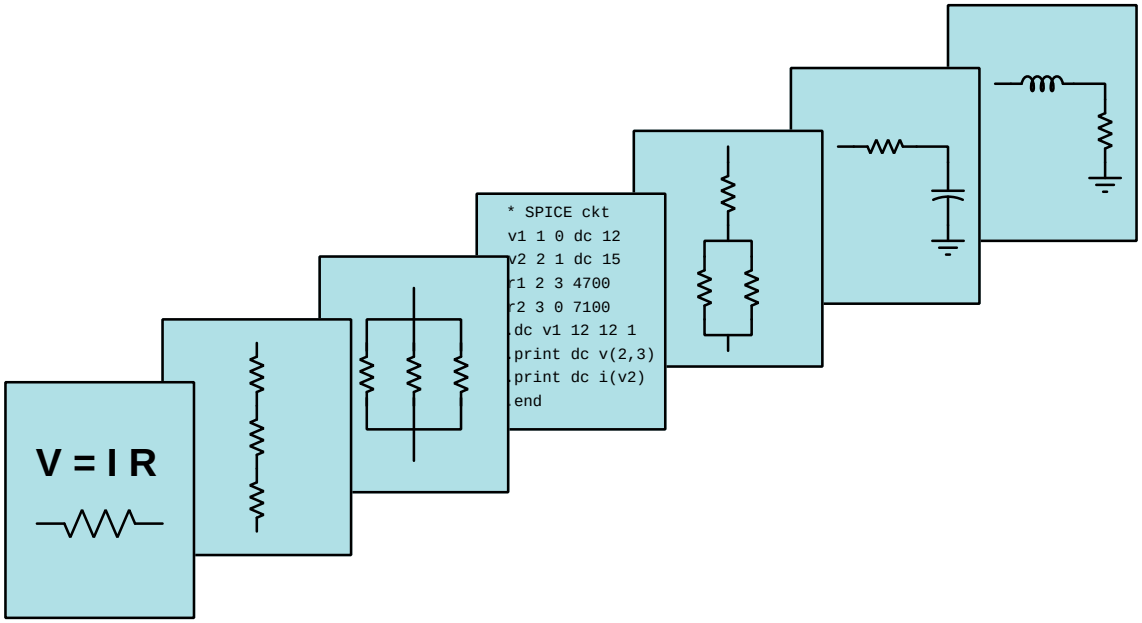
Modular Electronics Learning (ModEL) project



Transmission Lines

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Last update = 12 October 2026

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Chapter 1

Introduction

1.1 Recommendations for students

In this tutorial we will explore the response of long cables to short-duration electrical pulses. You will find that certain "fundamental" rules describing the behavior of DC circuits do not seem to apply to cases involving long conductor lengths and brief time periods. The reason for this is the fact that electric and magnetic fields do not propagate at infinite speed, but are actually limited by the speed of *light*. Not only will we explore these "strange" effects, but we will also learn how to compensate for their unintended consequences.

Important concepts related to transmission lines include the **speed of light**, **capacitance**, **inductance**, **parasitic effects**, **impedance**, **permittivity**, **velocity factor**, **Thévenin** equivalent resistance, the effects of **opens** versus **shorts**, **oscilloscope** usage, **Conservation of Energy**, **decibels**, **standing wave ratio**, **reflection coefficient**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to measure the length of a cable using reflectometry? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to measure the characteristics impedance of a cable using reflectometry? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What do you realize now about cables that you did not prior to studying the subject of transmission lines?
- What causes parasitic capacitance within a cable?
- What causes parasitic inductance within a cable?
- How does the construction of a cable affect its parasitic capacitance and inductance?

- How and why is characteristic impedance affected if cable conductors are brought into closer proximity with each other?
- How and why is characteristic impedance affected if cable conductors are spaced farther apart from each other?
- What is a "lumped" parameter model?
- Why can we not use an ohmmeter to measure the characteristic impedance of a cable?
- What test equipment *is* able to determine characteristic impedance?
- What happens, step by step, when we apply a voltage to one end of an open-ended cable?
- What happens, step by step, when we apply a voltage to one end of a cable terminated by a short?
- Why do "echo" signals exist in a cable?
- Why might "echo" signals be undesirable in a signal cable?
- How do we mitigate those echoes?
- What constitutes a "discontinuity" in a cable?
- Where might echoes form along a cable other than at the ends?
- What is *velocity factor*, and what physical parameters affects it?
- How are "standing waves" formed along a transmission line?
- What factors determine the "reflection coefficient" for a transmission line?
- How do we calculate a transmission line's standing wave ratio, and what practical significance does this figure have?
- How does reflection coefficient relate to standing wave ratio in a transmission line?

1.2 Challenging concepts related to transmission lines

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Conceptualizing the progression of a voltage or current wave** – some very important rules learned in DC and low-frequency AC circuits no longer apply in high-frequency and "pulse" circuits, namely that the effects of electricity are instantaneous across all portions of an electric circuit. In fact, they are not truly instantaneous, but rather propagate at (nearly) the speed of light. Thus, the establishment of a voltage or current at some given location in a circuit requires a certain amount of *time* following the application of a power source. In the most literal sense, those DC "rules" were not entirely true, but they were *close enough to being true* that they sufficed for DC and other low-frequency circuits. Certainly, basic circuit analysis would have been far more complicated for us if we had to view the circuit from the perspective of a wave traveling down the wires of that circuit. Instead, the rules learned for DC circuits represent the *final, steady-state conditions* of those circuits. Now, as we begin to study transmission lines we must come to terms with the limits of those rules and consider the circuit's behavior during the fleeting moments prior to reaching its steady-state condition.
- **Interpreting oscilloscope displays** – the graphical display of an oscilloscope represents a voltage measured at one particular location in a circuit (i.e. between two specified terminals, or points, in that circuit) over time. In using an oscilloscope to explore the behavior of a transmission line, we must satisfy ourselves with this single point of view as the wave-front progresses down (and back!) the length of the line.

The *Animations* chapter of this module contains flip-book animations helpful to understanding how signals propagate along the length of transmission lines, both properly terminated and unterminated.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of traveling waves to practical problems

  Assessment – Predict time-domain waveforms for healthy, open, and shorted cables; e.g. pose problems in the form of the "TDR cable fault testing" Conceptual Reasoning question.

  Assessment – Locate a fault along an electric power transmission line based on oscillography showing reflected waves following a fault; e.g. pose problems in the form of the "Traveling-wave fault location on a power line" Diagnostic Reasoning question.

- **Outcome** – Apply the concept of characteristic impedance to practical problems

  Assessment – Specify how to alter characteristic impedance for twin-line and/or stripline traces on a PCB; e.g. pose problems in the form of the "Impedance of lines on a printed circuit board" Conceptual Reasoning question.

- **Outcome** – Independent research

  Assessment – Read and summarize in your own words reliable source documents on the subject of *Bewley lattice diagrams*.

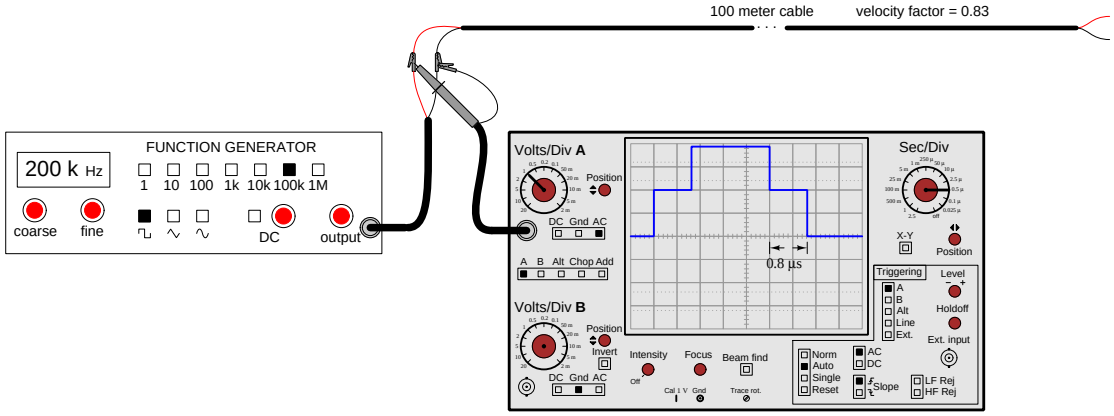
Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

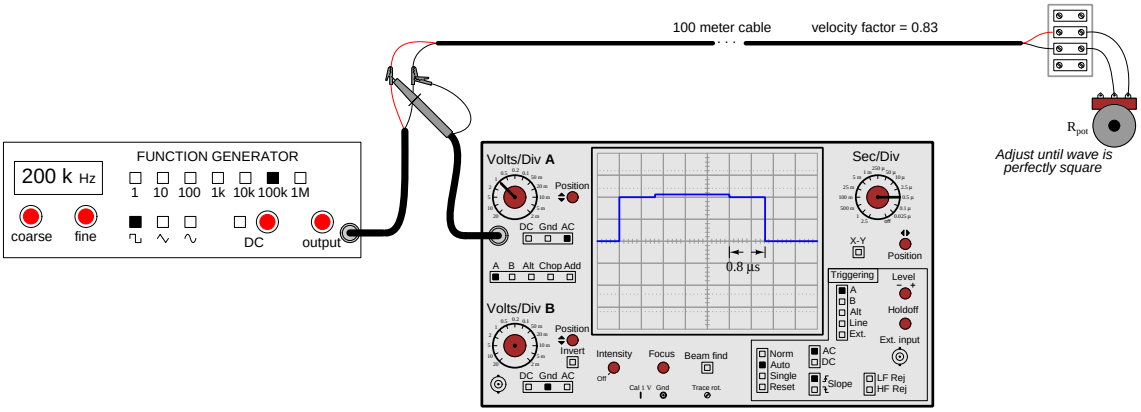
2.1 Example: testing a cable's length



The time duration of the "step" shown in the oscillograph represents the pulse's *round-trip time* up and down the entire length of the cable. Pulse velocity along the cable is equal to the speed of light in a vacuum ($c = 2.9979 \times 10^8$ meters per second) multiplied by the cable's velocity factor.

If a signal generator is not available, you may configure the oscilloscope for single-sweep operation and use *any* transient pulse (e.g. battery and pushbutton) as the source, making your measurements on the leading edge of that pulse.

2.2 Example: testing a cable's impedance

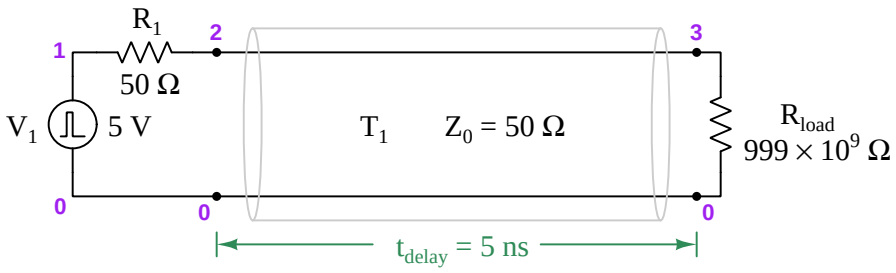


$Z_0 = R_{pot}$ when adjusted to make wave perfectly square

If a signal generator is not available, you may configure the oscilloscope for single-sweep operation and use *any* transient pulse (e.g. battery and pushbutton) as the source, making your measurements on the leading edge of that pulse.

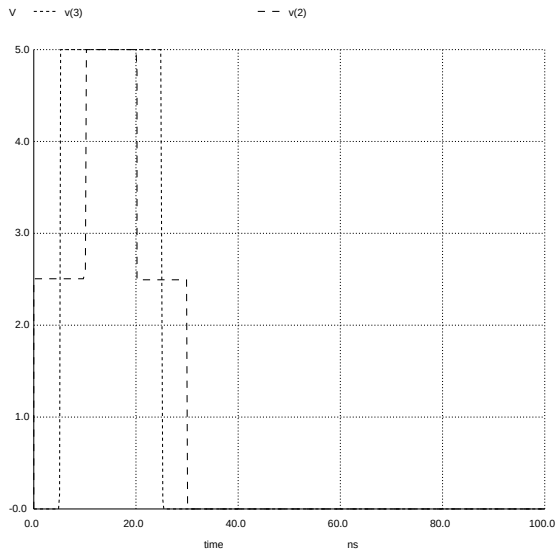
2.3 Example: SPICE simulation of open-ended line

The following SPICE simulation tests voltages at both ends of a transmission line resulting from a pulsed voltage source at one end and an open at the other. Note the immense load resistance value effectively acting as an *open*:



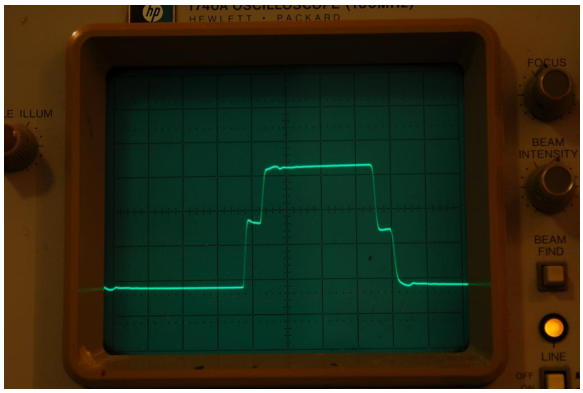

```
* SPICE simulation
v1 1 0 pulse 0 5 0 0.1n 0.1n 20n 100n
r1 1 2 50
t1 2 0 3 0 z0=50 td=5n
rload 3 0 999e9
.tran 0.25n 100n
.plot tran v(2) v(3)
.end
```

Here we instruct SPICE to generate a 0-Volt-to-5-Volt pulse at source V_1 starting immediately (0 ns) with a 0.1 ns rise time, 0.1 ns fall time, 20 ns duration, and 100 ns period. The transmission line is short enough that its time-of-flight delay is only 5 ns. This means the pulse will have time to travel all the way down the line, reach the open, and reflect all the way back before the source's pulse has elapsed. With these parameters we simulate typical cable testing using an inexpensive signal generator capable only of producing relatively slow (long) pulses compared to the cable's delay time.



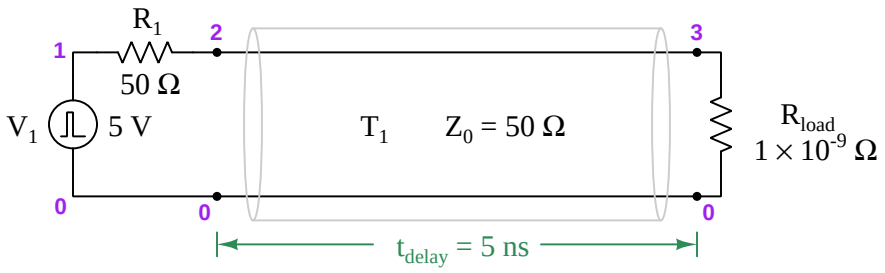
In this simulation voltage source V_1 outputs a 5-Volt pulse lasting 20 nanoseconds, which is seen at the far end of the transmission line (voltage between nodes 3 and 0) 5 nanoseconds later. At first the voltage between nodes 2 and 0 only goes to 2.5 Volts (not 5 Volts) because the transmission line is acting as a 50-Ohm load. When the reflected pulse returns back to the source-end of the transmission line (at the 10 nanosecond mark) the voltage between nodes 2 and 0 jumps from 2.5 Volts up to a full 5 Volts – it is now at this time that the source "sees" the cable's open end. At the source's falling edge the voltage between nodes 2 and 0 falls to 2.5 Volts (but not all the way to zero) because the reflected pulse is still returning. Not until the 30 nanosecond mark do we see the voltage between nodes 2 and 0 fall all the way to zero, after the reflected pulse has elapsed.

This result is similar to that shown in the Tutorial with an analog oscilloscope connected to the point where a signal generator connects to an open-ended cable. Instead of a clean square-shaped pulse signal, we see "steps" on the rising and falling edges, each step as wide as the total *round-trip* time for the pulse:



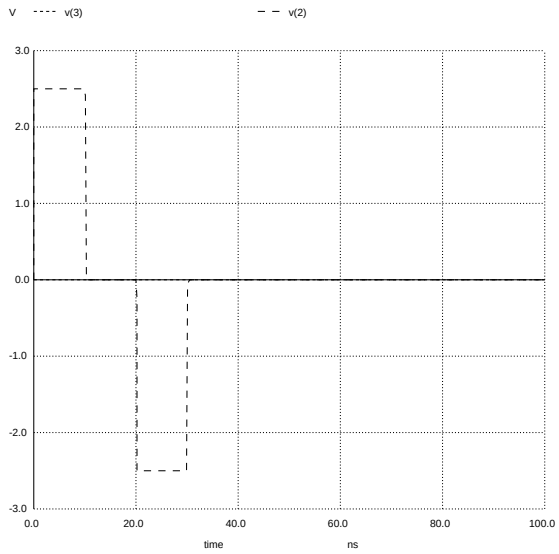
2.4 Example: SPICE simulation of short-ended line

The following SPICE simulation tests voltages at both ends of a transmission line resulting from a pulsed voltage source at one end and a short at the other. Note the miniscule load resistance value effectively acting as a *short*:



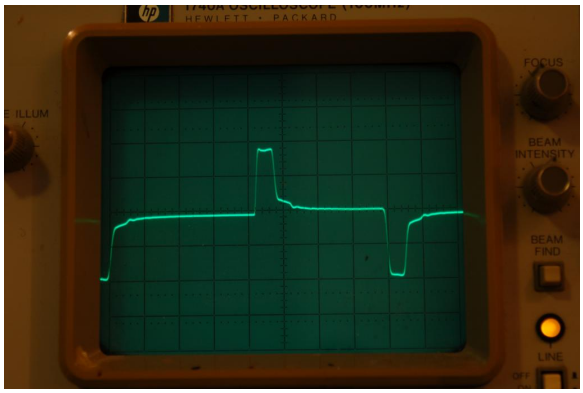

```
* SPICE simulation
v1 1 0 pulse 0 5 0 0.1n 0.1n 20n 100n
r1 1 2 50
t1 2 0 3 0 z0=50 td=5n
rload 3 0 1e-9
.tran 0.25n 100n
.plot tran v(2) v(3)
.end
```

Here we instruct SPICE to generate a 0-Volt-to-5-Volt pulse at source V_1 starting immediately (0 ns) with a 0.1 ns rise time, 0.1 ns fall time, 20 ns duration, and 100 ns period. The transmission line is short enough that its time-of-flight delay is only 5 ns. This means the pulse will have time to travel all the way down the line, reach the short, and reflect all the way back before the source's pulse has elapsed. With these parameters we simulate typical cable testing using an inexpensive signal generator capable only of producing relatively slow (long) pulses compared to the cable's delay time.



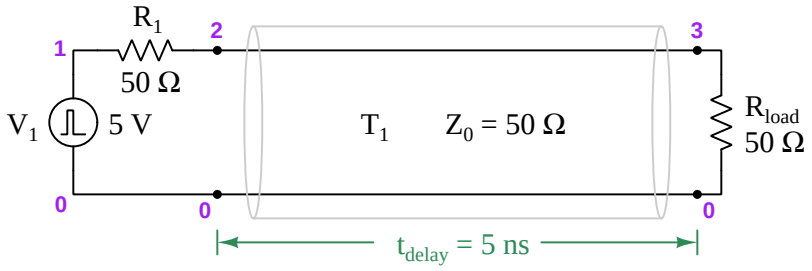
In this simulation voltage source V_1 outputs a 5-Volt pulse lasting 20 nanoseconds, which is never seen at the far end of the transmission line because the short prohibits any voltage there (between nodes 3 and 0). The voltage between nodes 2 and 0 only goes to 2.5 Volts (not 5 Volts) because the transmission line initially acts as a 50-Ohm load. When the reflected pulse returns back to the source-end of the transmission line (at the 10 nanosecond mark) the voltage between nodes 2 and 0 falls to 0 Volts – it is now at this time that the source "sees" the cable's shorted end. At the source's falling edge the voltage between nodes 2 and 0 falls to -2.5 Volts because the reflected pulse (polarity opposite that of the forward pulse) is still returning. Not until the 30 nanosecond mark do we see the voltage between nodes 2 and 0 return back to zero, after the reflected pulse has elapsed.

This result is similar to that shown in the Tutorial with an analog oscilloscope connected to the point where a signal generator connects to a shorted cable. Instead of a clean pulse signal, we see two shorter-duration "steps", one positive and one negative, each step as wide as the total *round-trip* time for the pulse:



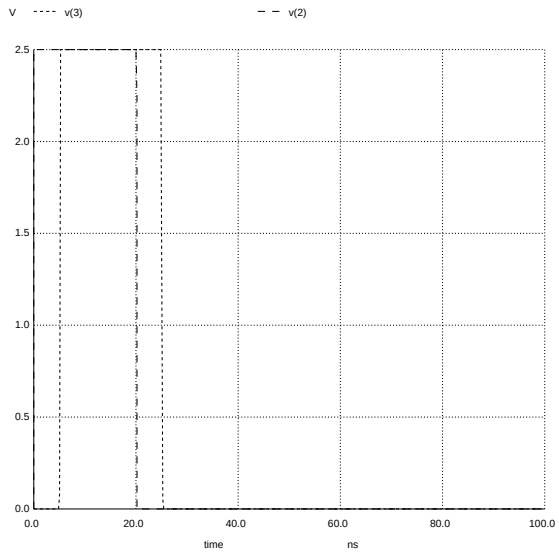
2.5 Example: SPICE simulation of properly terminated line

The following SPICE simulation tests voltages at both ends of a transmission line resulting from a pulsed voltage source at one end and a properly-sized resistance (to match the line's characteristic impedance) at the other:



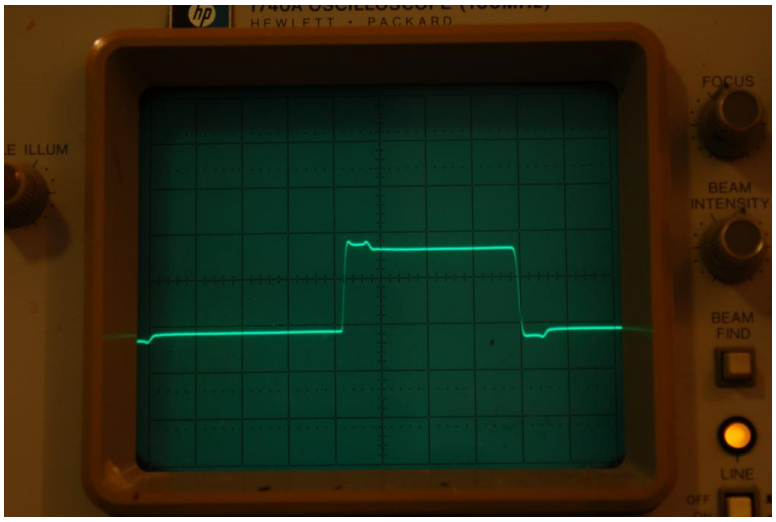

```
* SPICE simulation
v1 1 0 pulse 0 5 0 0.1n 0.1n 20n 100n
r1 1 2 50
t1 2 0 3 0 z0=50 td=5n
rload 3 0 50
.tran 0.25n 100n
.plot tran v(2) v(3)
.end
```

Here we instruct SPICE to generate a 0-Volt-to-5-Volt pulse at source V_1 starting immediately (0 ns) with a 0.1 ns rise time, 0.1 ns fall time, 20 ns duration, and 100 ns period. The transmission line is short enough that its time-of-flight delay is only 5 ns. This means the pulse will have time to travel all the way down the line and reach the end before the source's pulse has elapsed. With these parameters we simulate typical cable testing using an inexpensive signal generator capable only of producing relatively slow (long) pulses compared to the cable's delay time.



In this simulation voltage source V_1 outputs a 5-Volt pulse lasting 20 nanoseconds, which is seen at the far end of the transmission line (voltage between nodes 3 and 0) 5 ns later. The voltage between nodes 2 and 0 only goes to 2.5 Volts (not 5 Volts) because the transmission line acts as a 50-Ohm load. Proper termination ensures there is no reflection, and so we see the same 2.5 Volt pulse at the far end and no interference back at the source end.

This result is similar to that shown in the Tutorial with an analog oscilloscope connected to the point where a signal generator connects to a properly-terminated cable. We see a clean pulse signal at the source end with negligible interference:



2.6 Example: cable loss decibel calculations

If a radio transmitter sends 15 Watts of power into a coaxial cable having a loss rating of −0.16 dB per meter, and the cable is 20 meters in length, the power available at the far end of the cable must be **7.1795 Watts**.

If an RF signal source sends 10 mW into one end of a 5-meter cable and the other end of the cable outputs 8 mW of signal power, the cable must have a loss rating of **−0.194 dB per meter**.

An RF signal of 2 Watts exits a cable 3 meters in length having a loss rating of −0.14 dB per meter. The amount of power being input to this cable must be **2.203 Watts**.

An RF signal of 15 Watts enters one end of a cable having a loss rating of −0.21 dB per foot, and an RF signal of 10 Watts exits the other end of that same cable. The cable's length must be **8.38530 feet**.

A 50-Watt radio transmitter connected to a 12-foot cable having a loss rating of 0.041 dB/foot terminates at a lightning arrestor with a loss rating of 0.5 dB, and finally to an antenna. The amount of power received by this antenna will be **39.7896 Watts**.

Chapter 3

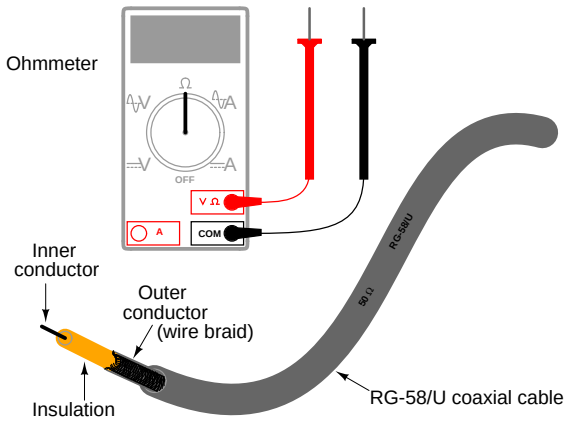
Tutorial

Many years ago, when I was first learning about electricity, I happened to discover a length of coaxial television cable with the rating "50 Ohm" printed on the jacket. Having my own ohmmeter, and wanting to know what the "50 Ohm" label referred to, I tried measuring the resistance of this cable to see where it would exhibit 50 Ohms of resistance. First I tried measuring between the two open-ended wires (the center conductor and the braided shield conductor), where my meter registered infinite resistance (no continuity). This is what I expected a normal cable would read when connected to nothing else. Next I tried connecting my meter from one end of the cable to the other: between both ends of the center conductor my meter registered 0 Ohms, and between both ends of the shield conductor my meter also read 0 Ohms. In fact, this is all I was ever able to measure with my ohmmeter: either 0 Ohms or infinite Ohms (open), no matter what I tried. I was confounded – how could this piece of cable possibly be called a "50 Ohm" cable if I could not measure 50 Ohms anywhere on it?

It was not until years later that I came across a reference in an old book to something called *surge impedance*, describing how lengths of cable responded to short-duration electrical pulses, that I finally understood the "50 Ohm" rating of that coaxial cable. What I failed to discover with my ohmmeter – in fact, what I never could have even seen with a plain ohmmeter – is that the cable did indeed act like a 50 Ohm resistor, *but only for extremely short periods of time*. This is an aspect of electrical theory I was completely unprepared to comprehend at that time, knowing only how DC circuits functioned. This is the subject we are about to explore now.

3.1 Surge impedance

If we test a length of type RG-58/U coaxial cable using an ohmmeter, we will find it behaves like any other two-conductor cable we might find: the end-to-end resistance of each conductor is negligible, and its conductor-to-conductor resistance is essentially infinite. Yet, a careful inspection of this cable's outer jacket typically reveals a rating of 50 Ω, which may seem strange because nowhere on this cable is it possible to measure a resistance of 50 Ohms using an ohmmeter. What might this rating mean?



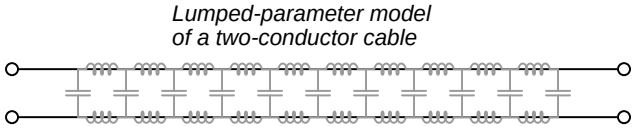
In basic DC electrical theory, students learn that open circuits always drop the full applied voltage and cannot have electric currents anywhere in them. Students likewise learn that short circuits drop negligible voltage while conducting full electrical current. They also learn that the effects of electricity occur instantaneously throughout the circuit: for example, if a functioning circuit suffers an "open" fault, current through that branch of the circuit halts immediately and everywhere. These basic rules of electricity are extremely useful for DC circuit analysis, indeed even essential for troubleshooting DC circuits, but they are not 100% correct. Like many of the scientific principles one initially learns, they are only *approximations* of reality.

The effects of electricity do *not* propagate instantaneously throughout a circuit, but rather spread at the speed of light (approximately 300,000,000 meters per second). Normally we never see any consequences of this finite speed because everything happens so fast, just as I never saw my ohmmeter register 50 Ohms when I tried to test that coaxial cable many years ago. However, when we test a circuit on a time scale of nano-seconds (*billionths* of a second!), we find some very interesting phenomena during the time electricity propagates along the length of a circuit: open circuits can indeed (temporarily) exhibit current, and short circuits can indeed (temporarily) exhibit voltage drops.

The key to understanding these phenomena is recognizing the natural capacitance and inductance existing as parasitic effects between separated conductors. Recall that capacitance is the ability to store energy within an electric field, and that inductance is the ability to store energy within a magnetic field. A pair of separated conductors, by virtue of the insulating medium separating those conductors from making direct contact with each other, may host an electric field and thereby store energy with any potential difference (i.e. voltage) between them. Similarly, each of those same

conductors generates a magnetic field as current passes through, and with the separation ensuring the conductors' magnetic fields will not cancel[1] one another, the conductor pair is able to magnetically store energy as well. Therefore, a simple pair of separated conductors is able to behave as an array reactive components.

We may crudely represent these parasitic effects as being sets of capacitors and inductors interspersed along the conductors' length, like so:



Of course, it is not as though tiny inductors and capacitors exist inside of a cable. These inductances and capacitances exist as parameters *continuously distributed* along the cable's length. Representing the cable's reactances as discrete L and C components is referred to as a *lumped-parameter model*, simply to make it easier for us to visualize: framing the cable's properties in terms of components we already understand.

When a pulse signal is applied to the beginning of a two-conductor cable, the reactive elements of that cable (i.e. capacitance between the conductors, inductance along the cable length) begin to store energy. This translates to a current drawn by the cable from the source of the pulse, as though the cable were acting as a (momentarily) resistive load. If the cable under test were infinitely long, this charging effect would never end, and the cable would indeed behave exactly like a resistor from the perspective of the signal source. However, real cables (having finite length) do stop "charging" after some time following the application of a voltage signal at one end, the length of that time being a function of the cable's physical length and the speed of light.

In honor of the cable's capacity to behave as a temporary load to any impressed signal, we typically refer to it as something more than just a cable. From the perspective of an electrical pulse, measured on a time scale of nanoseconds, we refer to any cable as a component in its own right called a *transmission line*[2]. All electrical cables act as transmission lines, but the effects just mentioned are so brief in duration that we only notice them when the cable is exceptionally long and/or the pulse durations are exceptionally short (i.e. high-frequency signals).

During the time when a transmission line is absorbing energy from a power source – whether this is indefinitely for a transmission line of infinite length, or momentarily for a transmission line of finite length – the current it draws will be in direct proportion to the voltage applied by the source. In other words, a transmission line behaves like a resistor, at least for a moment. The amount of "resistance" presented by a transmission line is called its *characteristic impedance*, or *surge impedance*, symbolized in equations as Z_0. Only after the pulse signal has had time to travel down the length of the transmission line and reflect back to the source does the cable stop acting as a load and begin acting as a plain pair of wires.

A transmission line's characteristic impedance is a function of its conductor geometry (wire

[1] Imagine two parallel conductors, each carrying the exact same amount of current but in opposite directions. The opposing directions of the currents set up opposing magnetic fields around the conductors, and if not separated from each other by some distance those magnetic fields would directly cancel each other out.

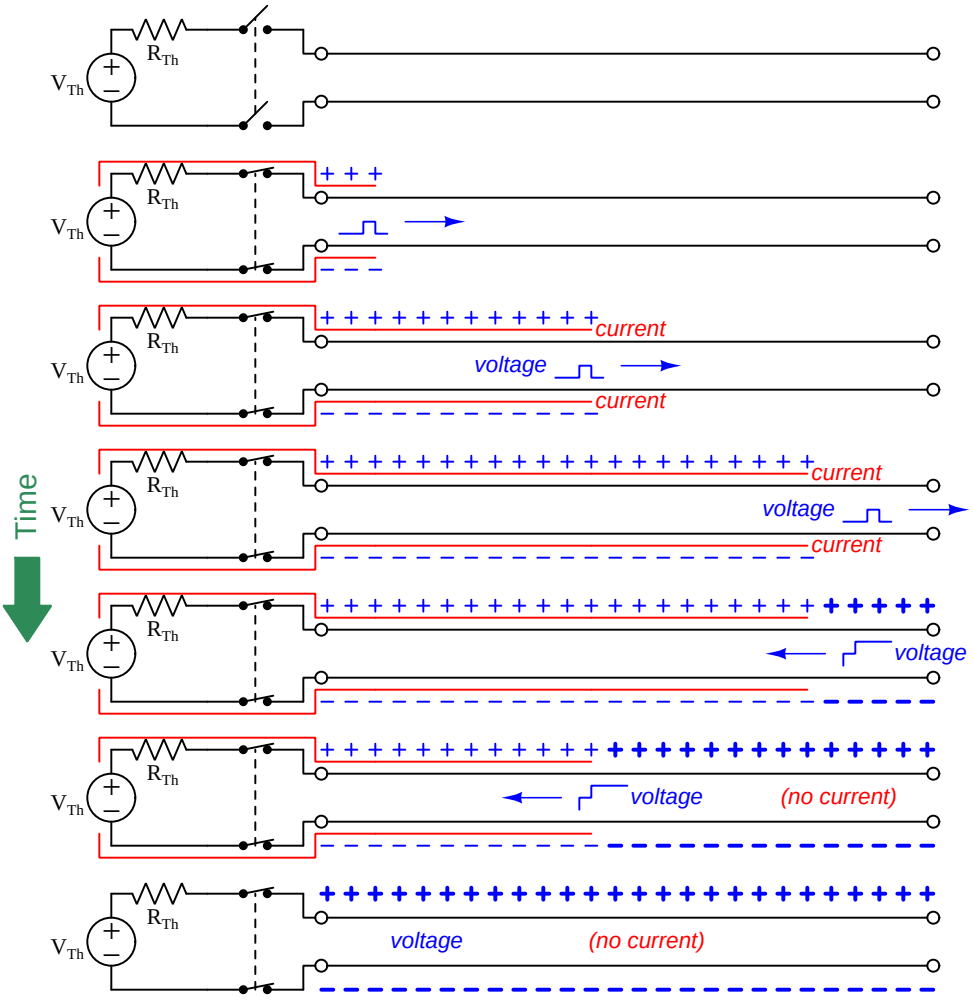
[2] This use of the term is entirely different from the same term's use in the electric power industry, where a "transmission line" is a set of conductors used to send large amounts of electrical energy over long distances.

diameter, spacing) and the permittivity of the dielectric separating those conductors. Z_0 *increases with increasing line inductance and decreases with increasing line capacitance.* The permittivity of the insulation separating conductors has a direct effect on capacitance, and therefore on Z_0. Also, the spacing of the conductors matters: increasing the distance of separation between conductors increases inductance (by providing less mutual cancellation of magnetic fields) and also decreases capacitance (more distance equates to a less intense electric field), both of which act to increase Z_0. Conversely, positioning the conductors closer together decreases inductance (i.e. greater cancellation of magnetic fields) and increases capacitance (stronger electric field), resulting in decreased Z_0.

It should be noted that the *length* of the transmission line has absolutely no bearing on characteristic impedance. A 10-meter length of RG-58/U coaxial cable will have the exact same characteristic impedance as a 10000 kilometer length of RG-58/U coaxial cable (50 Ohms, in both cases). The only difference is the *length of time* the cable will behave like a resistor to an applied voltage.

3.2 Open-ended transmission line

The following sequence illustrates the propagation of a voltage pulse forward and back (reflected) on an open-ended transmission line beginning from the time the DC voltage source is first connected to the left-hand end:

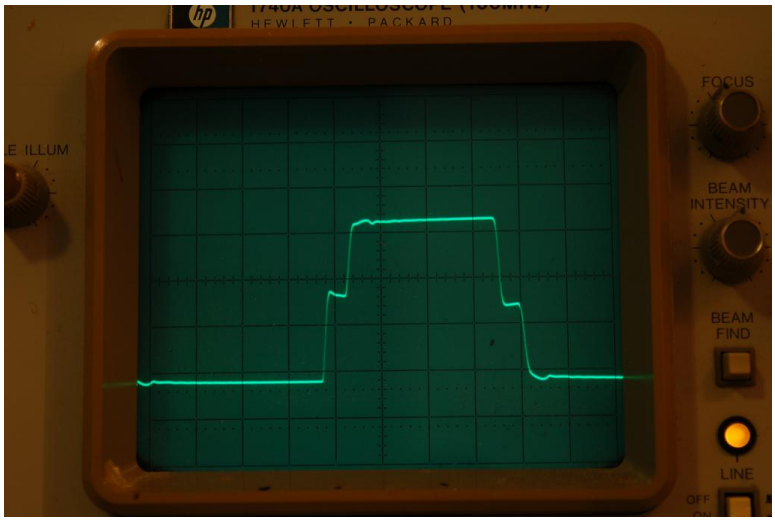


During the time the pulse travels along the cable away from the source, both the cable's inductance and cable's capacitance absorb energy from the source. As the pulse returns back toward the source after "reflecting" off the open end of the cable, energy formerly stored in cable inductance is transferred to cable capacitance, which explains why we see current ceasing (no more magnetic field) and voltage increasing (stronger electric field) in the wake of the returning pulse. In the end there is only energy stored capacitively, and not inductively, in the cable.

The end-result is a transmission line exhibiting the full source voltage, but no current – exactly

what we would expect in an open circuit. However, during the time it took for the pulse to travel down the line's length and back, it drew current from the source equal to the source's internal (Thévenin) voltage divided by the sum of the source's internal resistance and the cable's characteristic impedance ($I_{surge} = \frac{V_{Th}}{R_{Th}+Z_0}$). For a brief amount of time, the two-conductor transmission line acted as a *load* to the voltage source rather than an open circuit. Only after the pulse traveled down the full length of the line and back did the line finally act as a plain open circuit.

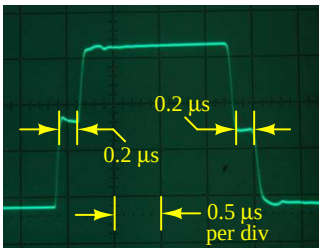
An experiment performed with a square-wave signal generator and oscilloscope[3] connected to one end of a long wire pair cable (with the other end of the cable open and connected to nothing) shows the effect of the reflected signal:



The waveform steps up for a brief time, then steps up further to full source voltage. The first step represents the voltage at the source during the time the pulse traveled along the cable's length, when the cable's characteristic impedance acted as a load to the signal generator (making its output voltage "sag" to a value less than its full potential). The next step represents the reflected pulse's arrival back to the signal generator, when the cable's capacitance is fully charged and is no longer drawing current from the signal generator (making its output voltage "rise"). A two-step "fall" appears at the trailing edge of the pulse, when the signal generator reverses polarity and sends an opposing pulse down the cable.

[3]The signal generator was set to a frequency of approximately 240 kHz with a Thévenin resistance of 118 Ohms to closely match the cable's characteristic impedance of 120 Ohms. The signal amplitude was just over 6 Volts peak-to-peak, and the cable happened to be a spooled length of twin-lead audio speaker cable.

The duration between these "step-changes" in voltage represents the time taken by the signal to propagate down the length of the cable *and* return to the source. As you can see, that total pulse propagation time is much shorter than the period of the pulse output by the square-wave signal generator. This oscilloscope's timebase was set[4] to 0.5 microseconds per division for this experiment, indicating a pulse round-trip travel time of approximately 0.2 microseconds according to the duration of the constant-voltage periods between steps. We can see this 0.2 μs time delay clearly annotated on the same oscilloscope screenshot viewed previously, each level period occupying approximately two minor divisions along the screen's horizontal axis and each major division containing five minor divisions:



This particular cable has a *velocity factor* of approximately 0.7, which means electrical impulses travel only 70% as fast as light travels through a vacuum, and so the round-trip distance based on that echo propagation time is approximately 42 meters:

$$\text{Distance} = \text{Velocity} \times \text{Time} = (70\%)(3 \times 10^8 \text{ m/s})(0.2\ \mu\text{s}) = 42 \text{ m}$$

Since 42 meters is the pulse's round-trip distance along the entire cable's length and back again, the cable itself must be half of that, or 21 meters in length:

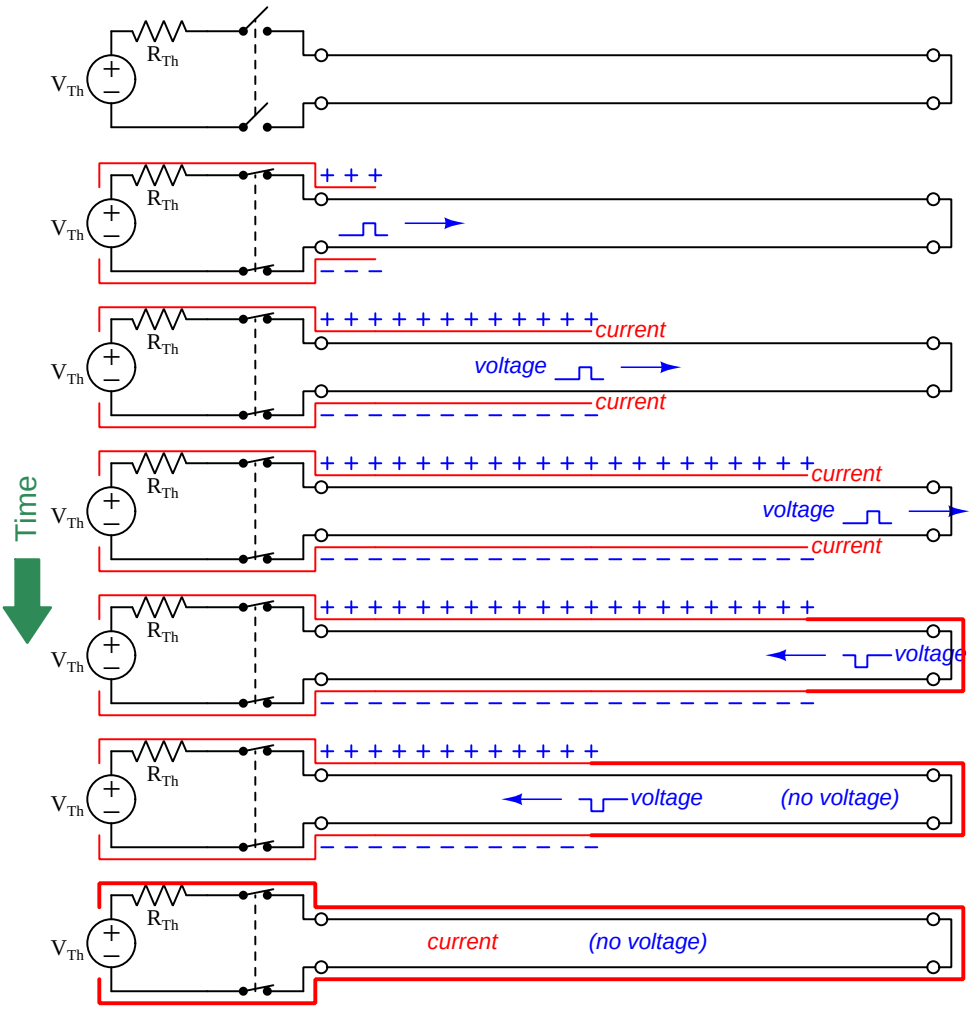
$$\text{Cable length} = \frac{1}{2}(\text{Distance traveled by pulse}) = (0.5)(42 \text{ m}) = 21 \text{ m}$$

To see a flip-book animation of a transmission line terminated by an open, refer to section 5.1 beginning on page 60.

[4] Although the amount of time required for the signal to travel down the cable's length and return to the source is a fixed function of cable length, the square-wave test signal frequency and the oscilloscope's timebase setting are not entirely arbitrary. Rather, they must both be properly configured in order to be able to reliably discern the loading effects of the cable. If the test frequency is too low and the timebase too slow, the "steps" will be so close together on the horizontal axis that we may not be able to see them in the rising and falling edges of the square wave. We need a square-wave signal frequency high enough, and a timebase set fast enough, to clearly show these steps on both rising and falling edges. A good way to predict reasonable signal frequency and timebase settings is to first estimate how long we suspect the cable is, calculate the approximate round-trip time for signal propagation along the cable, and then from that time figure determine an appropriate signal period and timebase setting to view the expected effects.

3.3 Shorted transmission line

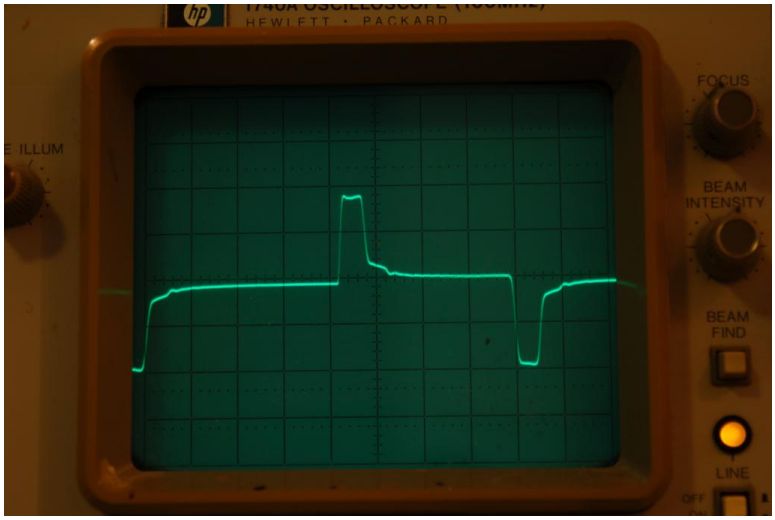
The following sequence illustrates the propagation of a voltage pulse forward and back (reflected) on a shorted-end transmission line beginning from the time the DC voltage source is first connected to the left-hand end:



During the time the pulse travels along the cable away from the source, both the cable's inductance and cable's capacitance absorb energy from the source. As the pulse returns back toward the source after "reflecting" off the shorted end of the cable, energy formerly stored in cable capacitance is transferred to cable inductance, which explains why we see voltage ceasing (no more electric field) and current increasing (stronger magnetic field) in the wake of the returning pulse. In the end there is only energy stored inductively, and not capacitively, in the cable.

The end result is a transmission line exhibiting full current resulting from source voltage divided by its internal (i.e. Thévenin) resistance ($I_{max} = \frac{V_{Th}}{R_{Th}}$), but no voltage – exactly what we would expect in a short circuit. However, during the time it took for the pulse to travel down the line's length and back, it drew current from the source equal to the source's internal voltage divided by the sum of the internal source resistance and the cable's characteristic impedance ($I_{surge} = \frac{V_{Th}}{R_{Th}+Z_0}$). For a brief amount of time, the two-conductor transmission line acted as a moderate *load* to the voltage source rather than a direct short. Only after the pulse traveled down the full length of the line and back did the line finally act as a plain short-circuit.

An experiment performed with the same signal generator and oscilloscope connected to one end of the same long wire pair cable (with the other end of the cable shorted and connected to nothing else) shows the effect of the reflected signal:



Here, the waveform steps up for a brief time, then steps down toward zero. As before, the first step represents the voltage at the source during the time the pulse traveled along the cable's length, when the cable's characteristic impedance acted as a load to the signal generator (making its output voltage "sag" to a value less than its full potential). The step down represents the (inverted) reflected pulse's arrival back to the signal generator, nearly canceling the incident voltage and causing the signal to fall toward zero. A similar pattern appears at the trailing edge of the pulse, when the signal generator reverses polarity and sends an opposing pulse down the cable.

Note the duration of the pulse on this waveform, compared to the duration seen in the open-circuited waveform. This pulse width represents the time taken by the signal to propagate down the length of the cable *and* return to the source. As with the open-circuited test, the pulse period lasts significantly longer than the round-trip travel time of the pulse's leading edge. This oscilloscope's timebase remained at 0.5 microseconds per division for this experiment as well, indicating the exact same pulse round-trip travel time of approximately 0.2 microseconds. This stands to reason, as the cable length was not altered between tests; only the type of termination (short versus open).

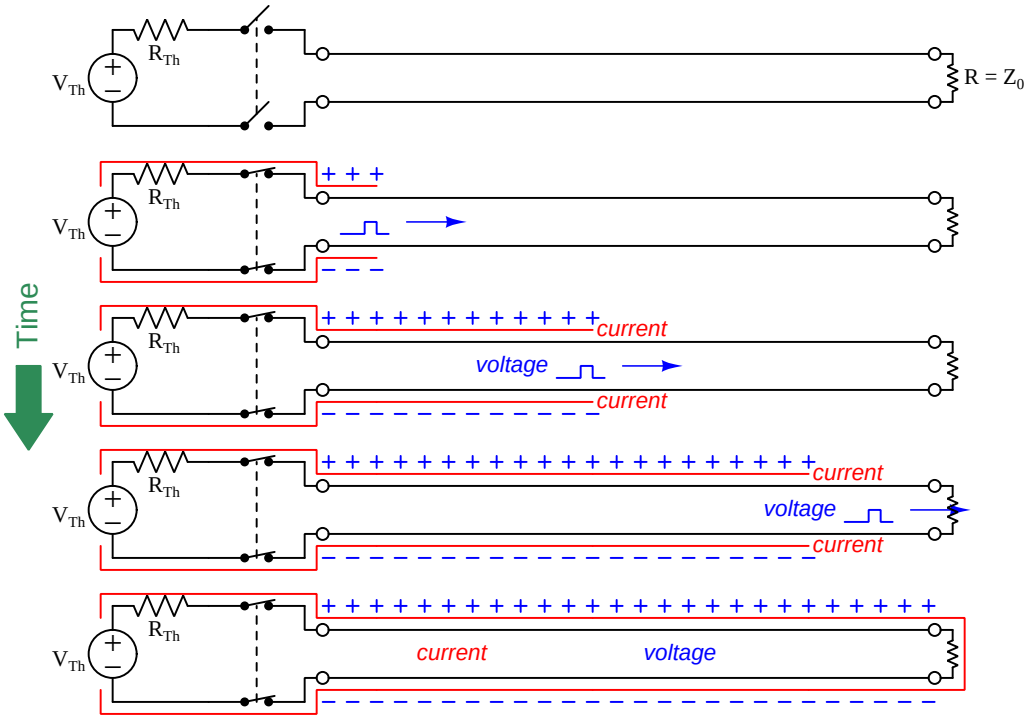
To see a flip-book animation of a transmission line terminated by a short, refer to section 5.2

beginning on page 76.

3.4 Properly terminated transmission lines

Proper "termination" of a transmission line consists of connecting a resistance to the end(s) of the line so that the pulse "sees" the exact same amount of load at the end as it did while propagating along the line's length. The purpose of the termination resistor is to completely dissipate the pulse's energy in order that none of it will be reflected back to the source.

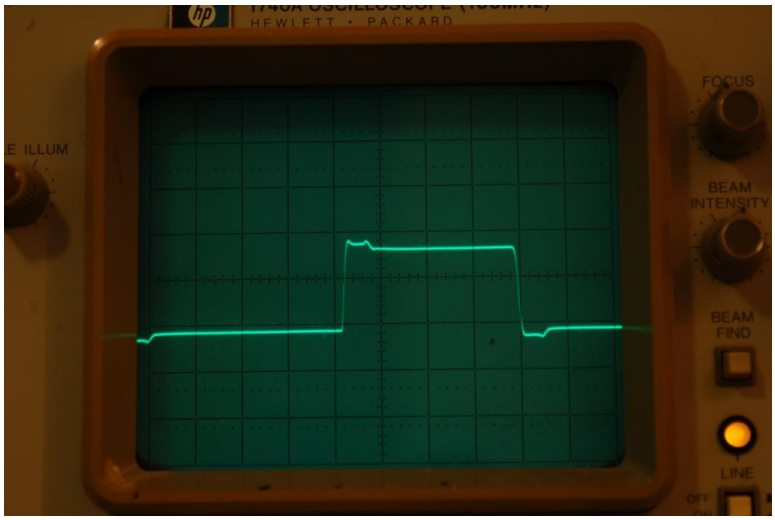
The following sequence illustrates the propagation of a voltage pulse on a transmission line with proper "termination" (i.e. a resistor matching the line's surge impedance, connected to the far end) beginning from the time the DC voltage source is first connected to the left-hand end:



From the perspective of the pulse source, this properly terminated transmission line "looks" the same as an unterminated line of infinite length. There is no reflected pulse, and the DC voltage source "sees" an unchanging load resistance the entire time.

It is worth noting that the phenomenon of reflected pulses is consistent with the Law of Energy Conservation: the energy absorbed by the transmission line cannot simply disappear, and must be accounted for. In a line terminated by either a short or an open, this energy is not dissipated at the end and so must return (at its maximum velocity) as another traveling wave. Only with a dissipative element at the end of a transmission line will the pulse's energy leave the transmission line entirely and not continue to "echo" along its length. Moreover, this terminating resistance must have voltage/current characteristics matching the cable itself in order to dissipate all the pulse's energy.

An experiment performed with a termination resistor in place shows the near-elimination of reflected pulses:



The pulse looks much more like the square wave it should be, now that the cable has been properly terminated[5]. With the termination resistor in place, a transmission line *always* presents the same impedance to the source, no matter what the signal level or the time of signal application. Another way to think of this is from the perspective of cable length. With the proper size of termination resistor in place, *the cable appears infinitely long* from the perspective of the power source because it never reflects any signals back to the source and it always consumes power from the source.

Data communication cables for digital instruments behave as transmission lines, and must be terminated at both ends to prevent signal reflections. Reflected signals (or "echoes") cause errors in received data in a communications network when those reflections destructively interfere with new data still being transmitted[6], which is why proper termination can be so important. For point-to-point networks (networks formed by exactly two electronic devices, one at either end of a single cable), the proper termination resistance is often designed into the transmission and receiving circuitry, and so no external resistors need be connected. For "multi-drop" networks where multiple electronic devices tap into the same electrical cable, excessive signal loading would occur if each and every device had its own built-in termination resistance, and so the devices are built with no internal termination, and the installer must place two termination resistors in the network (one at each far end of the cable).

[5] The termination shown here is imperfect, as evidenced by the irregular amplitude of the square wave. The cable used for this experiment was a length of twin-lead speaker cable, with a characteristic impedance of approximately 120 Ohms. I used a 120 ohm ($\pm$ 5%) resistor to terminate the cable, which apparently was not close enough to eliminate all reflections.

[6] This is analogous to attempting to hold a spoken conversation in a room with severe echoes: the sound of the echoed speech interferes with the new words being spoken, making it difficult for listeners to discern what is being said. Interestingly, one way to mitigate this problem is to simply *talk slower*, and in fact it may suffice to decrease the data rate in a digital communication system as a means of limiting interference. However, the best solution is to eliminate the reflected signals altogether, and this is best accomplished in a transmission line by proper resistive termination.

3.5 Discontinuities

A transmission line's characteristic impedance will be constant throughout its length so long as its conductor geometry and dielectric properties are consistent throughout its length. Abrupt changes in either of these parameters, however, will create a *discontinuity* in the cable capable of producing signal reflections. This is why transmission lines must never be sharply bent, crimped, pinched, twisted, or otherwise deformed[7].

This phenomenon has a very interesting practical application, and that is *radar*: the detection of remote objects by radio wave reflections. A radio wave propagating through empty space travels at the speed of light, and when encountering an object presenting a different amount of dielectric permittivity and/or magnetic permeability than empty space, some of the radio energy is reflected away from that object. A receiving antenna positioned to detect this reflected energy is then able to detect the presence of the object. If the time delay between radio pulse transmission and reflected wave reception is precisely measured, the distance to that object may be inferred, much the same as the length of a transmission line may be inferred from the echo time delay of any pulse signal imposed at one end.

