: 3 V ÷ 500 Ω = 6 mA. This will be a great opportunity to use the new power source circuit, as 3 Volts is well within the voltage adjustment range!

She first sets up her circuit to output 3 Volts precisely (turning the 10 kΩ potentiometer to the 50% position), measuring with her voltmeter as she did when initially testing the circuit. Then she connects the output leads to the two parallel resistors through her multimeter (configured as an ammeter), like this:



However, when she reads her ammeter display, the current only measures 1 mA, not 6 mA as she predicted. This is a very large discrepancy between her prediction and the measured value for current!

Why didn't her circuit behave as she predicted it would?

Challenges

- Explain what this student would have to do to use her adjustable-voltage power source circuit to properly demonstrate the lab circuit as assigned.
- Identify at least one circuit failure which would result in zero measured (ammeter) current.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

5 October 2025 – minor edits to the Tutorial chapter, including an error correction where I mistakenly wrote "maintain constant load resistance" instead of "maintain constant load voltage". Also corrected a typographical error, courtesy of Wyatt Gleason.

16 September 2025 – added some new instructor notes and also corrected a spelling error.

23 October 2024 – added a new Technical References section on applying Thévenin's and Norton's Theorems to time-delay circuits.

8 October 2024 – fixed the R_{Th} error in equivalent network #3 of the "Example: Thévenizing series-parallel networks" section of the Case Tutorial chapter.

2 October 2024 – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

15-17 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors. Also added active-reading suggestions to the Tutorial chapter on analyzing the example equivalent networks, as well as changed the phrase "equivalent circuit" to "equivalent network".

5 March 2024 – added notes to images anim_thevenin_24 and anim_thevenin_25 explaining the use of the voltage divider equation to calculate V_{R3}.

10 October 2023 – converted a Case Tutorial section into a new (regular) Tutorial section, focusing on equivalent electrical networks using a "sealed box" analogy.

12 June 2023 – added Derivations and Technical References chapter, with a section showing a derivation of Millman's Theorem.

6-7 Mar 2023 – minor edits to the Tutorial for readability, including capitalizing the word "Theorem" when used with Thévenin's or Norton's.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

10-11 October 2022 – minor additions to the Tutorial chapter, providing alternative conceptualizations for the disabling of current sources and voltage sources. Also fixed a typographical error ("short-circuit voltage" should have been "short-circuit current") courtesy of Kobe Wessels. Also made minor additions to the Introduction chapter. Also corrected a mathematical error in one of the instructor comments.

28 February 2022 – added some page breaks to the "Example: Thévenizing series-parallel networks" Case Tutorial section for cleaner appearance.

15-17 February 2022 – minor additions to the Introduction and Tutorial chapters, as well as some edits to illustrations in the "Norton equivalent of an electric arc welder" Quantitative Reasoning question. Also added a Conceptual Reasoning question on Thévenizing a solar cell, and added some new Case Tutorial sections.

12 October 2021 – added "A" and "B" terminal labels to image_2038 to make it clear that the two equivalent networks are referencing the terminals shown in the original network. Also, colored the DMM digits shown in image_3524 and image_3513 and image_3514.

23 September 2021 – minor edits to a schematic diagram in the "Thévenizing a loaded voltage divider" Quantitative Reasoning question.

16 September 2021 – added section to Tutorial elaborating on the practical applications of these theorems.

28 August 2021 – commented out or deleted empty chapters, as well as added more questions.

23 March 2021 – added Case Tutorial sections showing a multi-source network being Thévenized and Nortonized.

12-13 October 2020 – minor additions to the Introduction chapter, and minor additions to instructor notes.

1 October 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

27 September 2020 – added a Case Tutorial chapter.

7 May 2020 – completed the Foundational concepts section.

30 April 2020 – added many questions.

12 September 2019 – continued writing Tutorial.

11 September 2019 – continued writing Tutorial.

6 September 2019 – continued writing Tutorial.

5 September 2019 – document first created.

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