3.6 Velocity factor

The speed at which an electrical signal propagates down a transmission line is never as fast as the speed of light in a vacuum, owing to the permittivity of the space separating the conductors typically being greater than that of a vacuum. A value called the *velocity factor* expresses the propagation velocity as a ratio to light, and its value is always less than one:

$$\text{Velocity factor} = \frac{v}{c} = \frac{1}{\sqrt{\epsilon_r}}$$

Where,

v = Propagation velocity of signal traveling along the transmission line

c = Speed of light in a vacuum ($\approx 3.0 \times 10^8$ meters per second)

ϵ_r = Relative permittivity of the insulation (compared to that of a vacuum)

Velocity factor is a function of dielectric constant, but not conductor geometry. A greater permittivity[8] value results in a slower velocity (lesser velocity factor).

[7] A colleague of mine was once building a house for himself to live in, and while installing electrical wiring in the unfinished walls he decided to also install coaxial cable for television usage. By doing this at construction time instead of afterward (as cables are often retrofitted into older homes), he reasoned that it would make a much neater installation. Unfortunately he was not aware of the need to avoid discontinuities, and decided to staple the coaxial cable into place just as he was doing to the electrical "Romex" cable. The result was that after his house was built, he could not obtain a reliable television signal through his stapled cable! All that work and investment in cable while building his house was for naught.

[8] Technically, both *magnetic permeability* and dielectric permittivity affect the velocity at which electromagnetic waves propagate. However, since the only substances with magnetic permeability values greater than that of empty space also happen to be electrically conductive, we would never expect them to inhabit the space between conductors inside a cable, and therefore we ignore permeability in the velocity factor formula.

3.7 Cable losses

Ideally, a transmission line is a perfectly loss-less conduit for electrical energy. That is, every watt of signal power entering the transmission line is available at the end where the load is connected. In reality, though, this is never the case. Conductor resistance, as well as losses within the dielectric (insulating) materials of the cable, rob the signal of energy[9].

For transmission lines, power loss is typically expressed in units of decibels per 100 feet or per 100 meters. A "decibel," as you may recall, is ten times the logarithm of a power ratio:

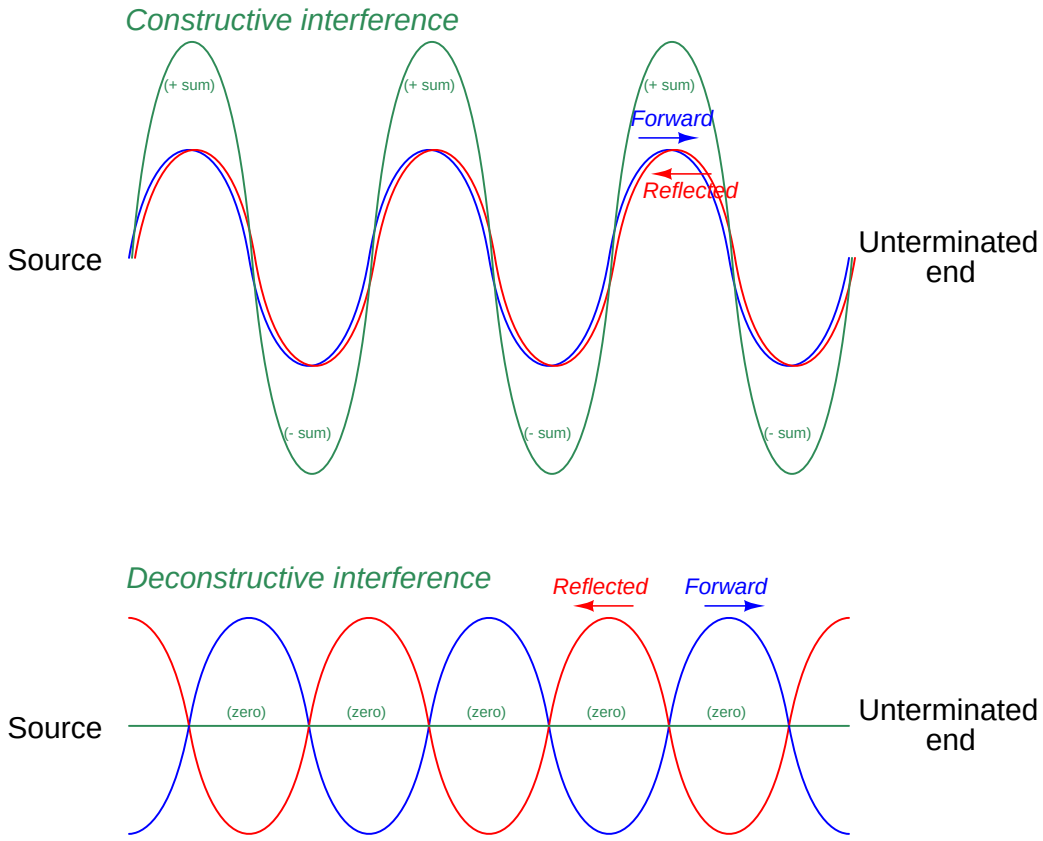
$$\text{Gain or Loss in dB} = 10 \log \left(\frac{P}{P_{ref}} \right)$$

Thus, if a transmission line receives 25 milliwatts of signal power at one end, but only conveys 18 milliwatts to the far end, it has suffered a 1.427 dB loss ($10 \log \frac{0.018}{0.025} = -1.427$ dB) from end to end. Power loss in cables is strongly dependent on frequency: the greater the signal frequency, the more severe the power loss per unit cable length.

[9] The molecules of dielectric materials become physically stressed under the influence of an electric field, and this in fact is why dielectric materials enhance capacitance: by stressing under an electric field, they act much like mechanical springs being stressed under force, and thereby help store more energy. However, the action of stressing and unstressing these molecules occurring under the influence of *alternating* electric fields means molecular motion is imparted to the dielectric material when exposed to AC voltages. Molecular motion is what *temperature* means for any material, and so AC electric fields cause dielectric materials to heat up. This heat dissipates into the surrounding environment as thermal energy never to return to the circuit, and therefore the dielectric material acts as a resistive load to the AC circuit of which the transmission line is a part.

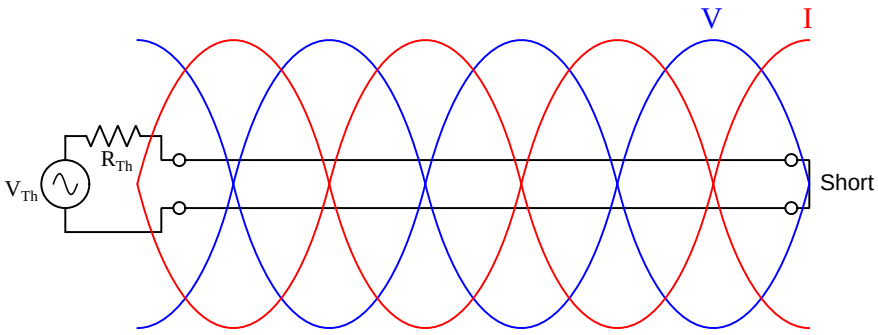
3.8 Standing waves

If reflections occur on a transmission line, the reflected wave interferes with the signal source's "forward" wave as the two waves travel in opposite directions. Every time the forward and reflected wave peaks are in-phase with each other, they add to create greater amplitude than either wave considered separately; every time the forward and reflected waves oppose in phase, they subtract and therefore diminish in amplitude. The fact that these two waves superimpose as they travel in opposite directions means their interference alternates between constructive and deconstructive (adding and subtracting). Since the two waves travel at the same speed in opposite directions, the zones of constructive and deconstructive interference tend to remain stationary at certain points along the transmission line's length at intervals of one wavelength:

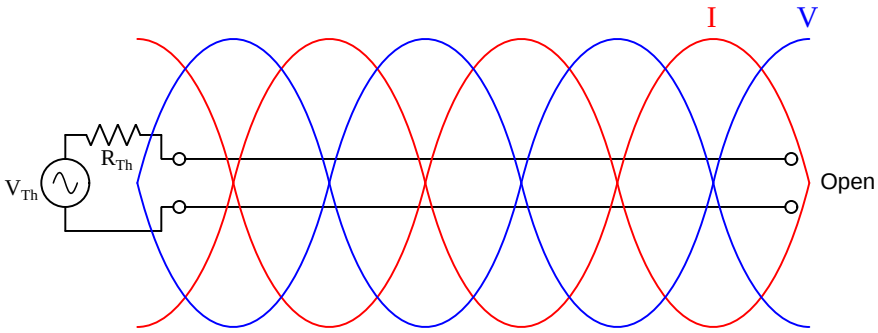


This phenomenon is known as a *standing wave*: regions of high alternating amplitude and low (or zero) amplitude that do not travel along the line in the same manner as either the forward or reflected wave but rather "stand still". To see a flip-book animation of how standing waves form, refer to section 5.4 beginning on page 101.

A short placed at the end of a transmission line creates a condition at that location where voltage cannot exist but current can. Therefore, the standing waves existing along a shorted transmission line are such that the voltage wave terminates at zero amplitude (called a *node*) while the current wave terminates at maximum amplitude (called an *antinode*):



Conversely, an open-ended transmission line exhibits standing waves where voltage is maximum at the end of the line but current is zero, because voltage can exist at an open but current cannot:



Even more interesting is the amount of impedance "seen" by the signal source as a result of these standing waves. Every quarter-wavelength along the transmission line the voltage and current standing waves alternate between nodes and antinodes (minimum and maximum amplitudes, respectively). Therefore, if the transmission line happens to be any whole-multiple of quarter-wavelengths' in length, the source will experience the exact opposite impedance to how the line actually terminates. The examples show above are illustrative of this, where we see 1-3/4 wavelengths between the end of the line and the source's R_{Th}. Note how the shorted line has no voltage and full current at the far end, but no current and full voltage at the source's end, which means from the source's perspective the line "appears" to be open rather than shorted. Note how the open-ended line has no current and full voltage at the far end, but no voltage and full current at the source's end, presenting a virtual short-circuit to the source.

When signal reflection is total, standing waves achieve twice the voltage and twice the current at their respective maxima (antinodes) compared to what the forward wave would be without any reflections. If only part of the forward wave gets reflected off the end of the line and returns to the source, the weaker reflected wave does not completely cancel out or double the amplitude of the forward wave through interference. This results in a standing wave with less peak amplitude and no point of zero magnitude anywhere on the line. To see a flip-book animation of standing waves forming as a result of half the forward signal strength being reflected back toward the source, refer to section 5.5 beginning on page 191.

An important measure of standing waves on any operating transmission line is the *standing wave ratio* or *SWR*. This is the ratio of the standing waves' maximum amplitude (at the antinodes) versus their minimum amplitude (at the nodes). For a line where 100% of the forward reflects back toward the source, the SWR is infinite. For real-world applications the SWR will be finite, but always greater than 1 which is the ideal condition achieved with perfect line termination (i.e. zero reflection). Expressed in terms of voltage, the standing wave ratio is empirically[10] defined as follows:

$$\text{VSWR} = \frac{V_{max}}{V_{min}}$$

Standing waves are usually undesirable because they "stress" the transmission line by creating large amounts of voltage and current at certain locations, resulting in excessive power dissipation and in some cases sufficient voltage and/or current to actually cause damage to the line or to equipment connected to it. Maintaining a low SWR value, therefore, can be important for efficient operation and long service life in applications where the amount of power conveyed along a transmission line is substantial, such as in high-power radio broadcast systems.

[10] An "empirical" definition is one rooted in real-world measurements, as opposed to something defined in terms of theoretical predictions. Here, the standing wave ratio is defined by means of actual measured voltage along the line, maximum versus minimum.

3.9 Reflection coefficient

When a signal reflects off the end of a transmission line and returns back toward the source, its magnitude and phase shift depend on the mis-match between the line's characteristic impedance and the terminating impedance (usually a load). If we measure the forward and reflected signals by their *voltages*, we may define their ratio as a quantity called the *reflection coefficient* (symbolized as Γ, the Greek capital letter "Gamma"). This coefficient, like the voltages themselves, must be a complex number (phasor) so that we are able to capture both the magnitudes and relative phase angles of the forward and reflected signals:

$$\Gamma = \frac{V_r}{V_f}$$

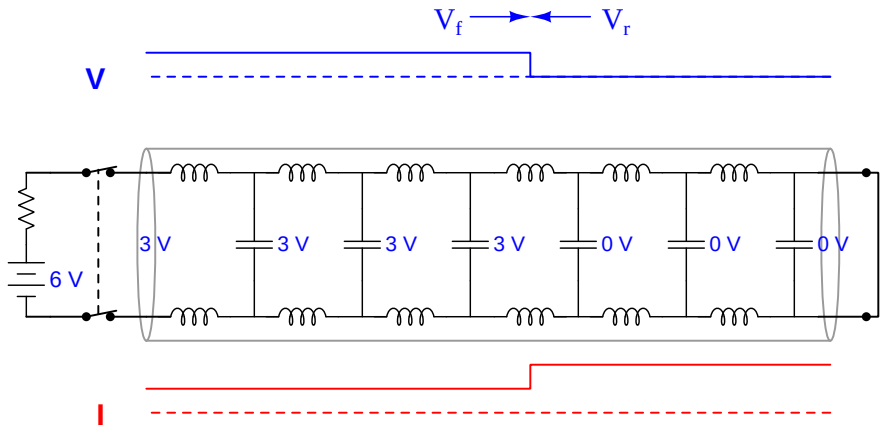
Where,

Γ = Reflection coefficient, unitless

V_r = Voltage value of reflected signal, in complex Volts

V_f = Voltage value of forward signal, in complex Volts

Let's consider the case of a short-circuited transmission line where a condition of zero voltage must[11] exist at the short. We may visualize this concept by imagining a 6-Volt DC source (with an internal Thévenin resistance R_{Th} equal to the line's impedance) suddenly being connected to one end of a transmission line, with the far end of that line being shorted. If we examine conditions along this transmission line at a time where the reflected signal has traveled about half-way back toward the DC source, we see negligible voltage at the far end of the line and half voltage (3 Volts) at the source end:



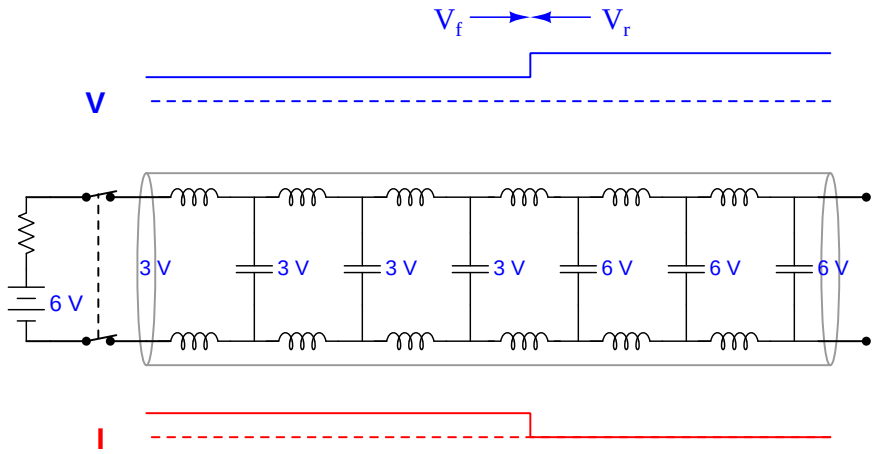
We know the short creates a zero-voltage condition at the line's far end, and over time this becomes a zero-voltage condition along the entire line. If we view that final condition as being the *superposition* of the forward and reflected pulses, we must conclude that the reflected voltage pulse has the opposite polarity (i.e. 180 degrees out of phase) of the forward pulse in order to cancel out and leave zero voltage as the two pulses overlap. This means $V_r = -V_f$, which means the line's reflection coefficient must be negative one:

$$\Gamma = \frac{V_r}{V_f} = \frac{-V_f}{V_f} = -1$$

Interestingly, the reflected *current* pulse does not cancel out the forward pulse as it does for voltage. Instead, the reflected current *adds* to the forward current to produce a larger current. Thus, the reflected current pulse must have the same polarity (sign) as the forward current pulse, which is to say the phase-shift for reflected current is 0 degrees.

[11] This is a foundational concept going all the way back to basic DC circuits: the guaranteed effect of a *short* is *zero voltage* because electric charges will not gain or lose any energy moving between points that are electrically common to each other.

Now let's consider an open-circuited transmission line subjected to the same test. Here, the open-circuit at the line's end forces a zero-current condition[12], which propagates all the way back to the source as the reflected wave returns toward it:



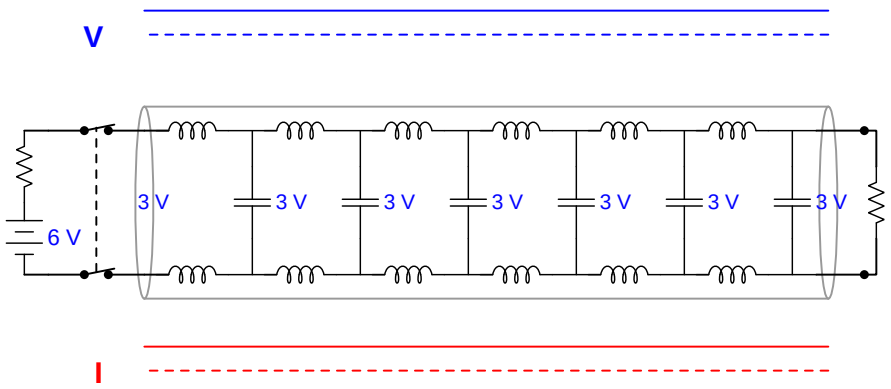
We know the open creates a full-voltage condition at the line's far end, and over time this becomes a full-voltage condition along the entire line. If we view that final condition as the *superposition* of the forward and reflected pulses, it means the reflected voltage pulse has the same polarity (i.e. 0 degrees out of phase) as the forward pulse in order to double the forward pulse's magnitude. This means $V_r = V_f$, which means the line's reflection coefficient must be positive one:

$$\Gamma = \frac{V_r}{V_f} = \frac{V_f}{V_f} = 1$$

This time, it is the *current* pulse that cancels out the forward pulse as it returns back to the source. Thus, the reflected current pulse must have the opposite polarity (sign) as the forward current pulse, which is to say the phase-shift for reflected current is 180 degrees.

[12] This is another foundational concept going all the way back to basic DC circuits: the guaranteed effect of an *open* is *zero current* because electric charges cannot move through an electrically insulating medium.

If the transmission line is properly terminated, there is no reflected signal at all, which is to say $V_r = 0$:



In this case, the reflection coefficient must be equal to zero as well:

$$\Gamma = \frac{V_r}{V_f} = \frac{0}{V_f} = 0$$

So, we see from these three examples that reflection coefficient is a complex ratio of the reflected versus forward voltage signals traveling along a transmission line. This coefficient may be empirically defined by measurements of reflected and forward voltage, or predicted by comparisons of the line and termination impedances as shown in the equation below:

$$\Gamma = \frac{Z_{term} - Z_{line}}{Z_{term} + Z_{line}}$$

Where,

Γ = Reflection coefficient, unitless

Z_{term} = Terminating impedance, complex Ohms

Z_{line} = Characteristic (surge) impedance, complex Ohms

In turn, we may predict the voltage standing wave ratio (VSWR) of any terminated line by using the following equations:

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{Z_{larger}}{Z_{smaller}}$$

Where,

$|\Gamma|$ = Magnitude (without phase angle) of the reflection coefficient, unitless

An elementary example serves to illuminate these formulae: a transmission line with a resistive characteristic impedance of 50 Ohms terminated by an 80 Ohm resistor would give us a reflection coefficient of 0.2308 and a standing wave ratio of 1.6. Note how we may alternatively calculate the voltage standing wave ratio (VSWR) simply by dividing the larger impedance value by the smaller impedance value, without having to calculate reflection coefficient.

Chapter 4

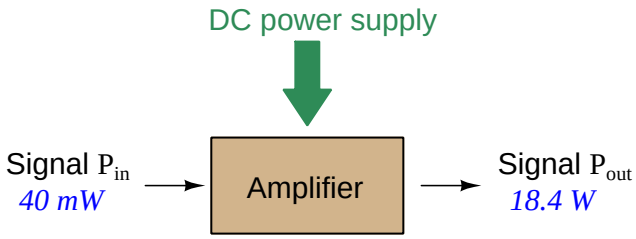
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the *common logarithm*, expressed as a measurement unit called the *decibel*. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of "decibel" you can think: *this is an expression of how much greater (or how much smaller) one power is to another*. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a *gain*. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{18.4\ W}{40\ mW} = 460$$

An alternative way to express the gain of this amplifier is to do so using the unit of the *Bel*, defined as the common logarithm of the gain ratio:

$$\log\left(\frac{P_{out}}{P_{in}}\right) = \log\left(\frac{18.4\ \text{W}}{40\ \text{mW}}\right) = 2.66276\ \text{B}$$

When you see an amplifier gain expressed in the unit of "Bel", it's really just a way of saying "The output signal coming from this amplifier is x powers of ten greater than the input signal." An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the "Bel" (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the *deci*Bel (1 dB = $\frac{1}{10}$ of a Bel). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

$$\text{dB} = 10 \log\left(\frac{P_{out}}{P_{in}}\right)$$

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid[1].

[1] It is interesting to note that although the "Bel" is a metric unit, it is seldom if ever used without the metric prefix

An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

$$\text{dB} = 10 \log(\text{Ratio})$$

If we wish to solve for the ratio, we must "undo" all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

$$\frac{\text{dB}}{10} = \frac{10 \log(\text{Ratio})}{10}$$

$$\frac{\text{dB}}{10} = \log(\text{Ratio})$$

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

$$10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}$$

$$10^{\frac{\text{dB}}{10}} = \text{Ratio}$$

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

$$\text{Ratio} = 10^{\frac{26.6276 \text{ dB}}{10}}$$

$$\text{Ratio} = 10^{2.66276 \text{ B}}$$

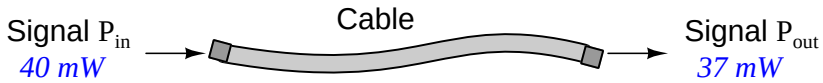
$$\text{Ratio} = 460$$

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

"deci" ($\frac{1}{10}$). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.

We may also use decibels to express power *losses* in addition to power *gains*. There are many practical applications of this in signaling systems, both electronic and optical. One such application is *filtering*, where a "filter" circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is *attenuation*, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: *cable loss*. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal's energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length[2], such that the power out is less than the power in:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{37\ mW}{40\ mW} = 0.925$$

$$10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{37 \text{ mW}}{40 \text{ mW}} \right) = -0.3386 \text{ dB}$$

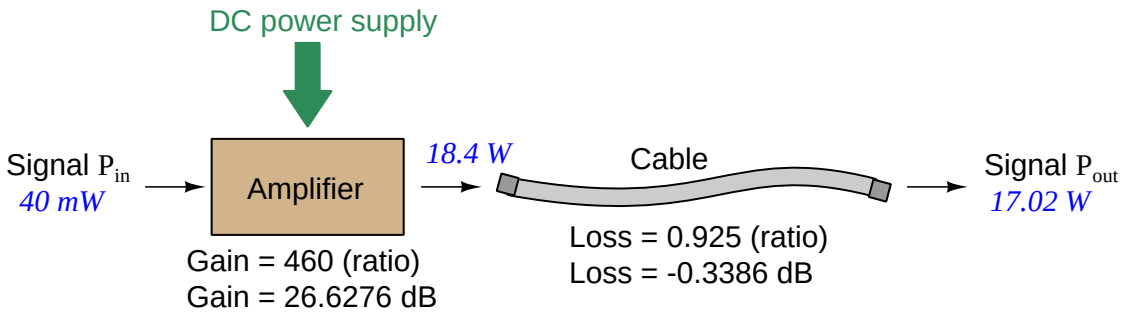
Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power *gain* is expressed as a *positive* decibel value, while any power *loss* is expressed as a *negative* decibel value. Any component outputting the exact same power as it takes in will exhibit a "gain" value of 0 dB (equivalent to a gain *ratio* of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of "greater than" and "less than". Positive values represent powers that are *greater* while negative values represent powers that are *lesser*. Zero Bel or decibel values represent *no change* (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of $10 \times 10 = 100$, whereas −30 dB represents a power ratio reduction of $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$. Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division or power ratio: e.g. +6 dB is approximately equal to a power ratio gain of $2 \times 2 = 4$, whereas −12 dB is approximately equal to a power ratio reduction of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$. We may combine ± 10 dB and ± 3 dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of $10 \times 10 \times 2 \times 2 = 400$.

[2] For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is *dielectric heating*, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.

Observe what happens if we combine a "gain" component with a "loss" component and calculate the overall power out versus power in:



The overall gain of this amplifier and cable system expressed as a ratio is equal to the *product* of the individual component gain/loss ratios. That is, the gain ratio of the amplifier *multiplied* by the loss ratio of the cable yields the overall power ratio for the system:

$$\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5$$

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the *sum* of the individual component decibel values. That is, the decibel gain of the amplifier *added* to the decibel loss of the cable yields the overall decibel figure for the system:

$$\text{Overall gain} = 10 \log \left(\frac{17.02 \text{ W}}{40 \text{ mW}} \right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}$$

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

$$425.5 \approx 400 = (10) \times (10) \times (2) \times (2)$$

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

$$(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}$$

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.

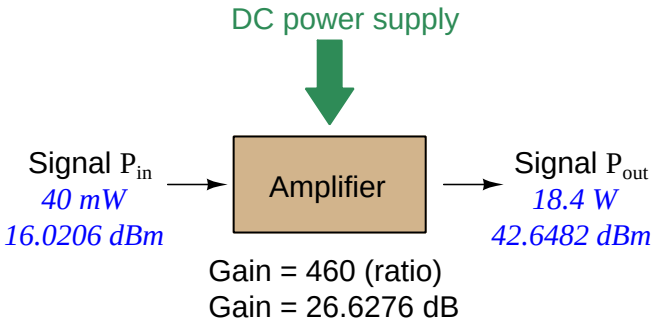
Decibels always represent comparisons of power, but that comparison need not always be P_{out}/P_{in} for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard "reference" power of exactly 1 milliWatt. The resulting decibel figure would be written as "dBm" in honor of the 1 *m*illiWatt reference:

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}$$

The unit of "dBm" literally means the amount of dB "greater than" 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}$$

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.



Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBm:

$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

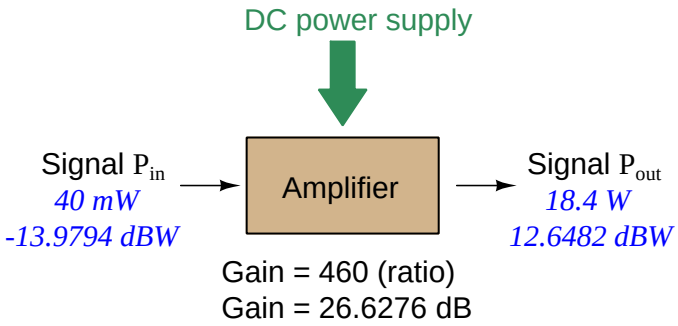
$$16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}$$

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.

We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of "dBW" (decibels greater than 1 Watt):

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW}$$

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW}$$



Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is *less* than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is *more* than the 1 Watt reference. A positive dB figure means "more than" while a negative dB figure means "less than."

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBW:

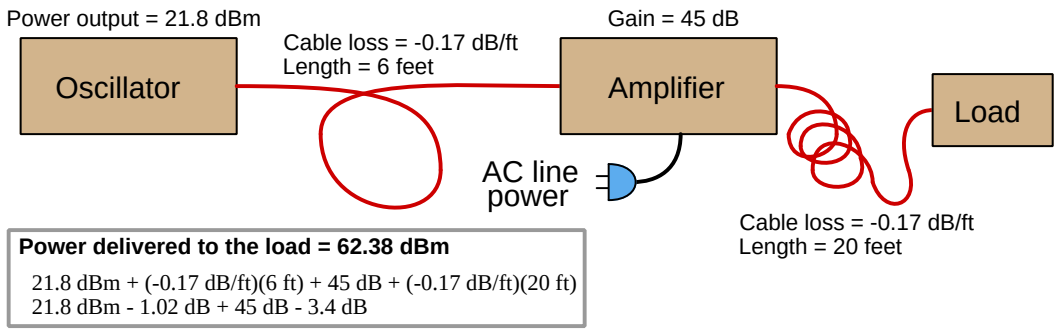
$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

$$-13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW}$$

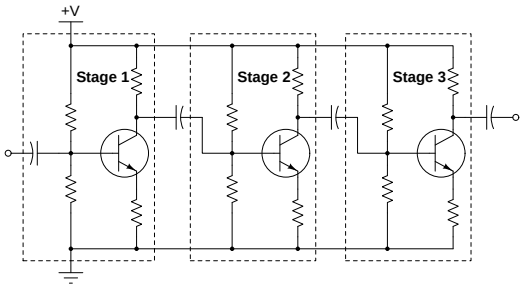
An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.

This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of *ratios* involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they "transform[3]" one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of -0.15 decibels per foot at a signal frequency of 2.4 GHz will suffer -15 dB over 100 feet, and -150 dB over 1000 feet. To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:



A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, *multiply* to make the over-all amplifier's power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, *add* to make the over-all amplifier's power gain (dB):



[3] In fact, logarithms are one of the simplest examples of a *transform function*, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the *Fourier transform* (converting a time-domain function into a frequency-domain function) and the *Laplace transform* (converting a differential equation into an algebraic equation).

Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a *power* ratio, we cannot simply substitute V or I for P in any of the formulae we've seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. V_{out} is twice as large as V_{in}), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load's power dissipation will be more than double when energized by a voltage twice as large. Joule's Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power *quadrupling* because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule's Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left(\frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

$$\log x^y = y \log x$$

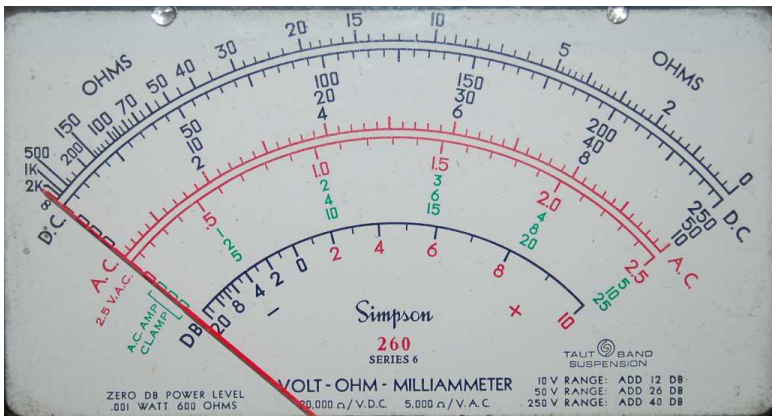
Therefore, we may simplify the decibel formula for voltage gain by removing the "2" power and making it a multiplier:

$$10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = (2)(10) \log \left(\frac{V_{out}}{V_{in}} \right) = 20 \log \left(\frac{V_{out}}{V_{in}} \right)$$

$$10 \log \left(\frac{I_{out}}{I_{in}} \right)^2 = (2)(10) \log \left(\frac{I_{out}}{I_{in}} \right) = 20 \log \left(\frac{I_{out}}{I_{in}} \right)$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.

We can see the practicality of using decibels to represent something other than electrical *power* by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter's face, calibrated in decibels (DB):



Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB[4]. A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC *voltage*. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically *undefined*[5]. Practically, it means $-\infty$ dB, which is why the needle at the 0 Volt position "falls off" the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the *nonlinear* nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the *logarithm* function which is nonlinear.

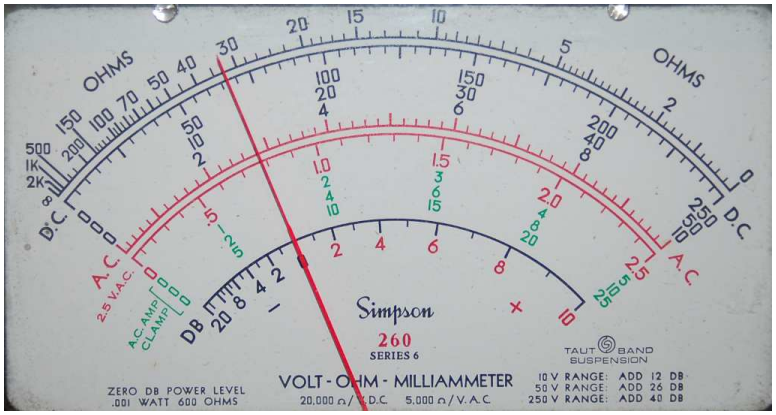
[4] Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

[5] Your electronic calculator will complain if you attempt to take the logarithm of zero!

Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

$$V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts}$$

Setting the VOM to the 2.5 VAC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):



In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range *four times* as great), the meter's needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

$$20 \log \left(\frac{2.5}{10} \right) = -12.04 \text{ dB}$$

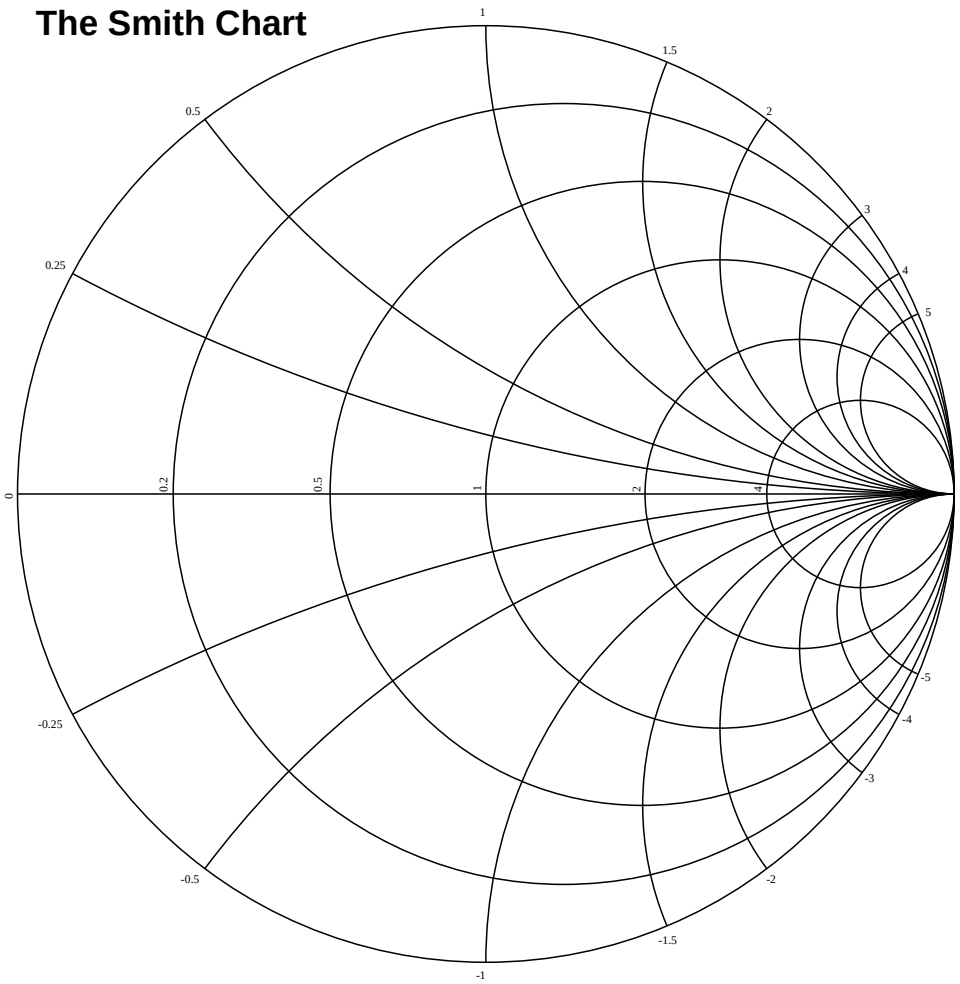
Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

$$20 \log \left(\frac{2.5}{50} \right) = -26.02 \text{ dB}$$

$$20 \log \left(\frac{2.5}{250} \right) = -40.0 \text{ dB}$$

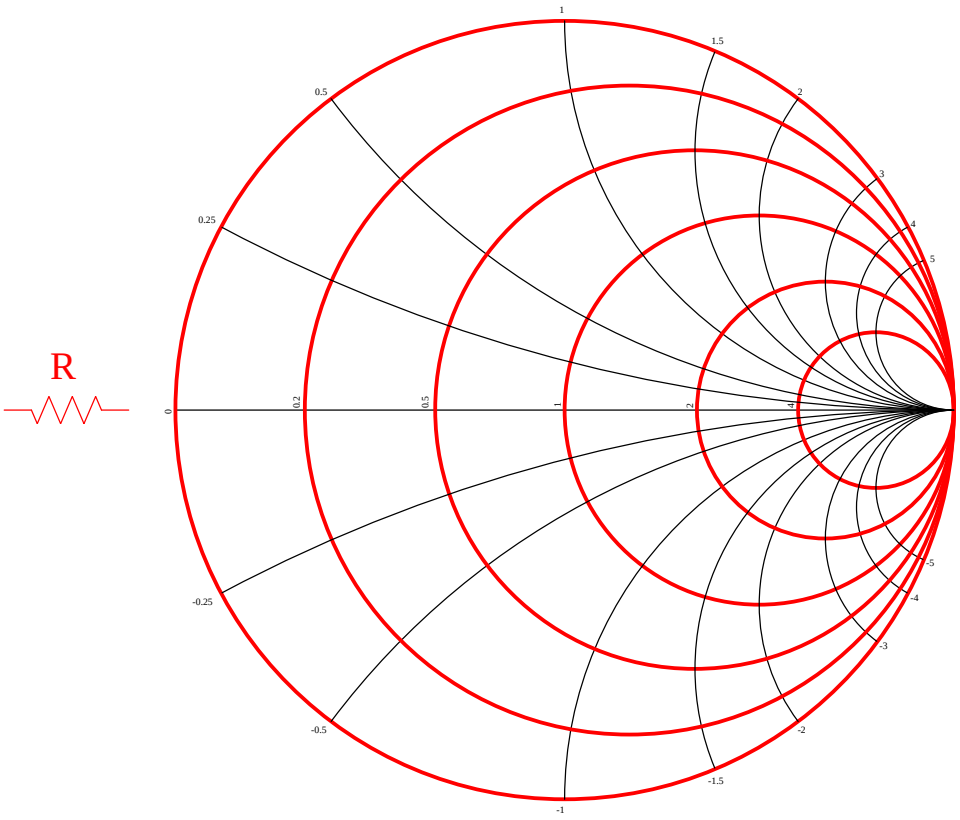
4.2 Smith charts

The *Smith chart* is a nomograph[6] useful for transmission line calculations where a mismatched load terminates the line. It is formed by two sets of intersecting circles, one set of circles centered along the horizontal axis representing normalized resistance values (i.e. termination resistance as a ratio of transmission line characteristic impedance) and another set of circles centered along the vertical axis representing normalized reactance values (i.e. termination reactance as a ratio of transmission line characteristic impedance):

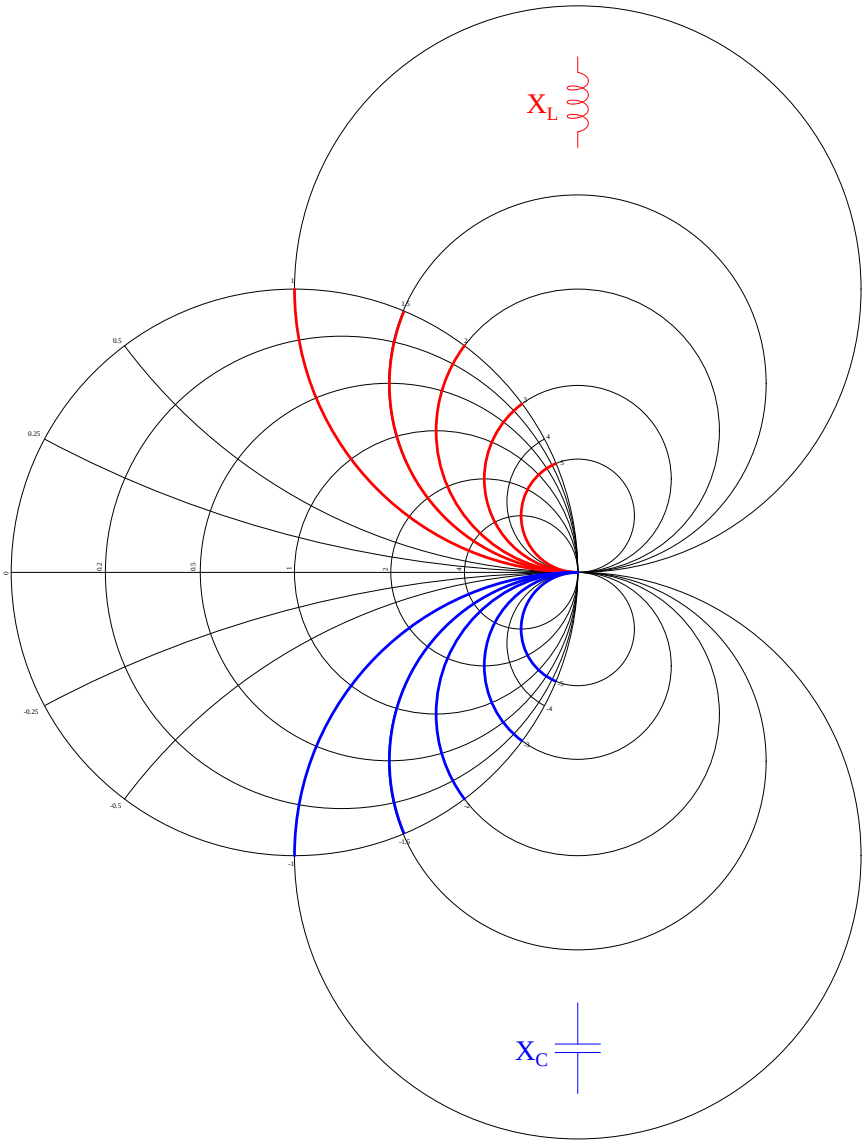


[6] A "nomograph" is any form of graphical calculating aid, where quantities are represented by positions along axes.

The resistance circles of a Smith chart lie along the horizontal axis and intersect at the far-right end of that axis, each one having a radius proportional to $\frac{1}{R+1}$ where R is the *normalized* resistance of the terminating impedance (i.e. the terminating impedance's resistance value divided by the transmission line's characteristic impedance value of 50 Ω):

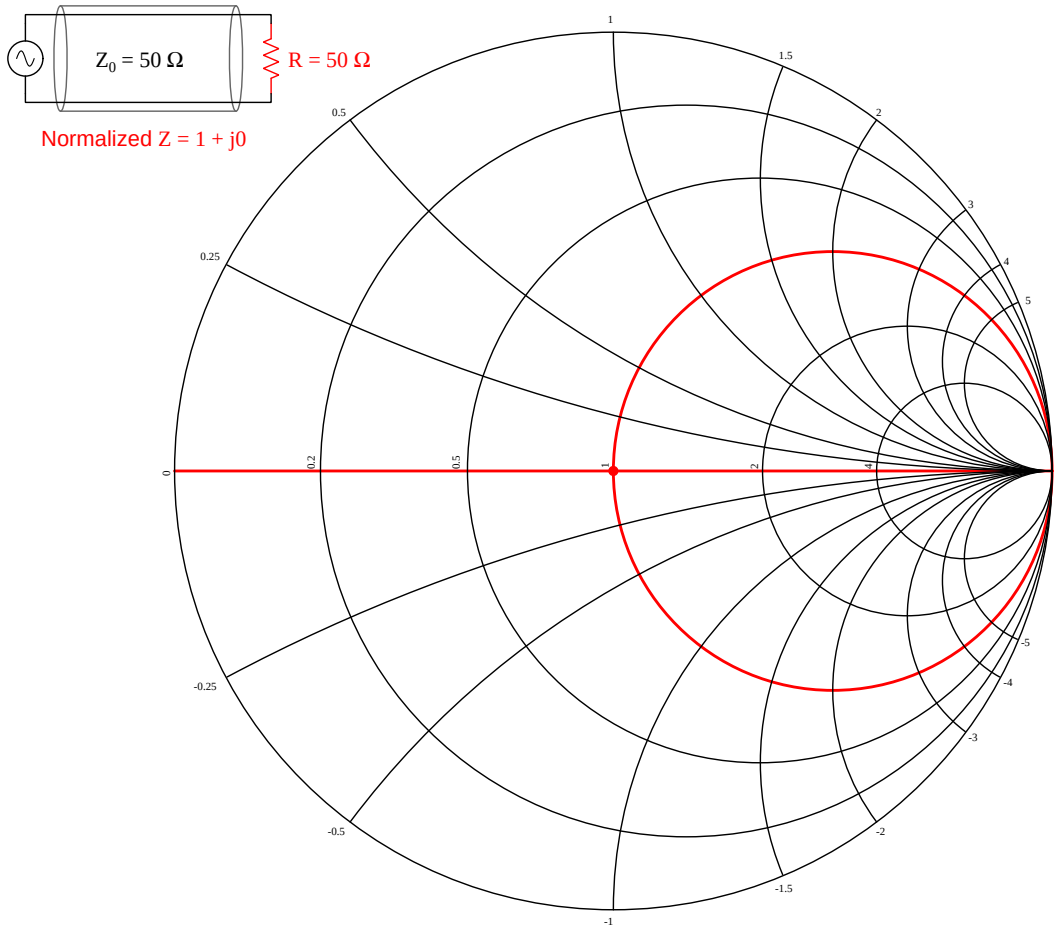


The reactance circles of a Smith chart lie along the vertical axis and intersect at the far-right end of the horizontal axis, each one having a radius proportional to $\frac{1}{X}$ where X is the *normalized* reactance of the terminating impedance (i.e. the terminating impedance's reactance value divided by the transmission line's characteristic impedance value of 50 Ω):



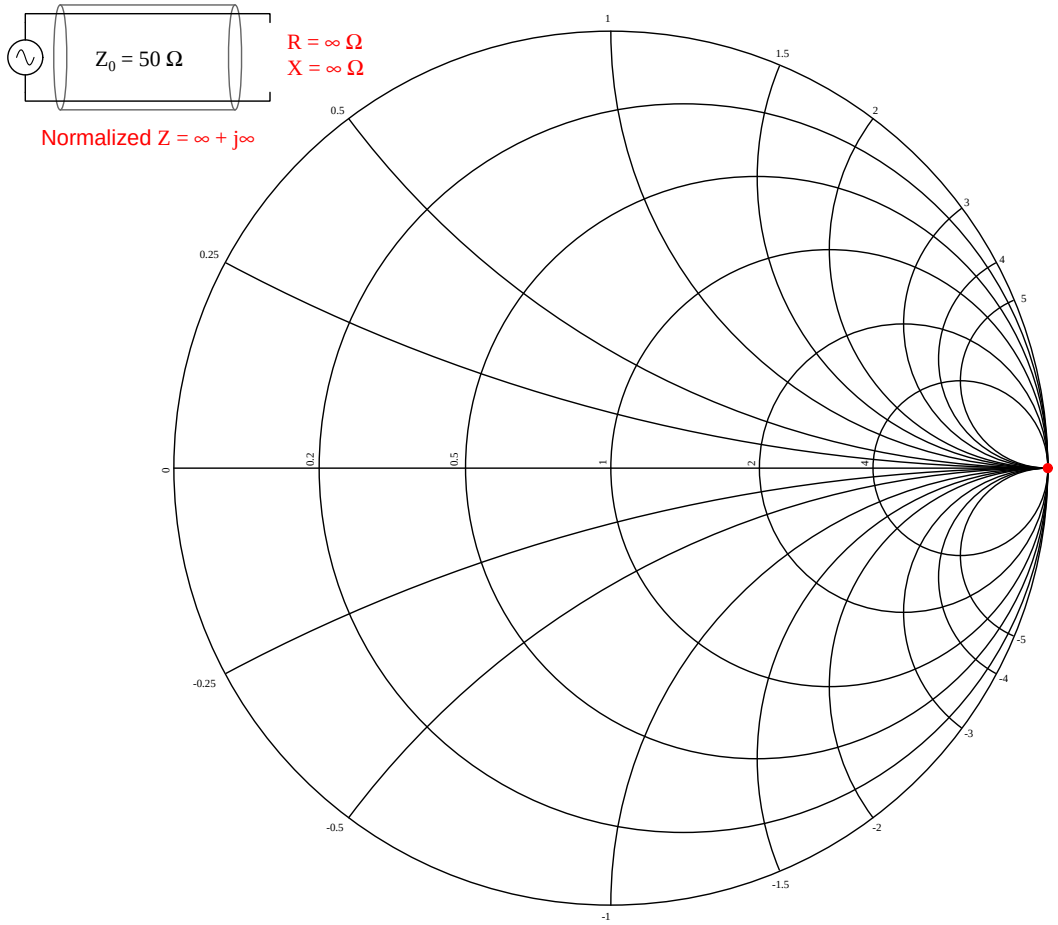
Inductive reactance values are positive (above the horizontal axis) while capacitive reactance values are negative (below the horizontal axis).

If the transmission line is perfectly terminated, in this case the 50 Ω line being terminated by a 50 Ω resistance, the normalized value of this terminating impedance will be $1 + j0$ which places it at the exact center of the Smith chart:



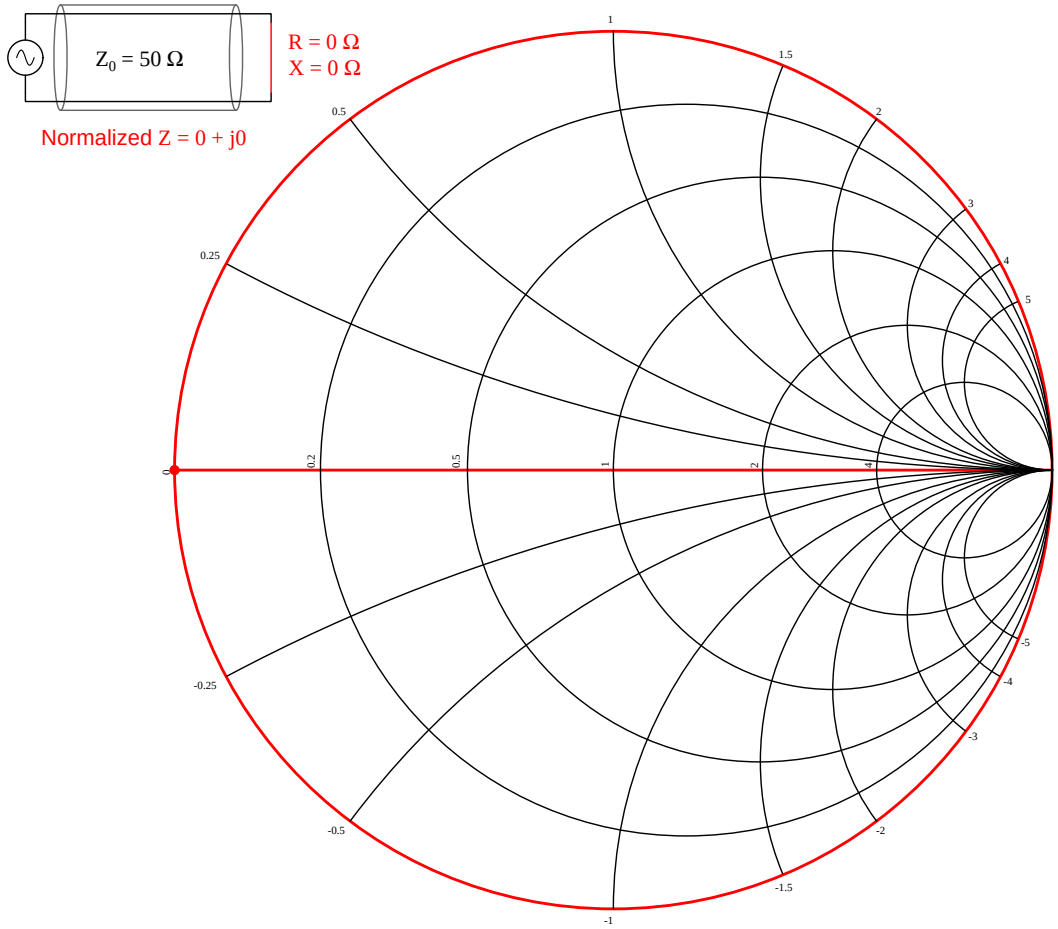
This is the point marked by the intersection of the "1" resistance circle and the "0" reactance circle (i.e. a "circle" with an infinite radius which is really a straight line overlapping the horizontal axis).

The limiting cases of an open-terminated transmission line is represented by a point with infinite resistance and reactance, located at the far-right end of the horizontal axis:



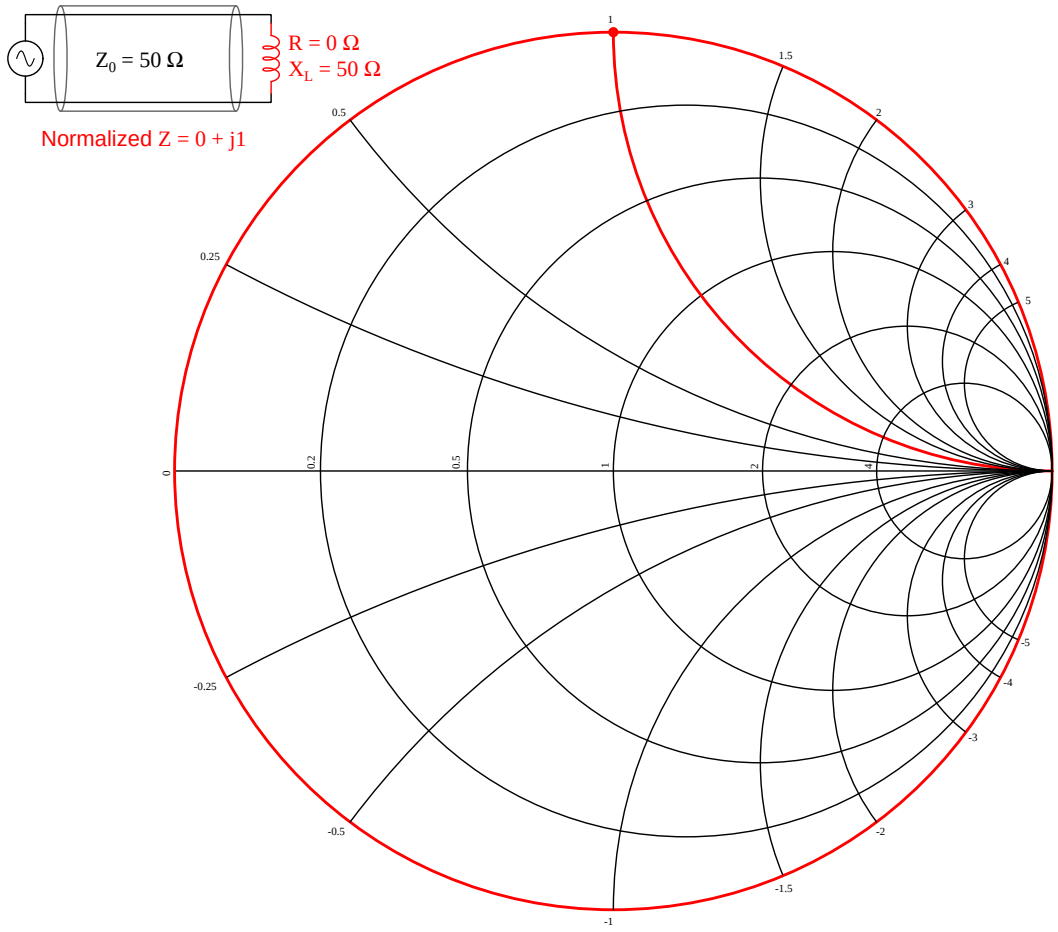
This is the point marked by the intersection of a zero-radius resistance circle and a zero-radius reactance circle.

The limiting cases of a short-terminated transmission line is represented by a point with zero resistance and reactance, located at the far-left of the horizontal axis:

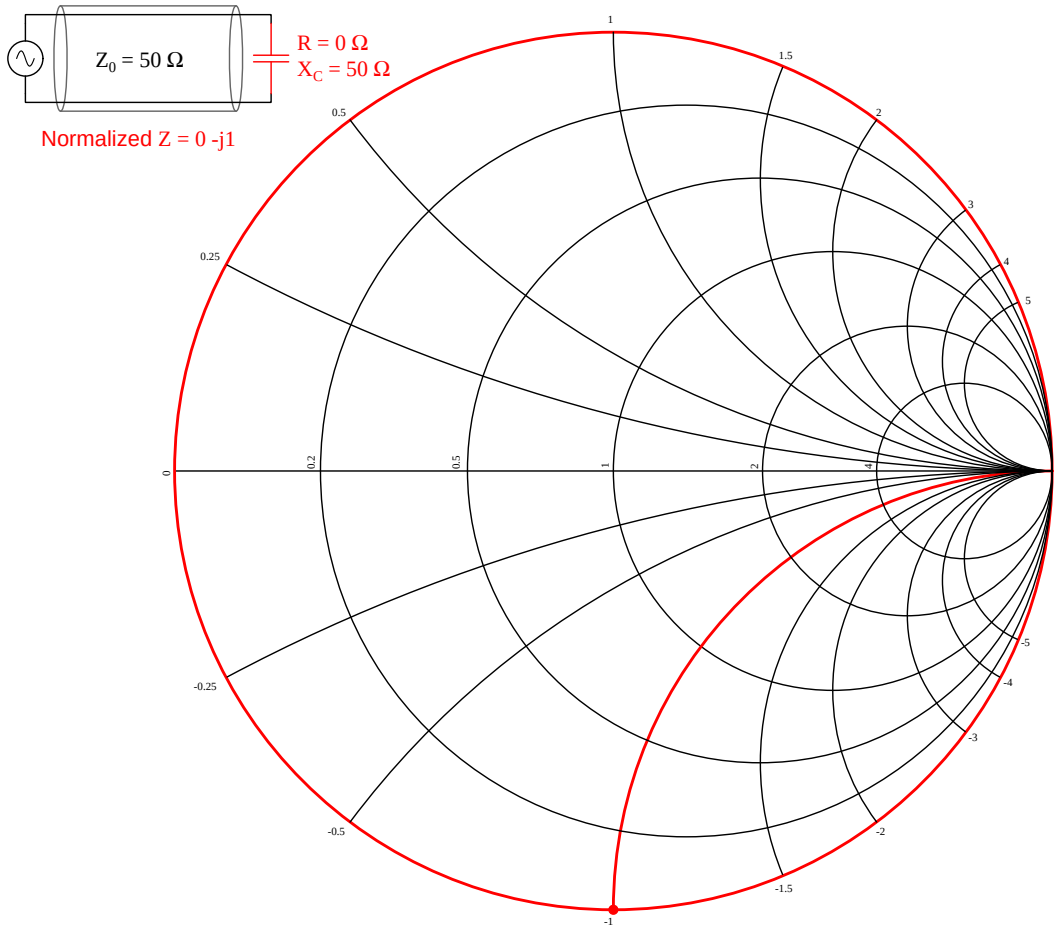


This is the point marked by the intersection of a full-radius resistance circle and a reactance circle having an infinite radius (i.e. a "circle" which is really a straight line overlapping the horizontal axis).

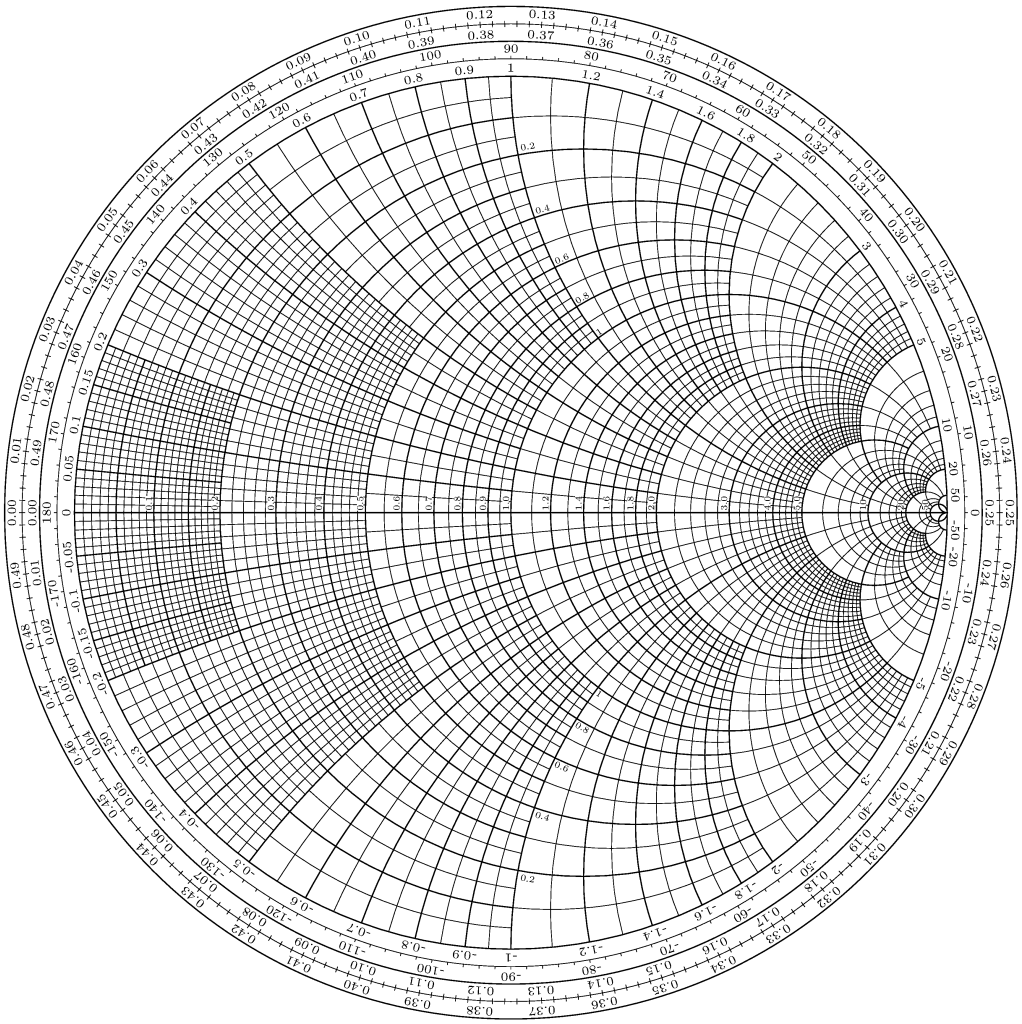
The upper half of the Smith chart is where all the inductive curves reside, which means if we terminate the 50 Ω transmission line with a pure inductance of 50 Ω the location will be where the +1 reactance circle intersects with the 0 resistance circle:



The lower half of the Smith chart is where all the capacitive curves reside, which means if we terminate the 50 Ω transmission line with a pure capacitance of 50 Ω the location will be where the −1 reactance circle intersects with the 0 resistance circle:



A practical Smith chart (courtesy of Germinal Camp[7]) is shown here:



[7]Germinal wrote code for a LaTeX document to draw this detailed Smith Chart, and from this source code one can generate a Postscript or PDF image. I took the Postscript output and converted that into an SVG image, which then imported well into Xcircuit where it is easier to overlay annotations.

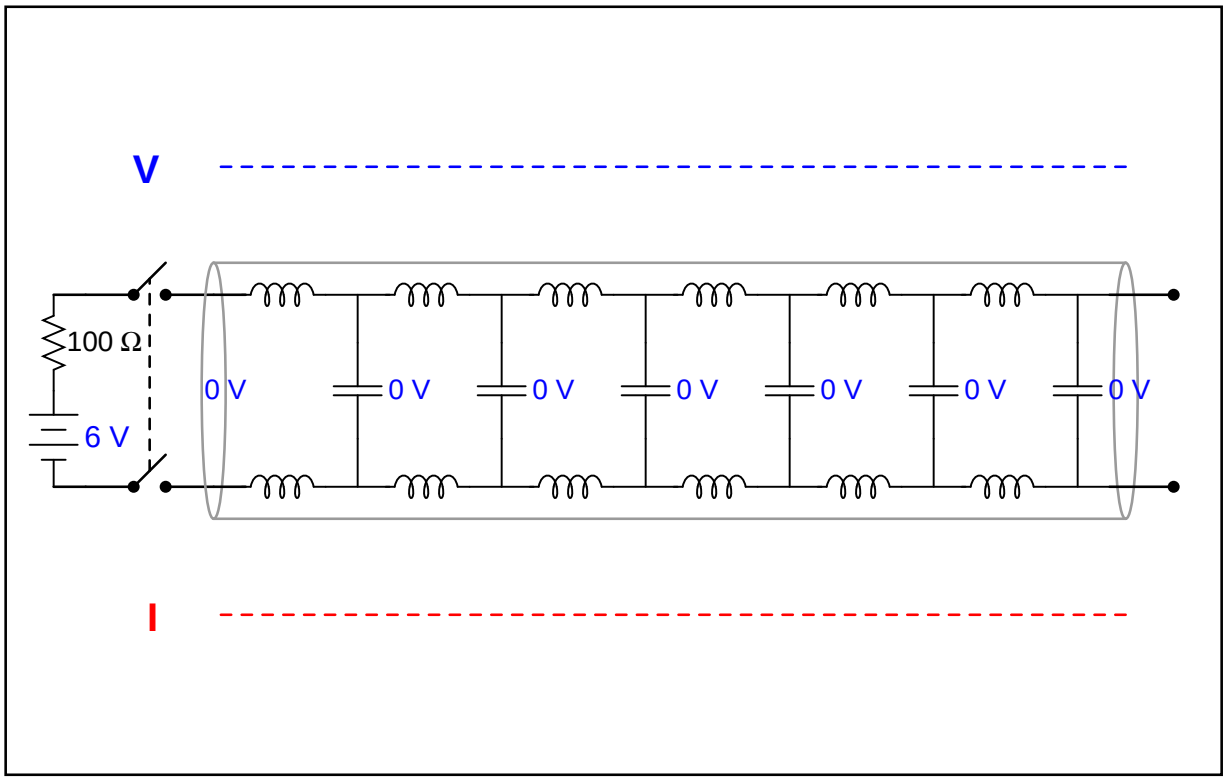
Chapter 5

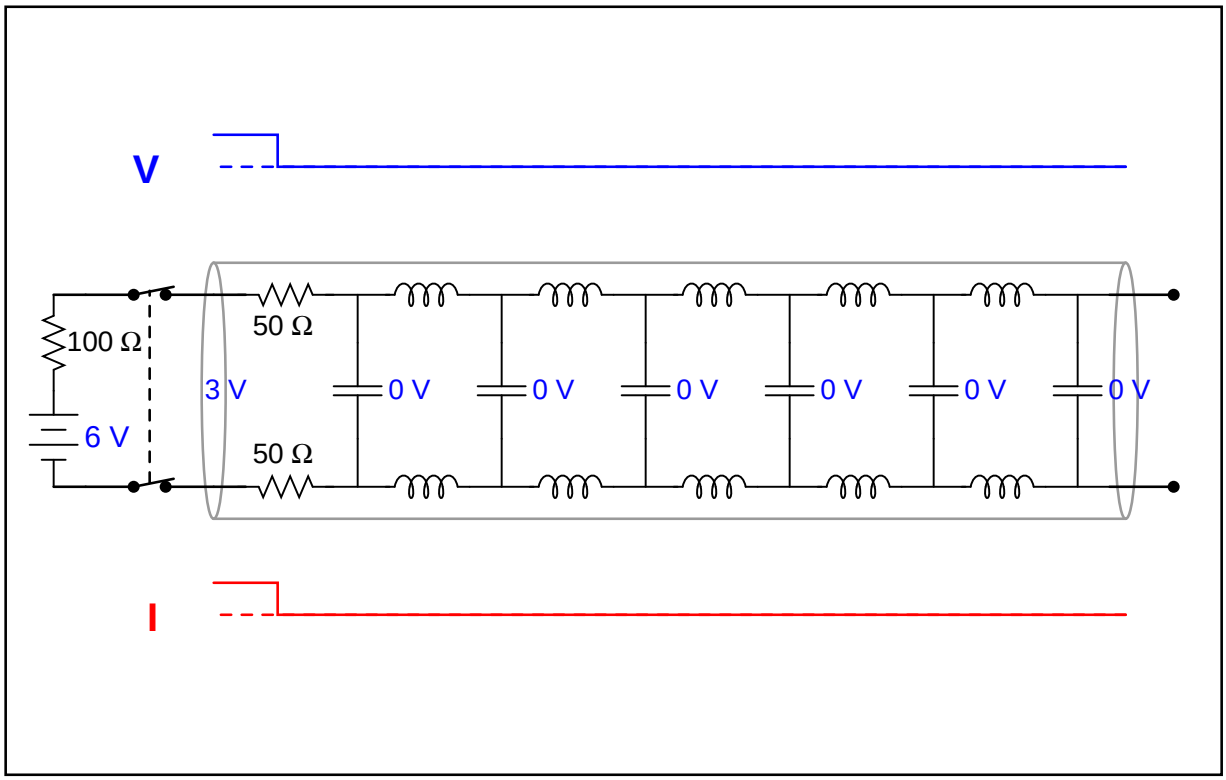
Animations

Some concepts are much easier to grasp when seen in *action*. A simple yet effective form of animation suitable to an electronic document such as this is a "flip-book" animation where a set of pages in the document show successive frames of a simple animation. Such "flip-book" animations are designed to be viewed by paging forward (and/or back) with the document-reading software application, watching it frame-by-frame. Unlike video which may be difficult to pause at certain moments, "flip-book" animations lend themselves very well to individual frame viewing.

5.1 Animation of an open transmission line

The following animation shows the progression of voltage and current down the length of a 100 Ohm transmission line while energized by a 6 Volt DC source, that line being terminated by an open-circuit.


V
100 Ω
6 V
0 V
0 V
0 V
0 V
0 V
0 V
0 V
I


V
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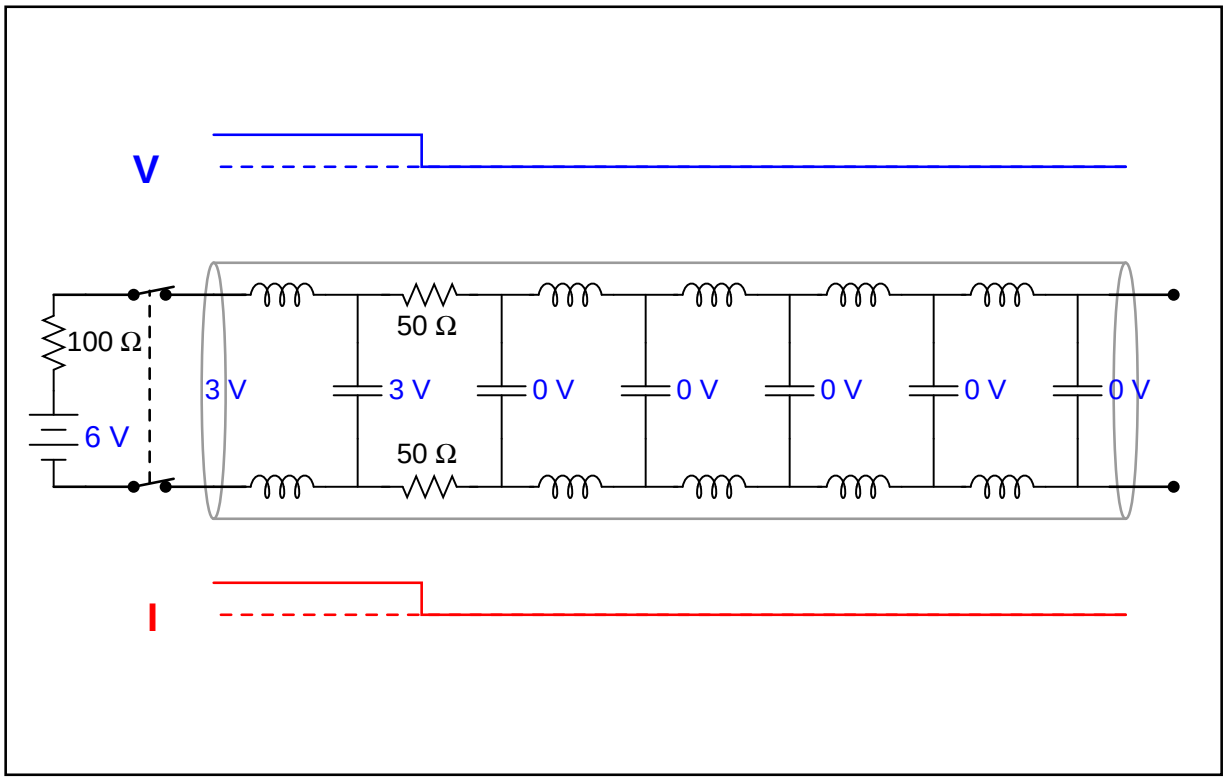
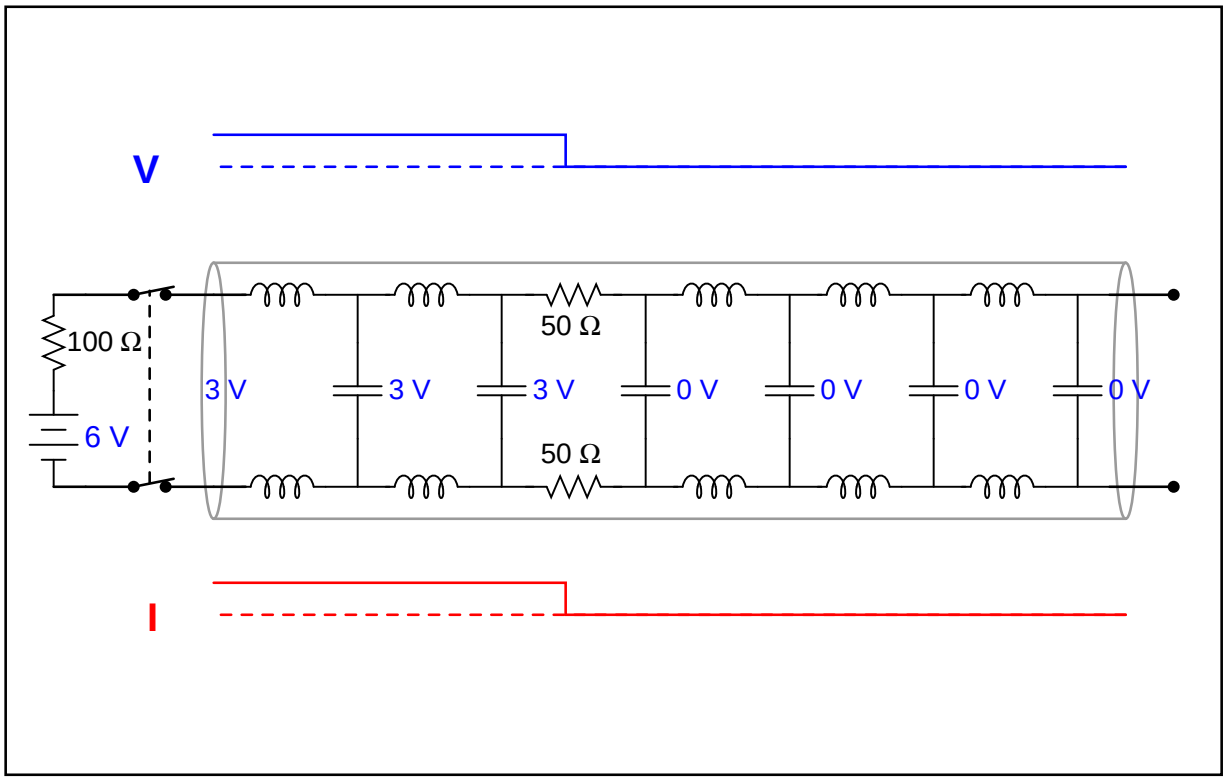
6 V
50 Ω
3 V
0 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I


V
100 Ω
6 V
50 Ω
3 V
3 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I


V
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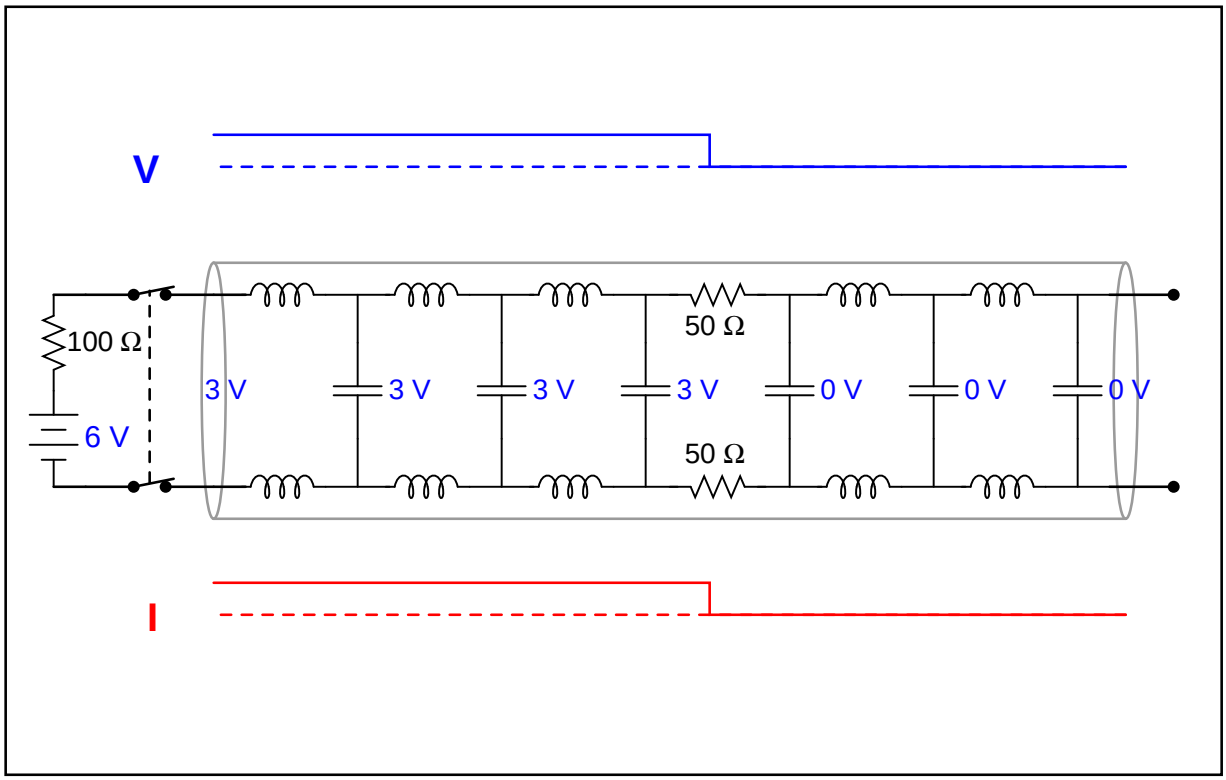
6 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
0 V
I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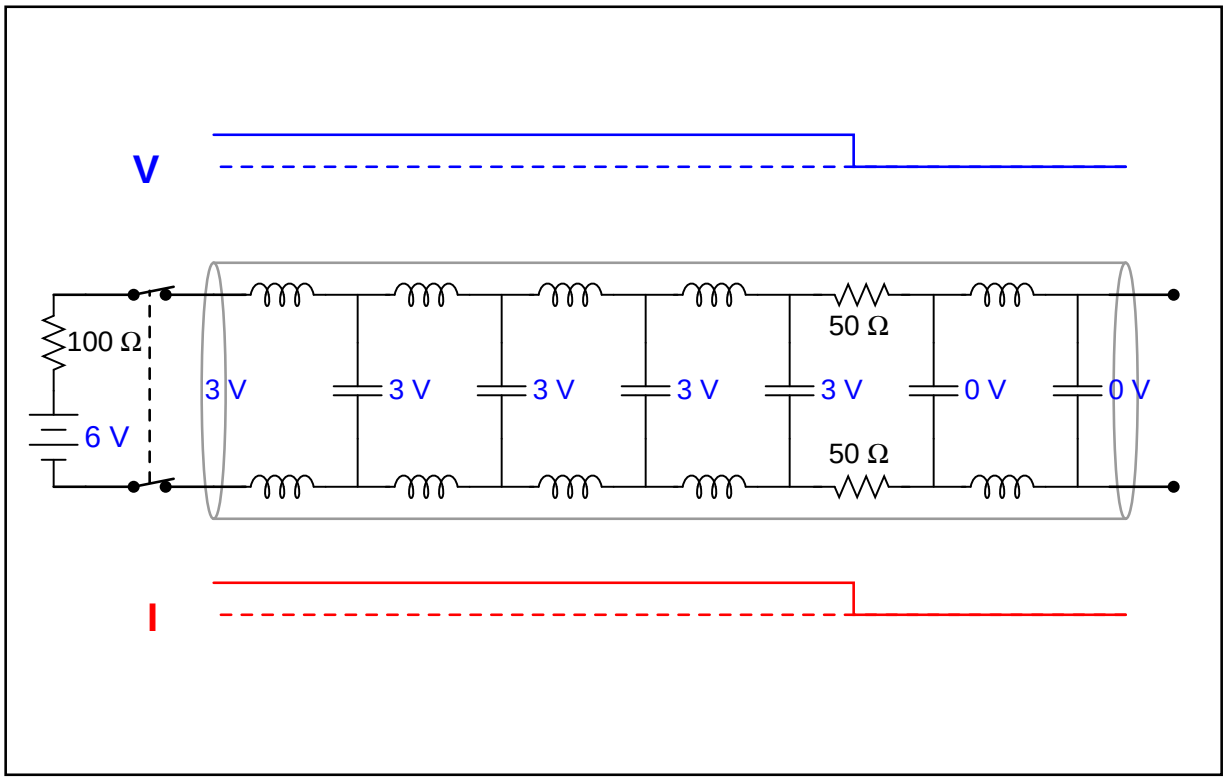
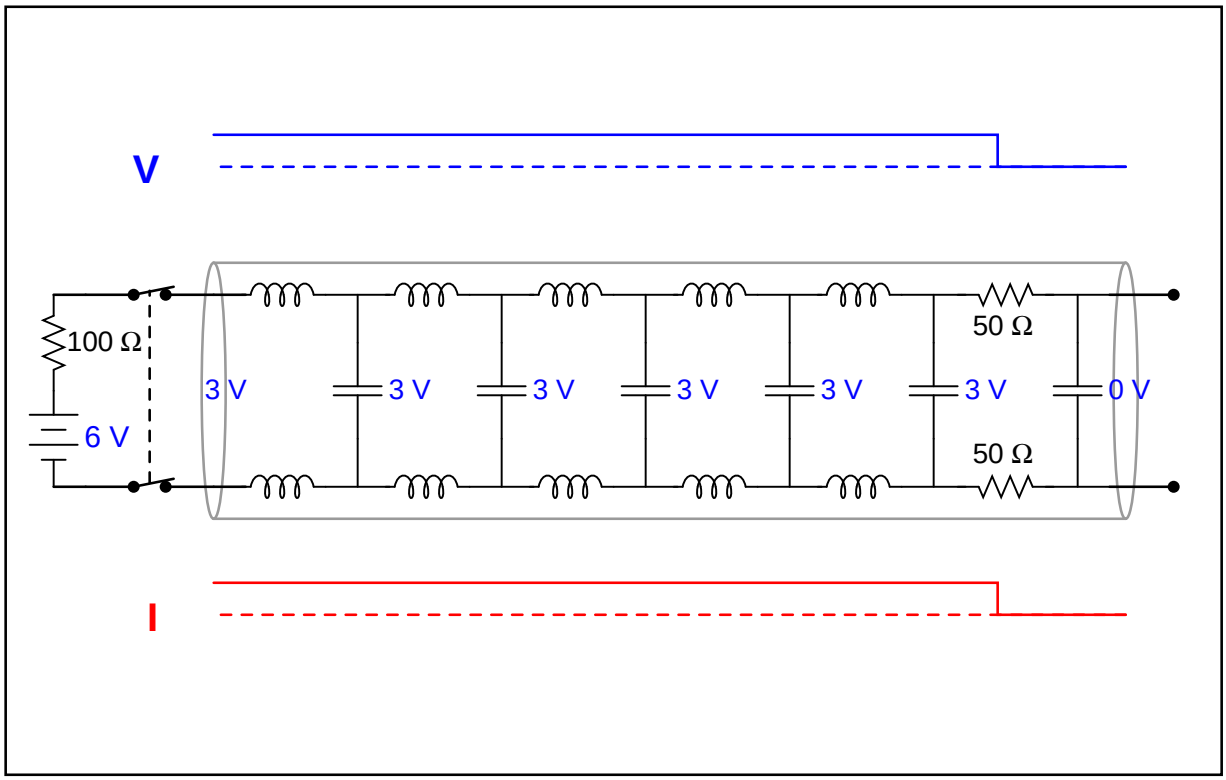
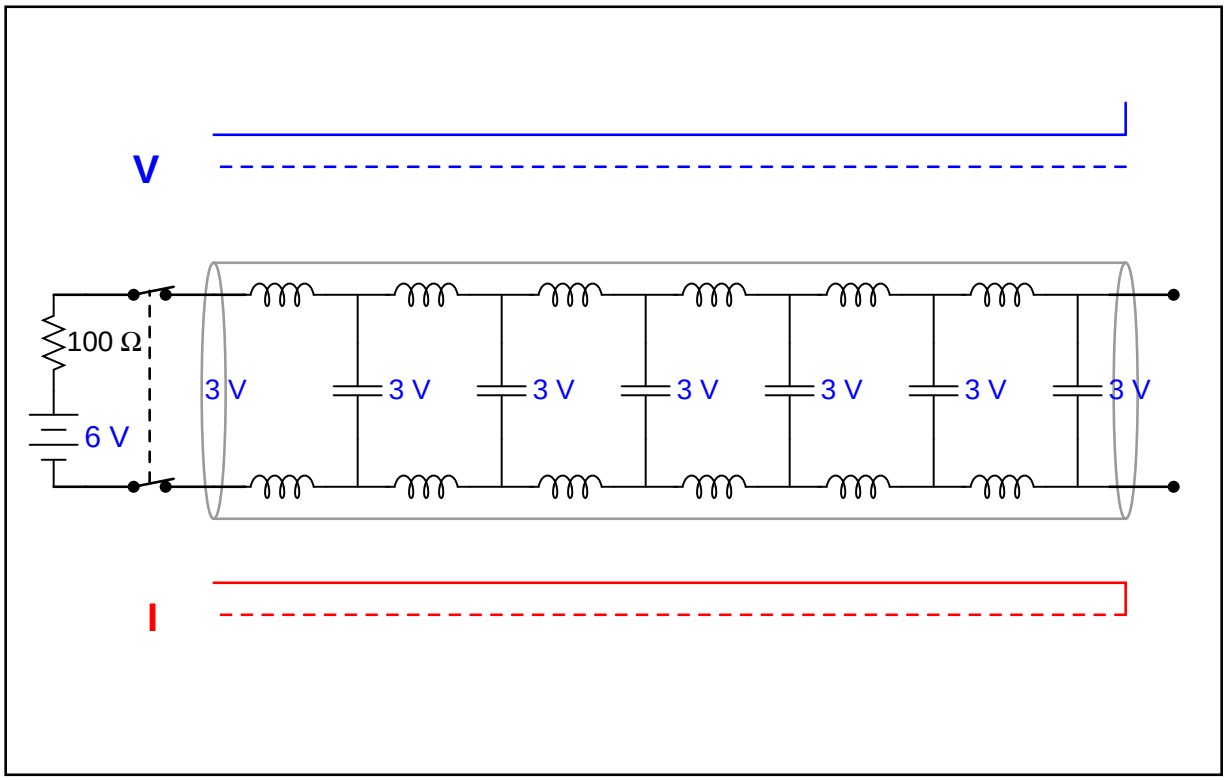
6 V
3 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
50 Ω
0 V
0 V
50 Ω
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
3 V
0 V
50 Ω
50 Ω
I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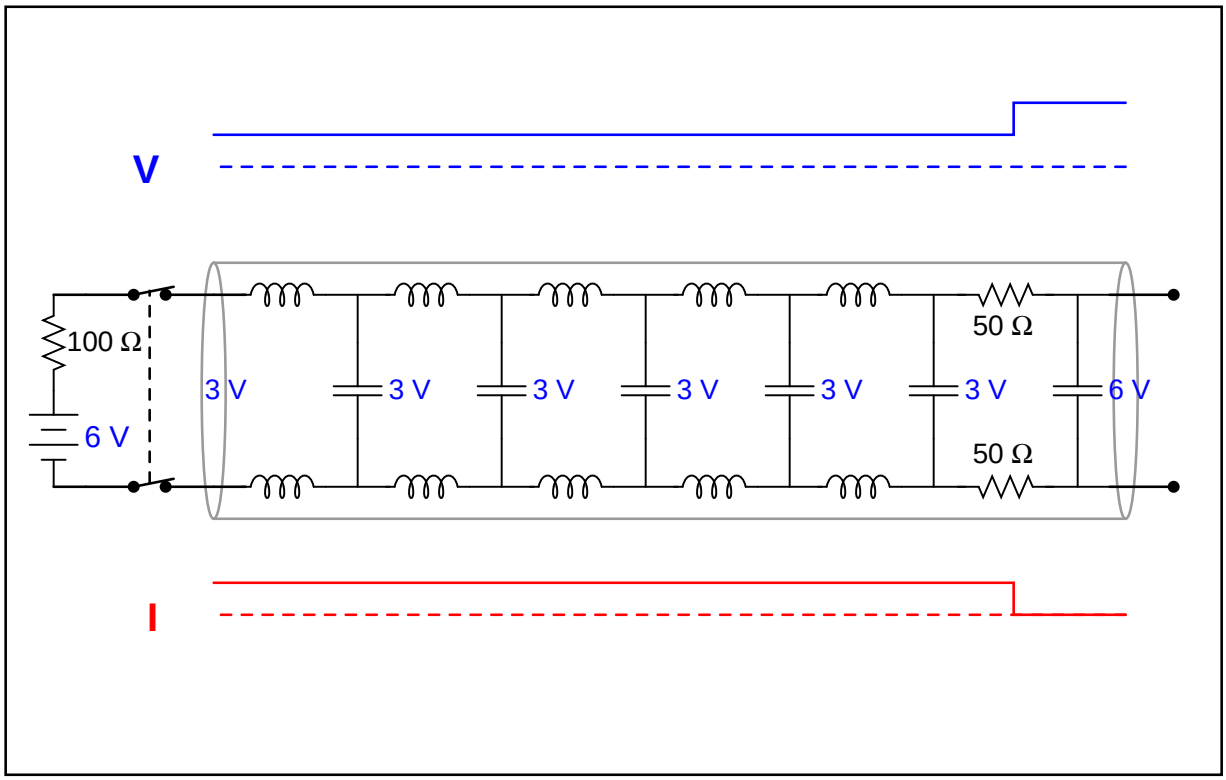
3 V
3 V
3 V
3 V
3 V
3 V
3 V
I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6 V
50 Ω
50 Ω
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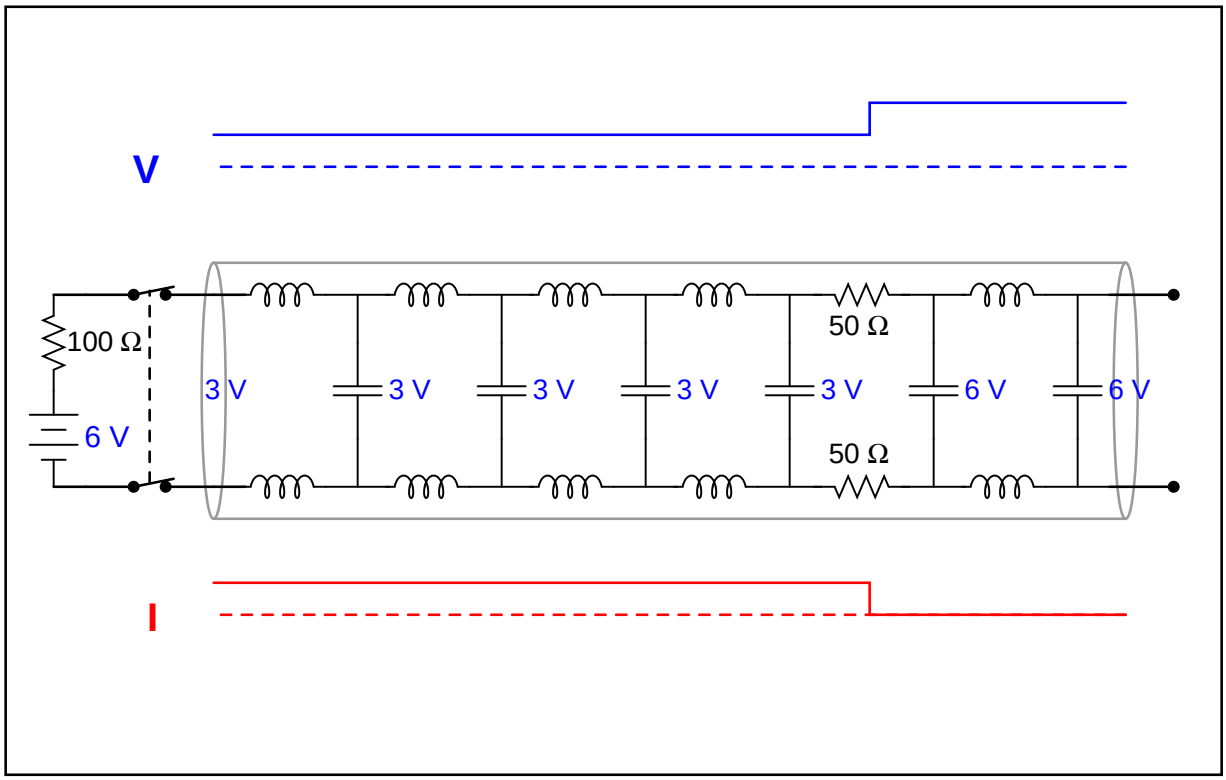
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3 V
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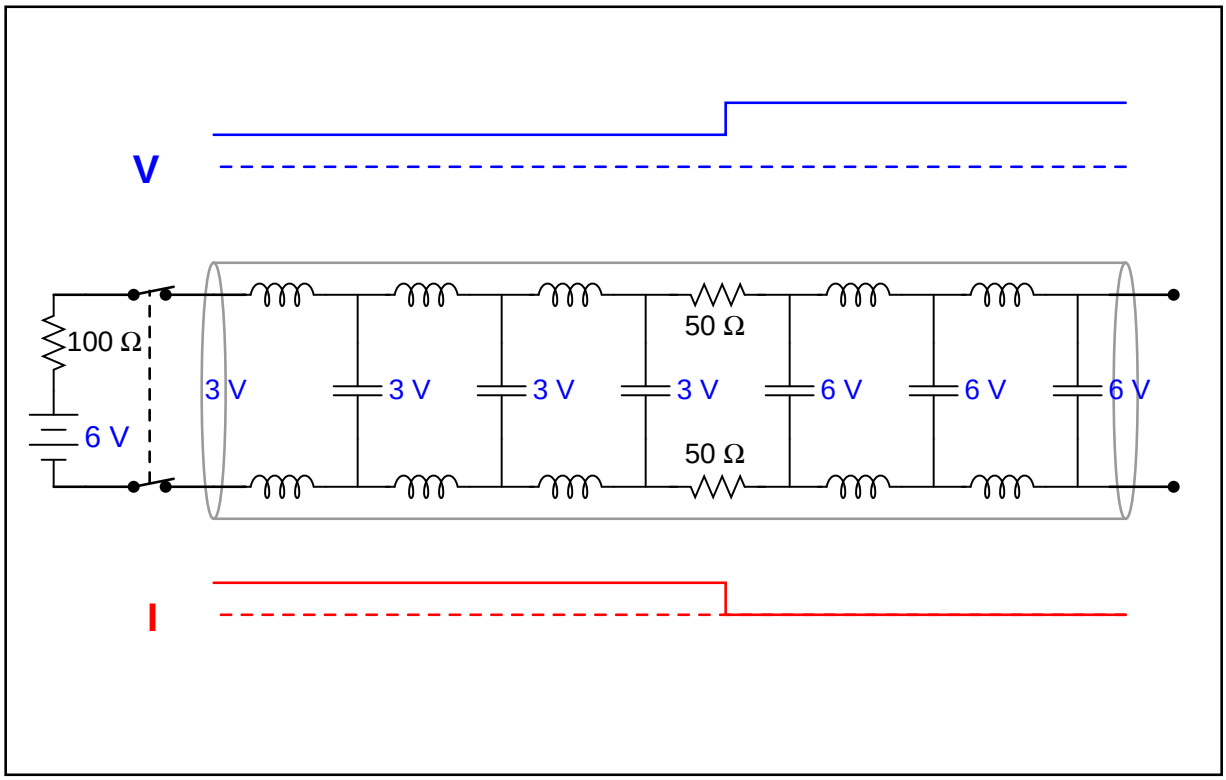
6 V
6 V
50 Ω
I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3 V
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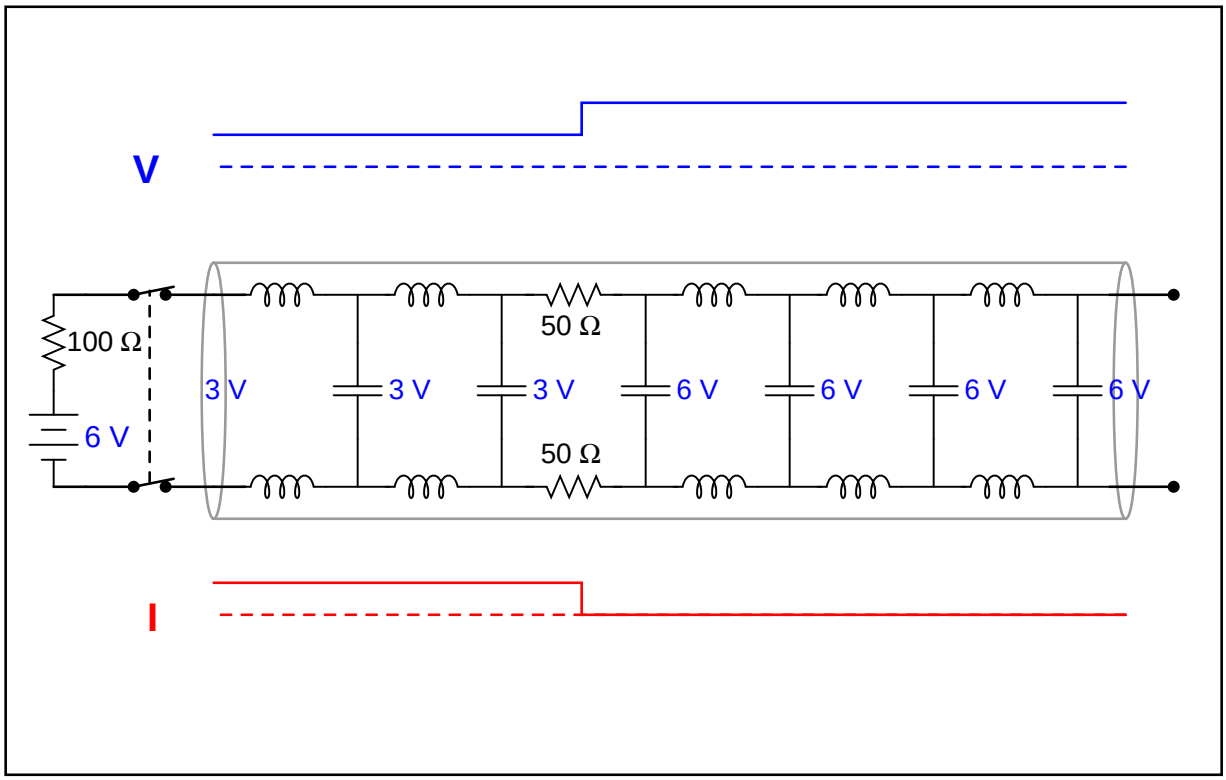
50 Ω
50 Ω
6 V
6 V
6 V
I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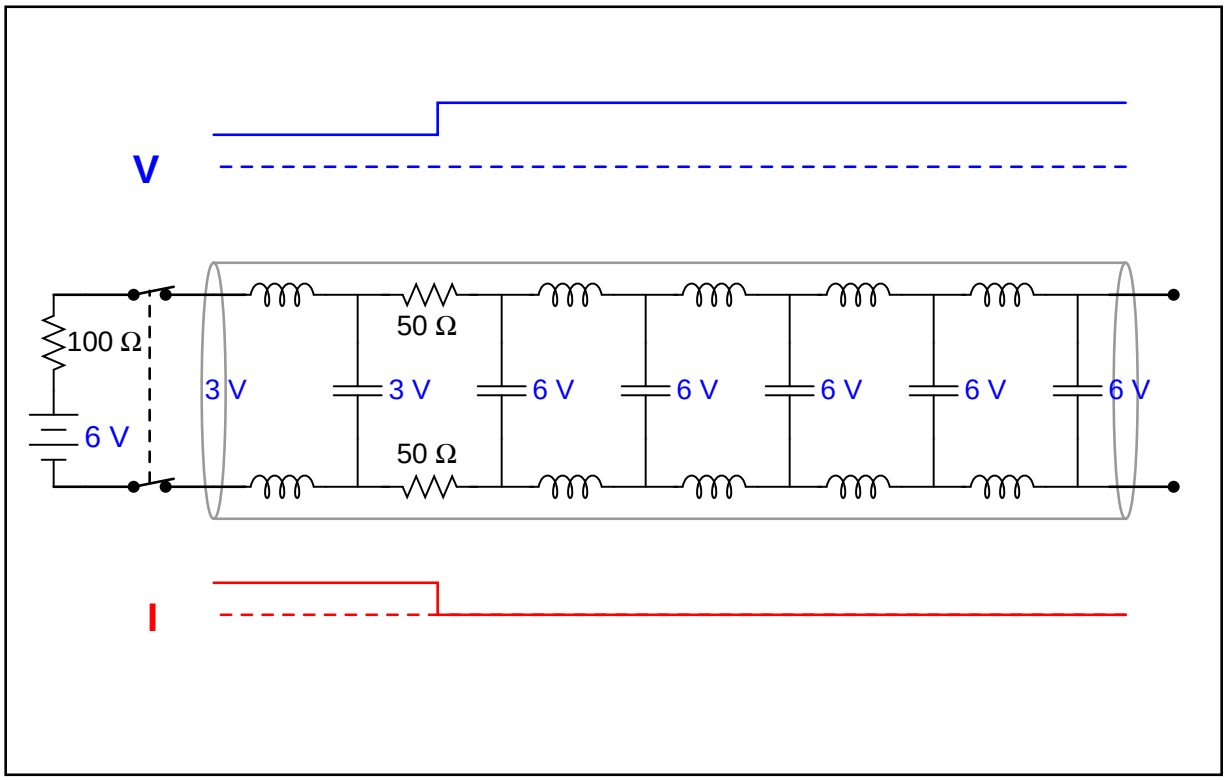
3 V
3 V
50 Ω
6 V
6 V
6 V
6 V
50 Ω
I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3 V
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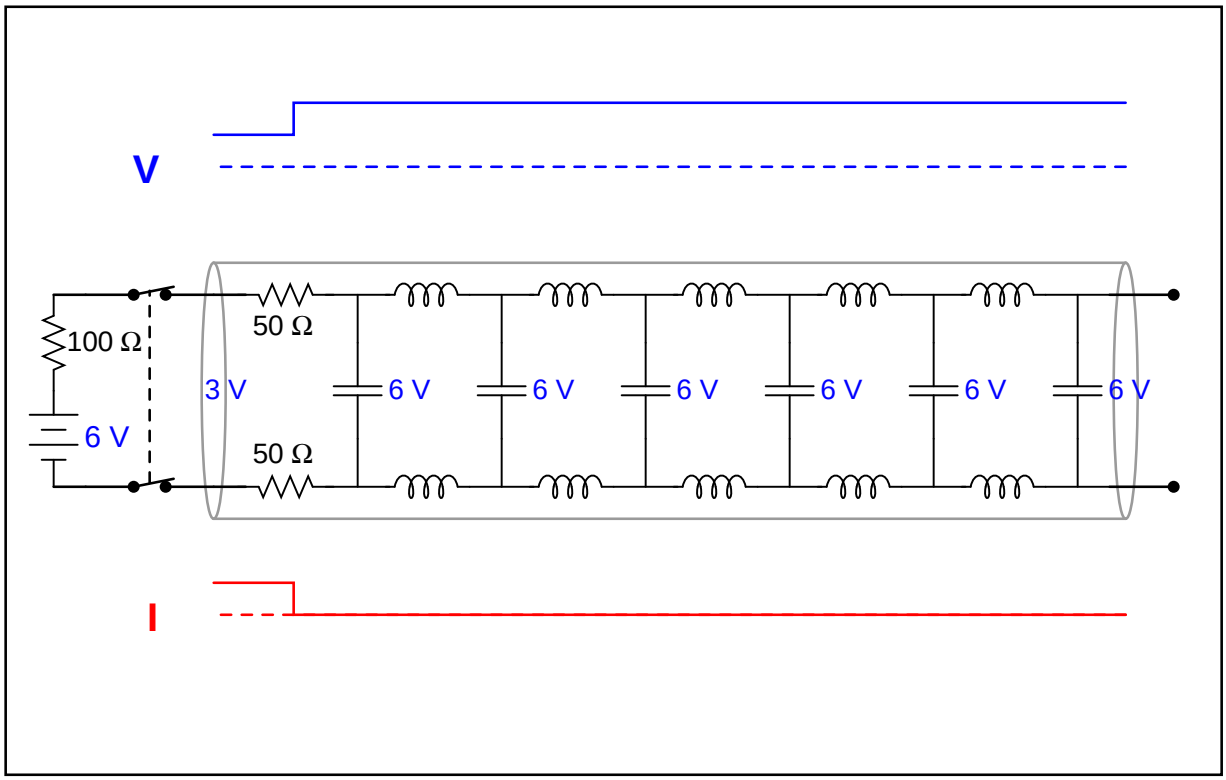
6 V
6 V
6 V
6 V
50 Ω
I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6 V
50 Ω
3 V
6 V
6 V
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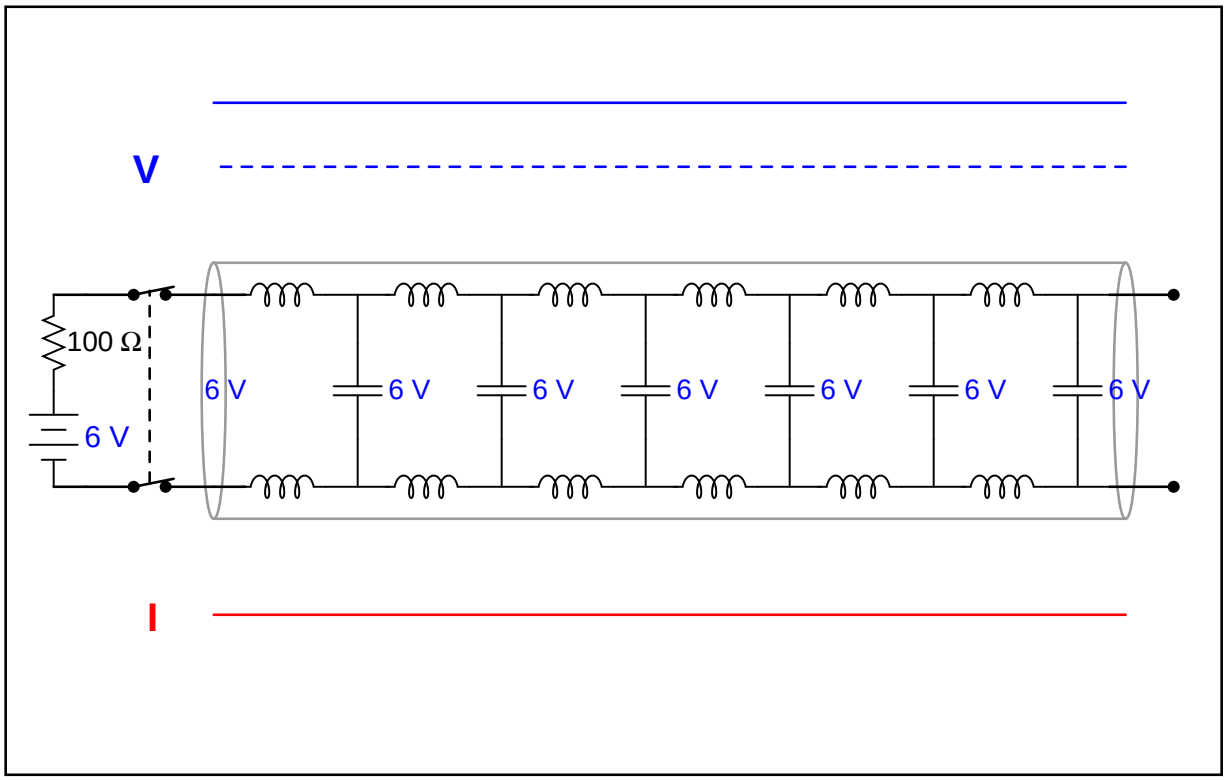
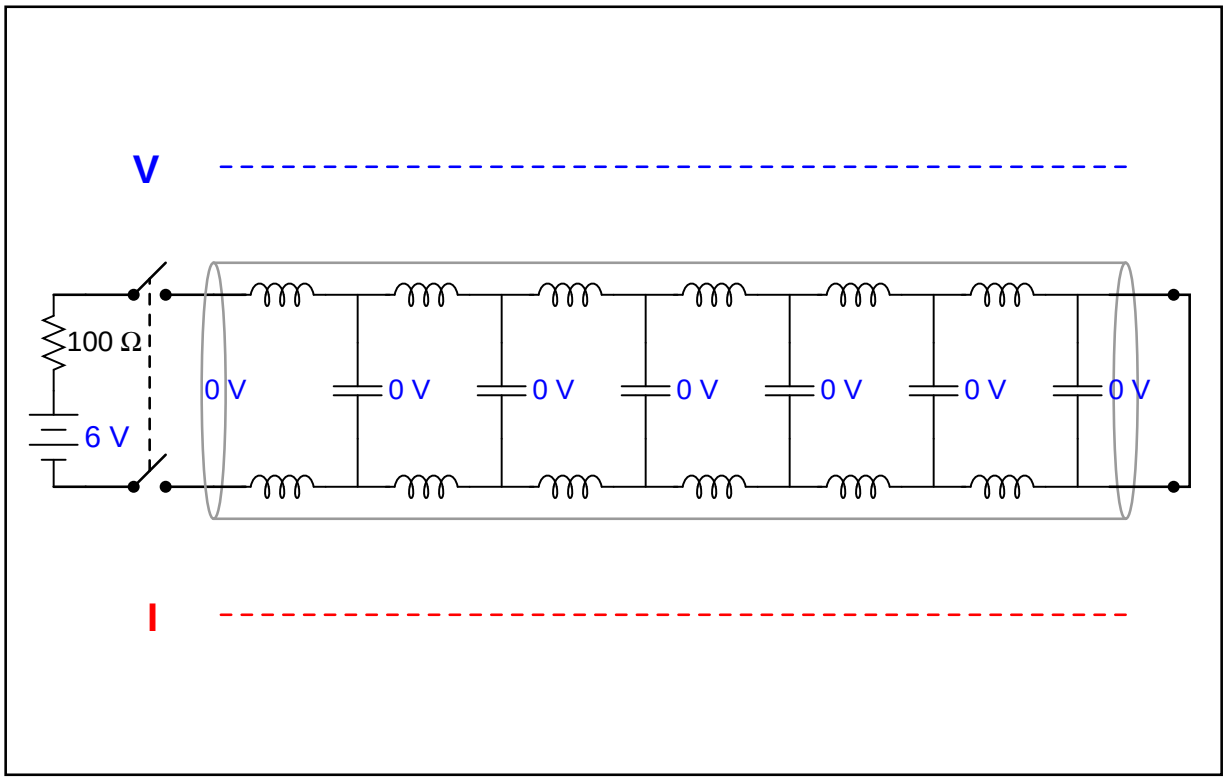
6 V
6 V
6 V
50 Ω
I


V
100 Ω
6 V
6 V
6 V
6 V
6 V
6 V
6 V
6 V
I

5.2 Animation of a shorted transmission line

The following animation shows the progression of voltage and current down the length of a 100 Ohm transmission line while energized by a 6 Volt DC source, that line being terminated by a short-circuit.


V
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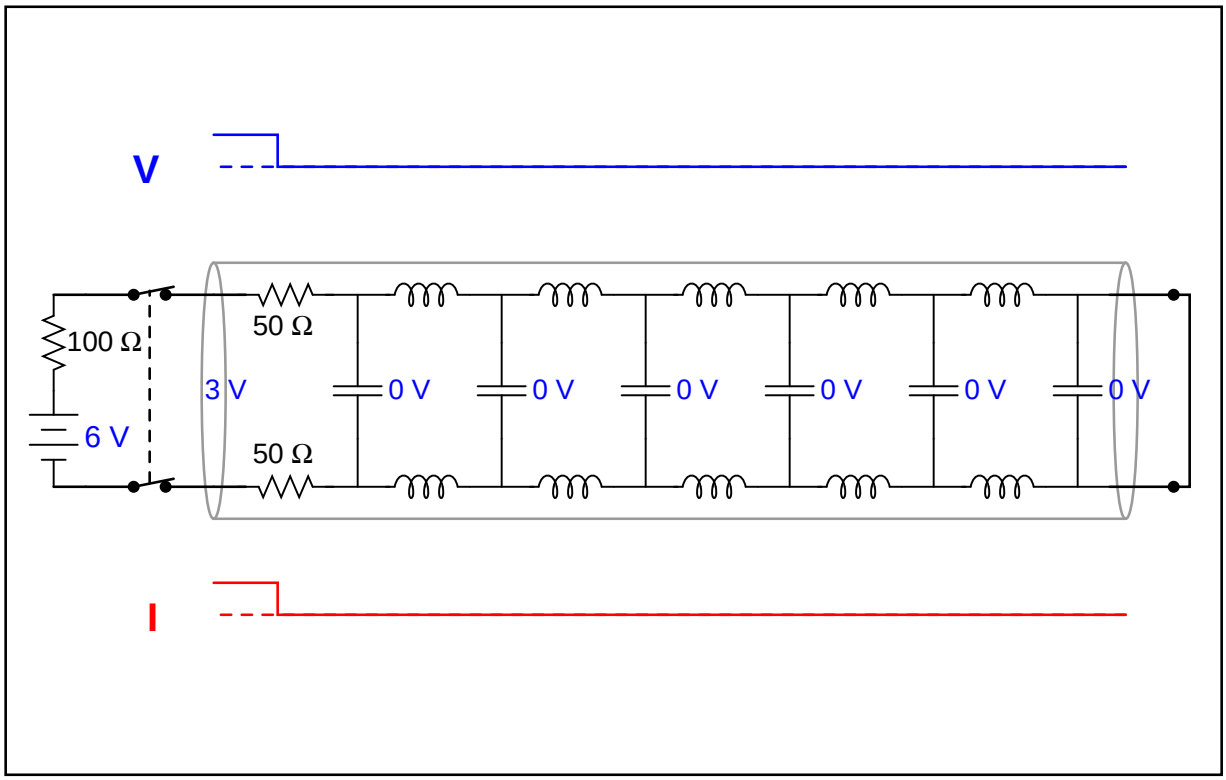
6 V
0 V
0 V
0 V
0 V
0 V
0 V
0 V
I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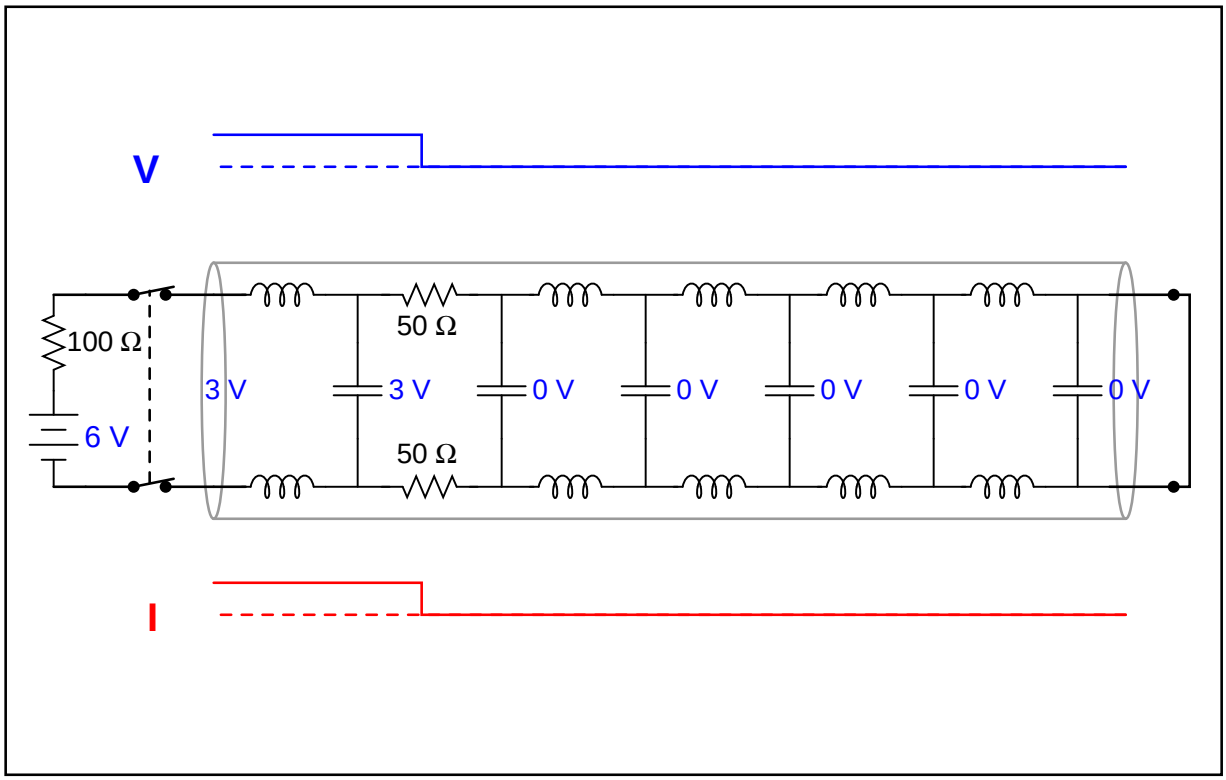
6 V
50 Ω
3 V
0 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I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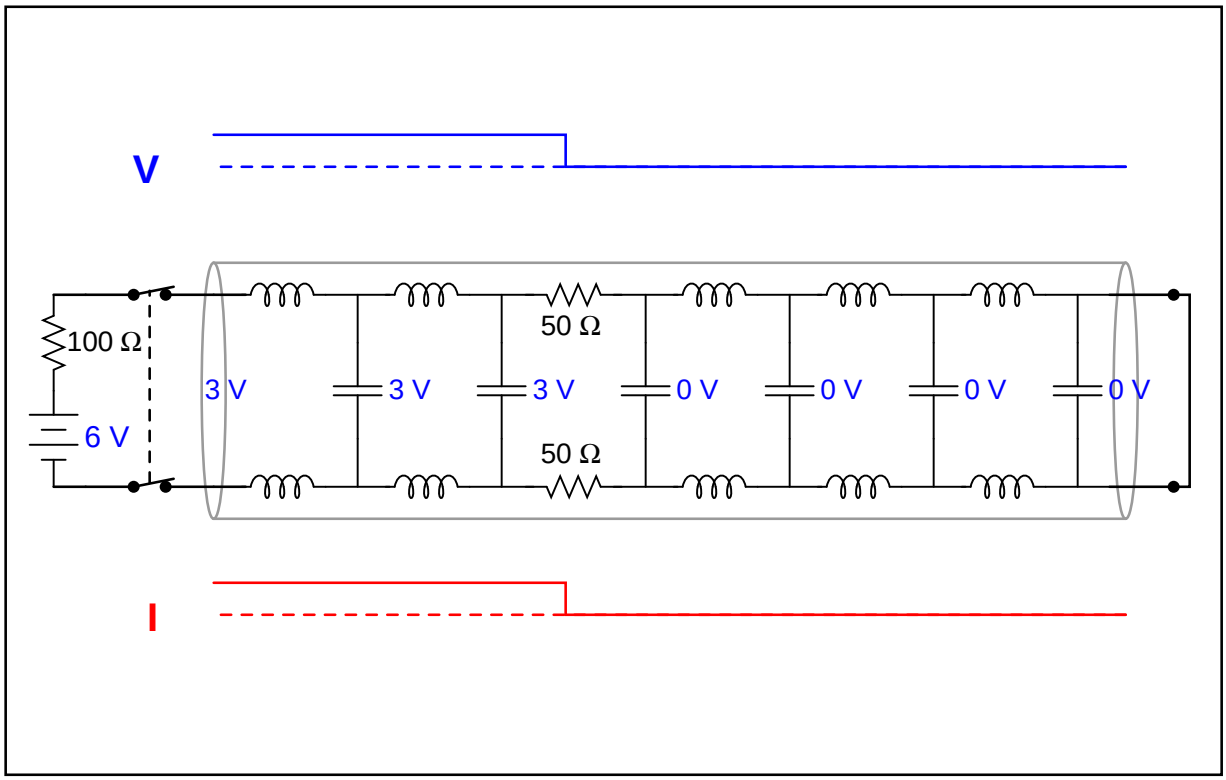
50 Ω
3 V
3 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I


V
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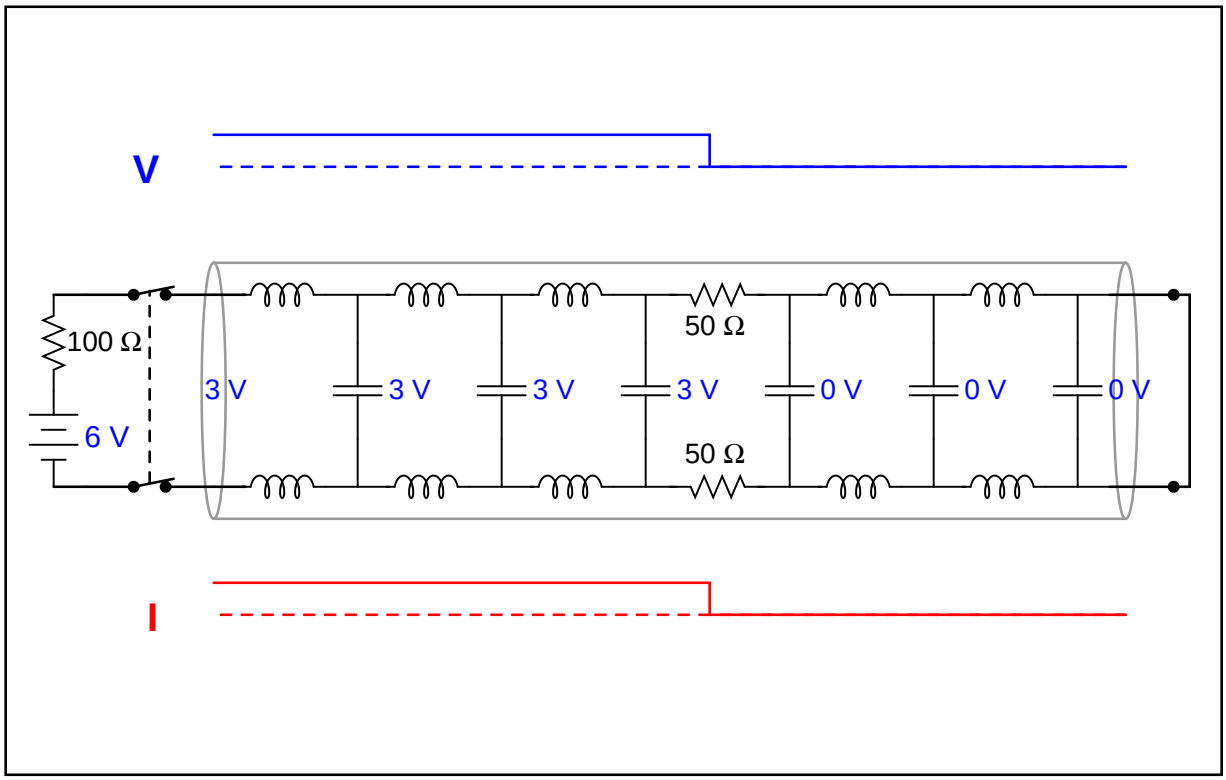
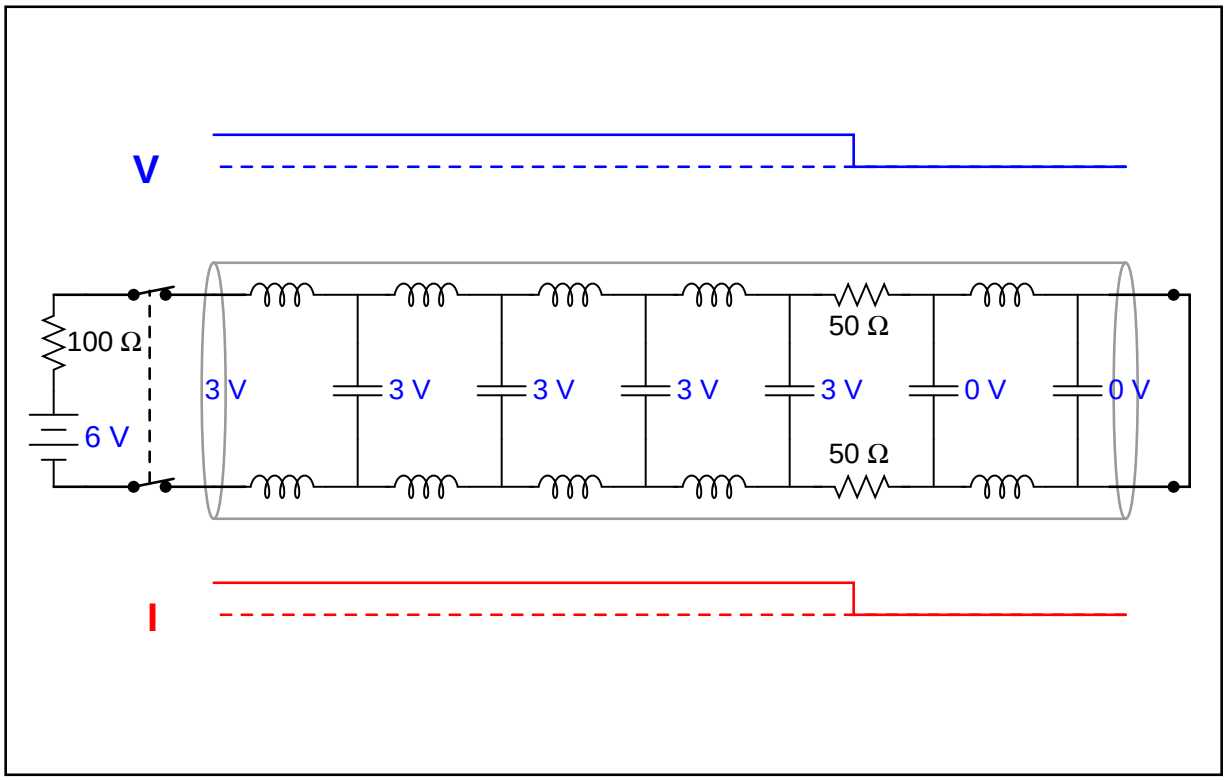
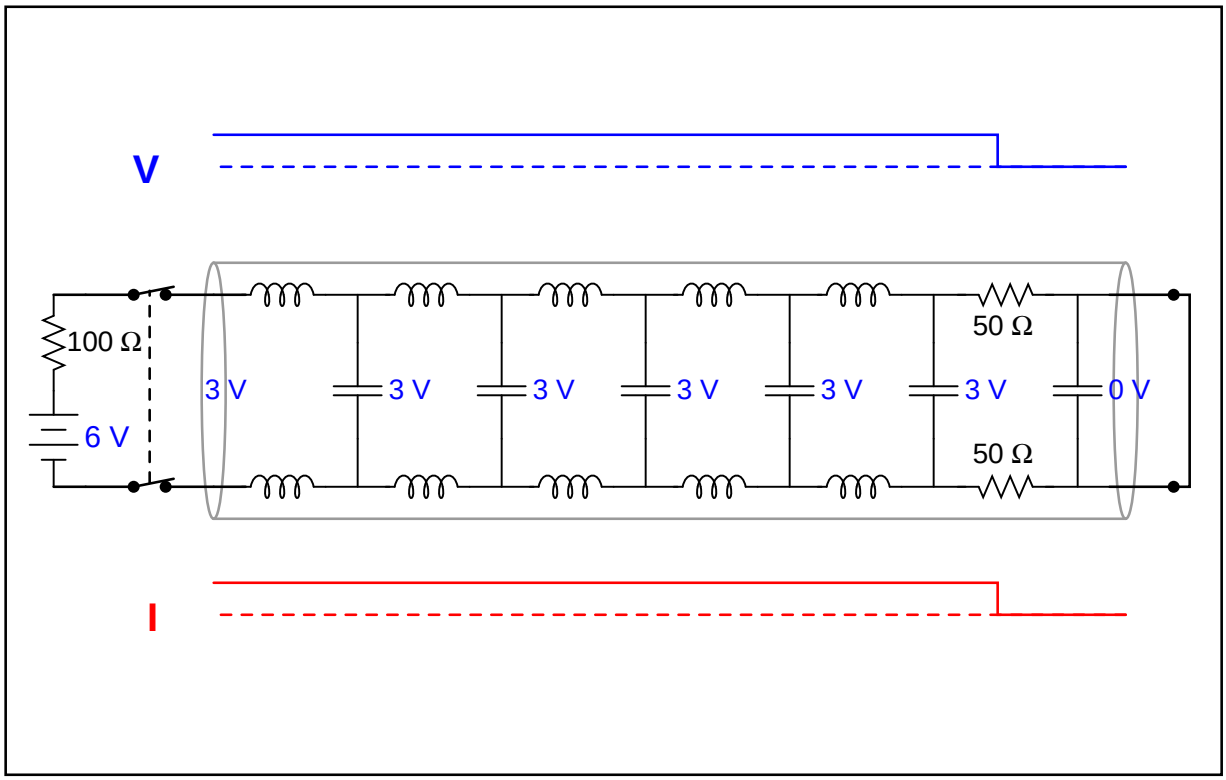
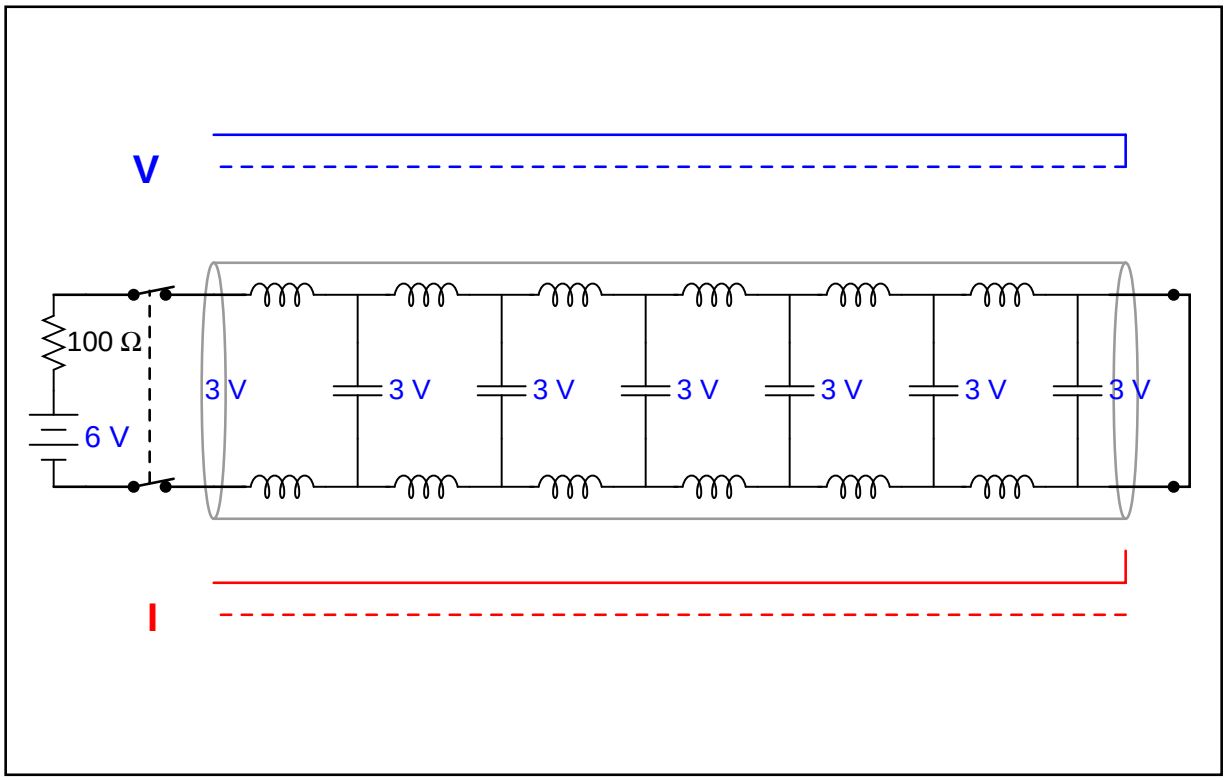
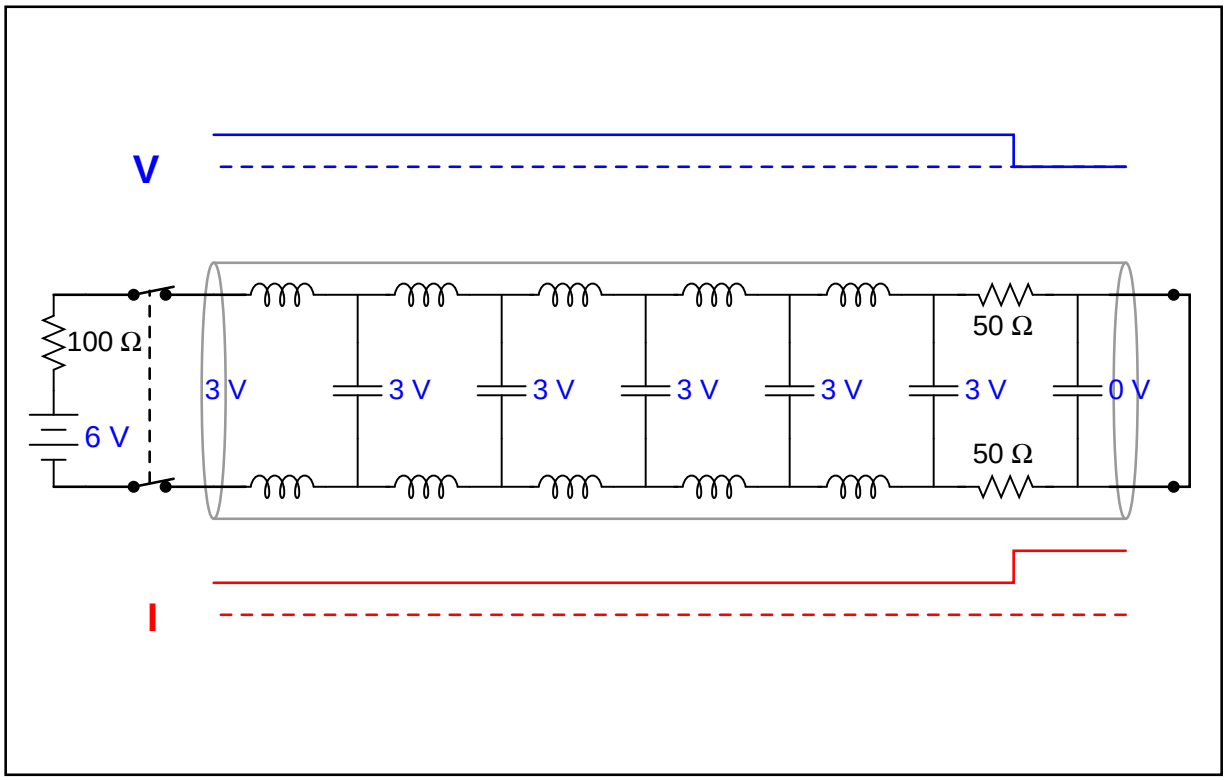
6 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
0 V
I

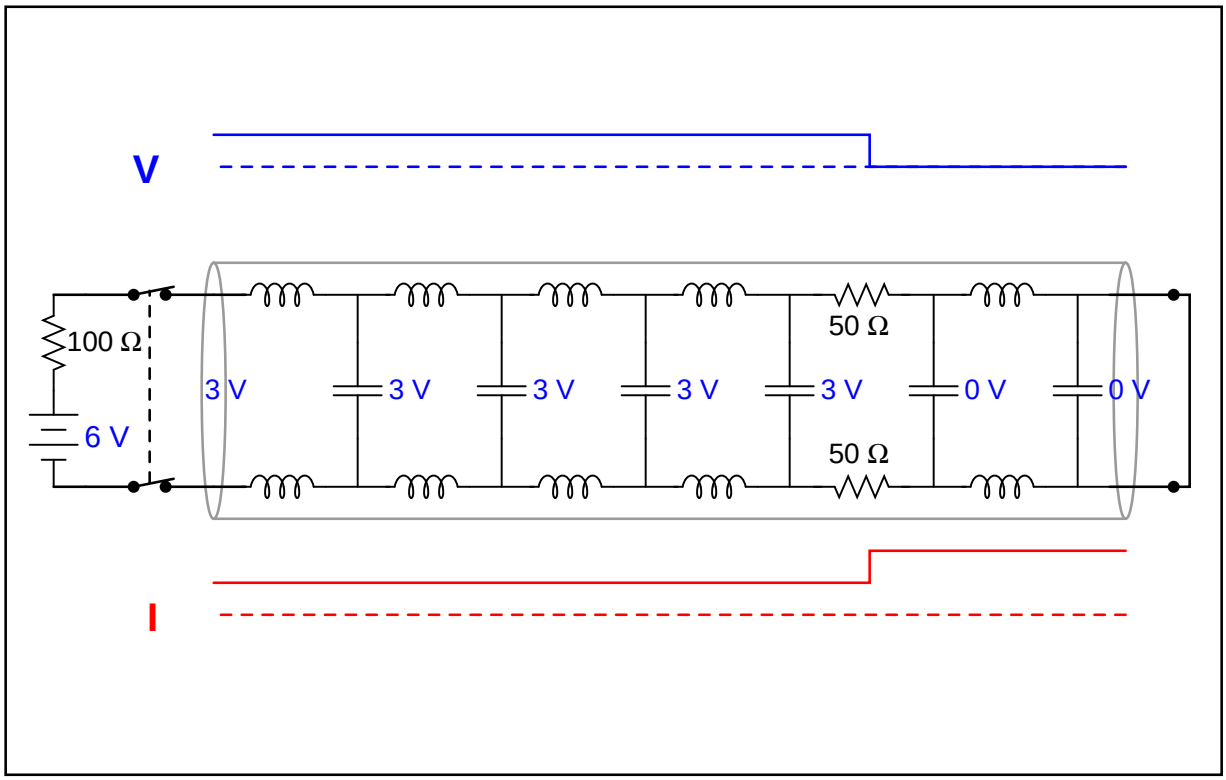

V
100 Ω
6 V
3 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
I

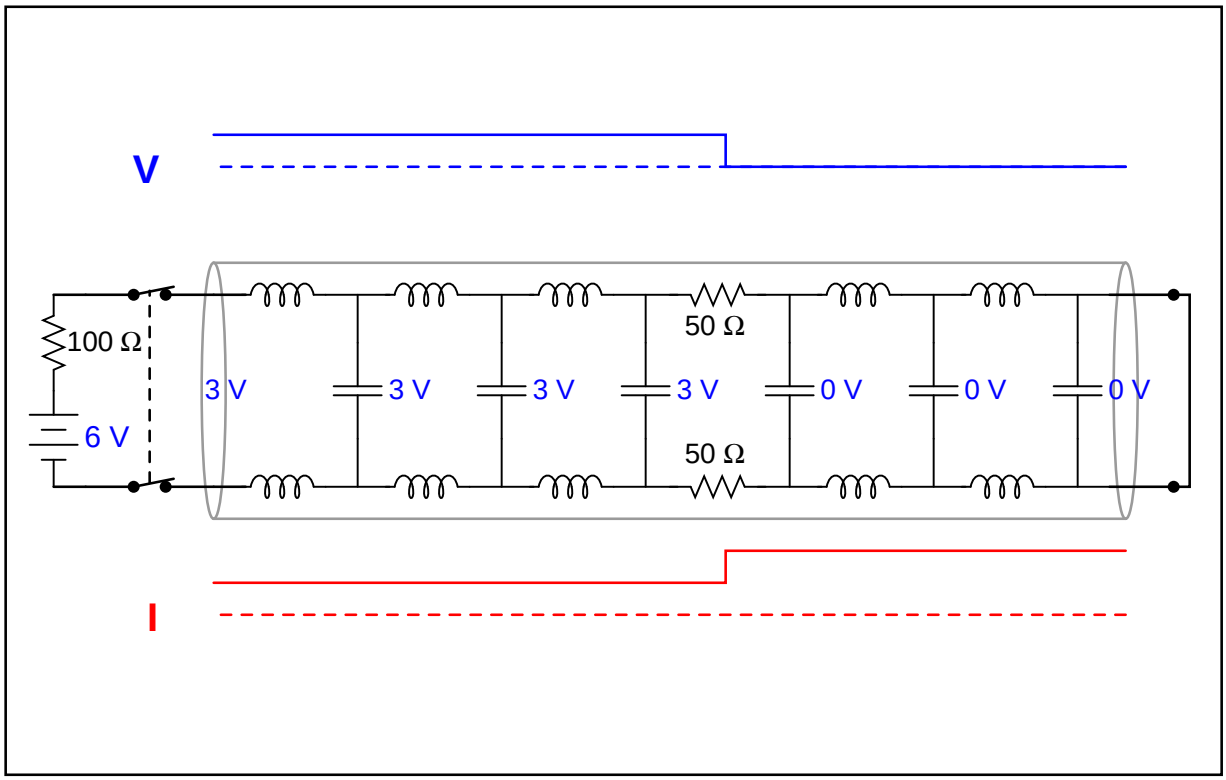

V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
50 Ω
0 V
0 V
50 Ω
I

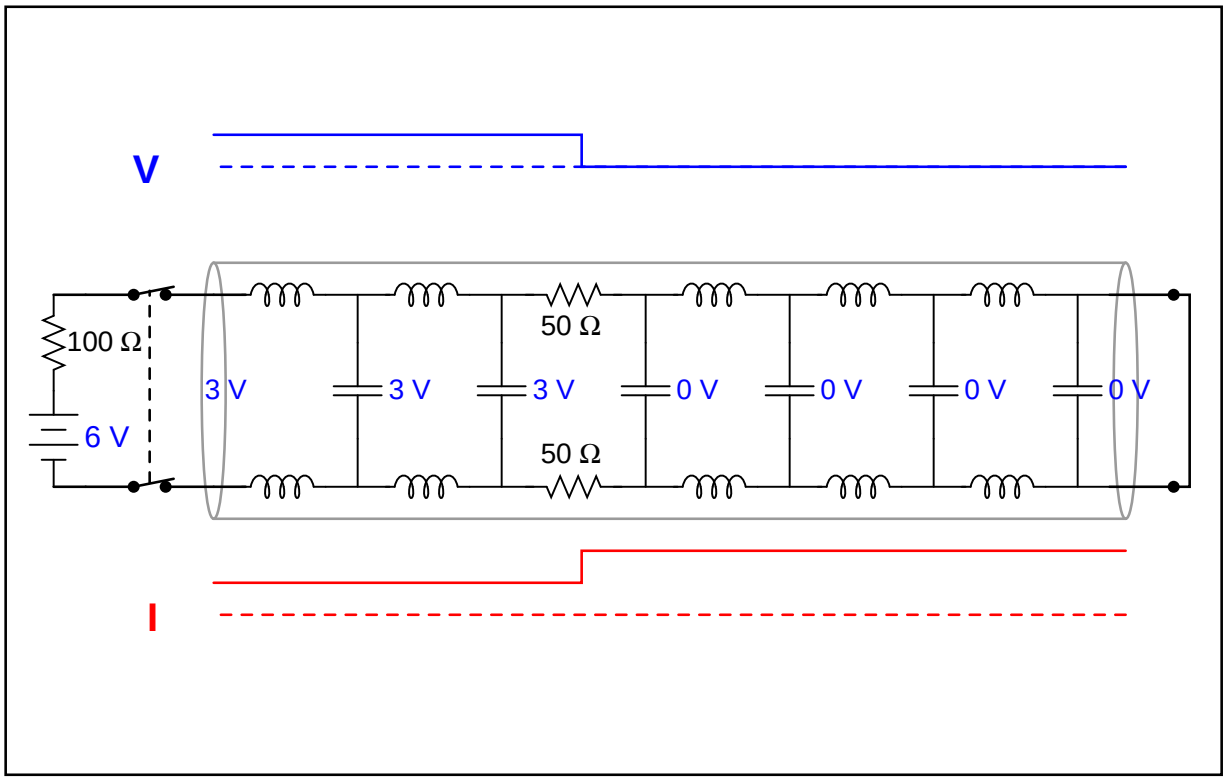

V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
3 V
0 V
50 Ω
50 Ω
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
3 V
3 V
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
3 V
0 V
50 Ω
50 Ω
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
3 V
50 Ω
0 V
0 V
50 Ω
I


V
100 Ω
6 V
3 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
I


V
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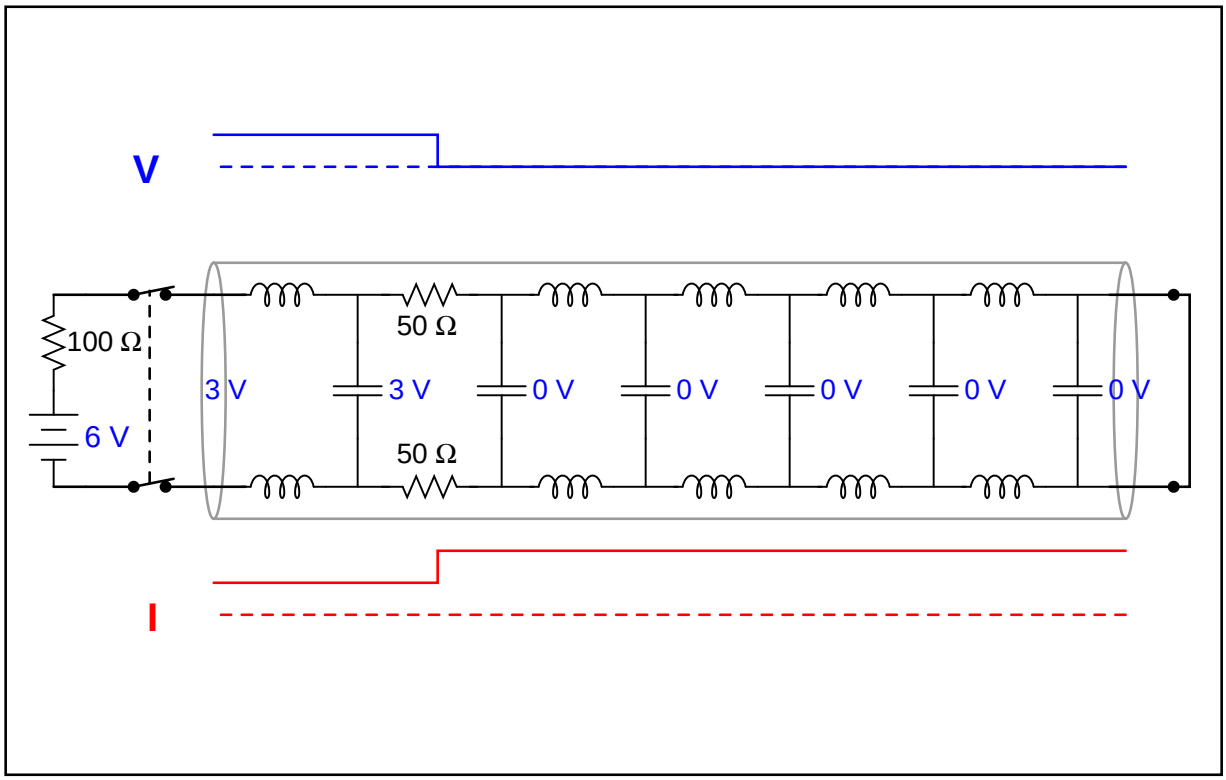
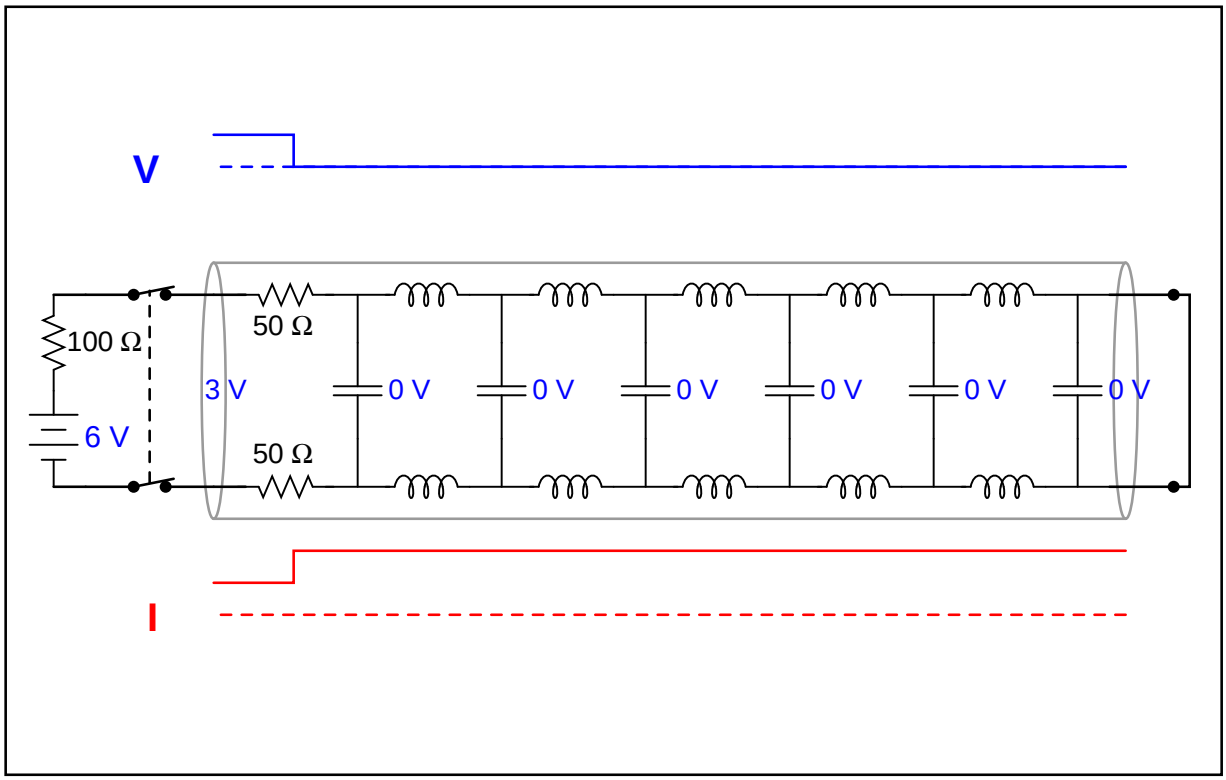
6 V
3 V
3 V
3 V
50 Ω
50 Ω
0 V
0 V
0 V
0 V
I


V
100 Ω
6 V
50 Ω
3 V
3 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I


V
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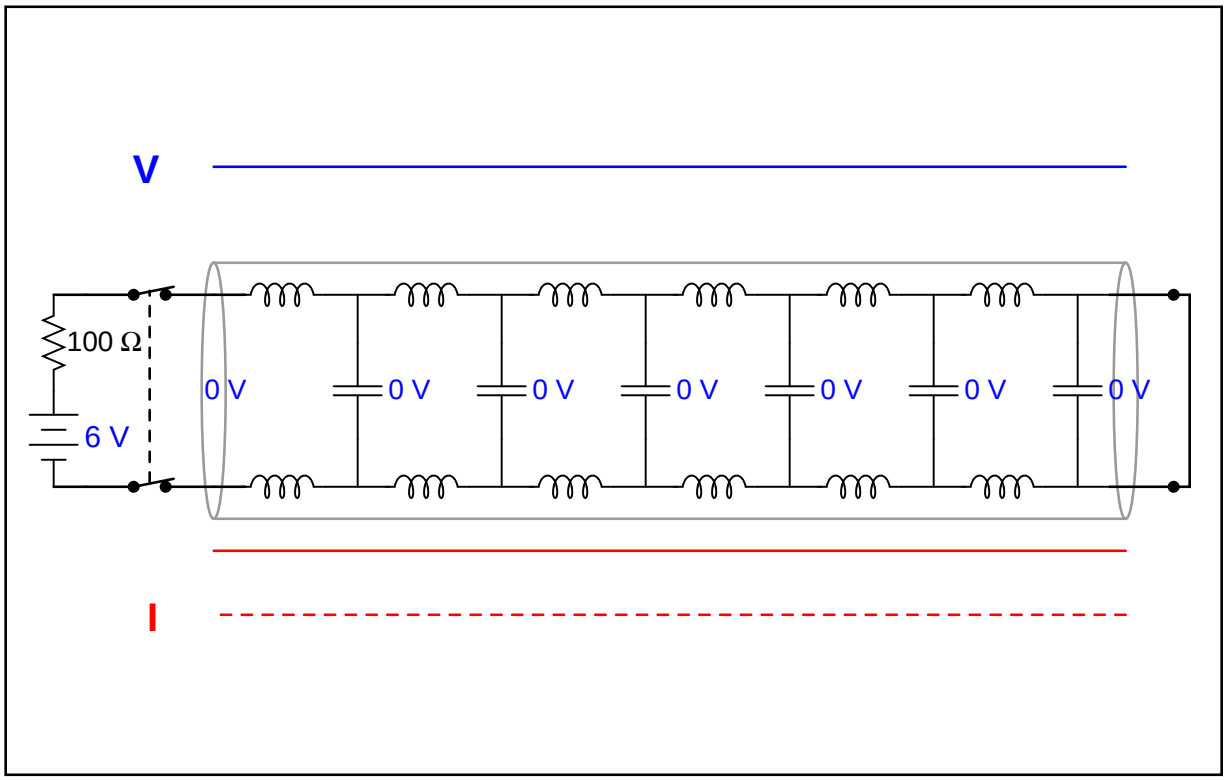
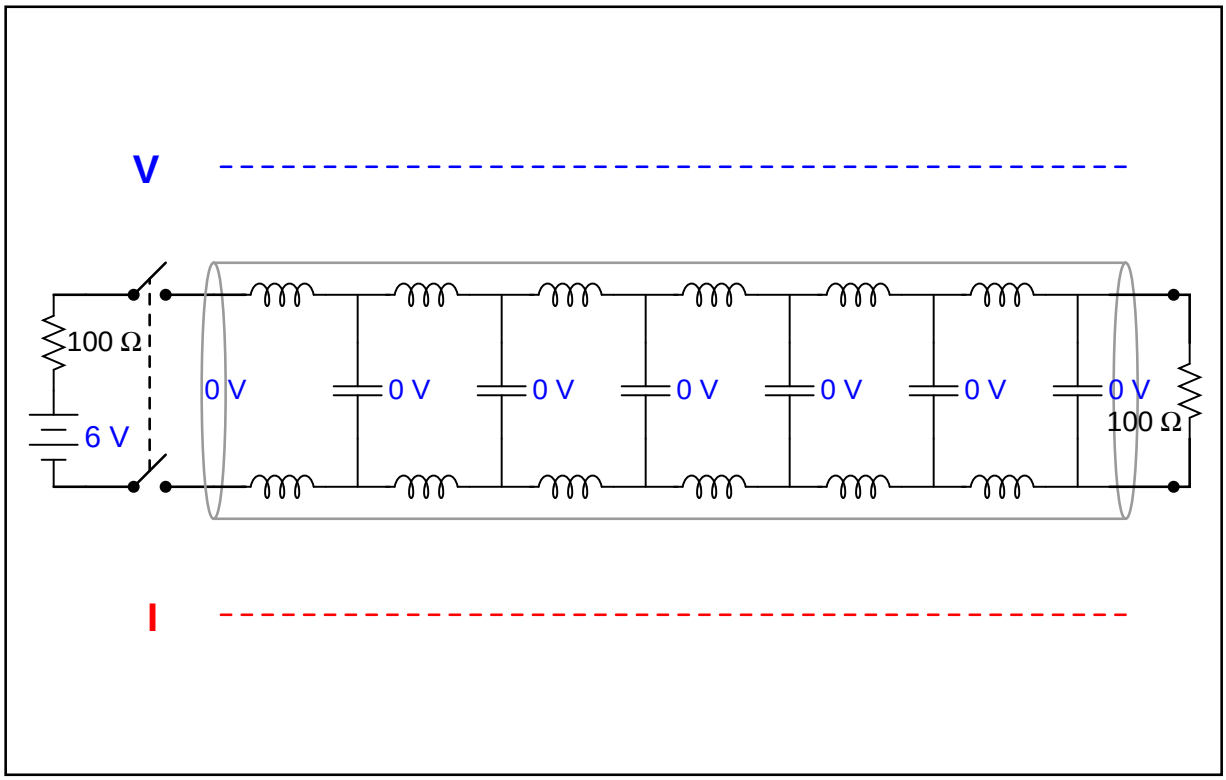
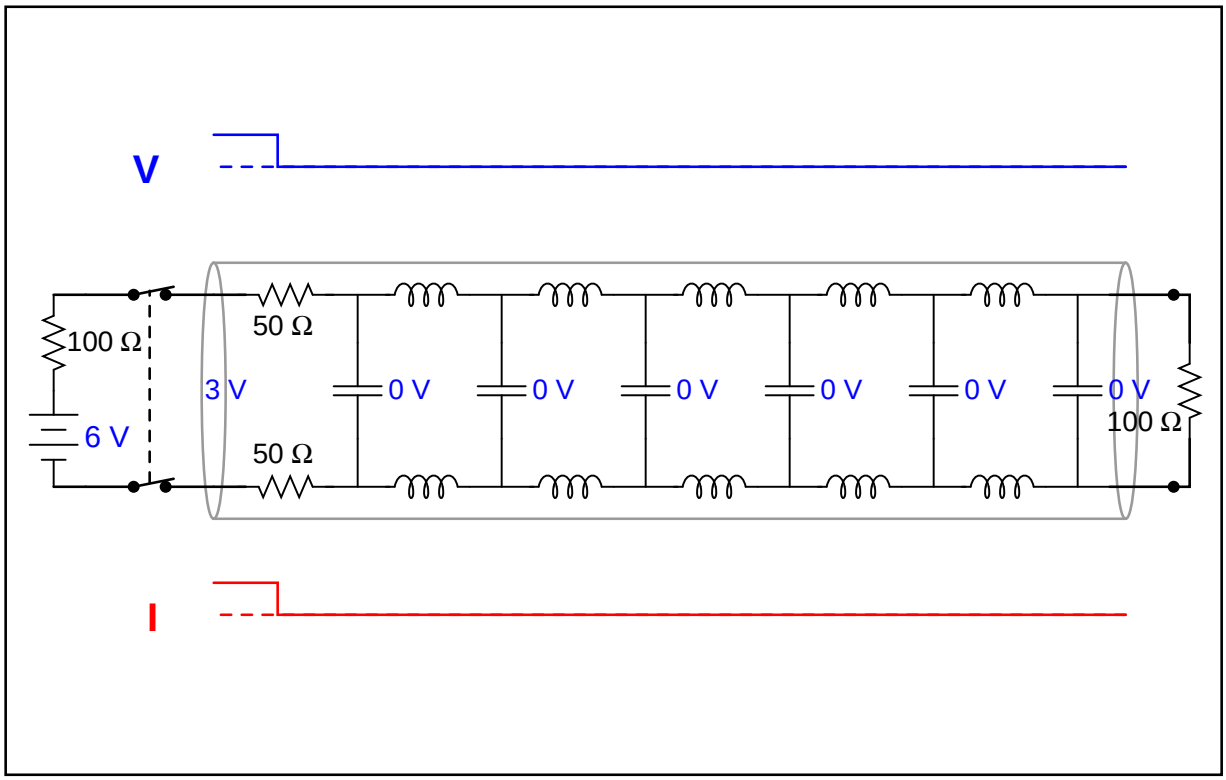
6 V
50 Ω
3 V
0 V
0 V
0 V
0 V
0 V
0 V
50 Ω
I

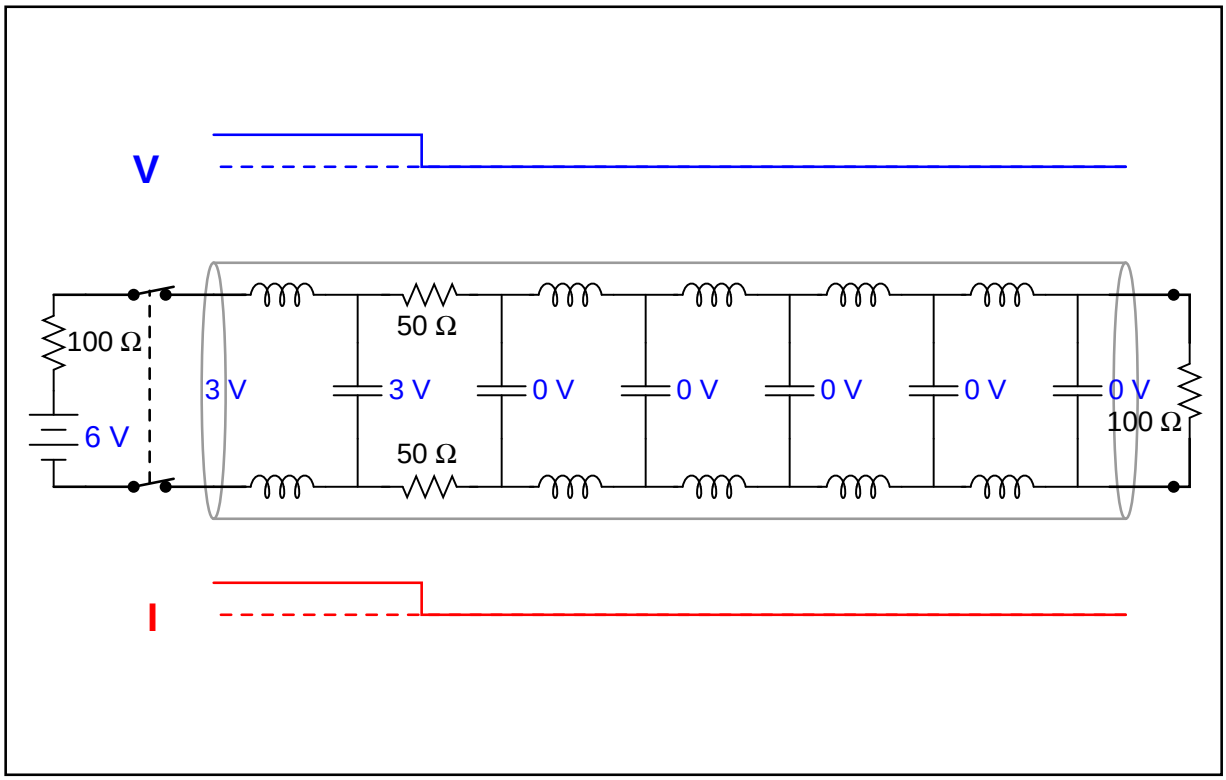

V
100 Ω
6 V
0 V
0 V
0 V
0 V
0 V
0 V
0 V
I

5.3 Animation of a terminated transmission line

The following animation shows the progression of voltage and current down the length of a 100 Ohm transmission line while energized by a 6 Volt DC source, that line being properly terminated by a 100 Ω resistor.


V
100 Ω
6 V
0 V
0 V
0 V
0 V
0 V
0 V
0 V
100 Ω
I


V
100 Ω
6 V
50 Ω
3 V
0 V
0 V
0 V
0 V
0 V
0 V
50 Ω
100 Ω
I


V
100 Ω
6 V
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0 V
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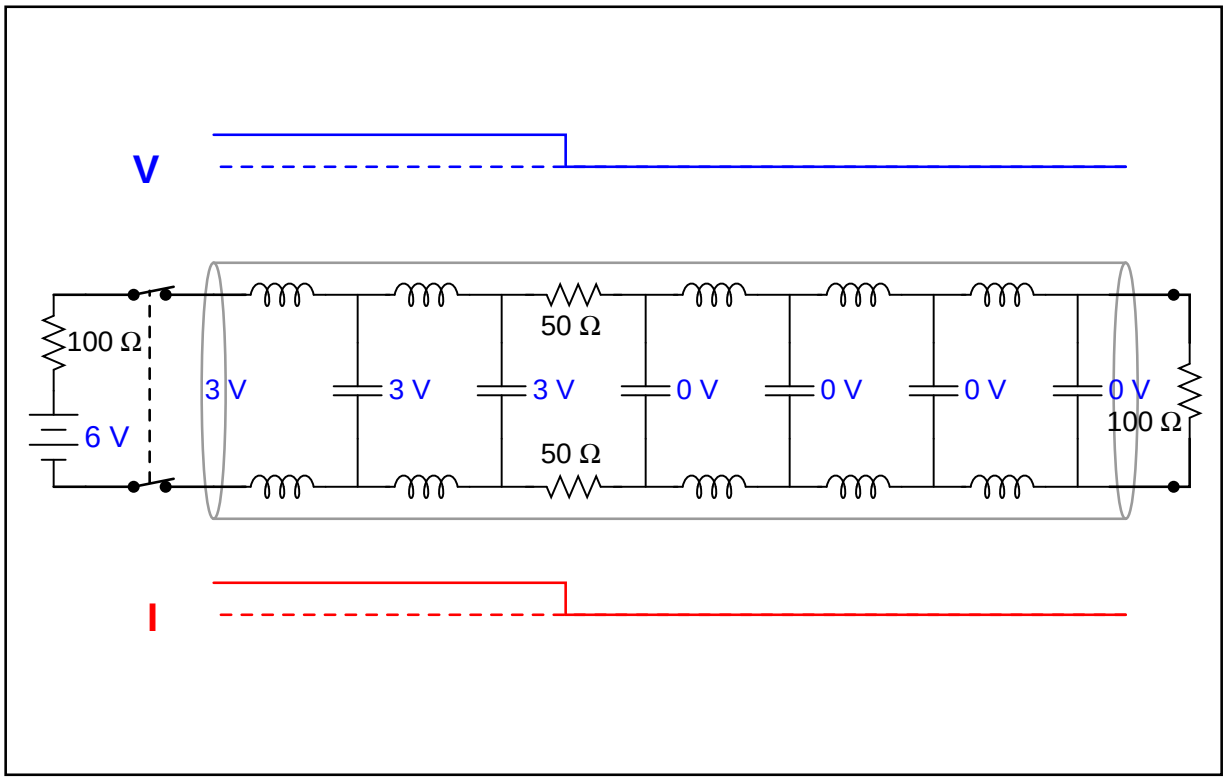
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3 V
50 Ω
50 Ω
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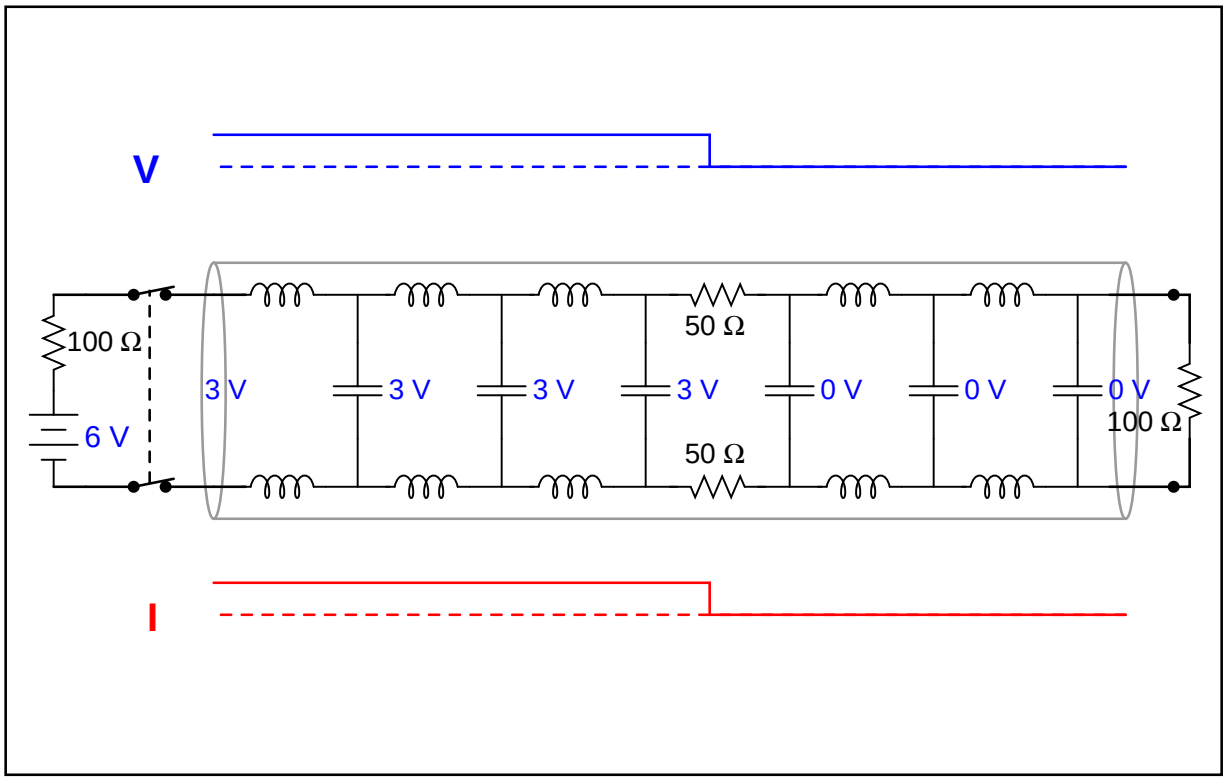
100 Ω
I


V
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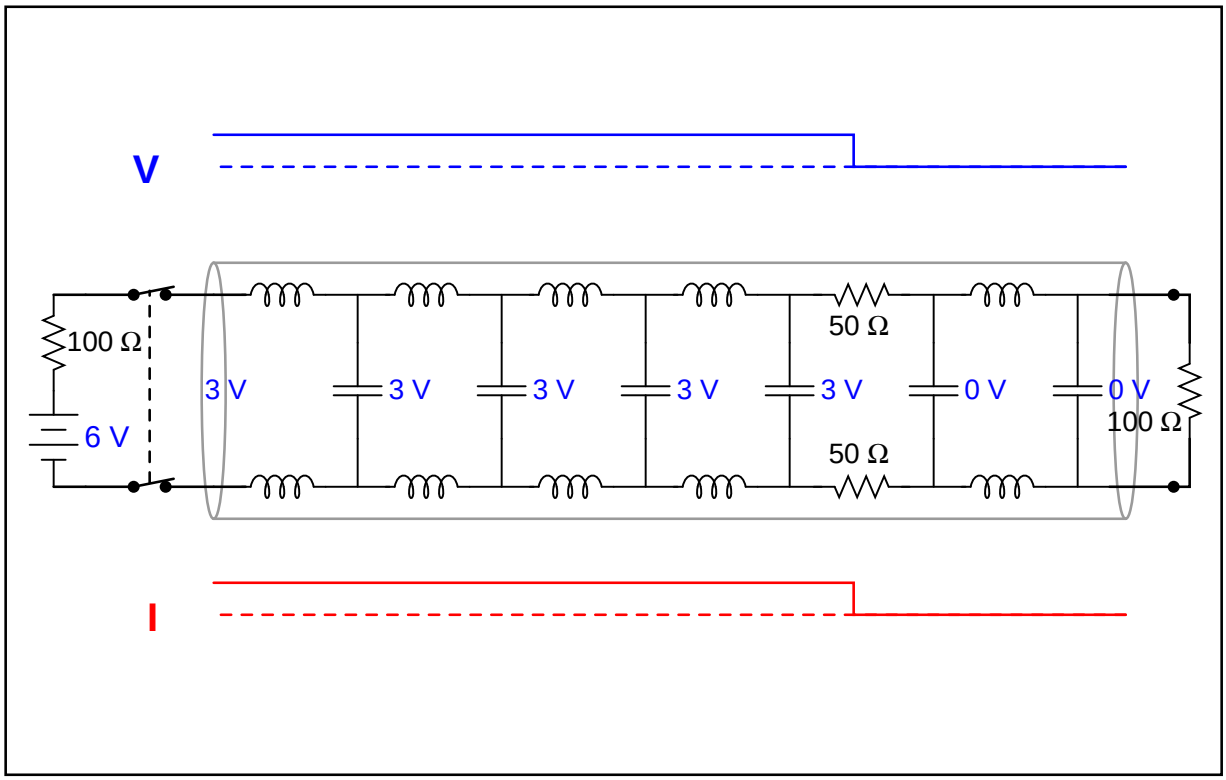
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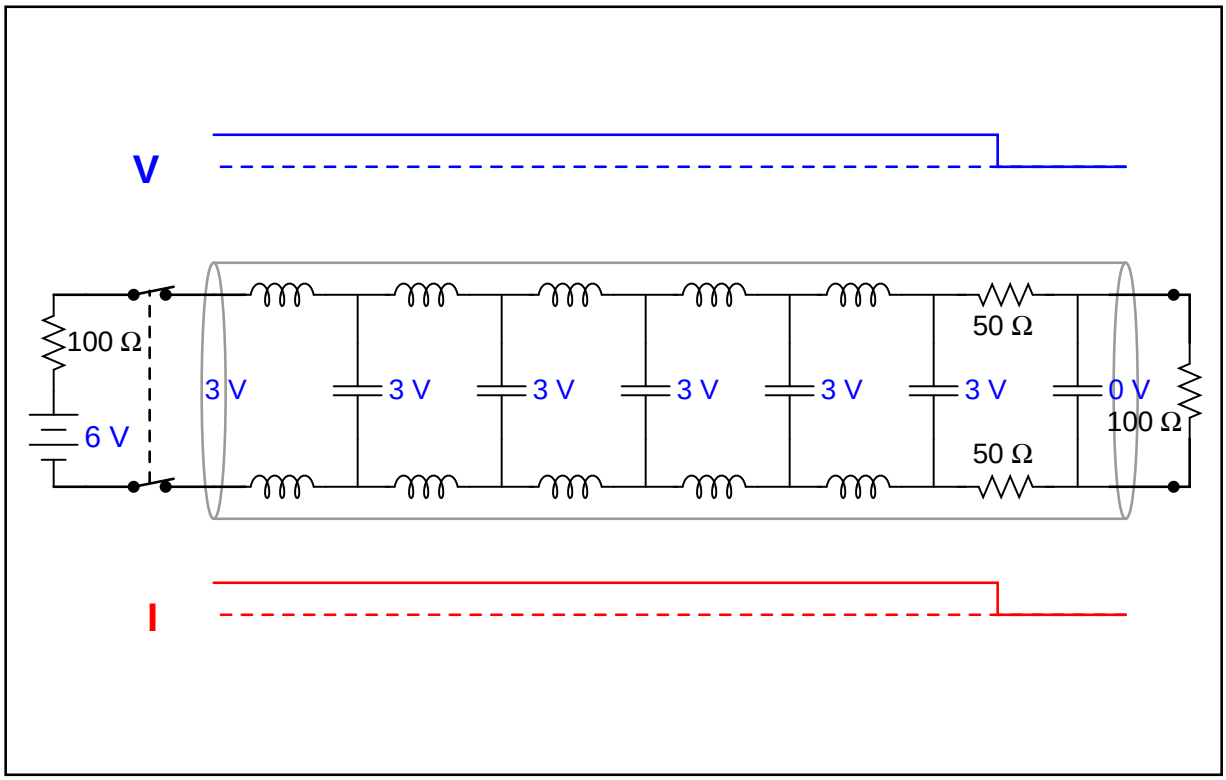
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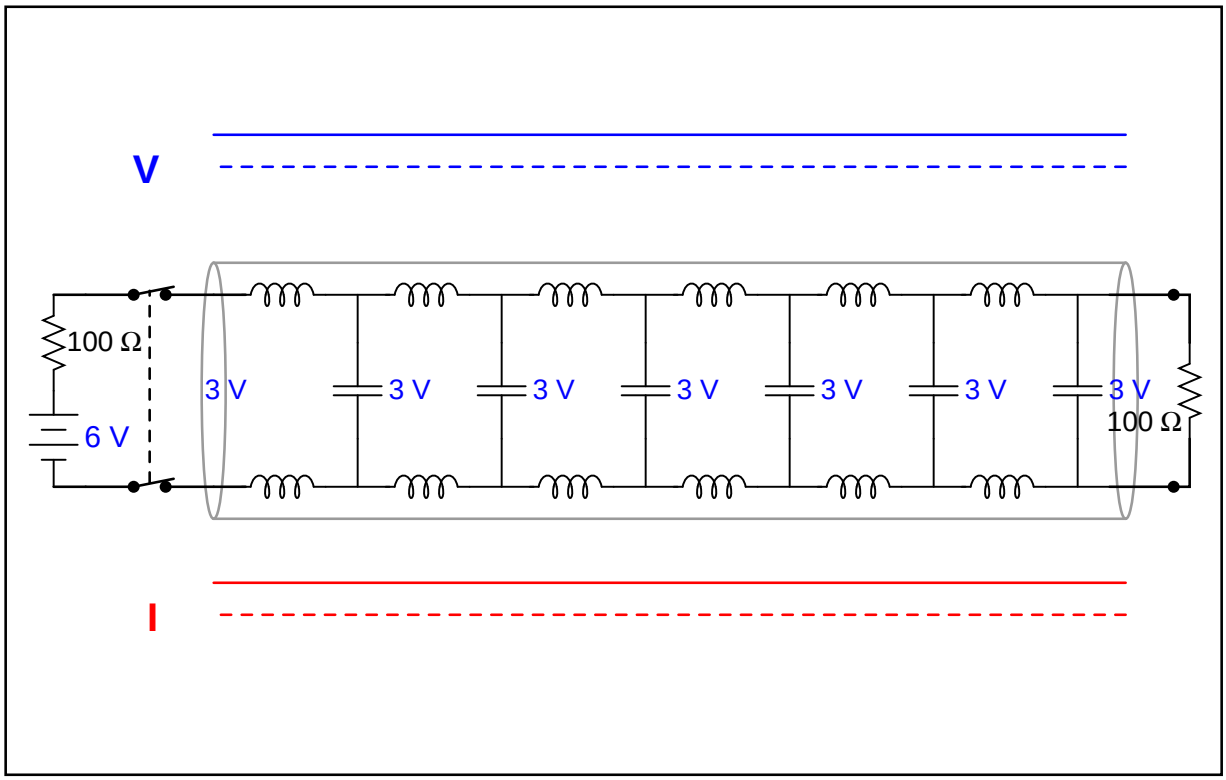
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100 Ω
6 V
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3 V
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3 V
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100 Ω
I

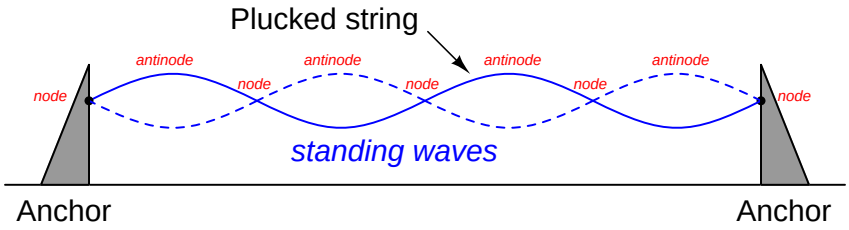
5.4 Animation of a standing wave with full reflection

This series of animations shows waveforms traveling along a transmission line, the incident (or "forward") waveform shown in red and the reflected waveform shown in blue. It is assumed that the signal source is on the left-hand side of the graph, while the transmission line terminates on the right-hand side of the graph.

If the transmission line is terminated in such a way as to completely reflect the forward waveform at its end, the reflected wave will have the exact same amplitude as the forward wave at the line's end but negative in value. This represents the state of affairs for voltage waves at the end of a shorted transmission line, or current waves at the end of an open-ended transmission line. The concept here is that the line's termination does not permit any amplitude at that end-point (e.g. no voltage at a short, no current at an open), and so the reflected wave must perfectly cancel the forward wave at that location.

However, the really interesting phenomenon is what happens as the reflected wave travels back toward the source as the source continues to output a forward wave. These two moving sinusoids superimpose with each other to produce a resultant waveform. This resultant wave does not "travel" along the transmission line like the forward and reflected waves do, but rather "stands" in place while oscillating – what is known as a *standing wave*.

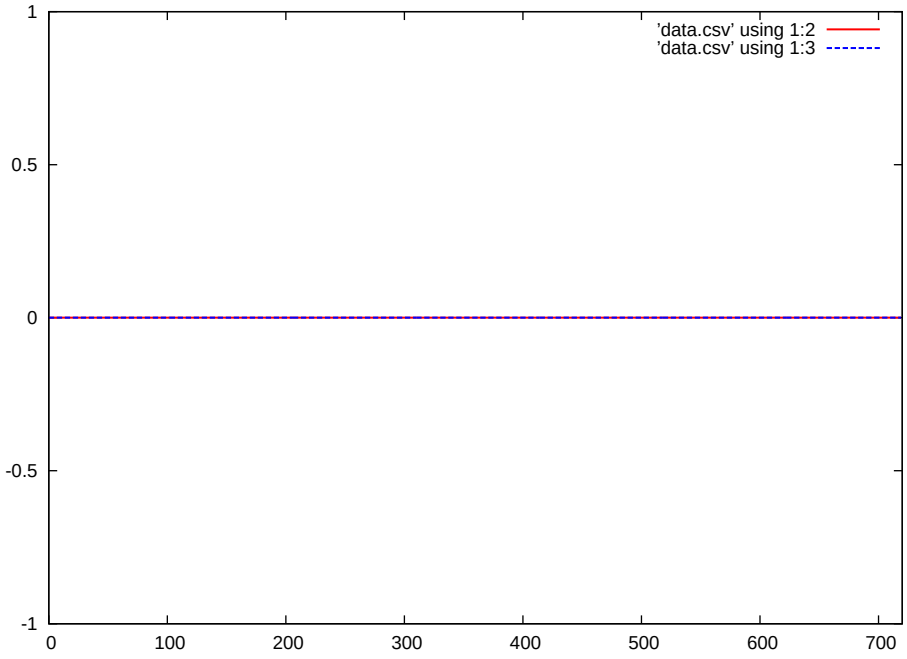
The rather non-intuitive result of standing waves is that some points along the transmission line will experience greater AC voltage than others, and some points will experience greater AC current than others. This phenomenon has many mechanical equivalents, including the existence of standing waves in plucked strings where certain points along the string's length oscillate more violently than others:



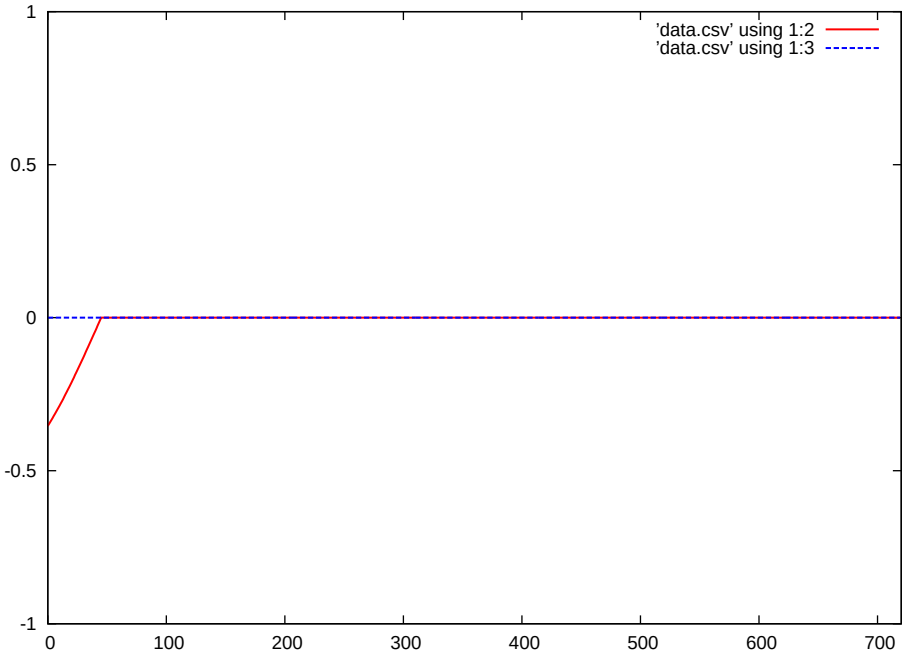
In the following flip-book animations, the first animation shows only the forward and reflected waves along the line for 1440 degrees of rotation. The second animation shows this same sequence of forward and reflected waves, but in an adjacent image shows the superposition of these two waveforms in a different color. The third animation shows that same superposition, but after the forward and reflected waves have traveled the full length of the transmission line to create an established "standing wave" along the line.

Standing wave ratio or *SWR* for any transmission line is defined as the maximum amplitude of the superposition wave divided by its minimum amplitude. In this case, where the superposition of the forward and reflected waves varies from full to zero, the SWR is infinite.

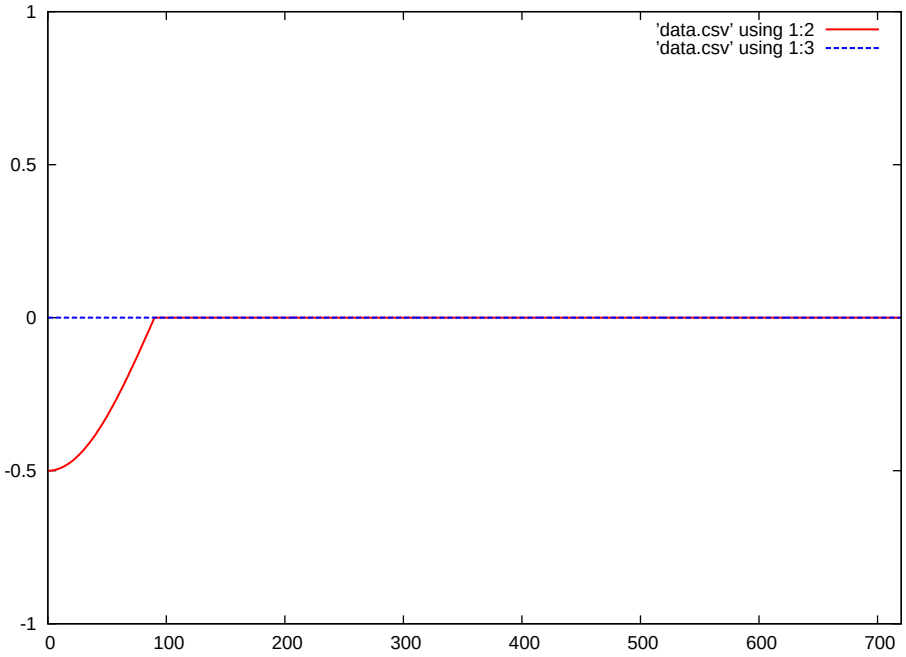
Red = forward wave Blue = reflected wave



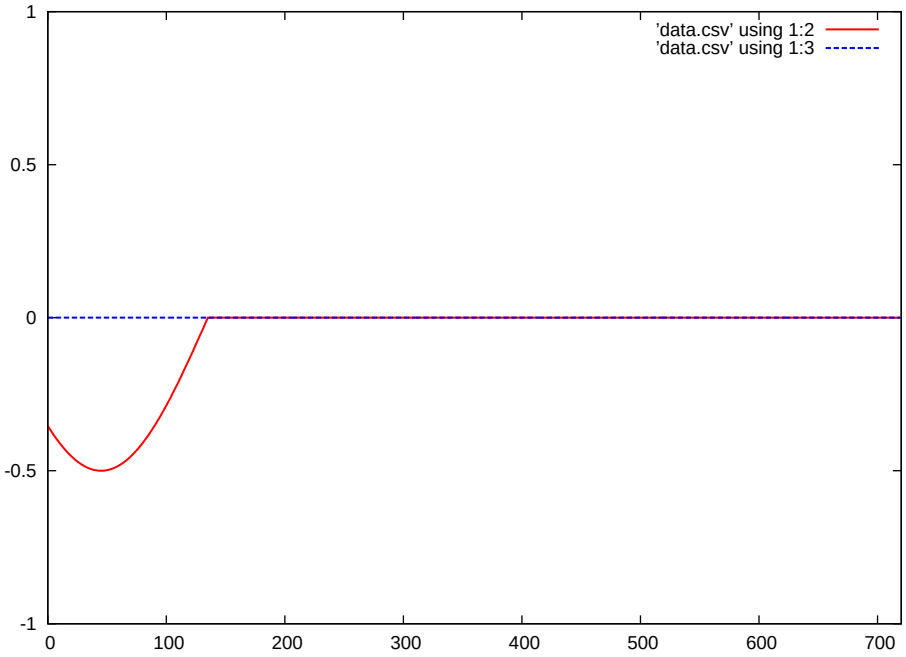
Red = forward wave Blue = reflected wave



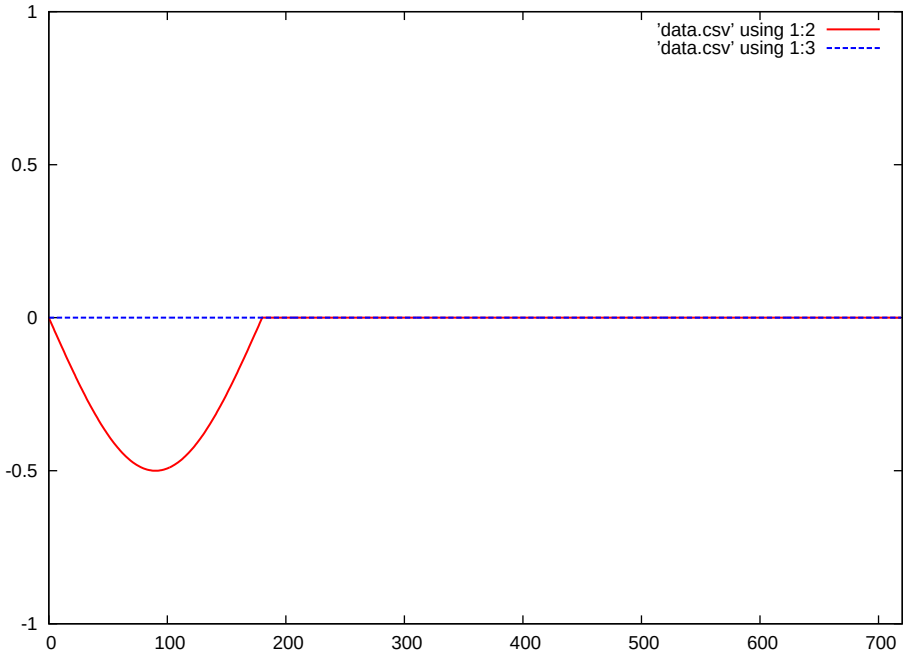
Red = forward wave Blue = reflected wave



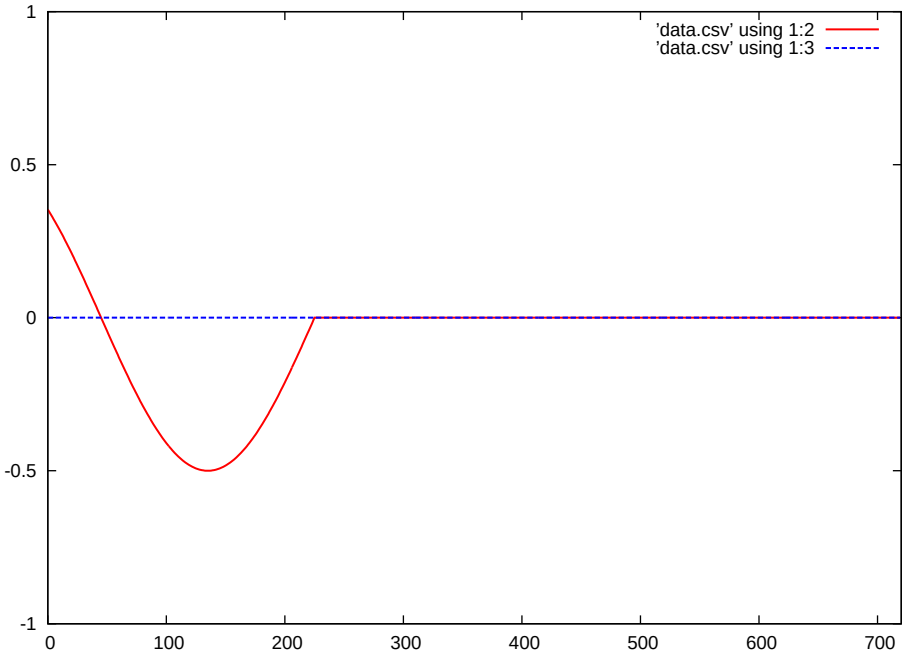
Red = forward wave Blue = reflected wave



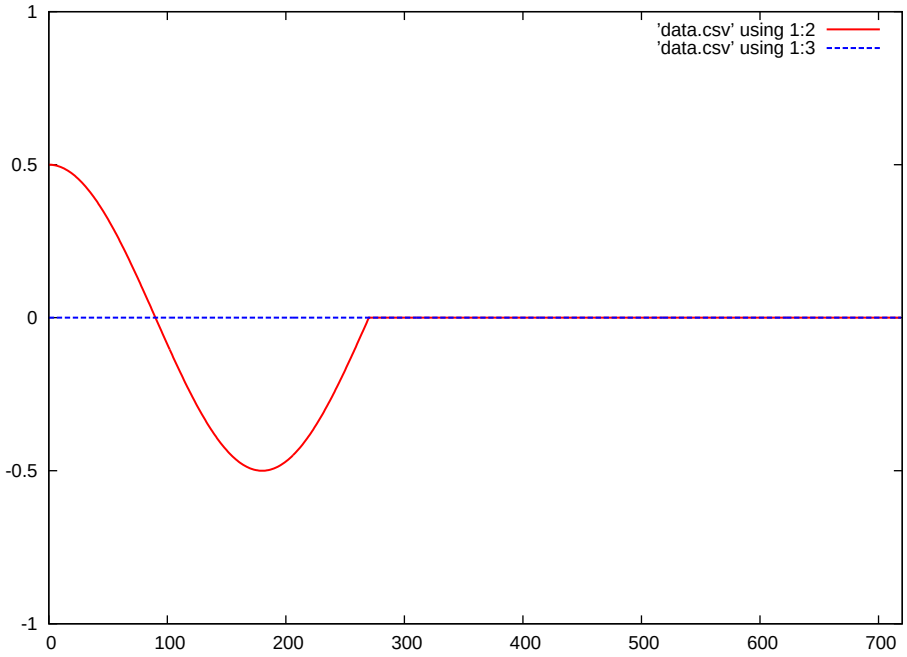
Red = forward wave Blue = reflected wave



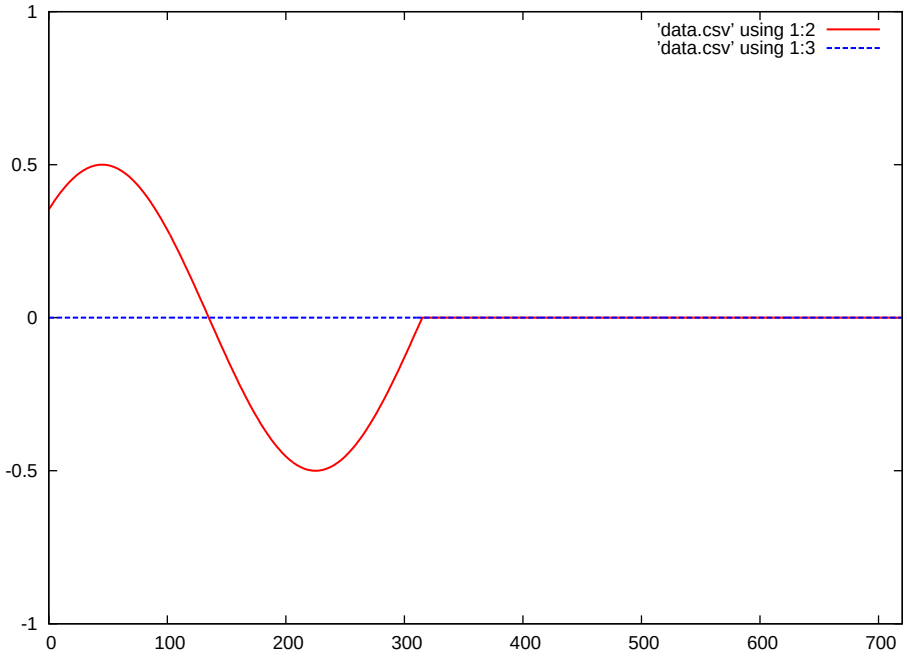
Red = forward wave Blue = reflected wave



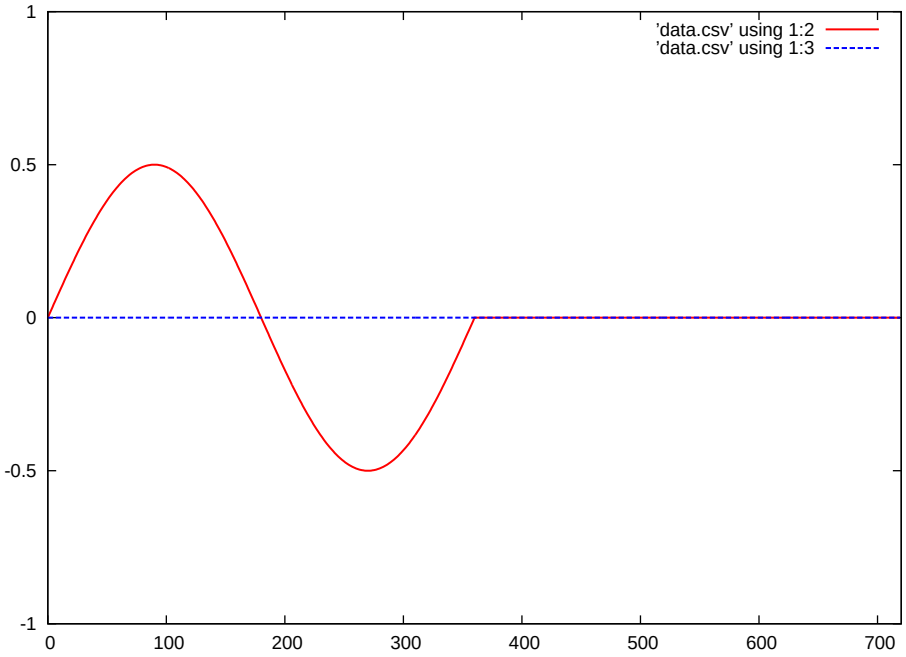
Red = forward wave Blue = reflected wave



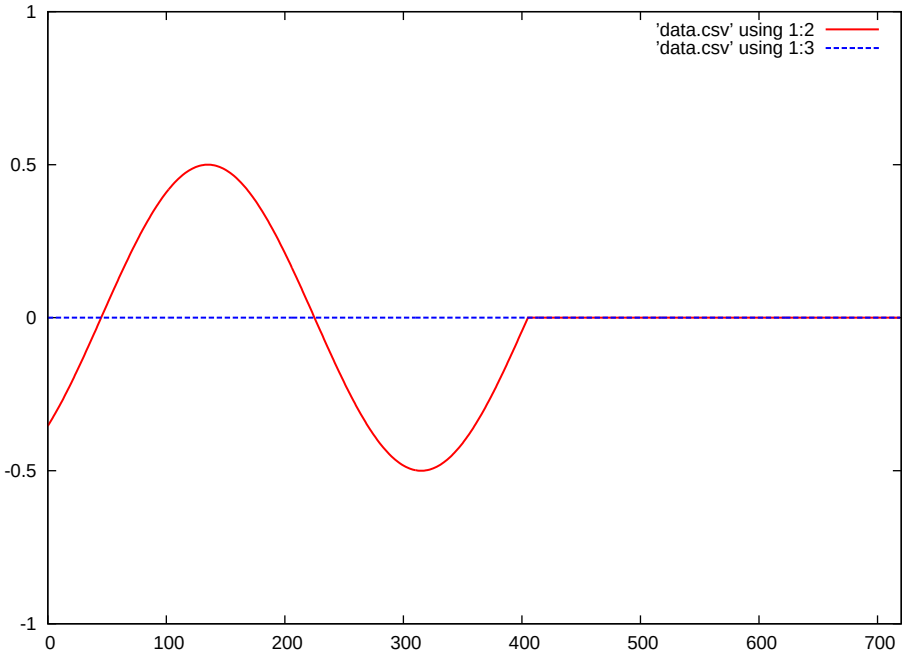
Red = forward wave Blue = reflected wave



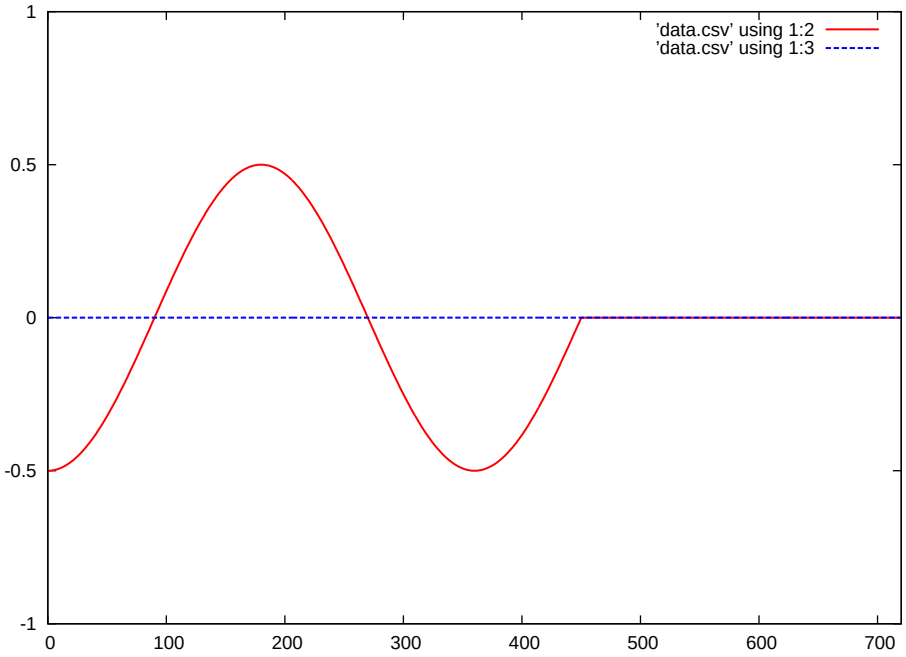
Red = forward wave Blue = reflected wave



Red = forward wave Blue = reflected wave



Red = forward wave Blue = reflected wave



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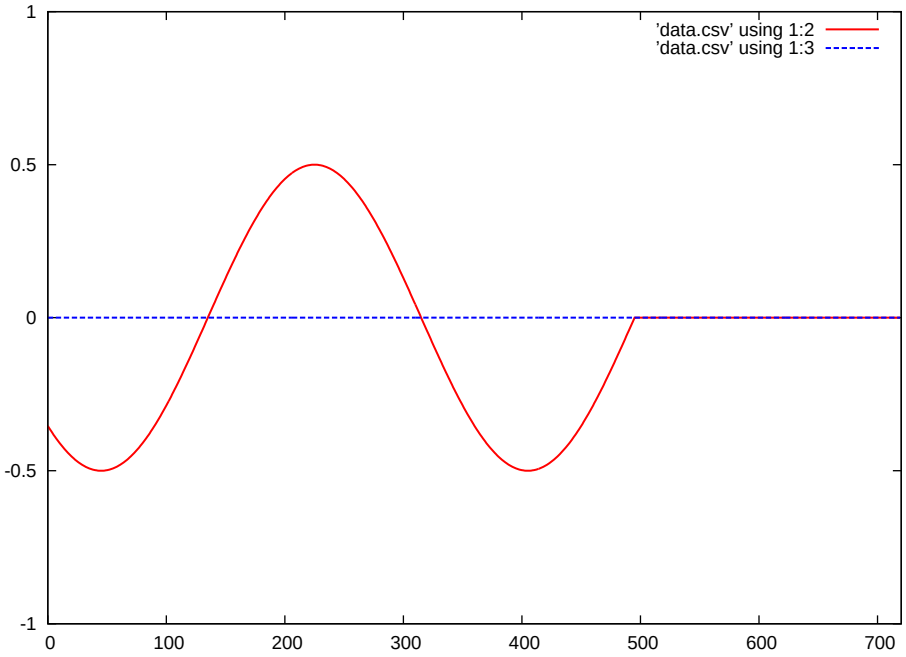

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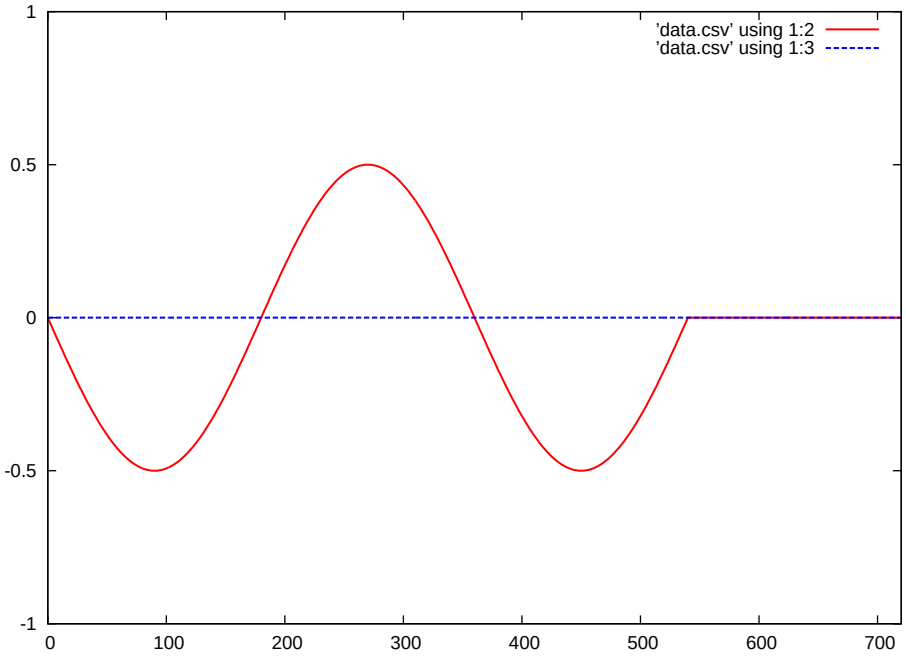
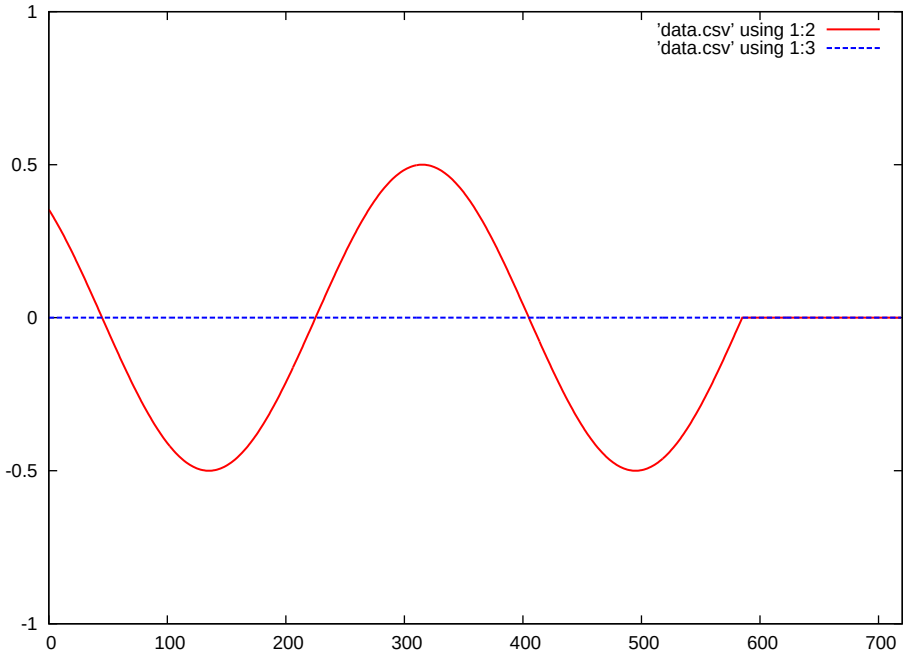
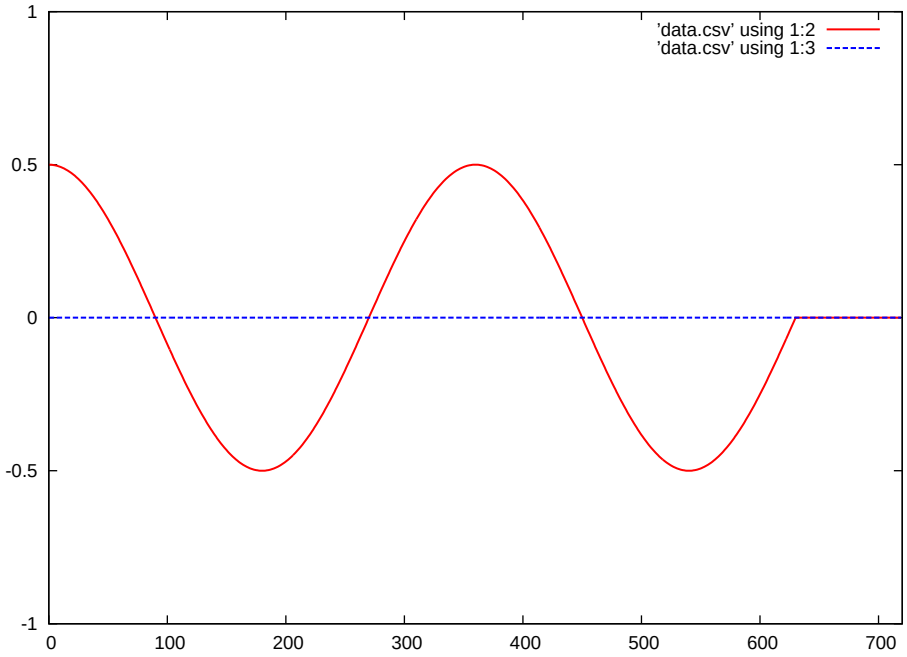


Red = forward wave Blue = reflected wave



Red = forward wave Blue = reflected wave



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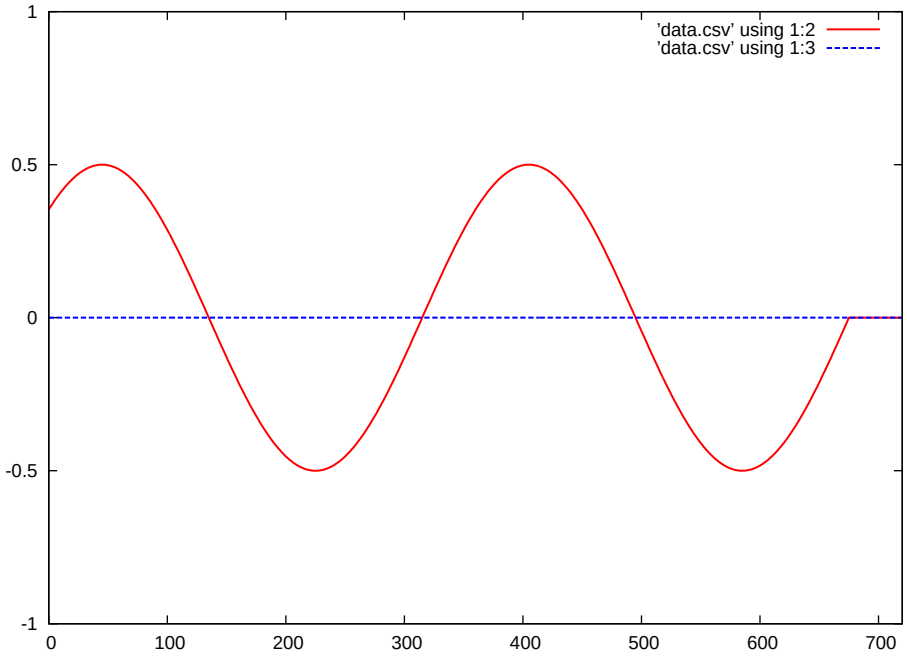
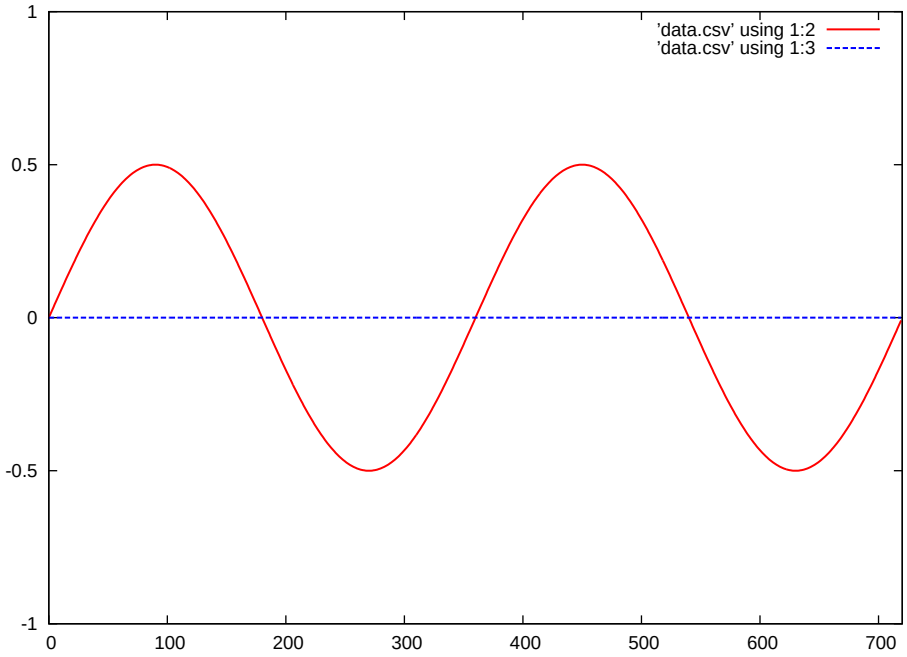


Red = forward wave Blue = reflected wave



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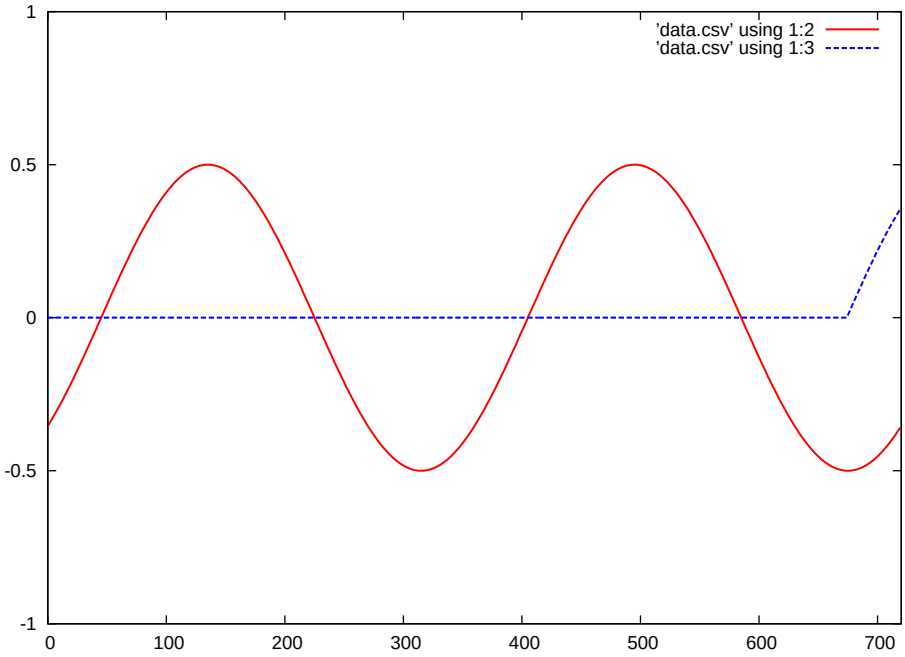

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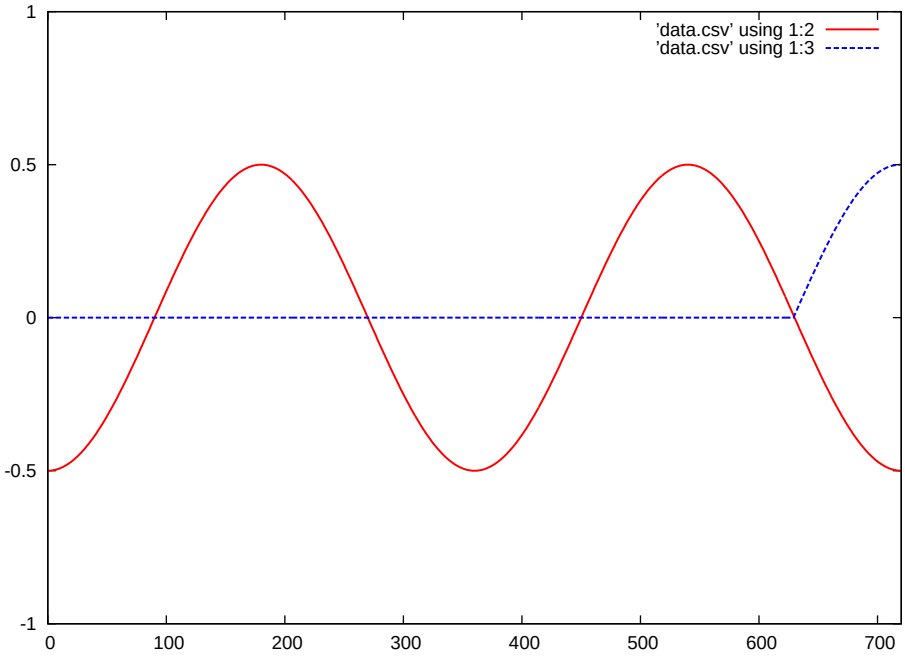

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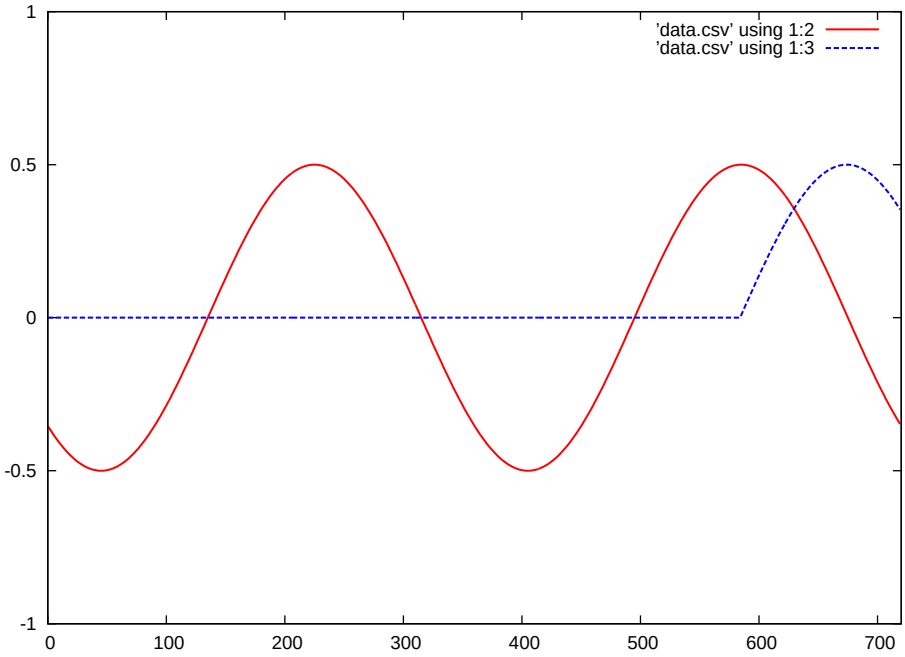

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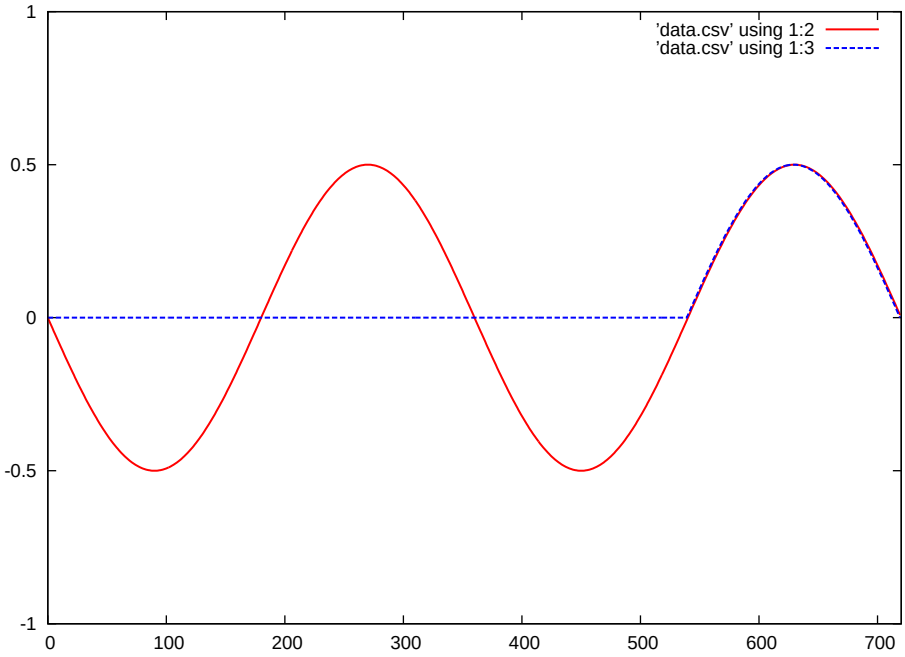

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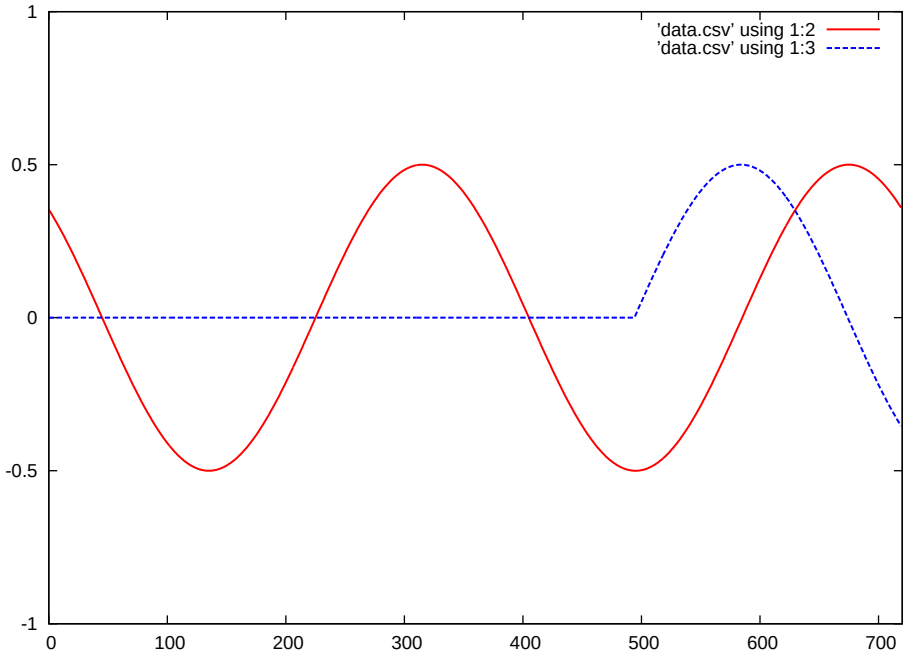

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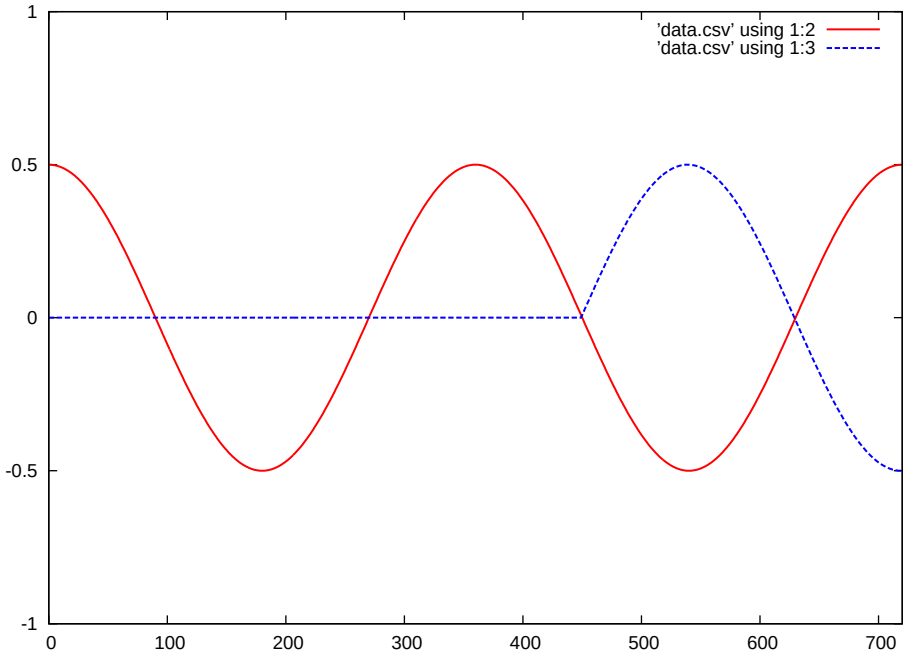

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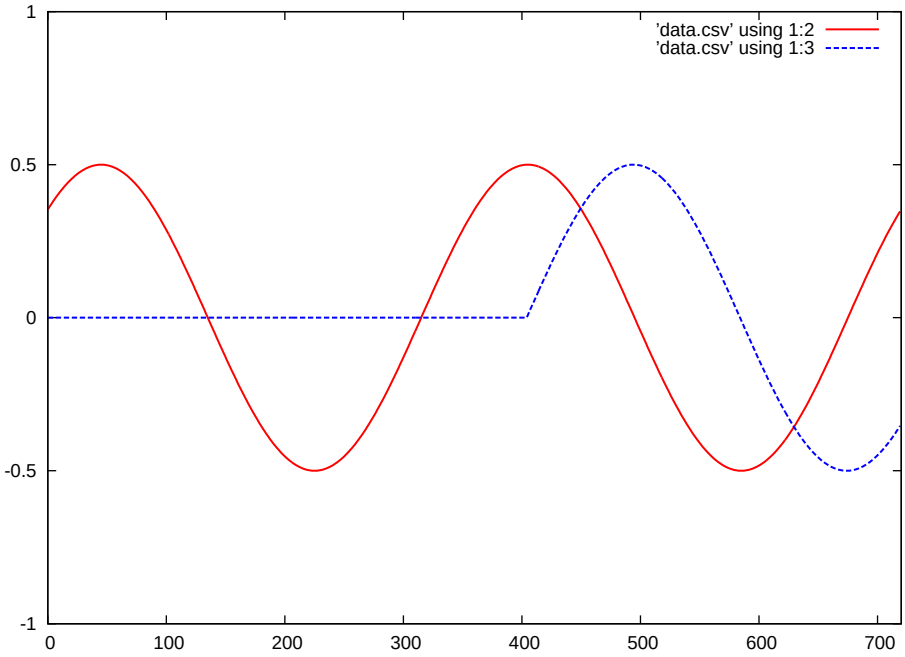

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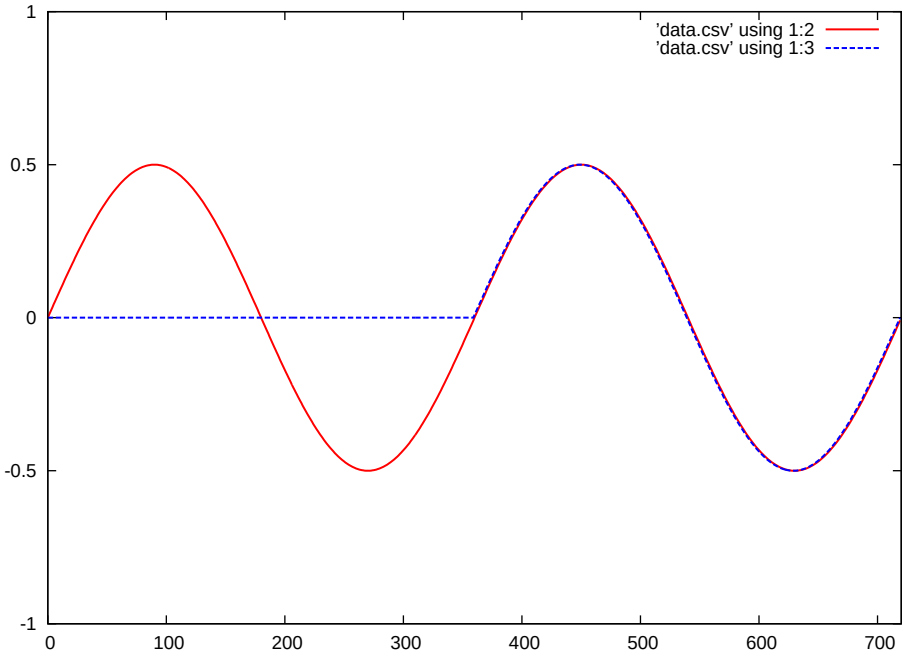
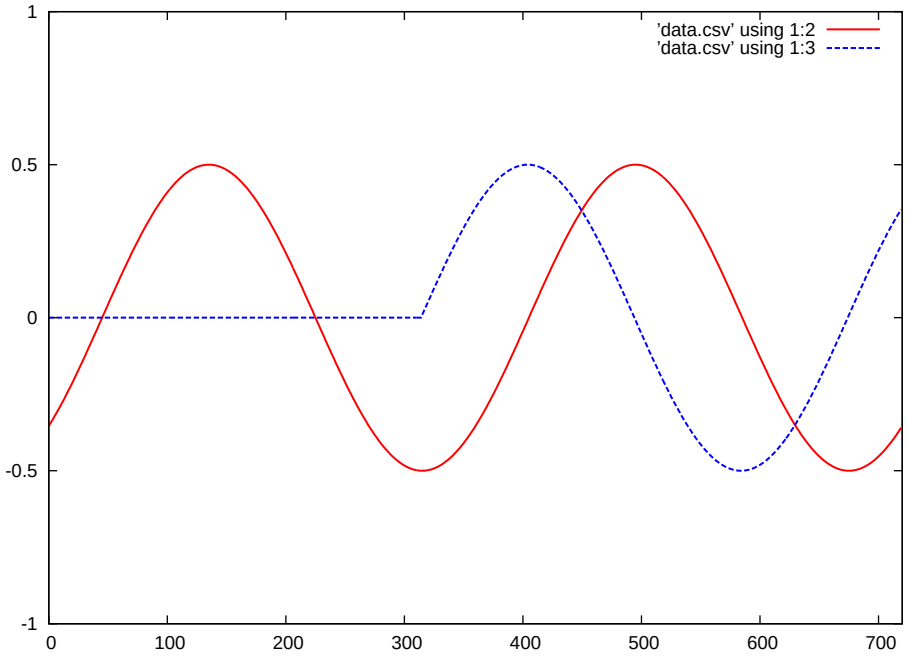
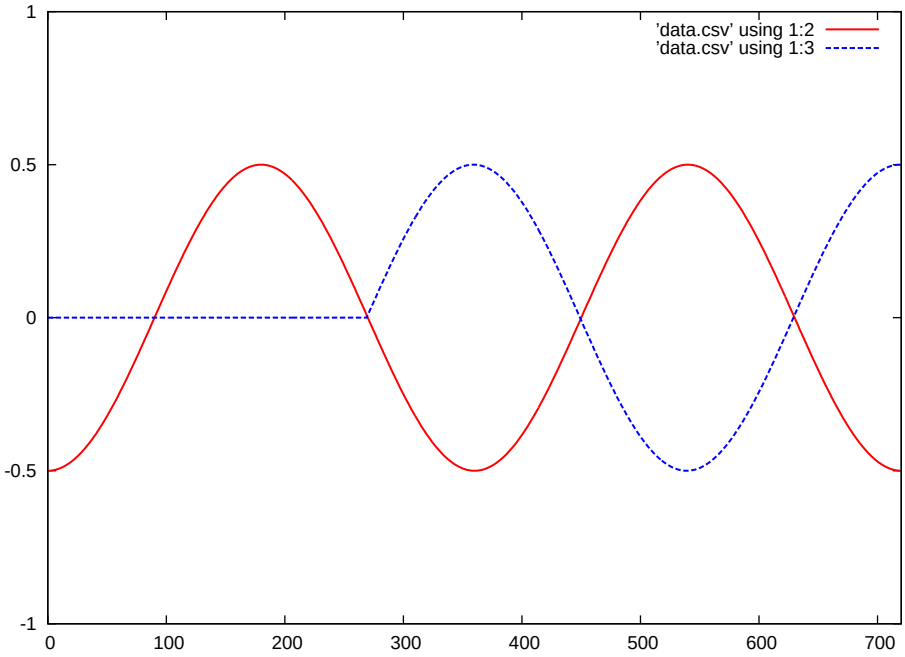


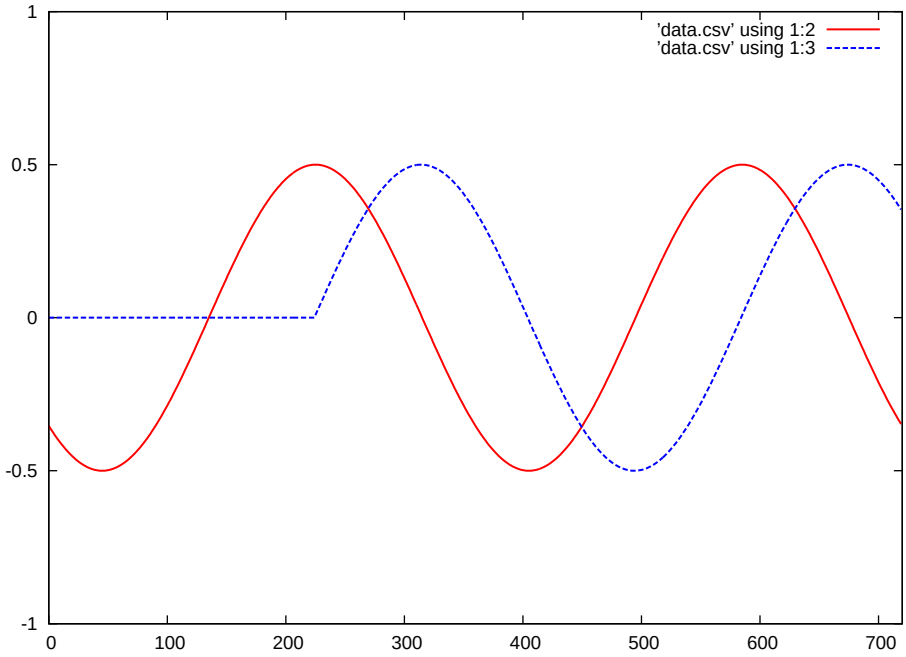
Red = forward wave Blue = reflected wave



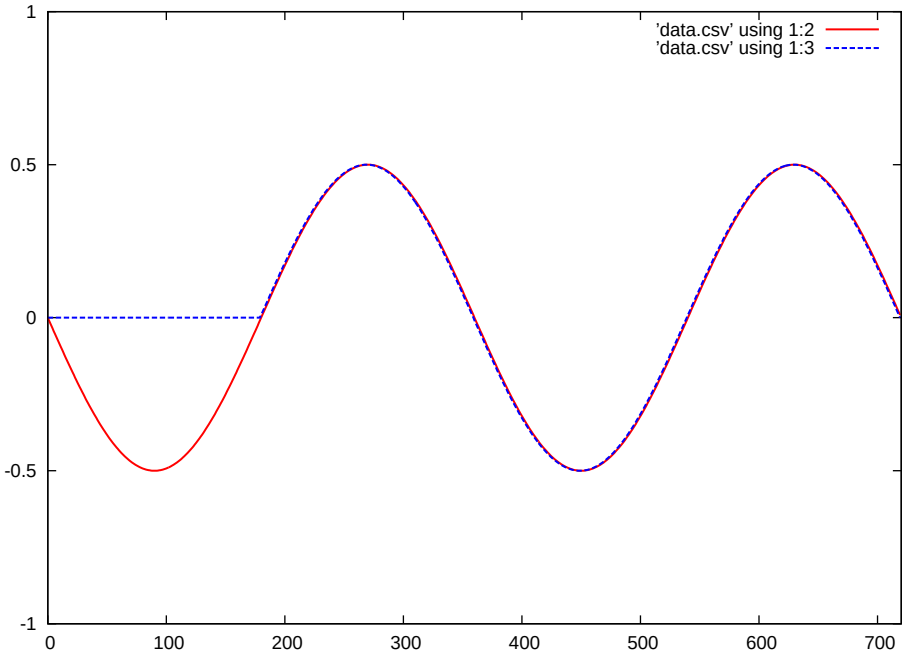
Red = forward wave Blue = reflected wave



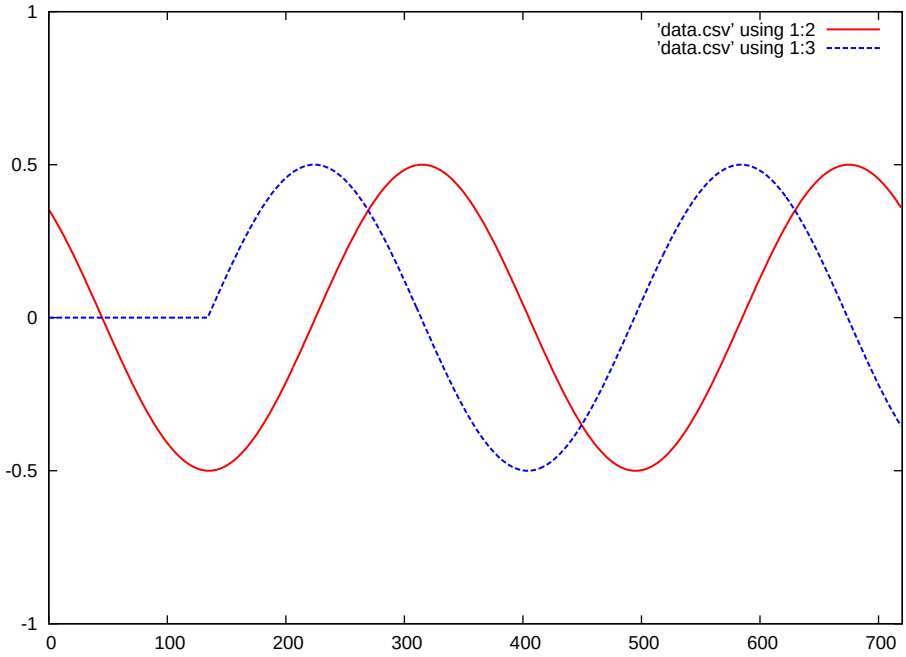
Red = forward wave Blue = reflected wave



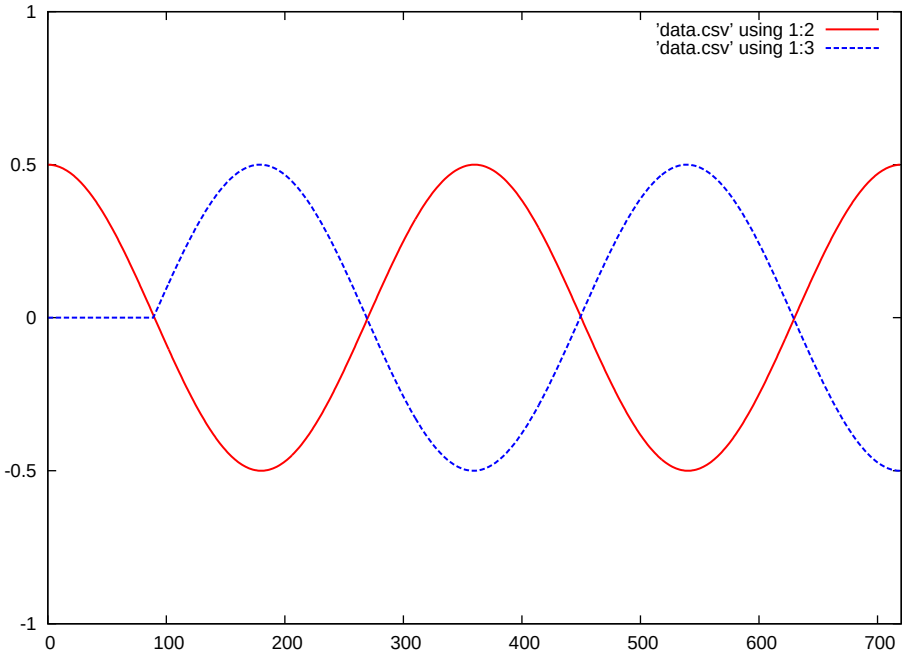
Red = forward wave Blue = reflected wave



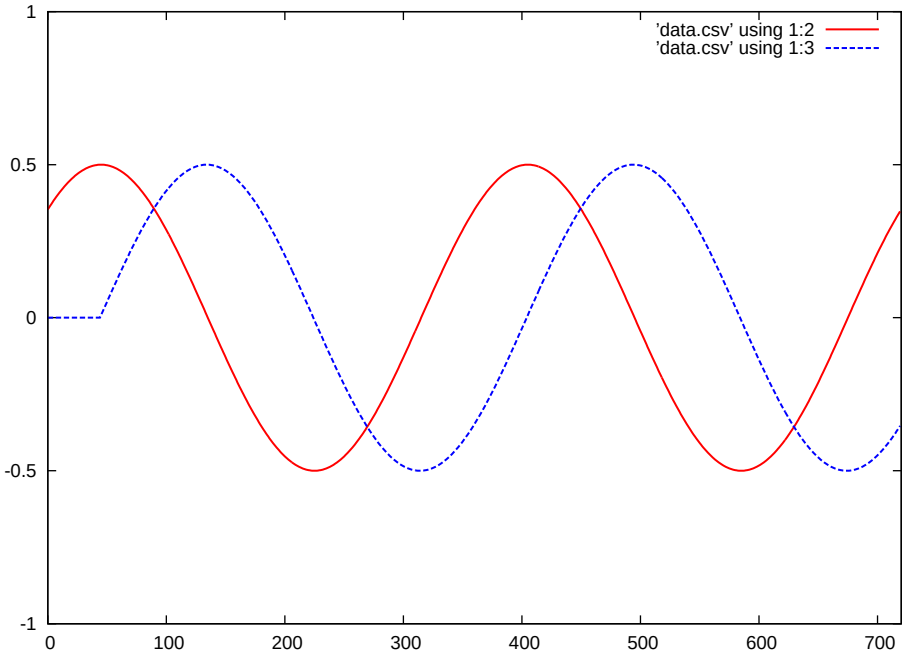
Red = forward wave Blue = reflected wave



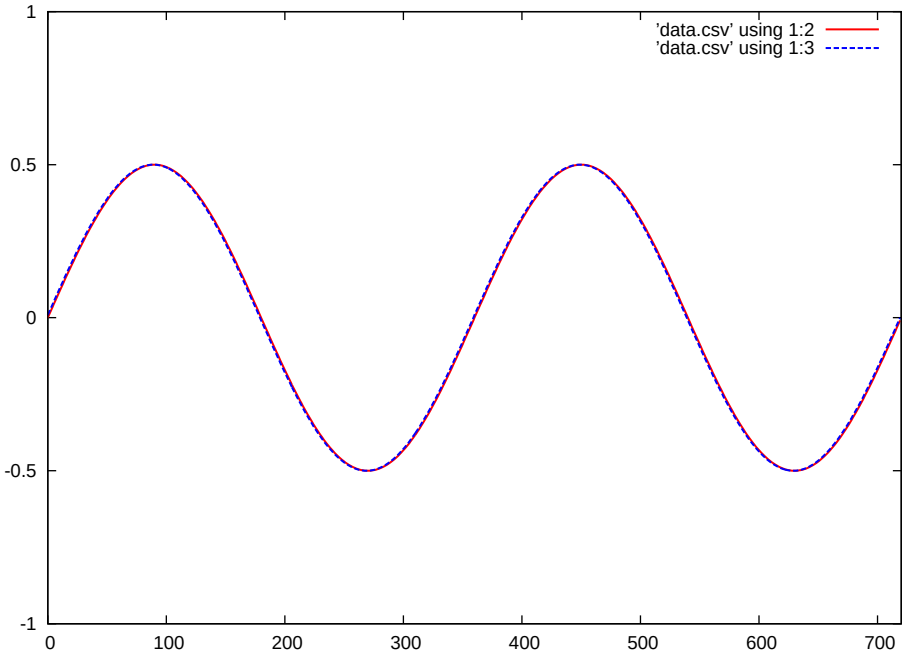
Red = forward wave Blue = reflected wave



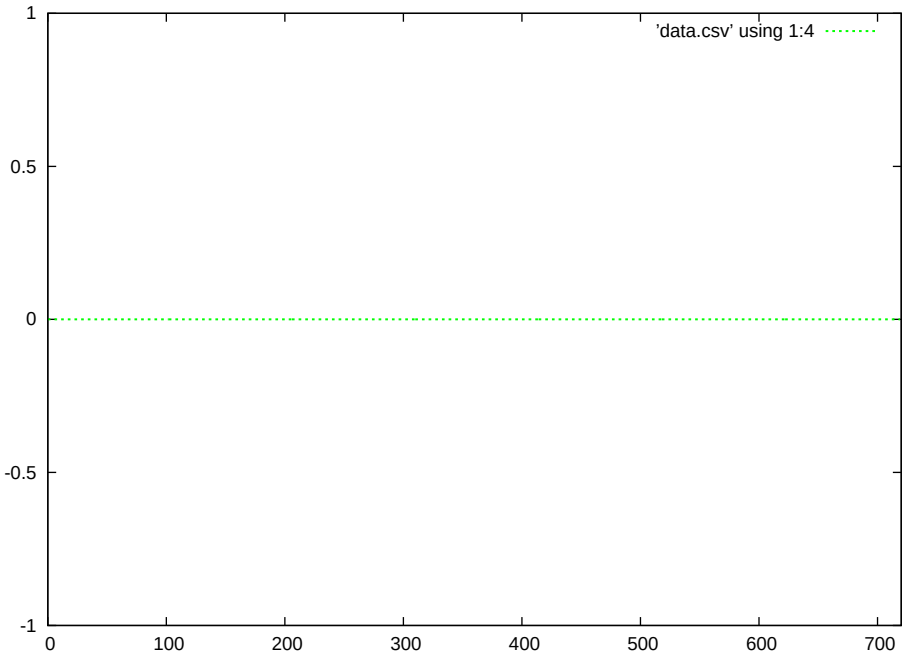
Red = forward wave Blue = reflected wave



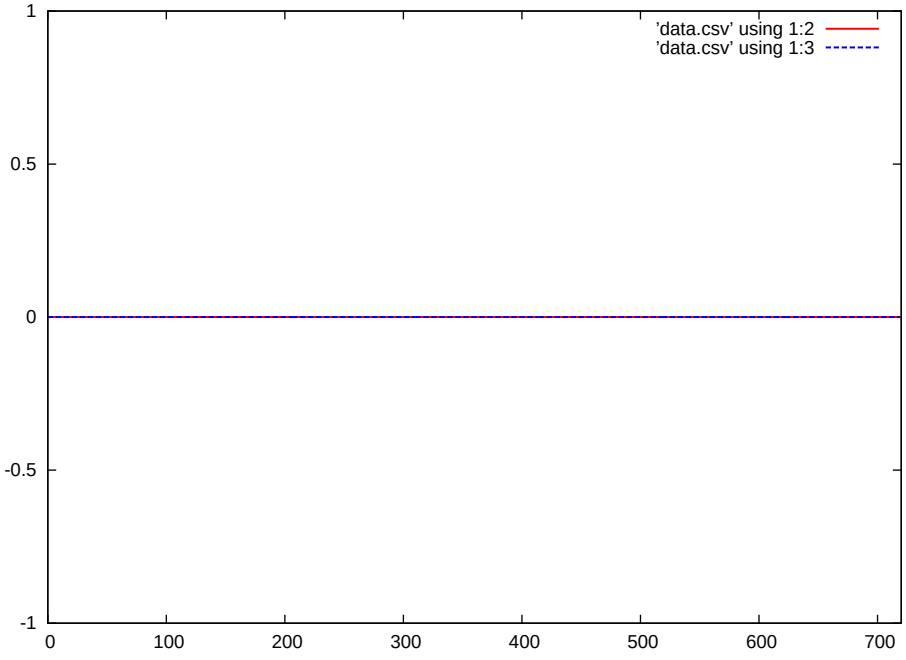
Red = forward wave Blue = reflected wave



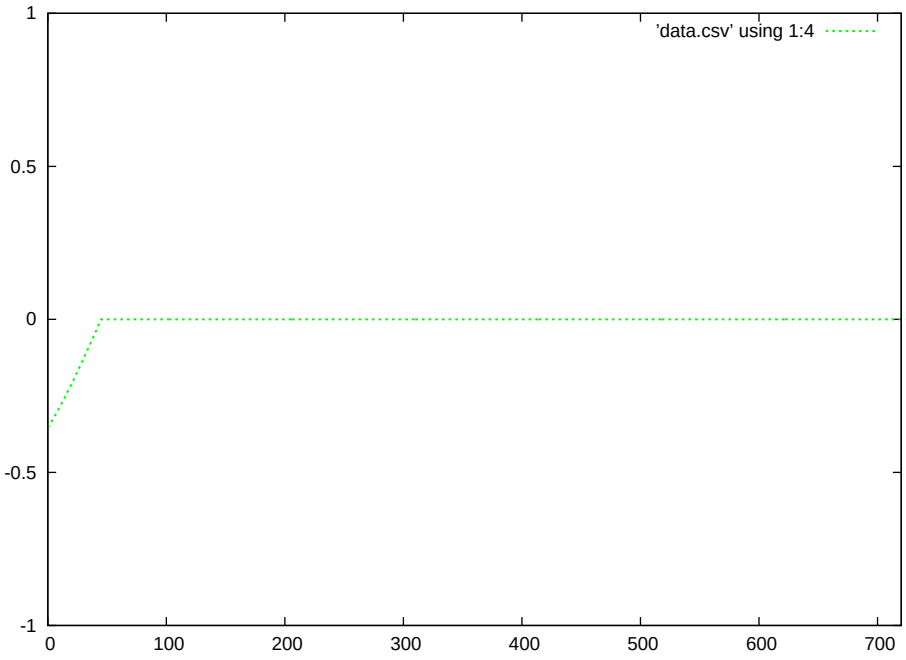
Green = superposition of the forward and reflected waves



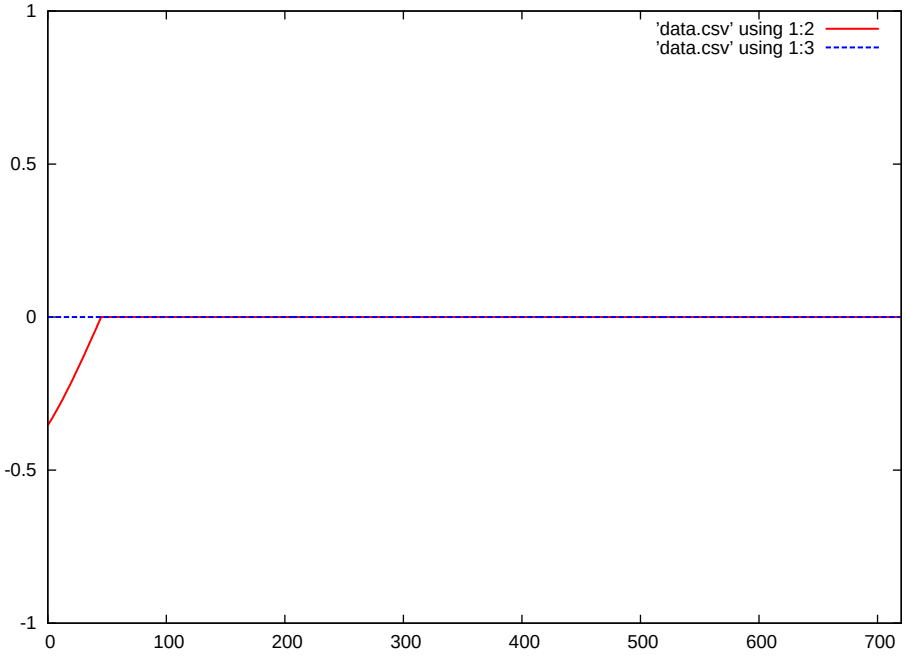
Red = forward wave Blue = reflected wave



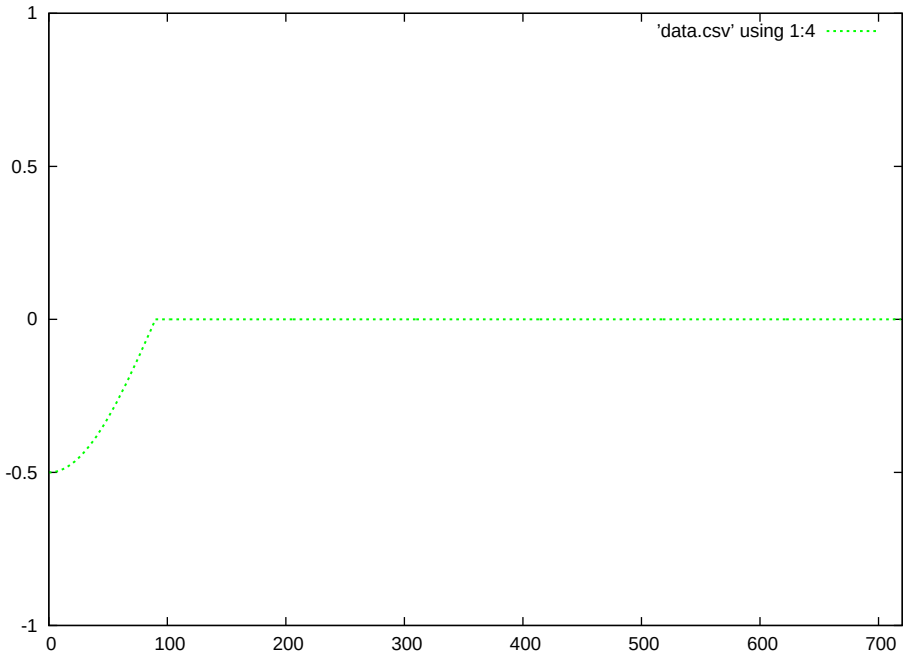
Green = superposition of the forward and reflected waves



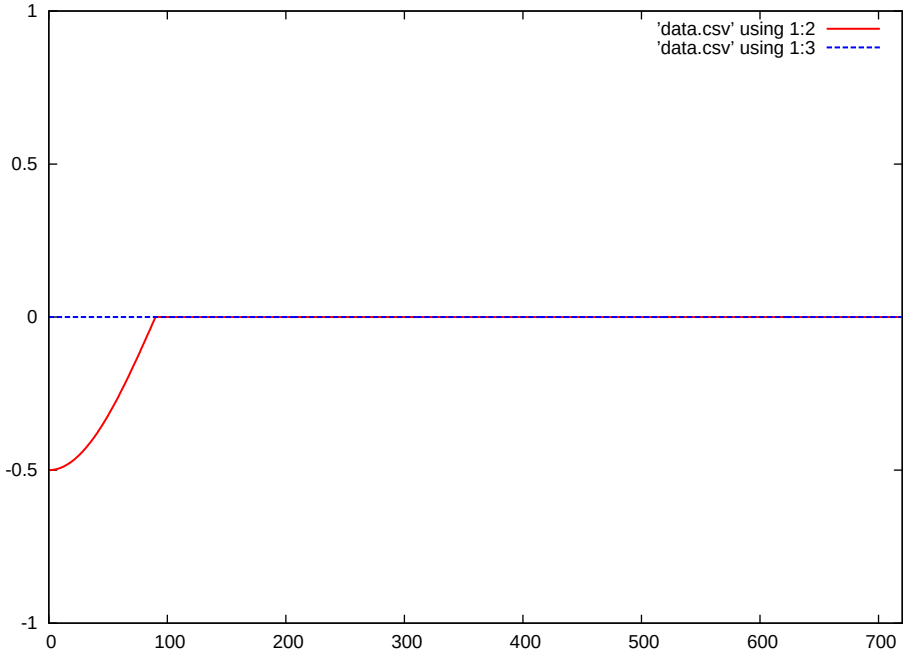
Red = forward wave Blue = reflected wave



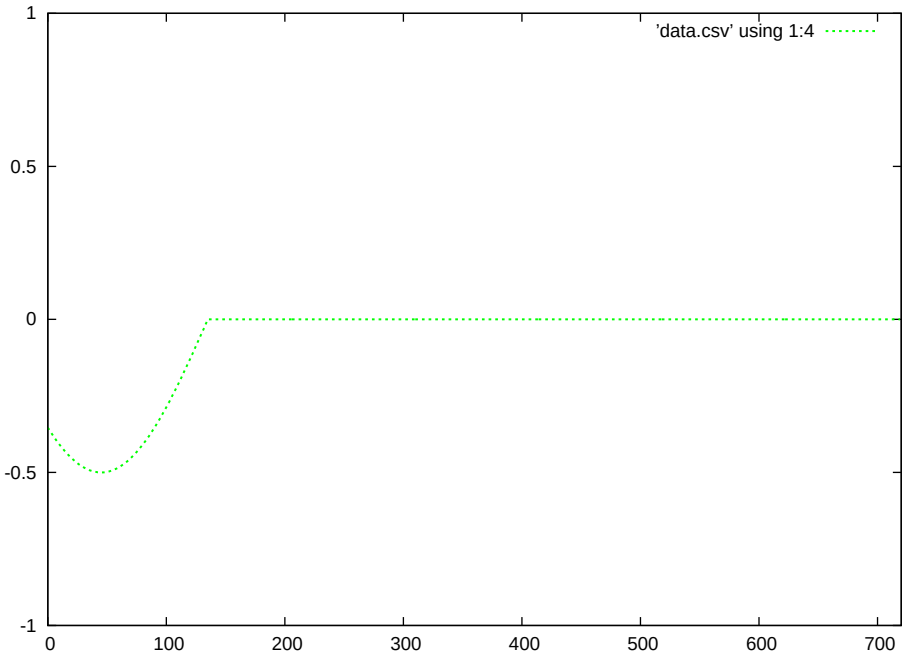
Green = superposition of the forward and reflected waves



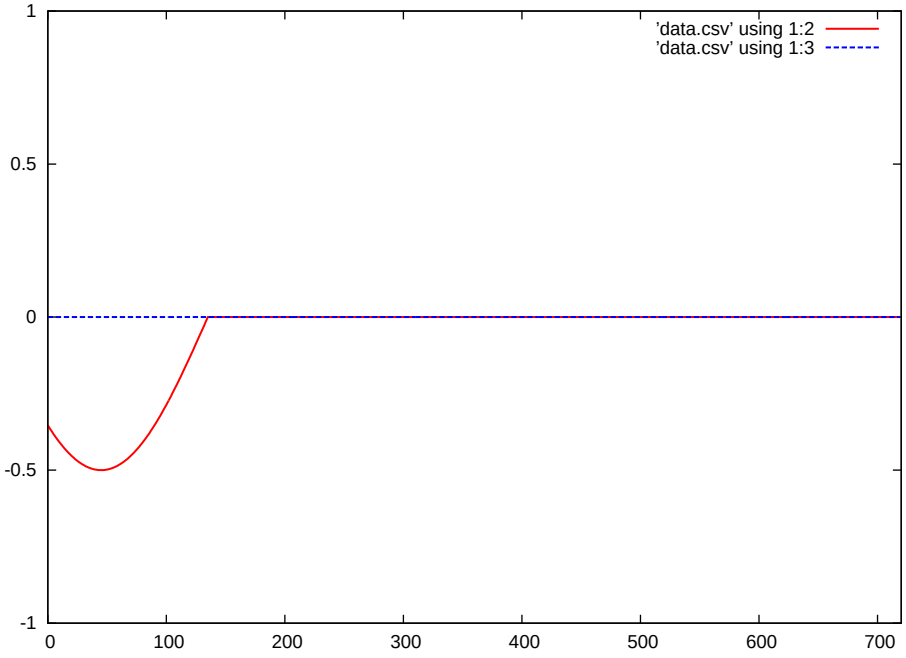
Red = forward wave Blue = reflected wave



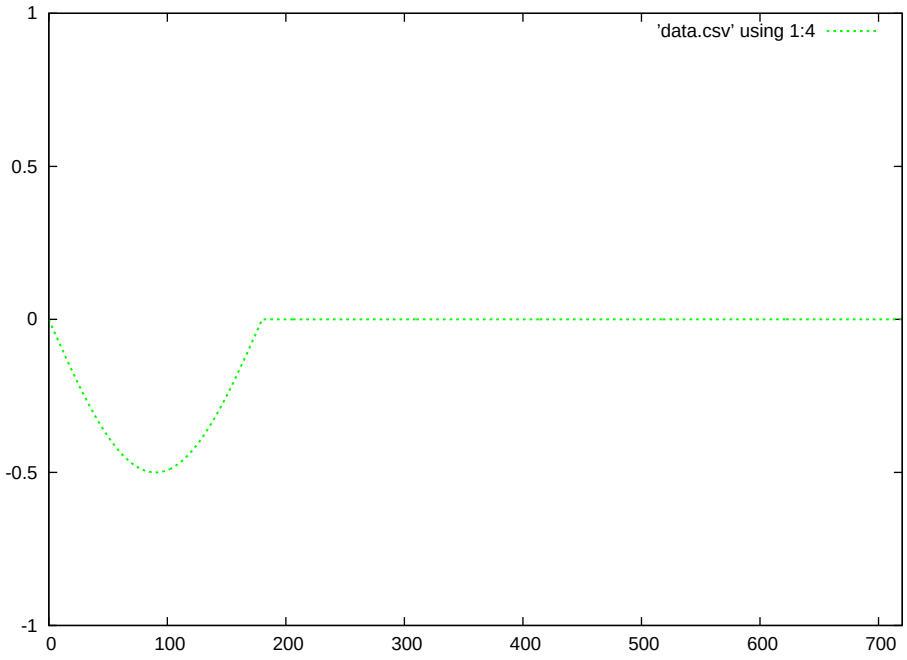
Green = superposition of the forward and reflected waves



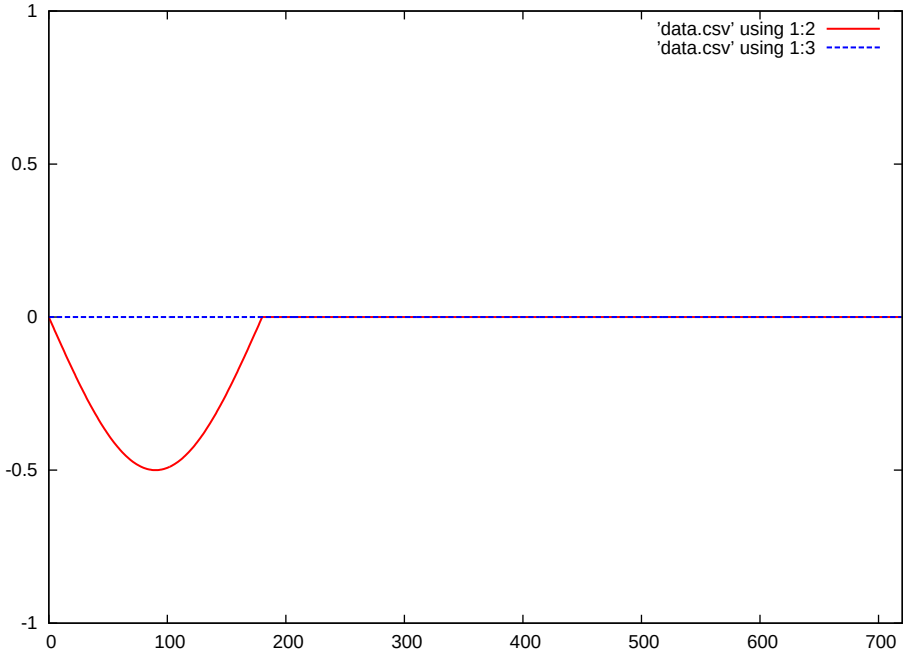
Red = forward wave Blue = reflected wave



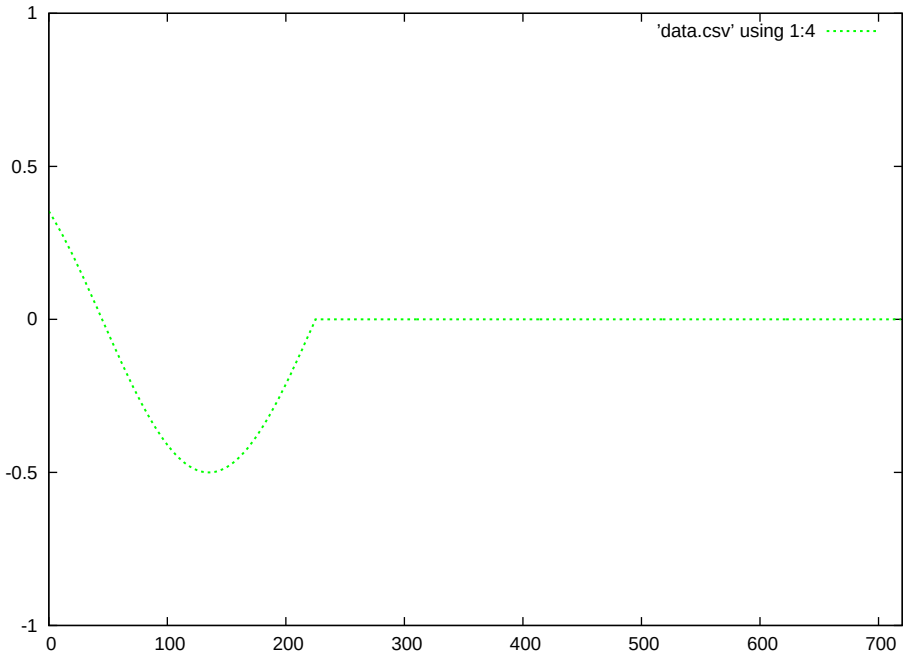
Green = superposition of the forward and reflected waves



Red = forward wave Blue = reflected wave



Green = superposition of the forward and reflected waves



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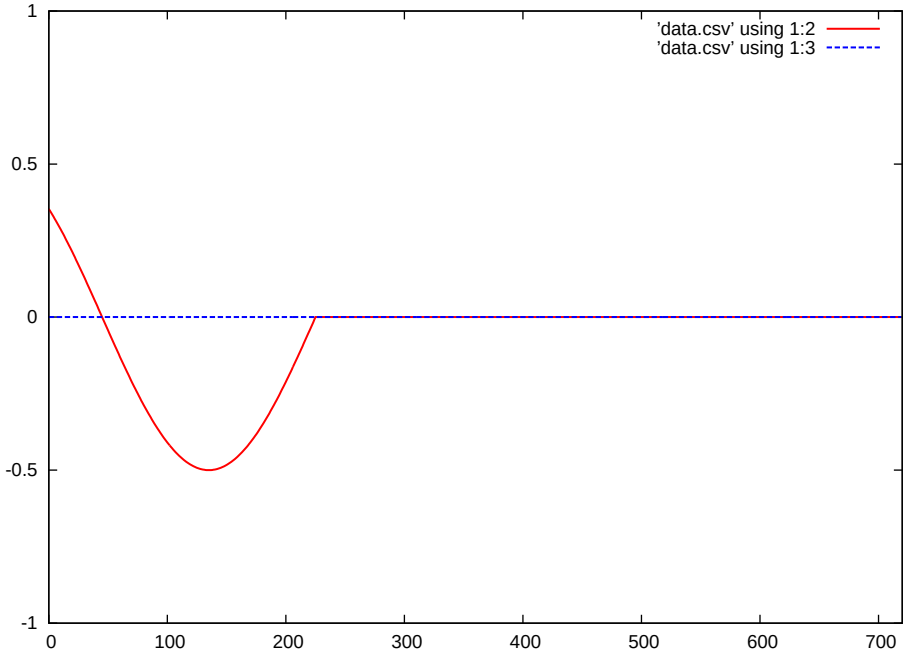
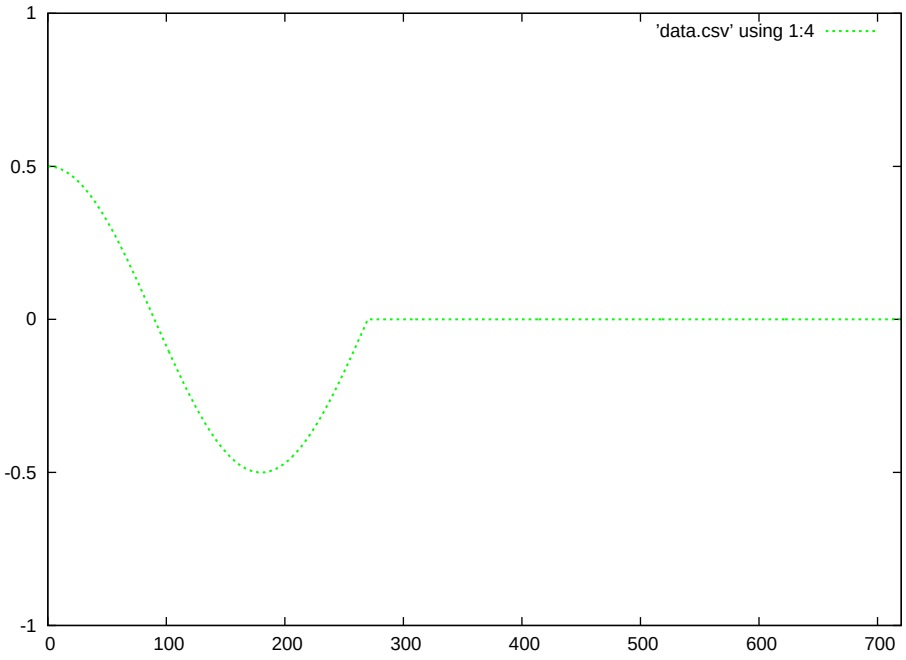


Green = superposition of the forward and reflected waves



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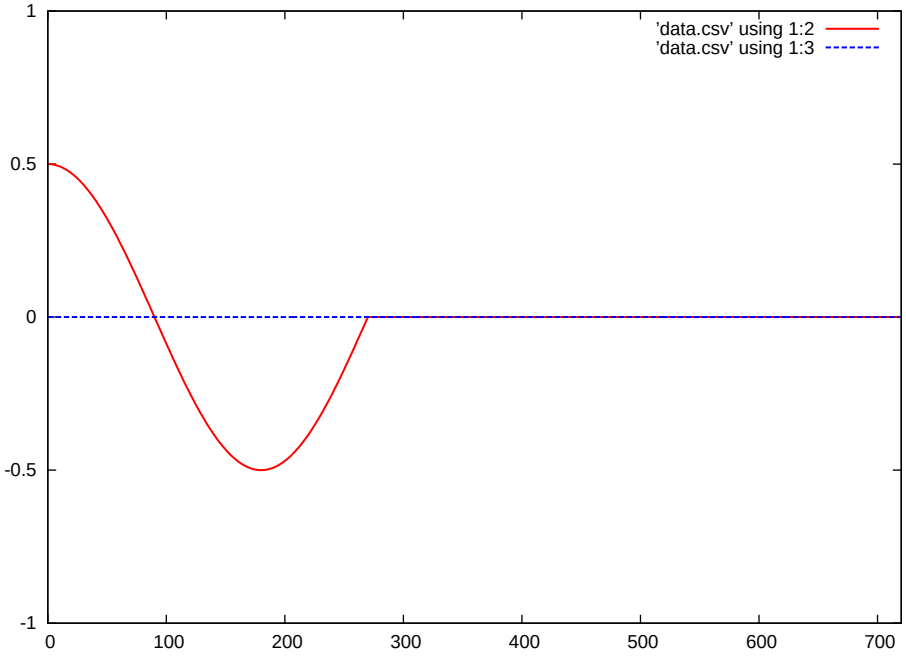
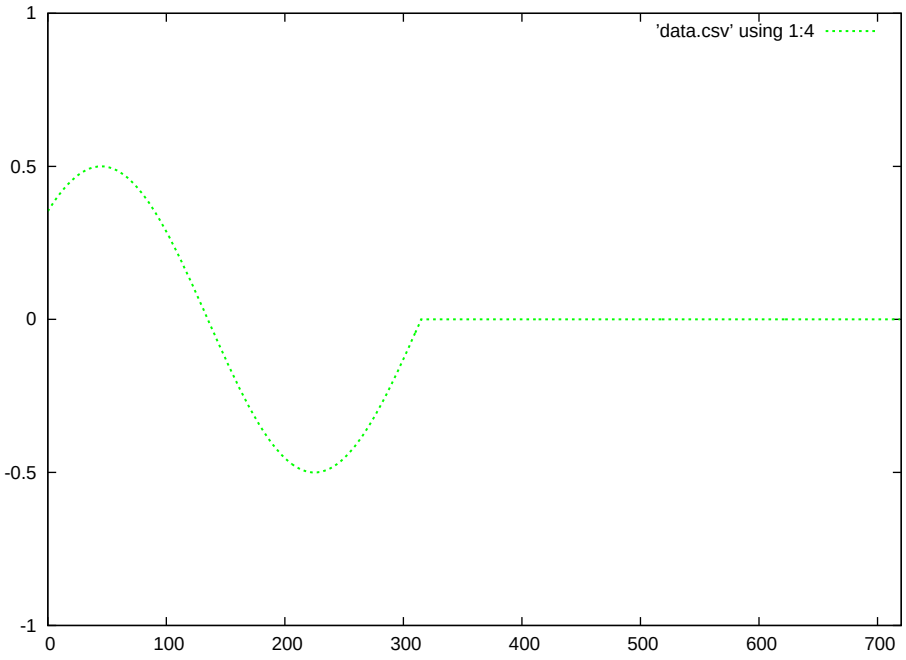


Green = superposition of the forward and reflected waves



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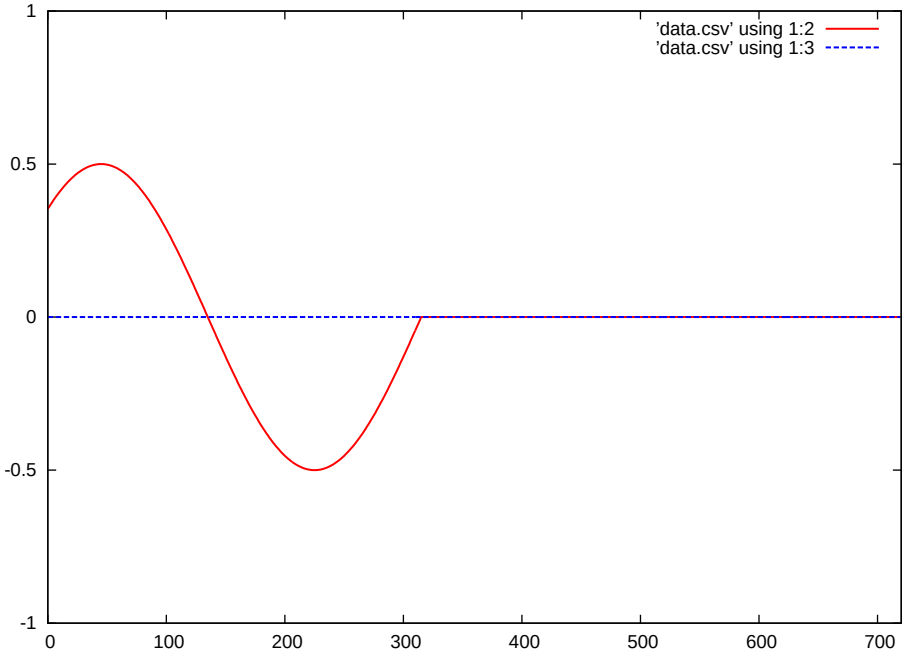

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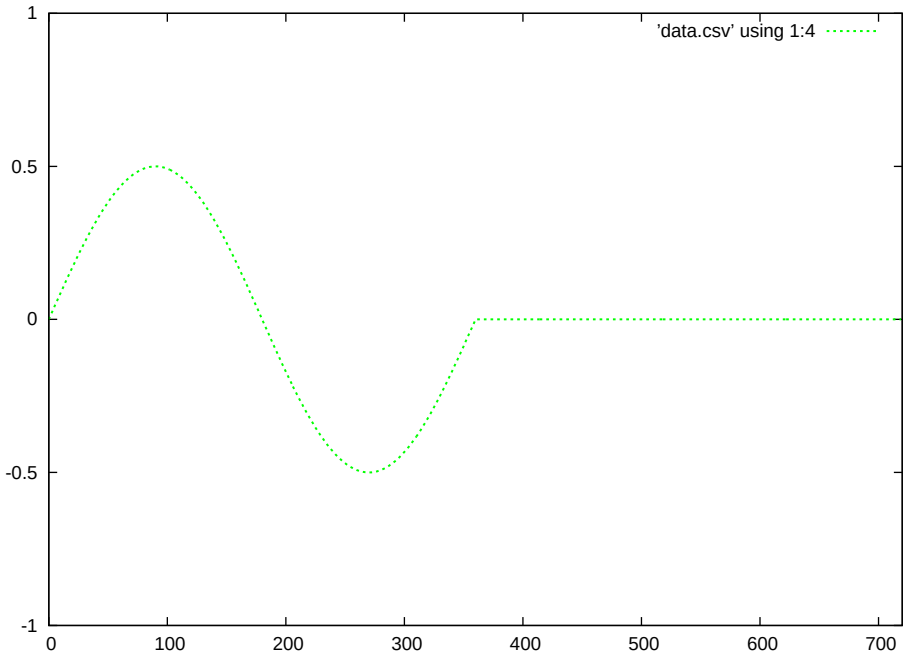

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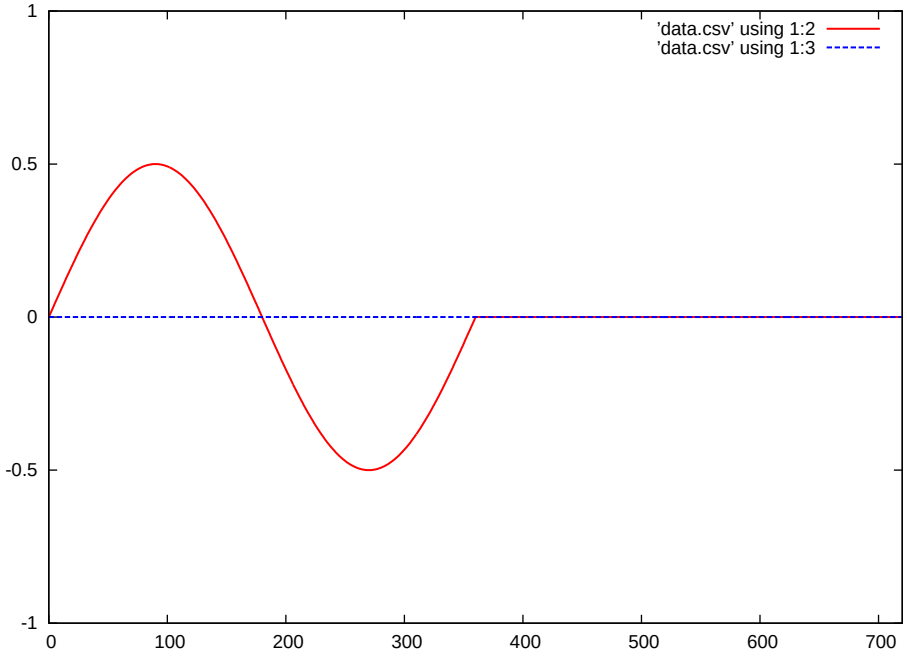

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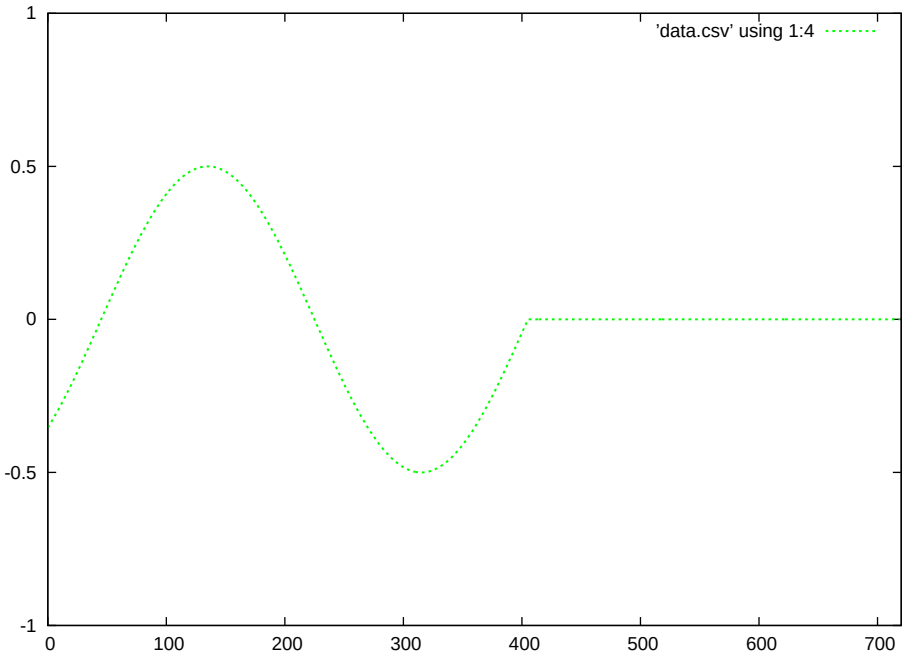

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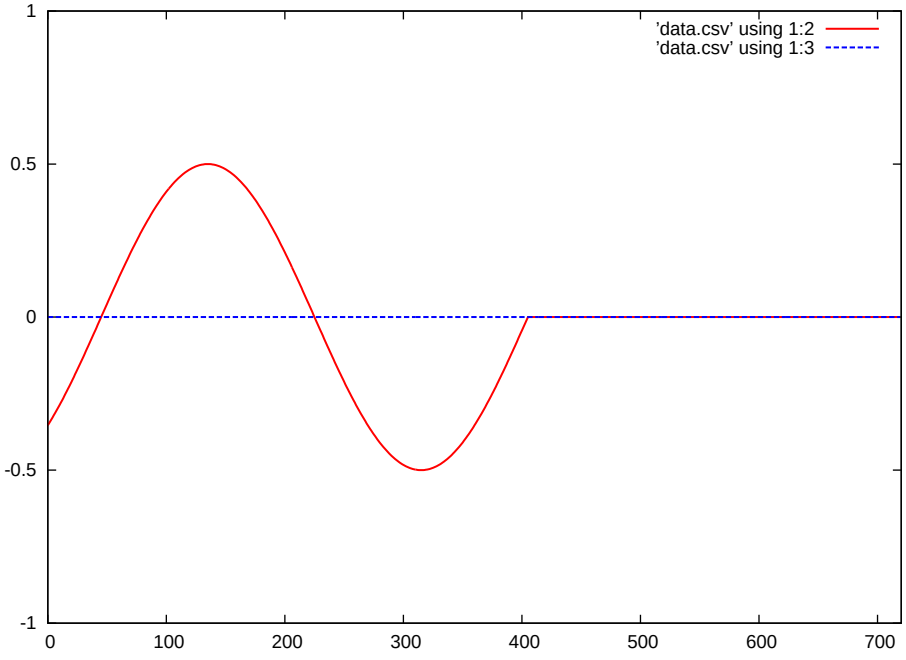

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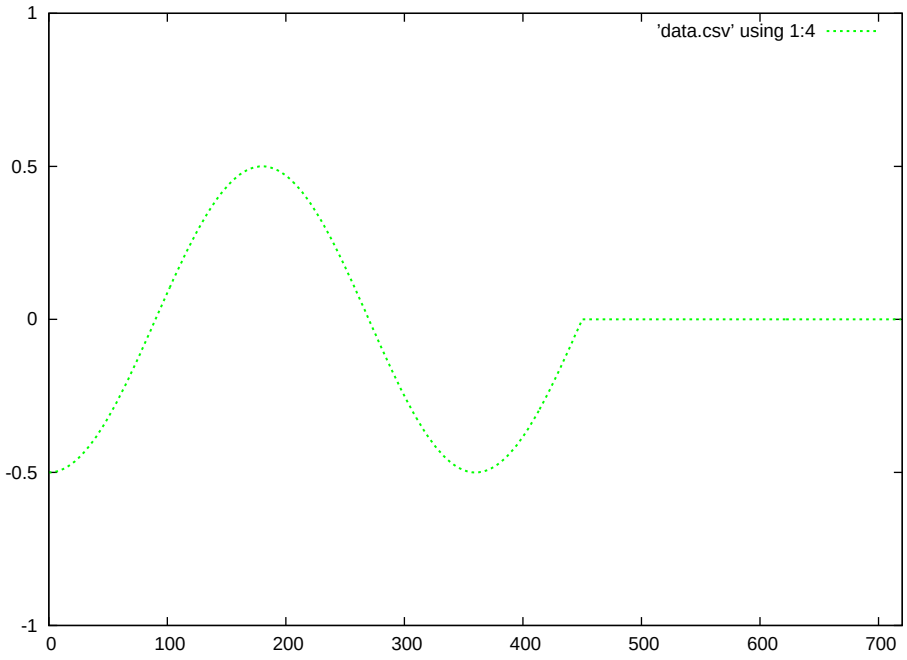

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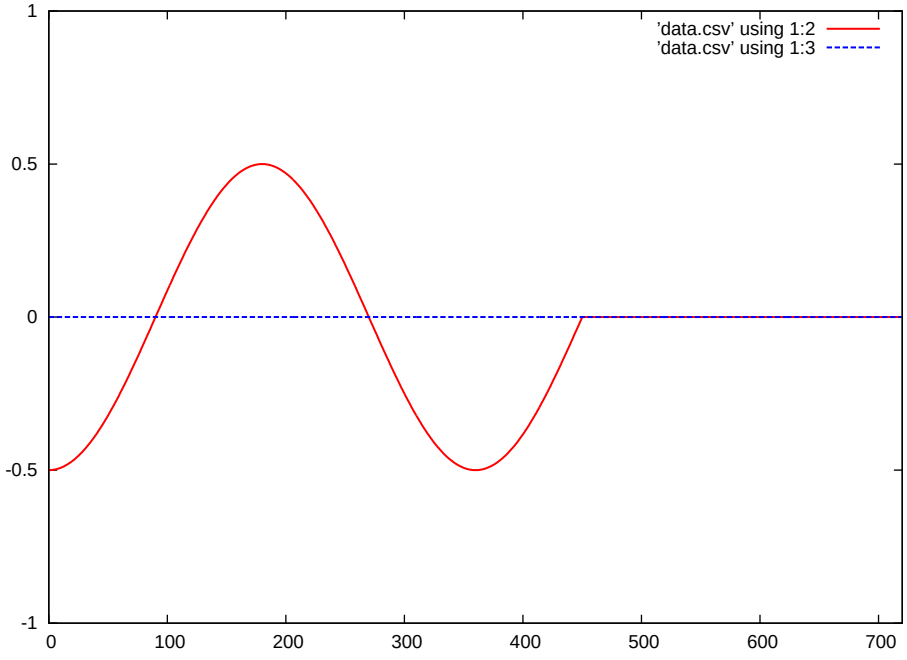

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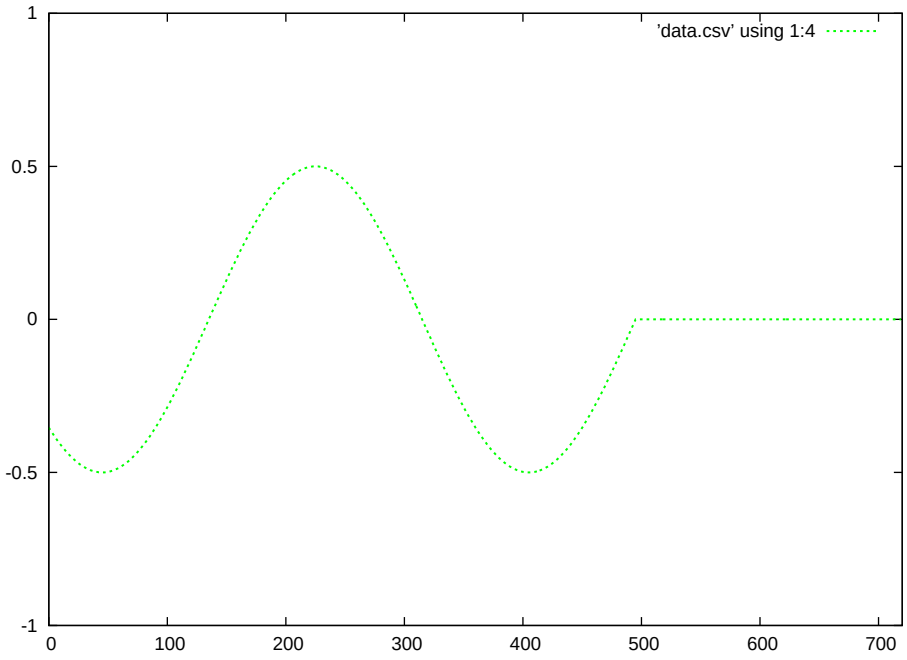

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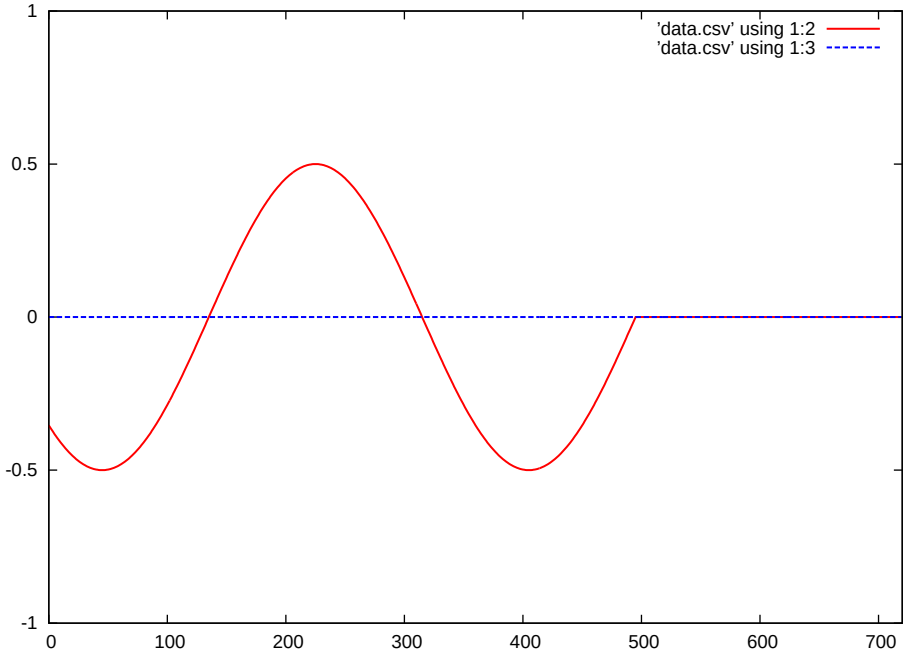

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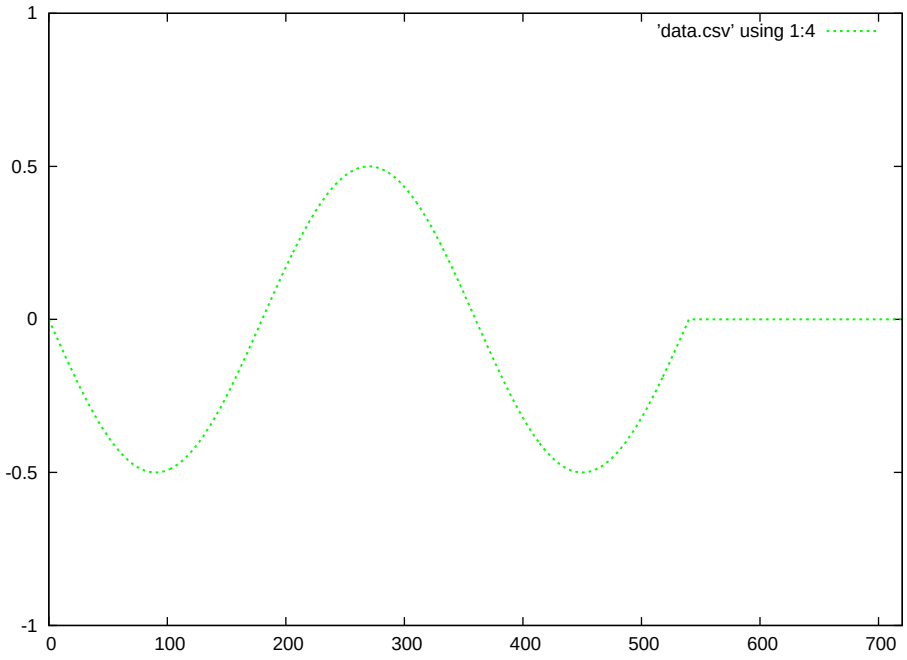

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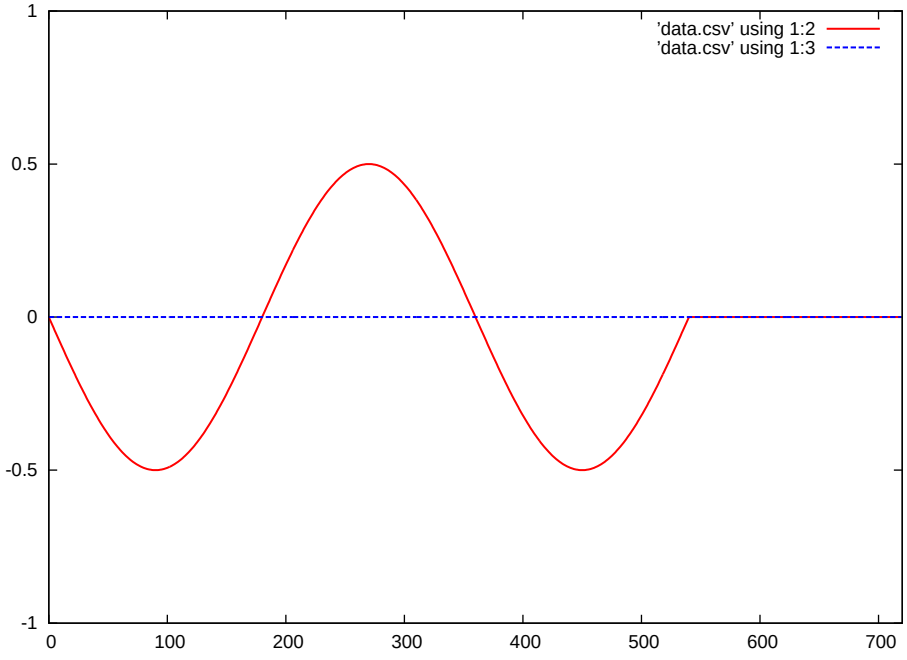

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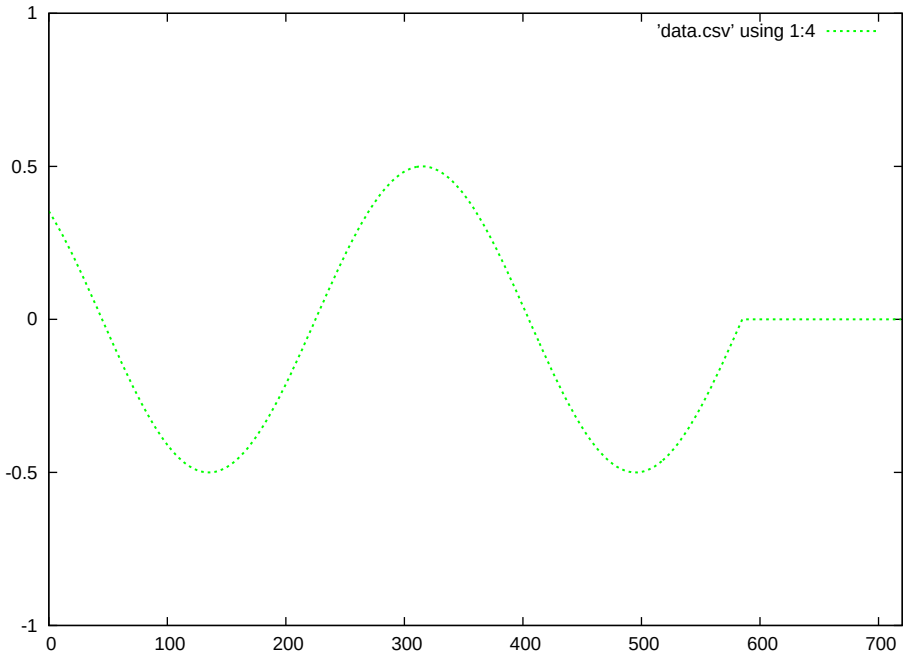

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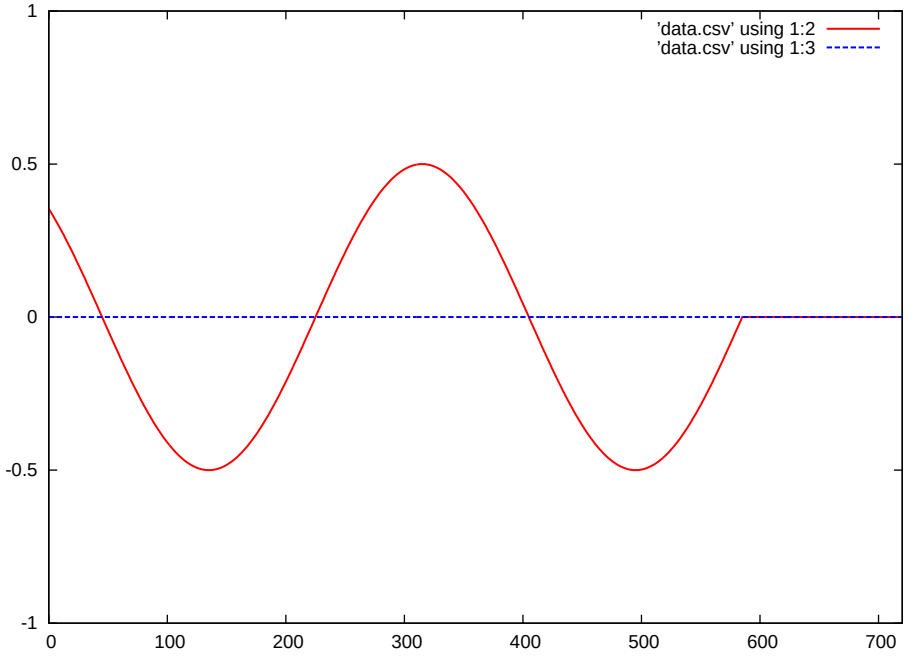

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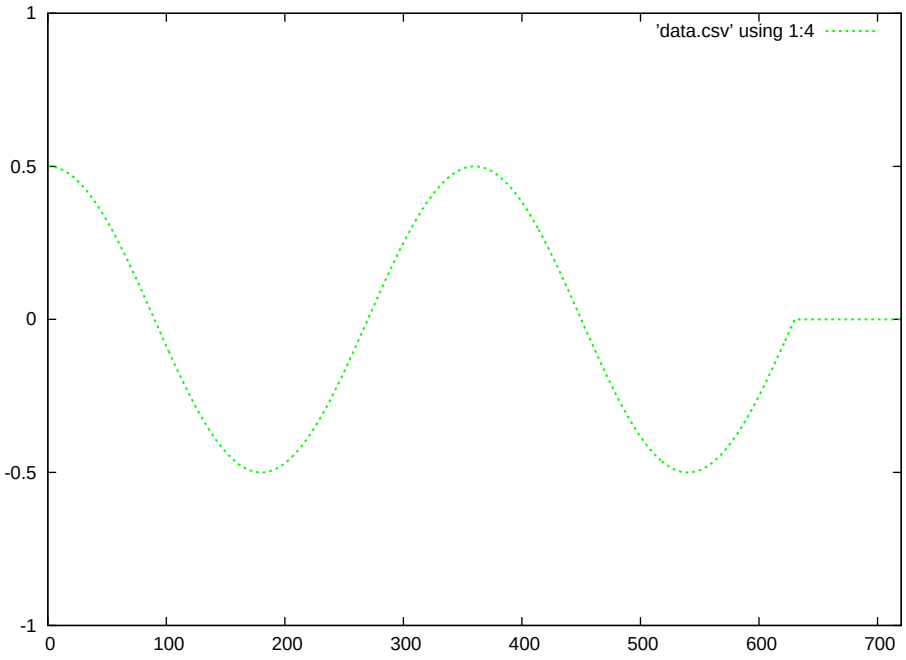

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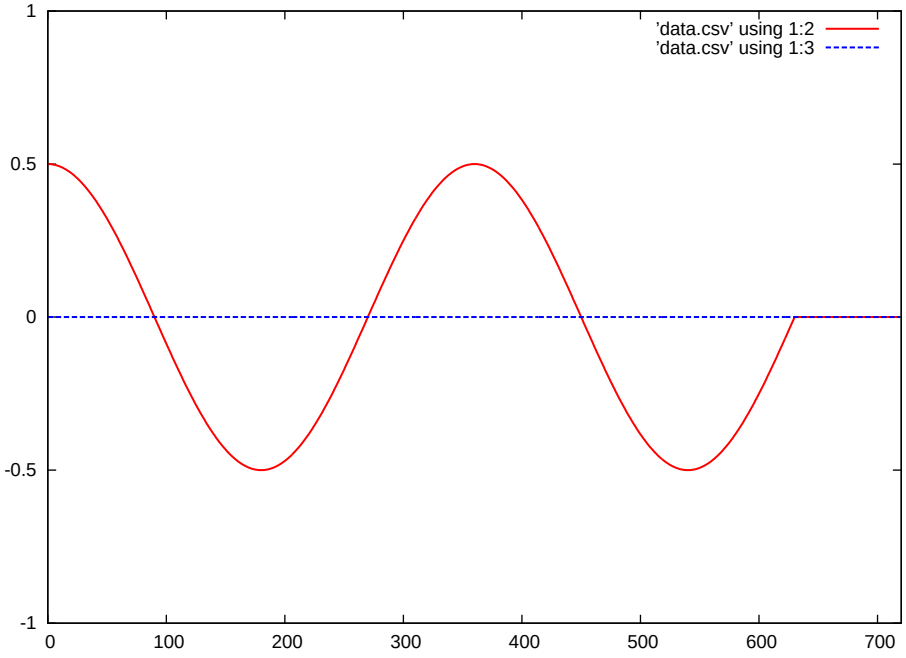

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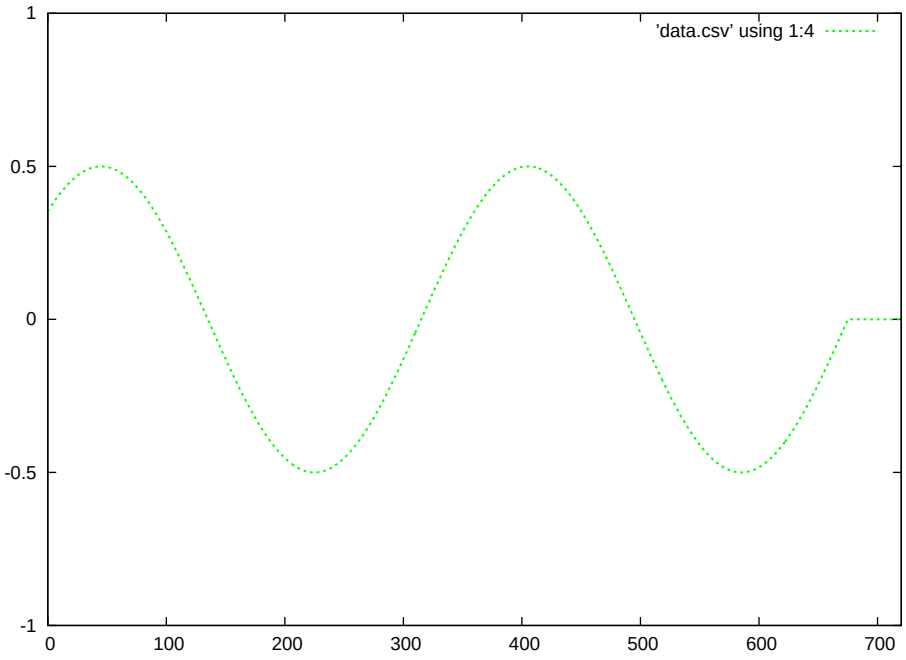

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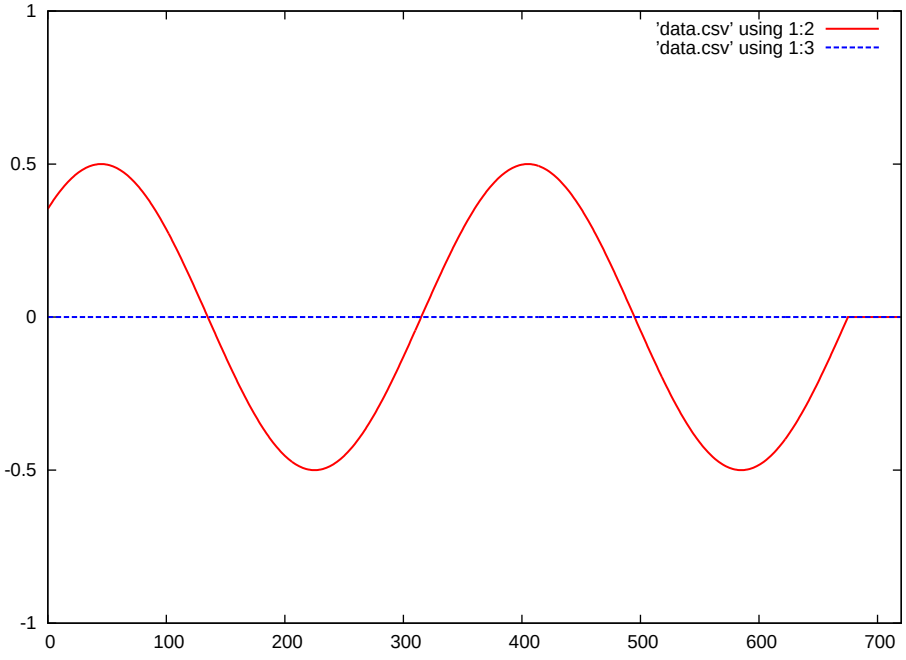

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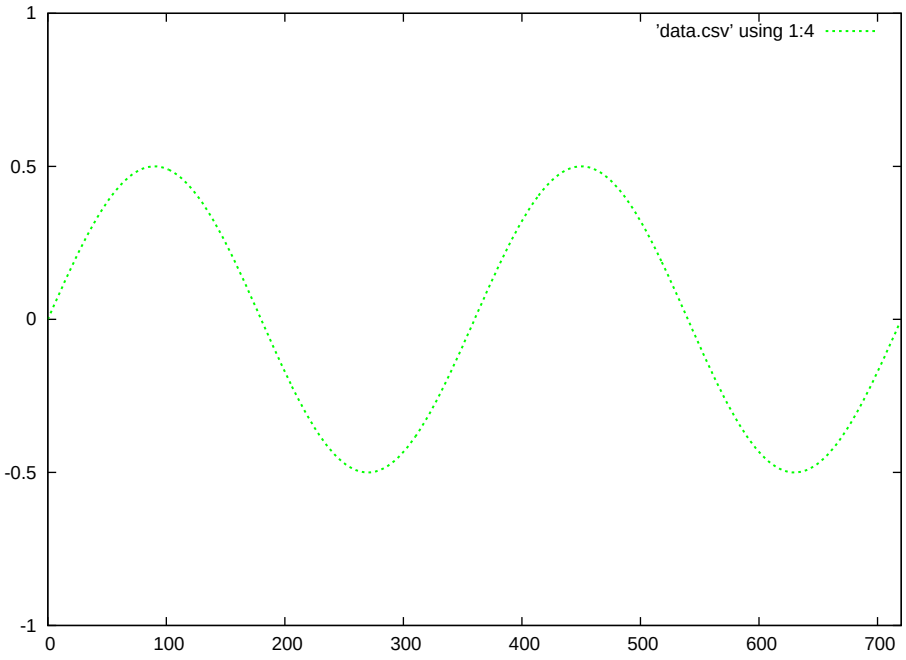

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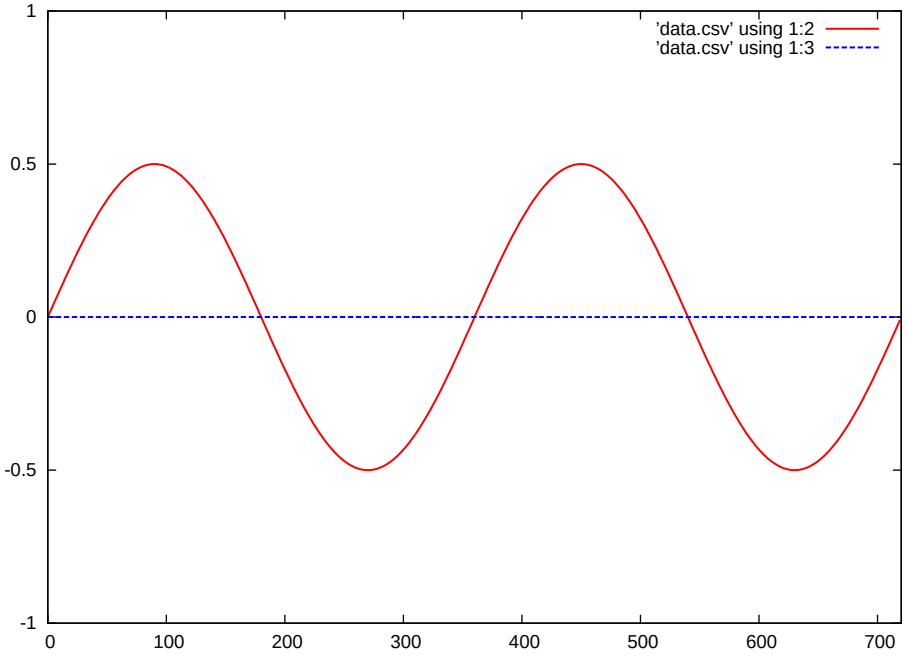

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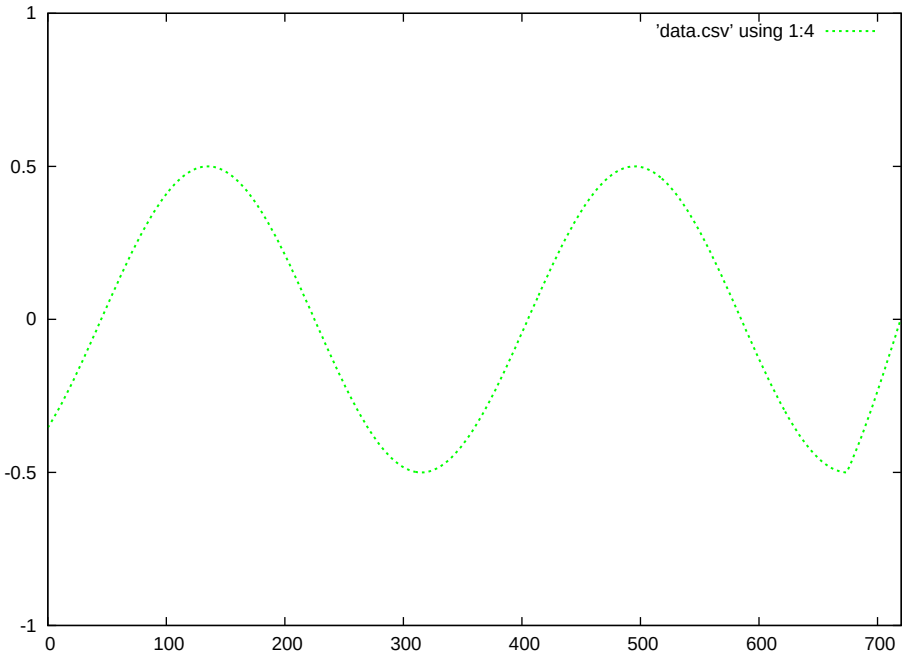

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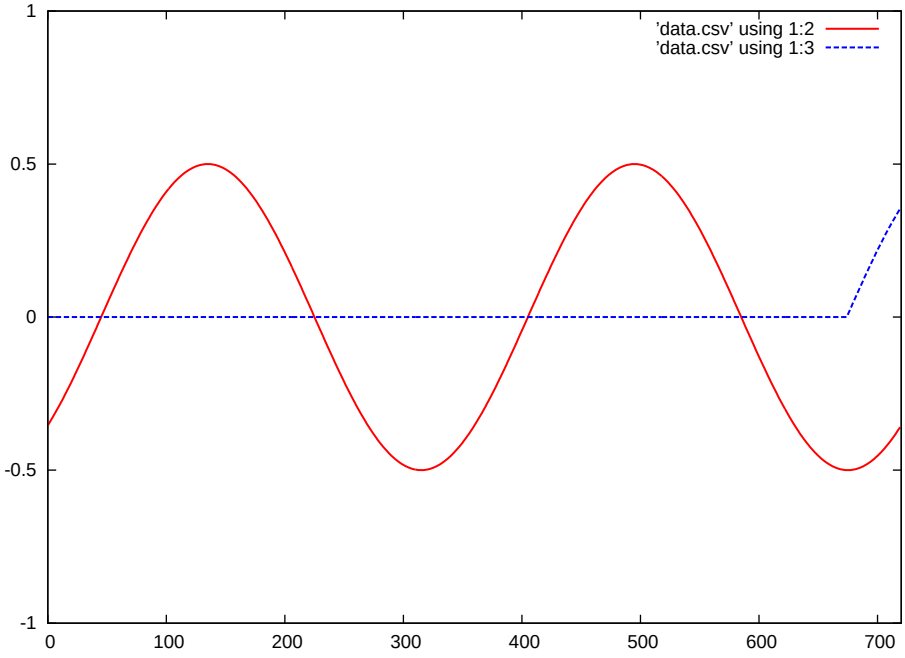

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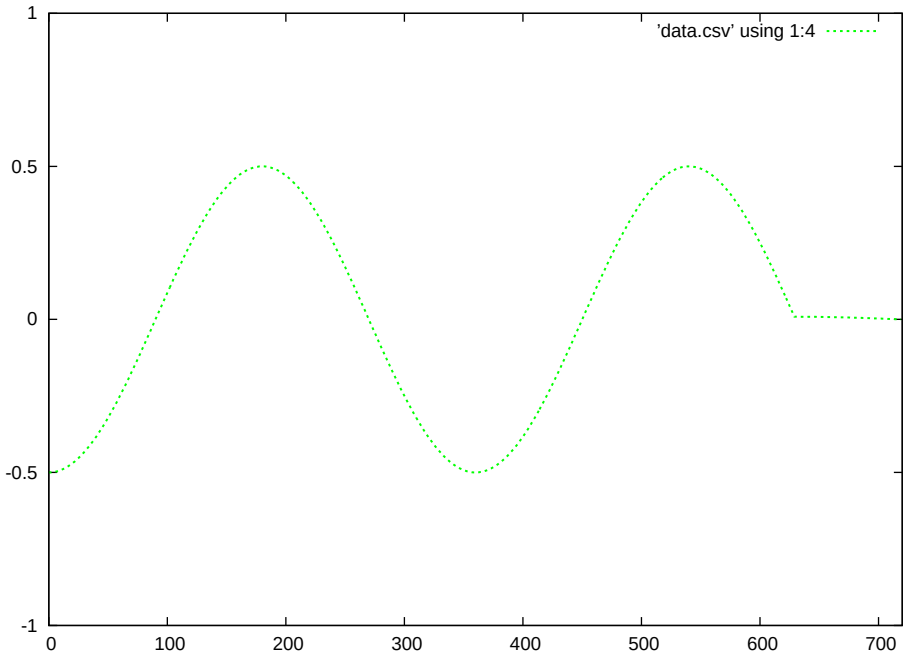

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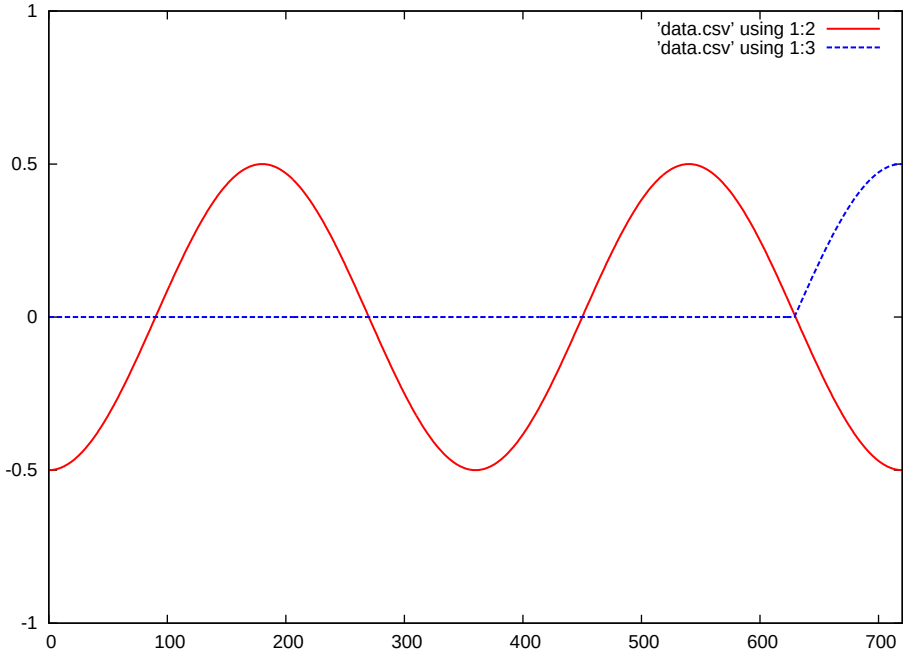

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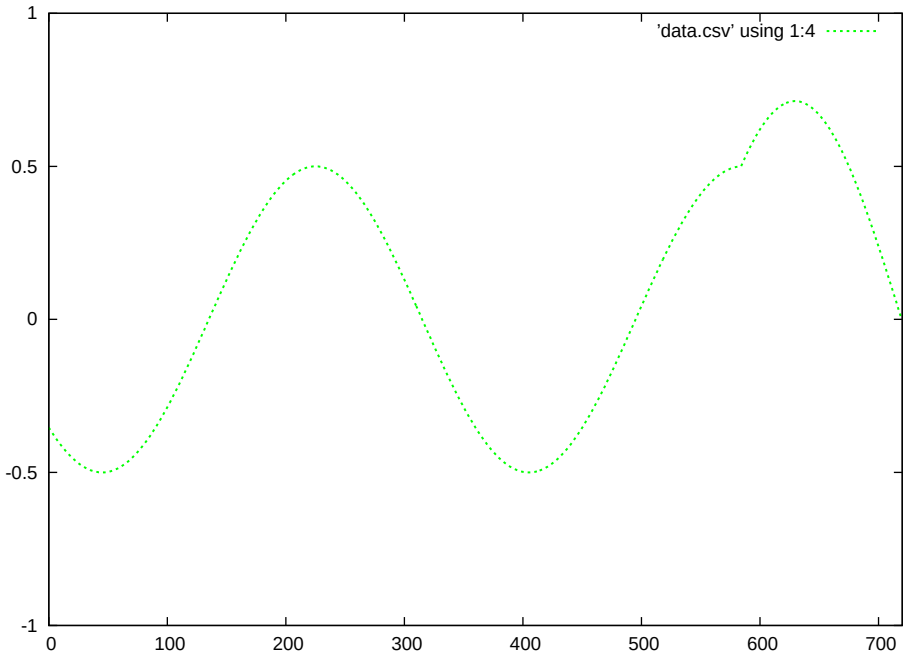

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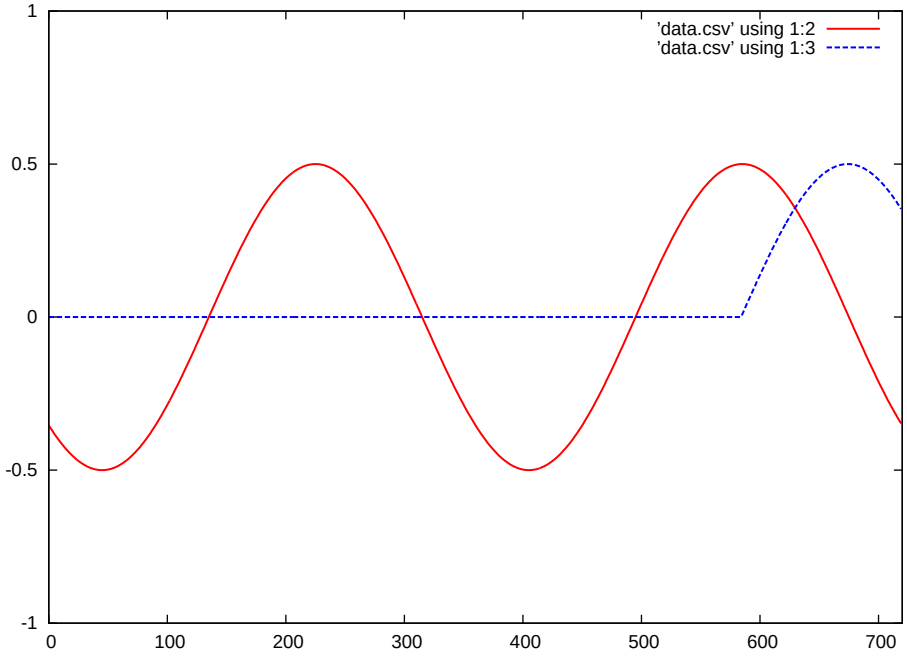

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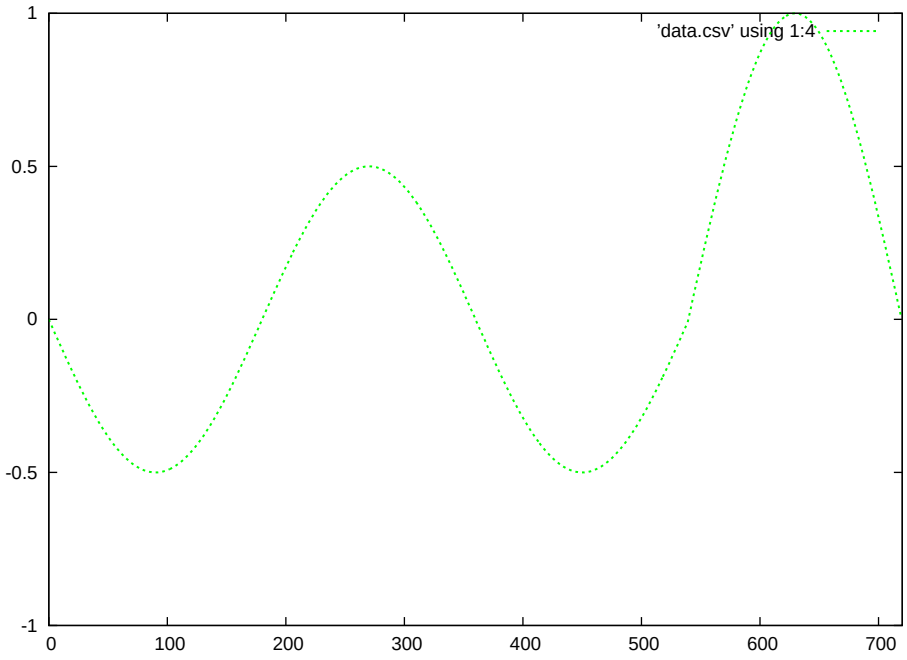

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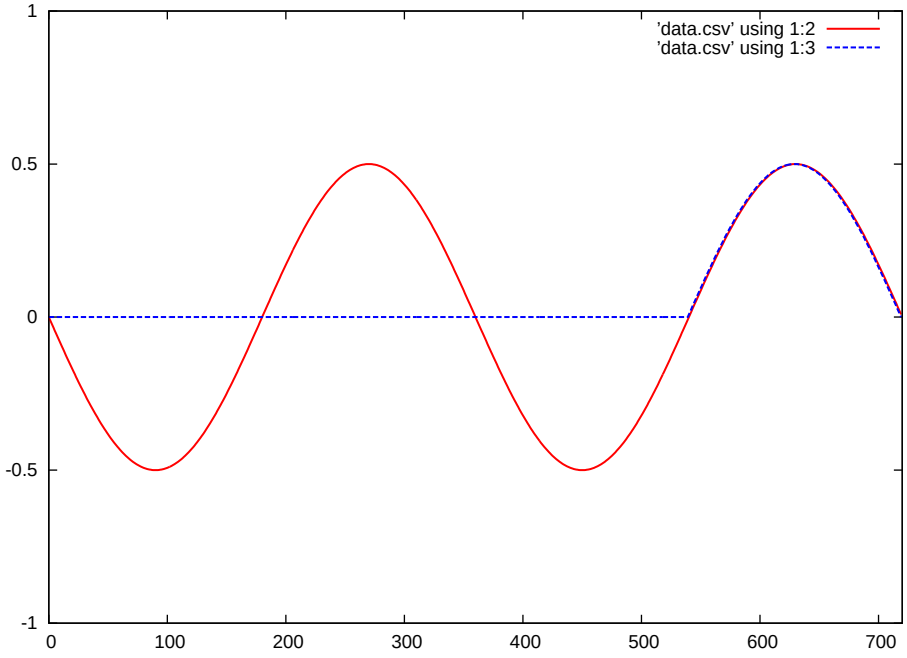

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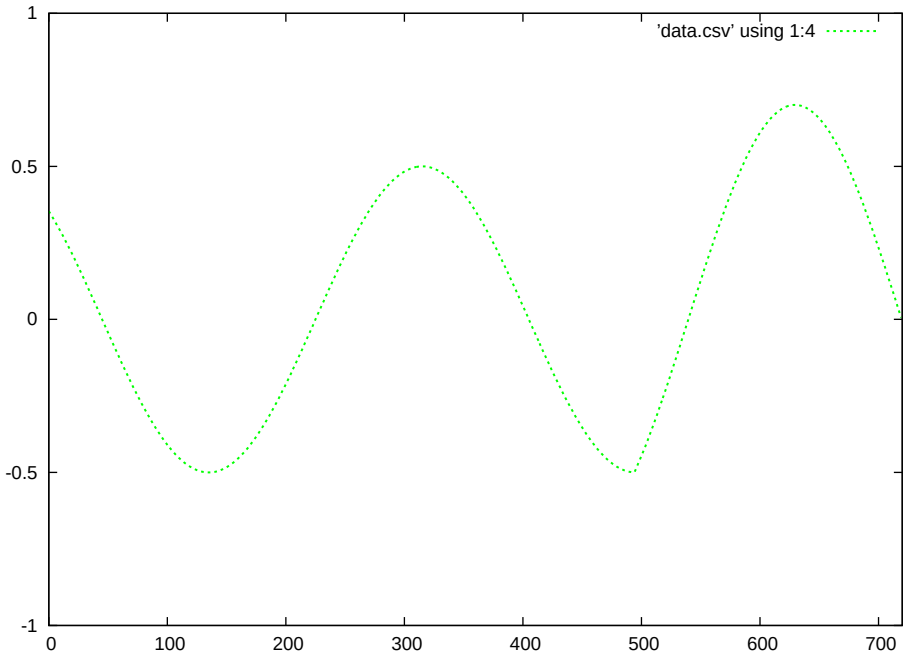

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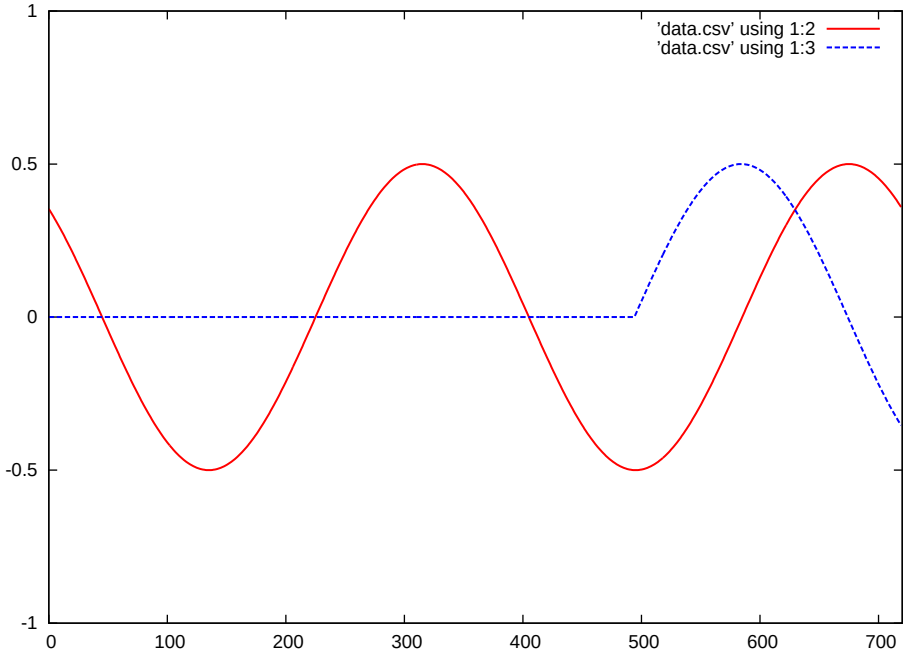

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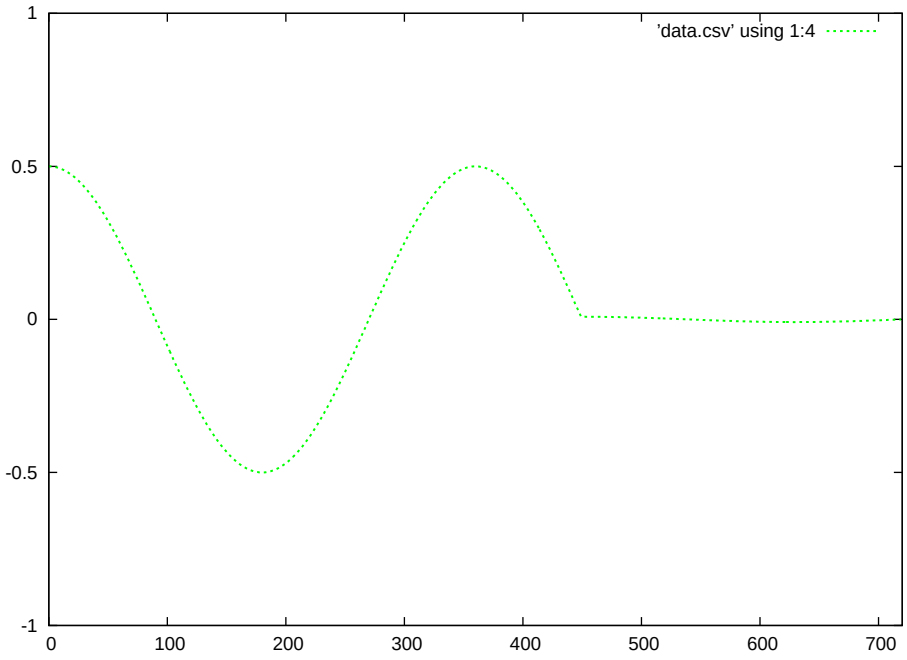

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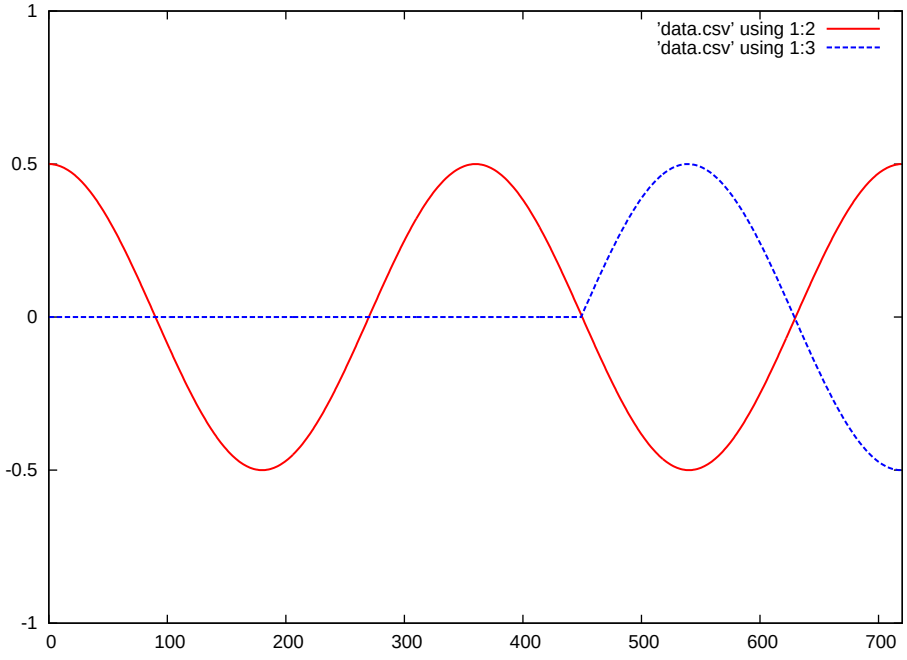

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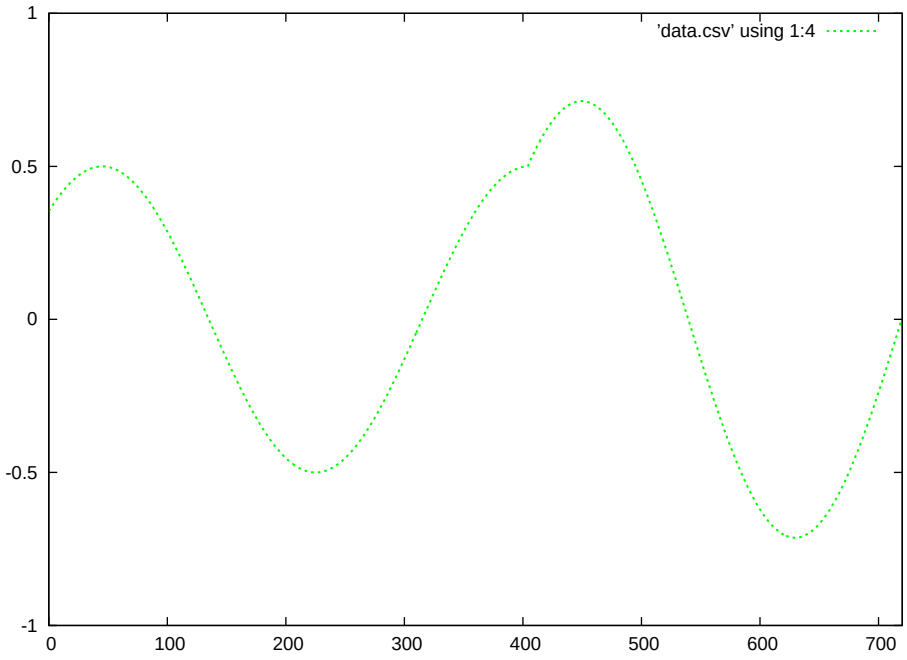

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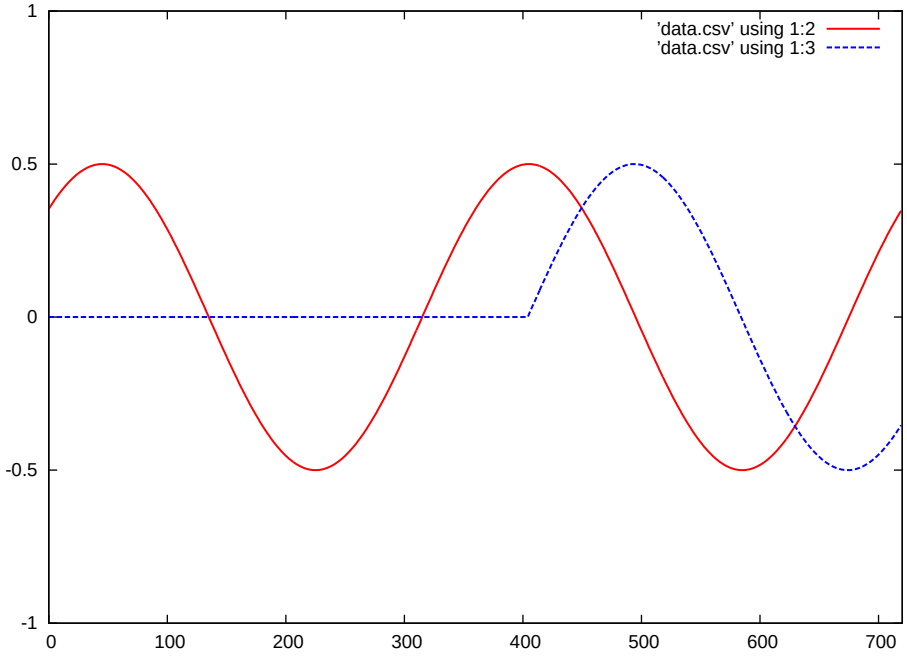

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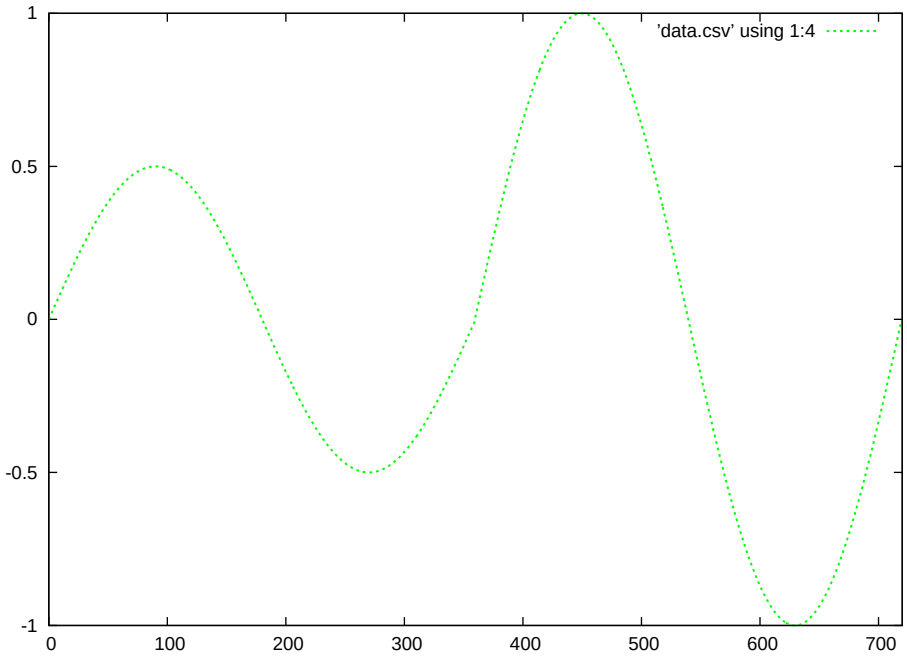

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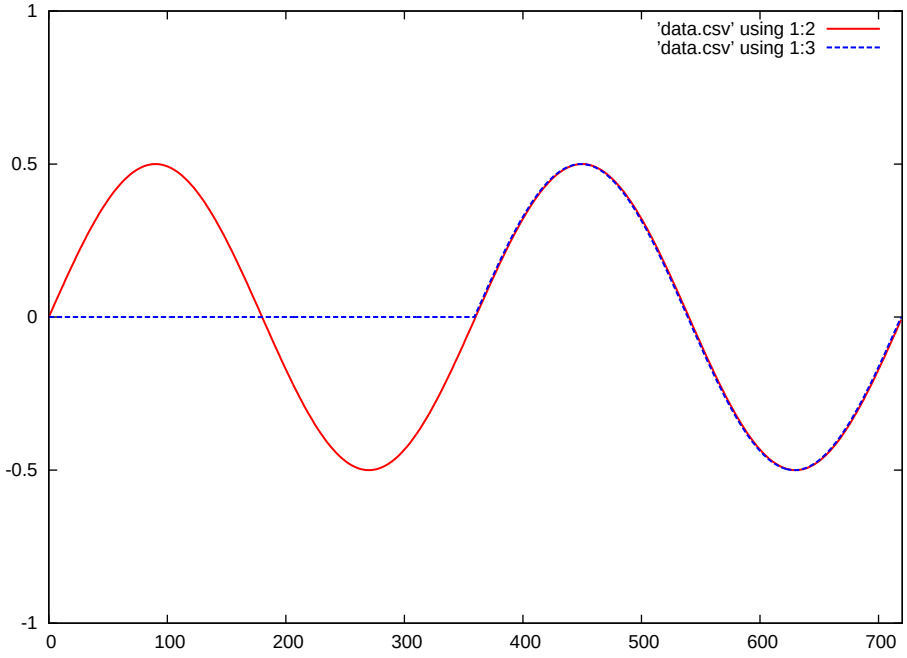

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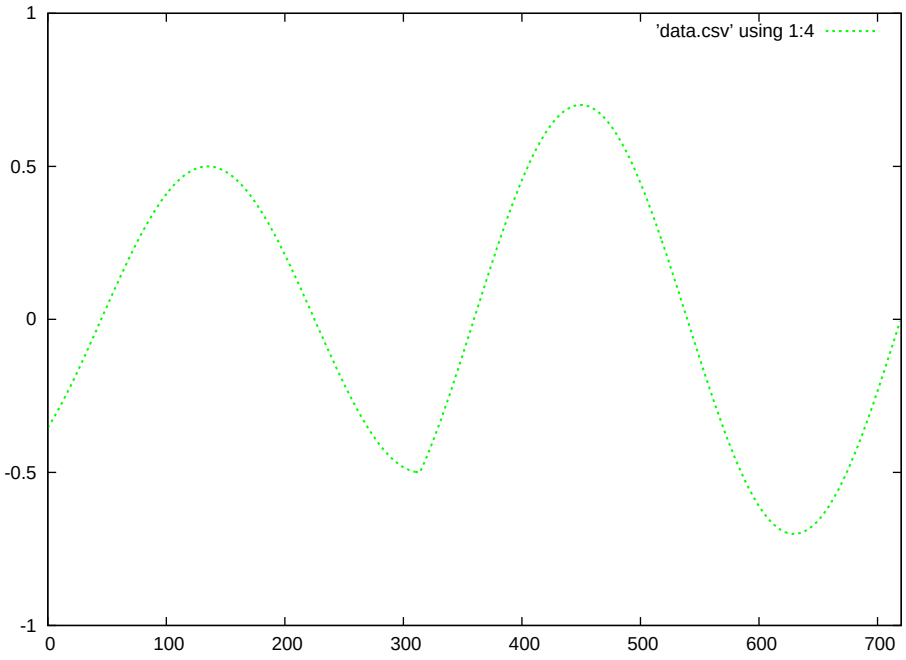

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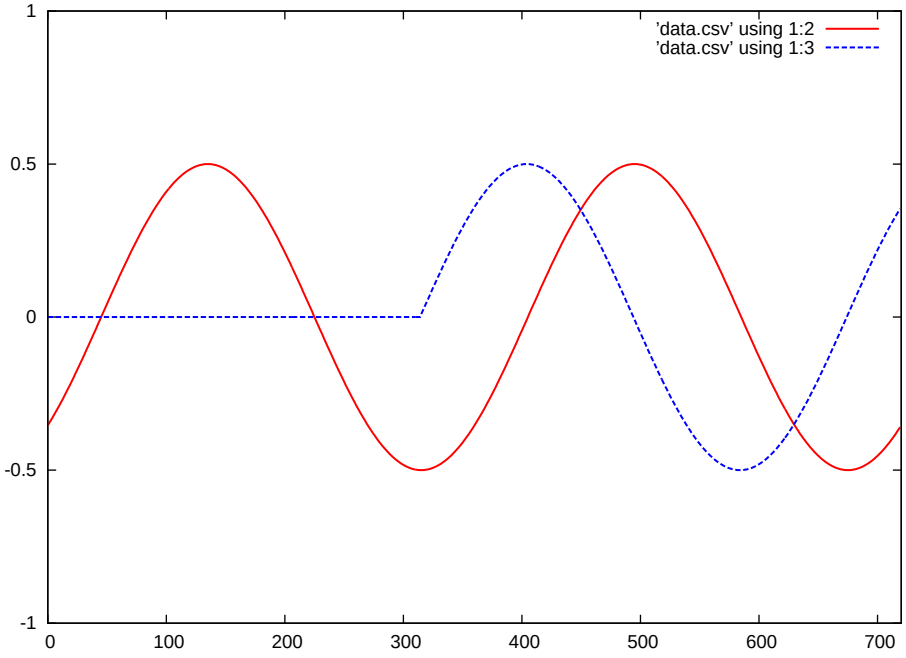

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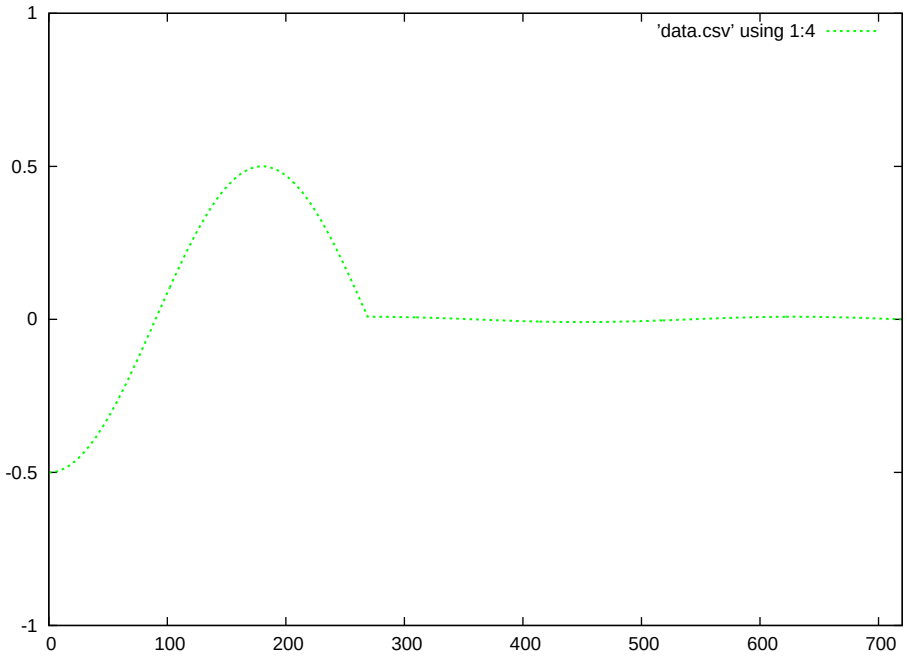

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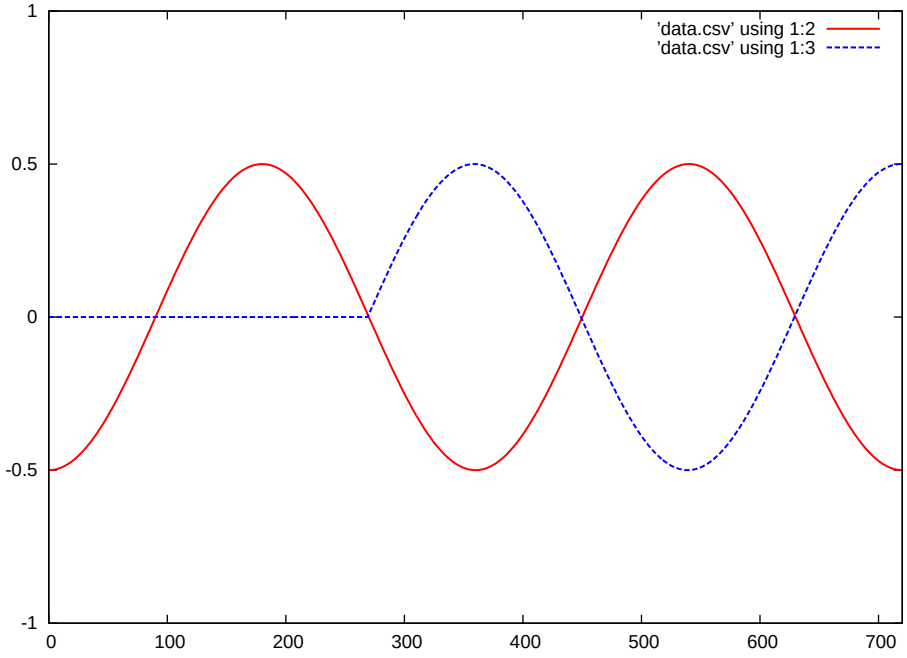

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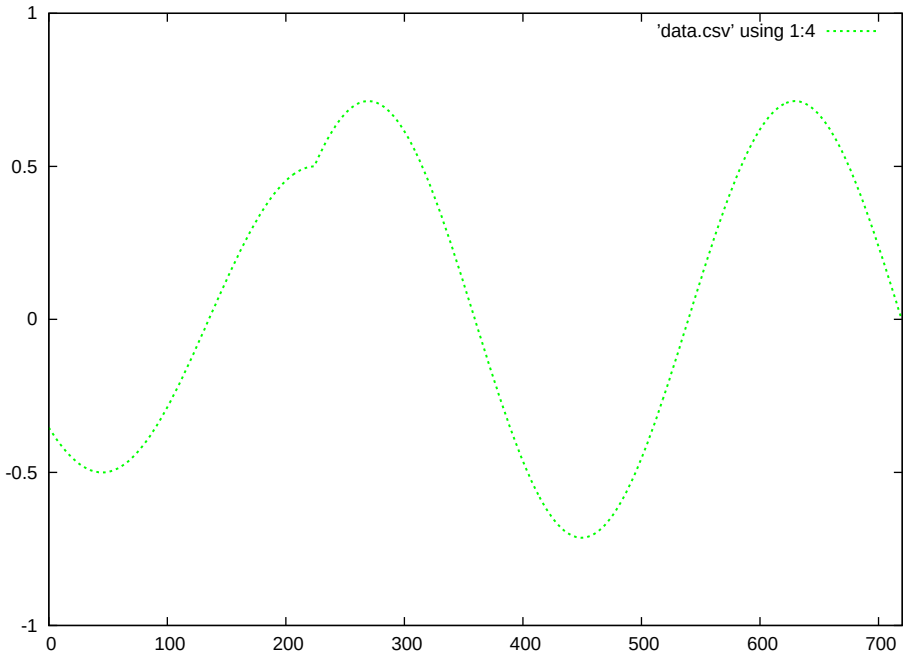

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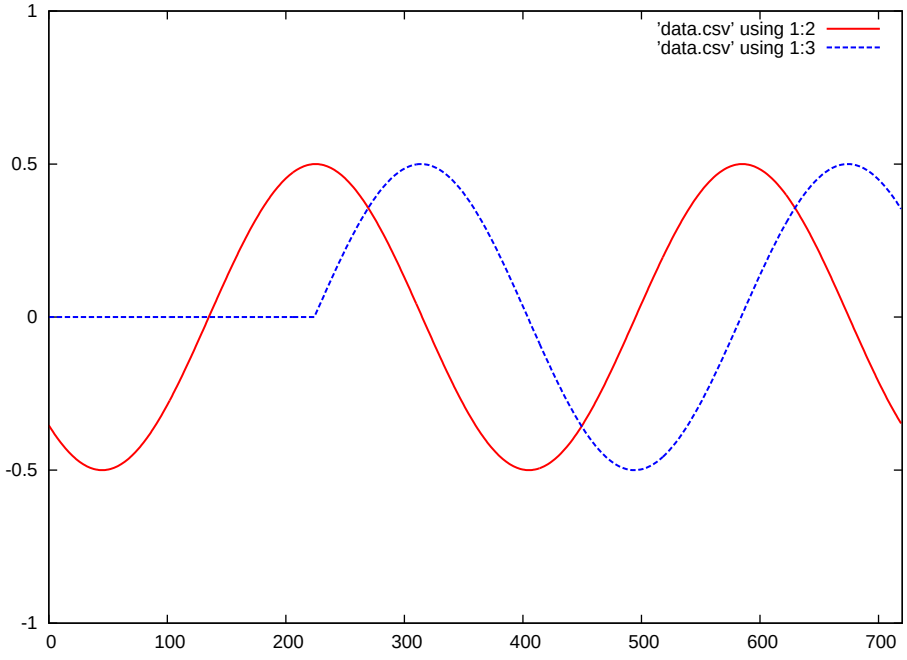

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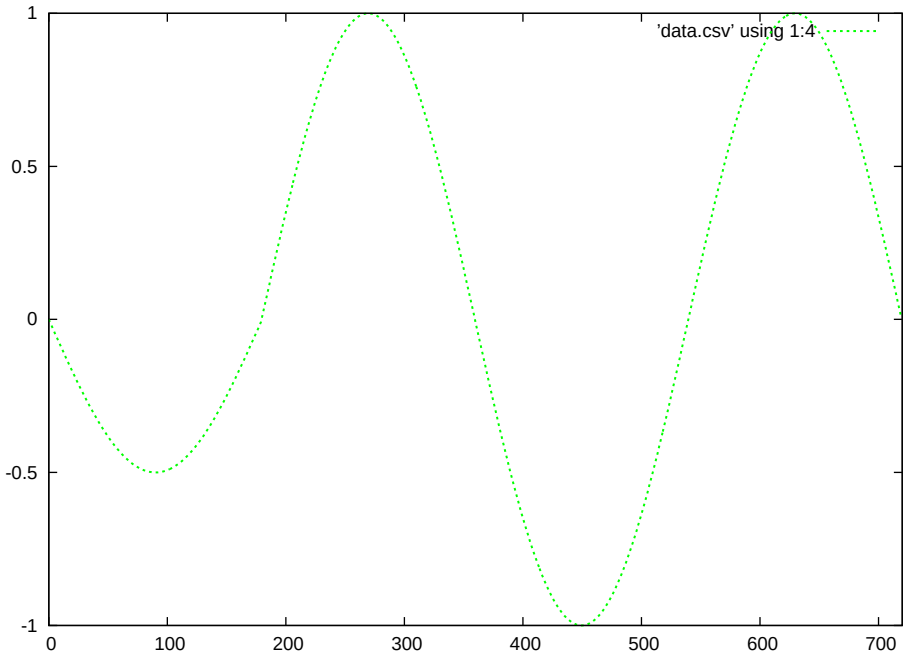

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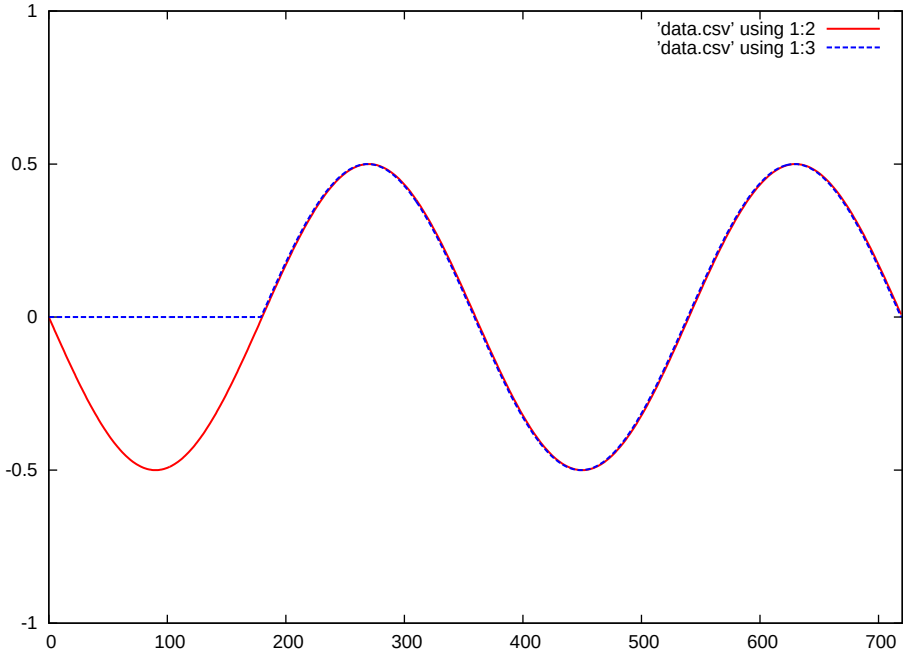

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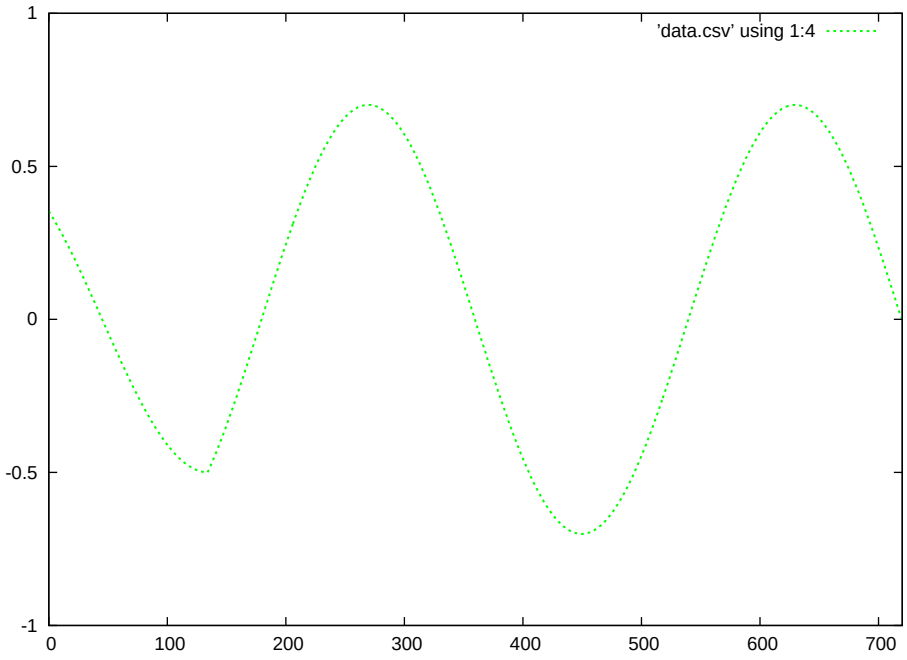

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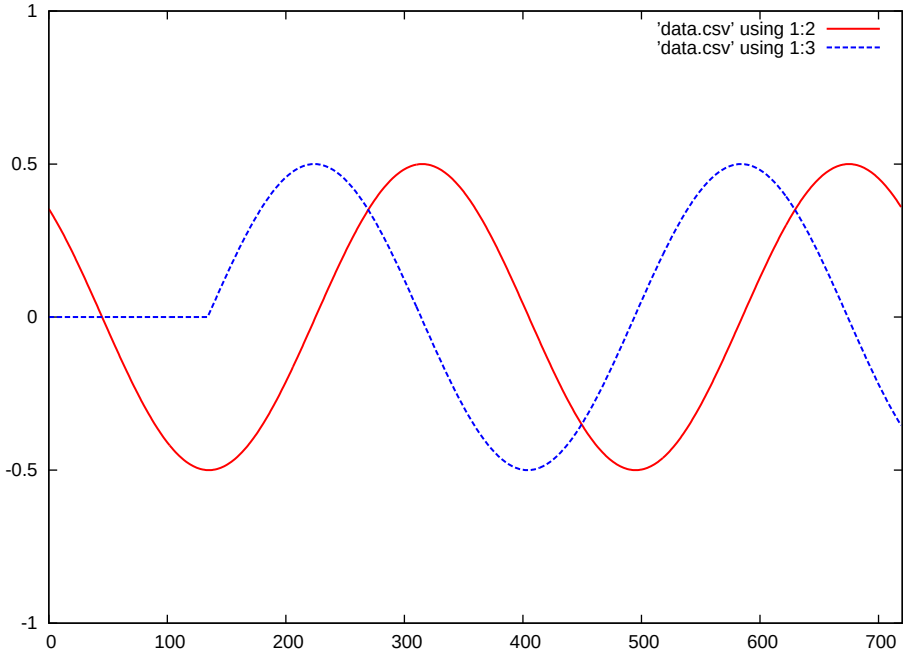

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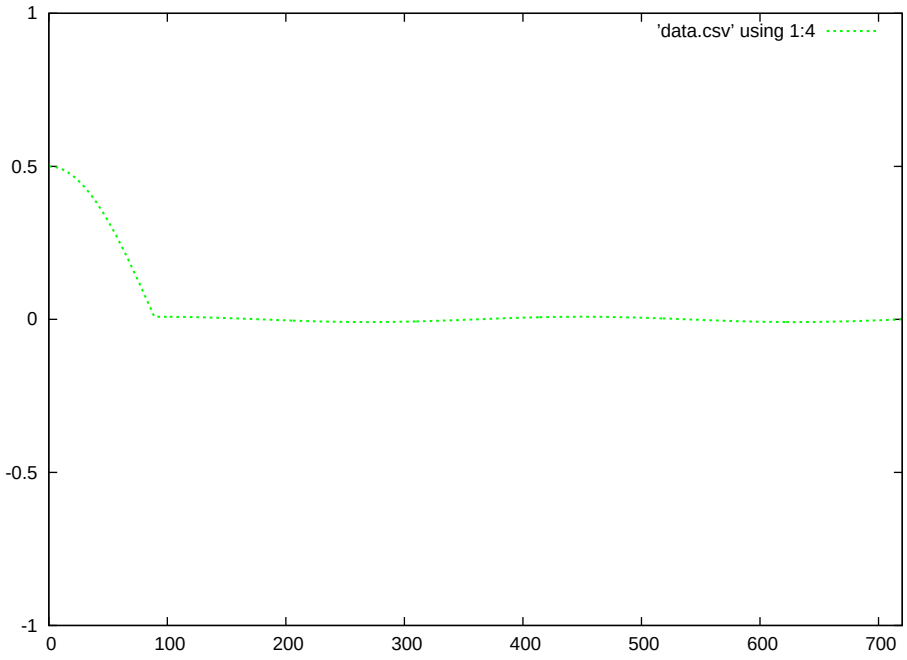

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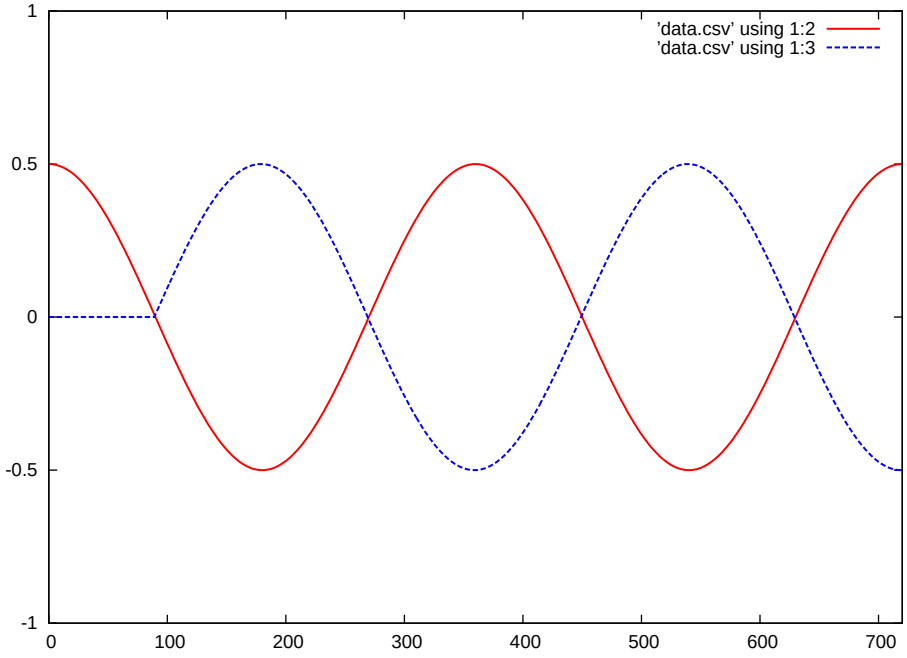
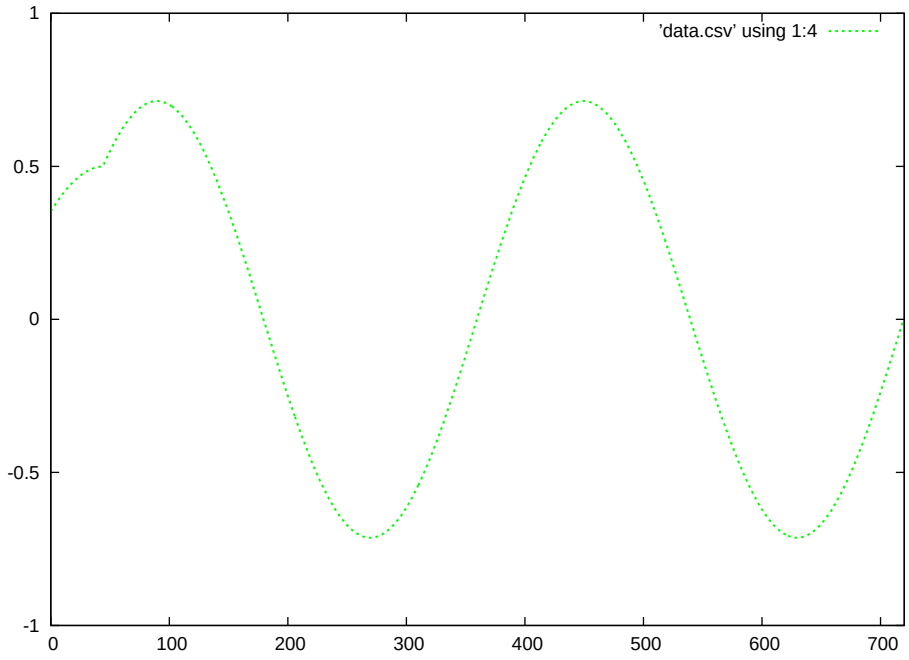
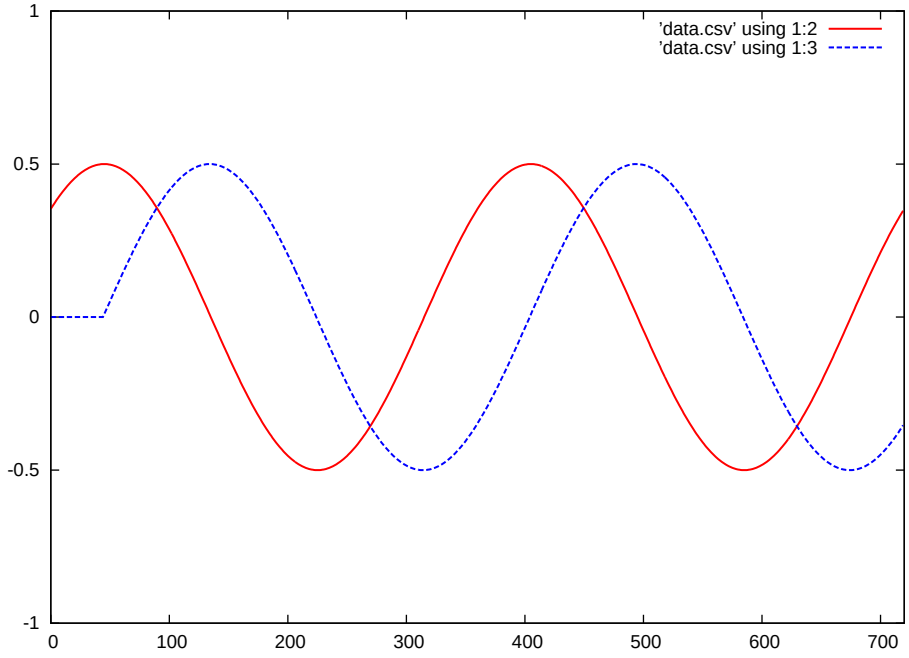


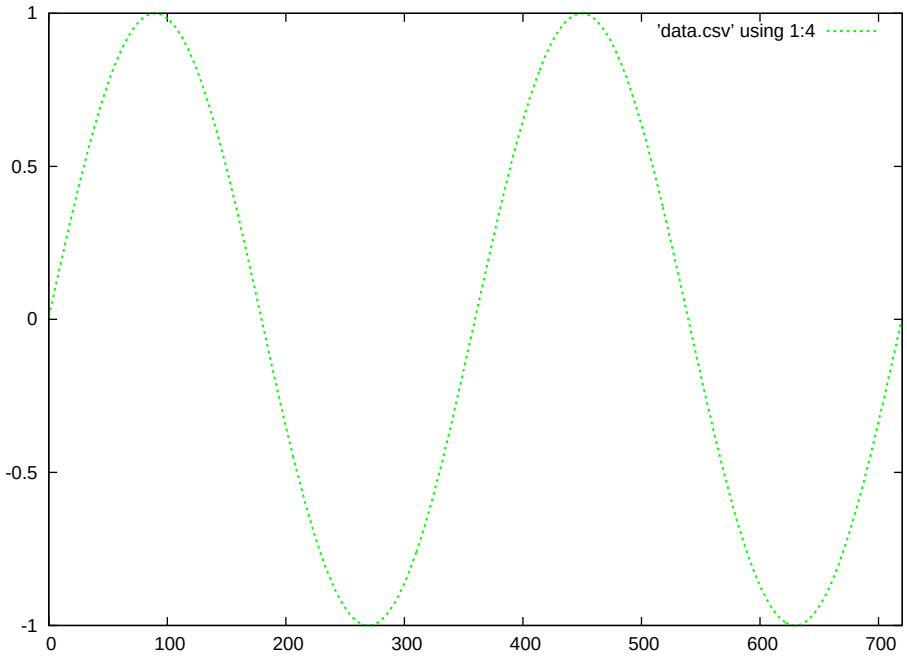
Green = superposition of the forward and reflected waves



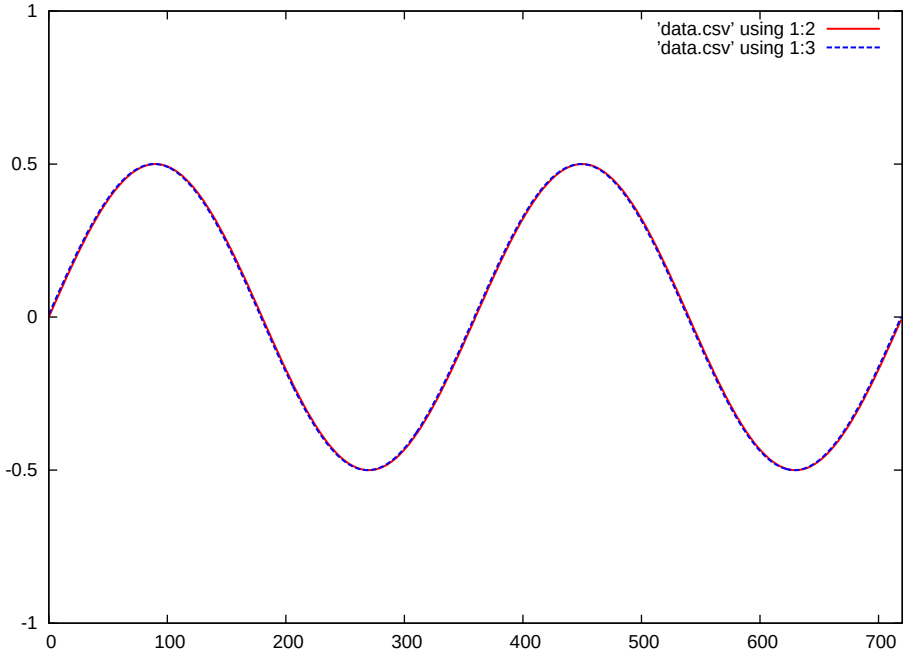
Red = forward wave Blue = reflected wave



Green = superposition of the forward and reflected waves

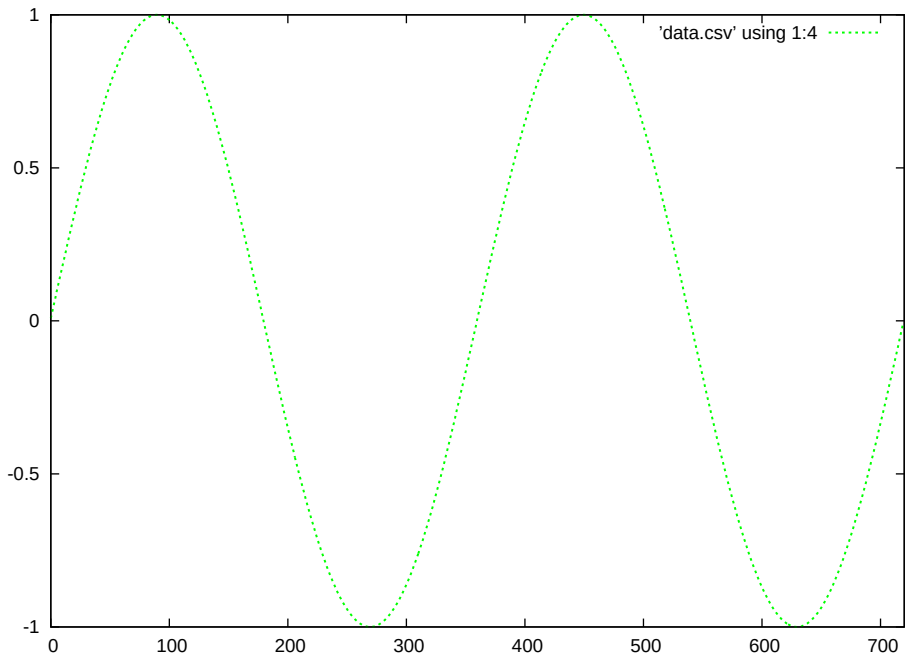


Red = forward wave Blue = reflected wave

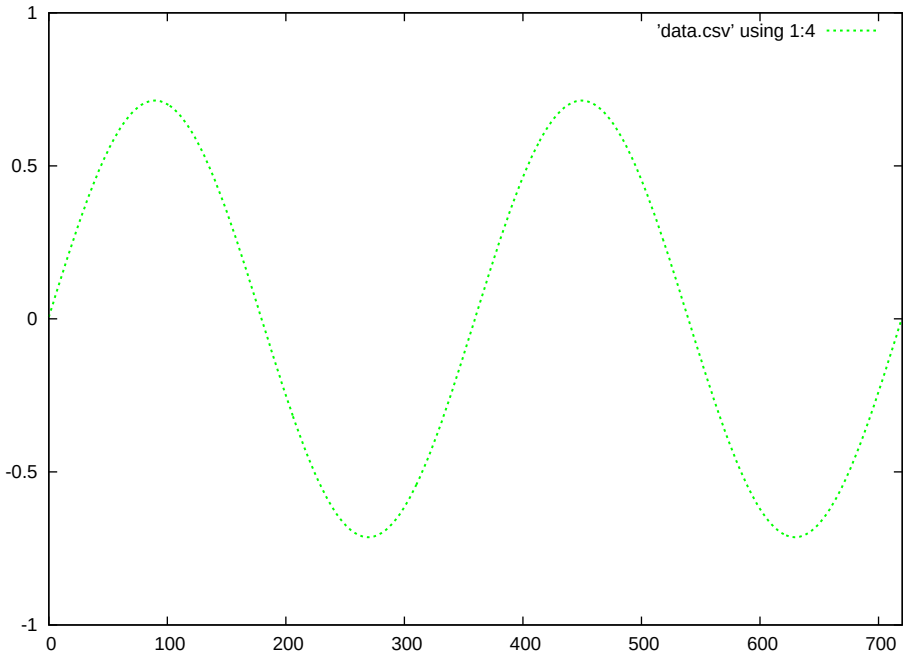


After the reflected wave has traveled all the way back to the source (left-hand end of the transmission line), the superposition of those two continuous waves results in a standing wave.

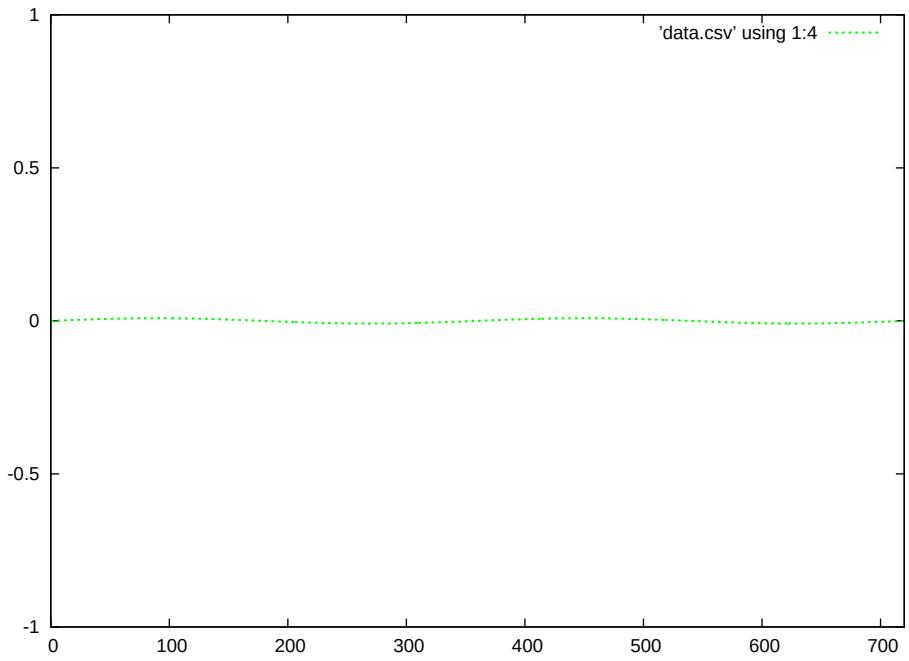
Green = standing wave



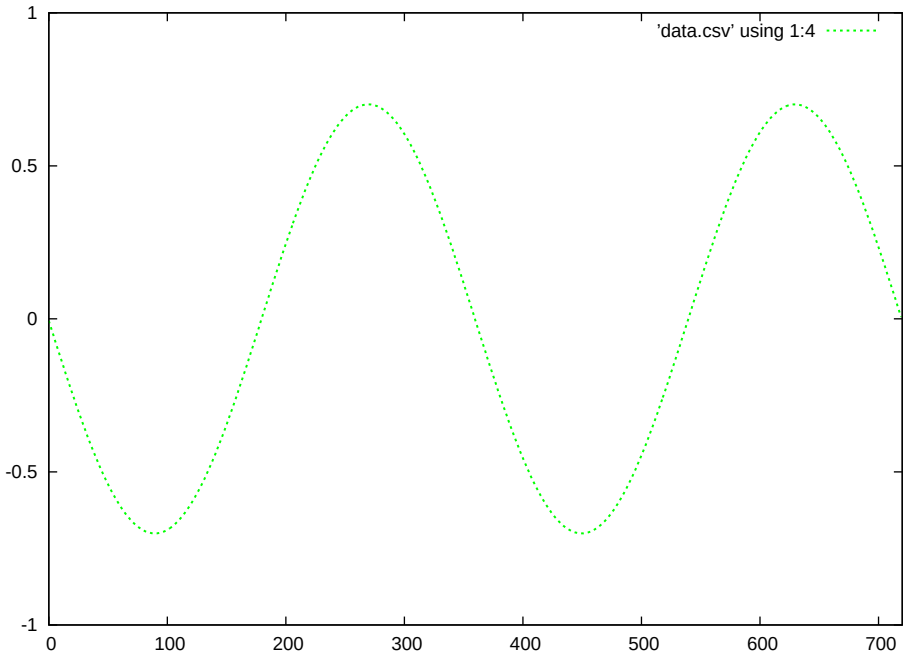
Green = standing wave



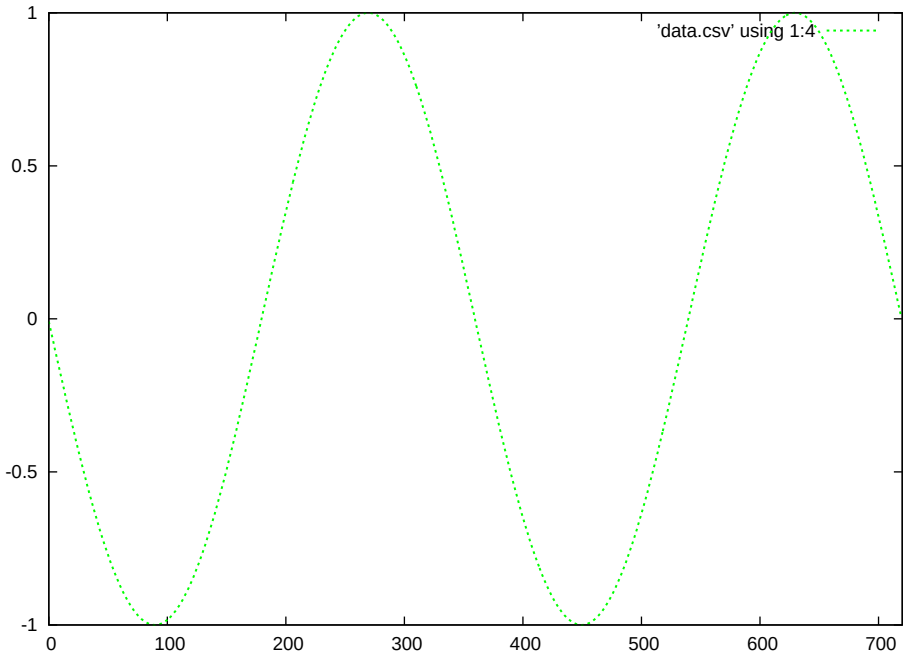
Green = standing wave



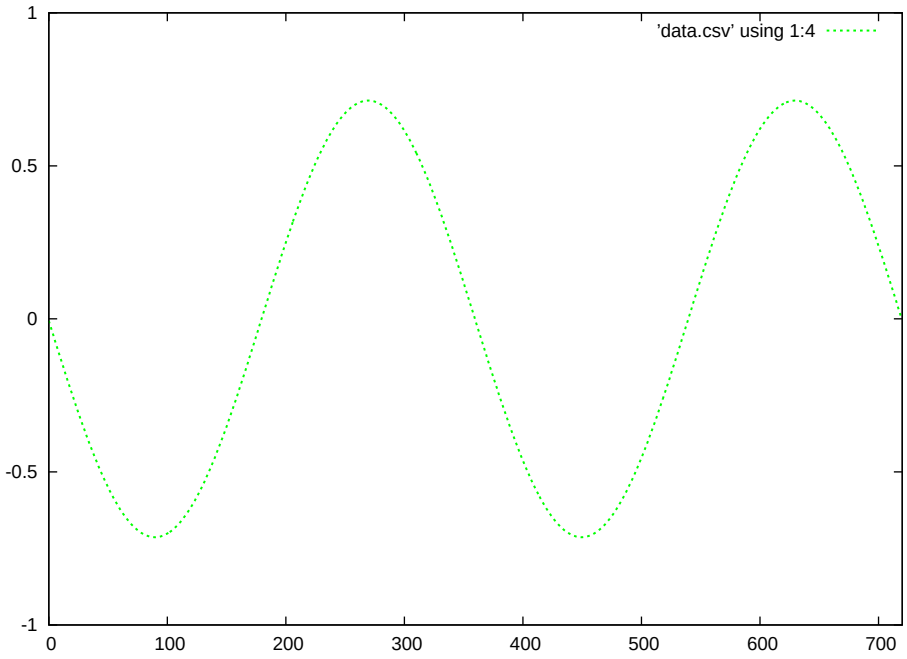
Green = standing wave



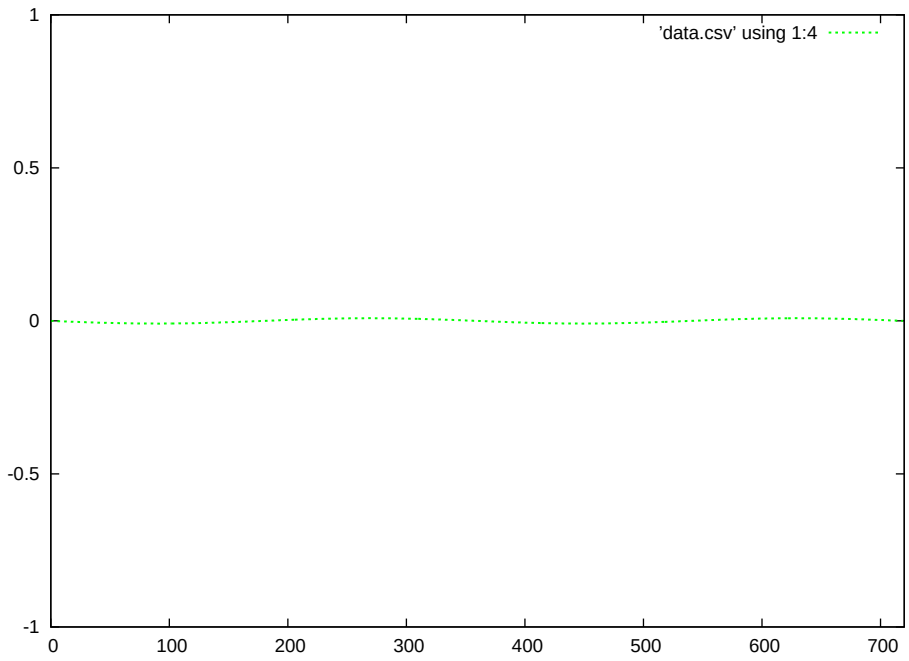
Green = standing wave



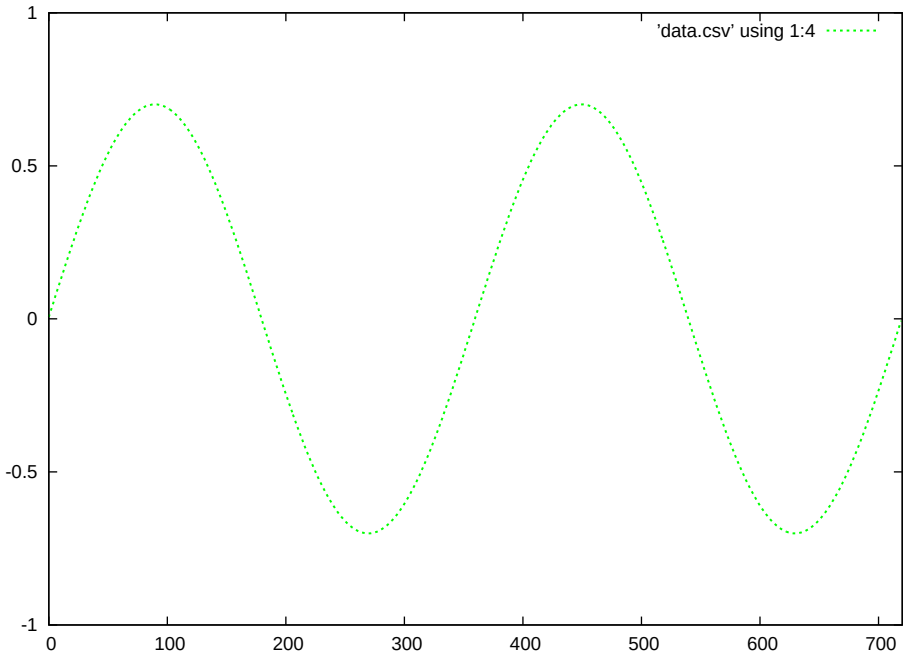
Green = standing wave



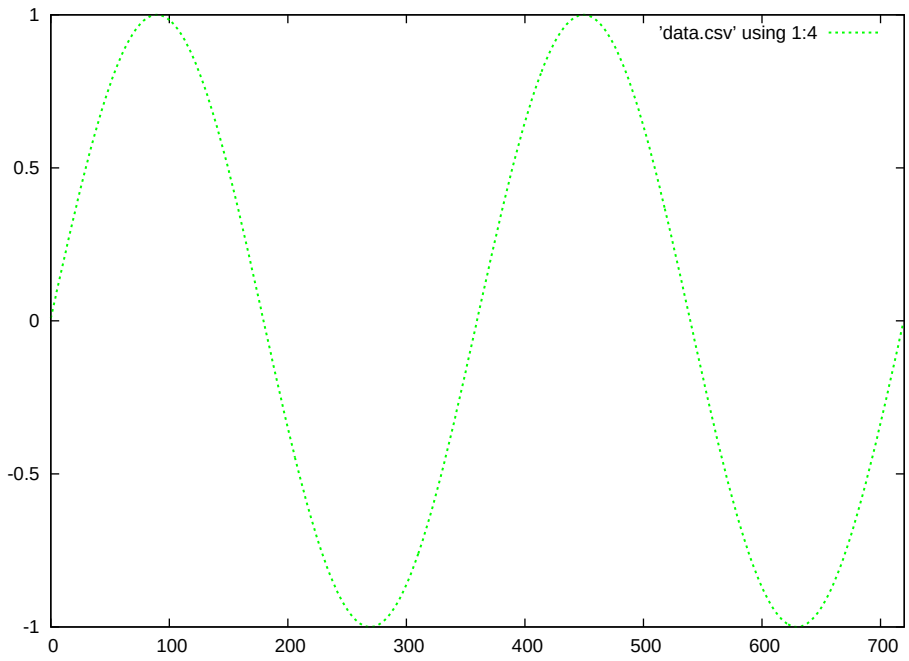
Green = standing wave



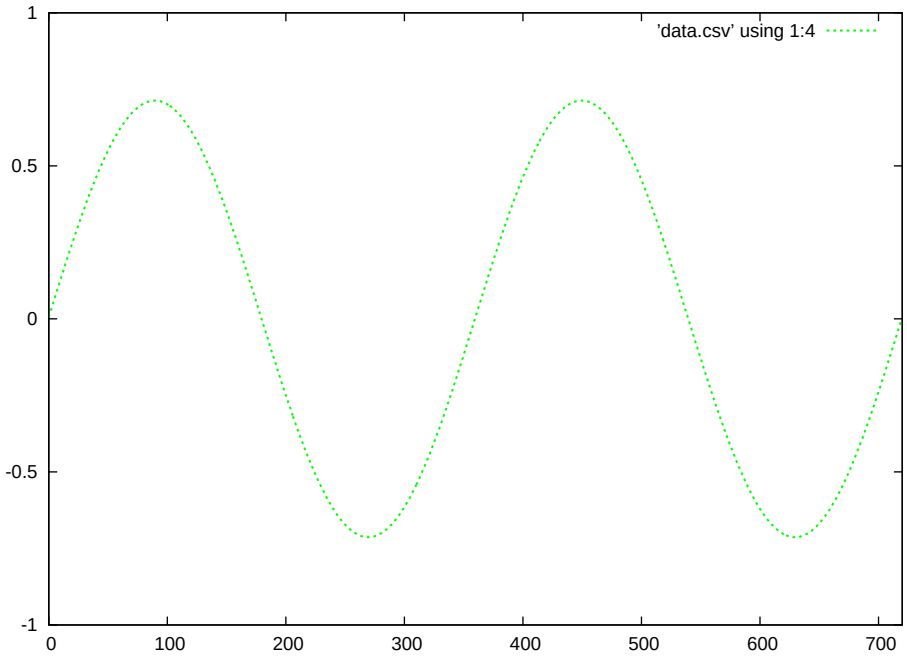
Green = standing wave



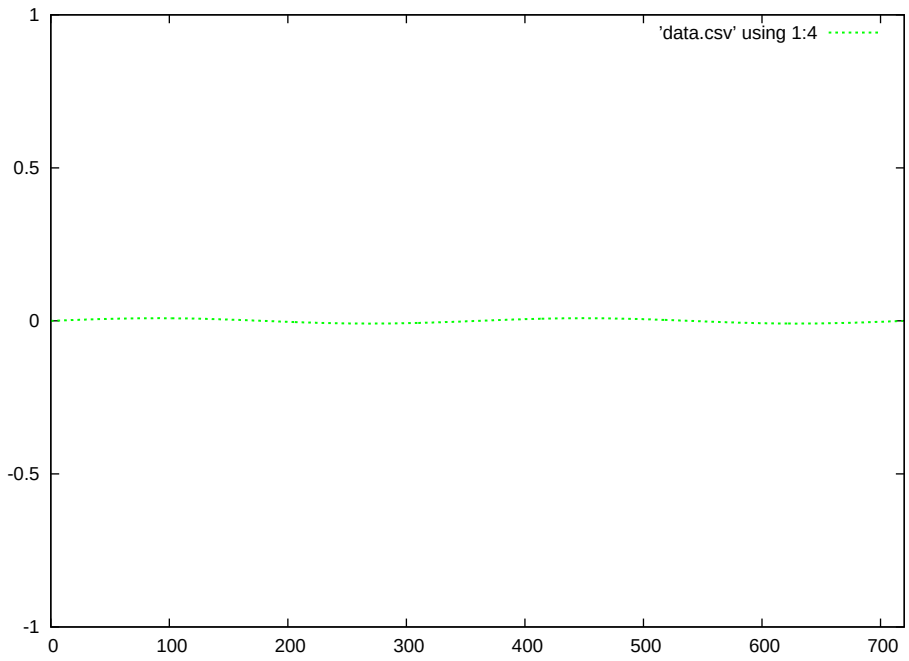
Green = standing wave



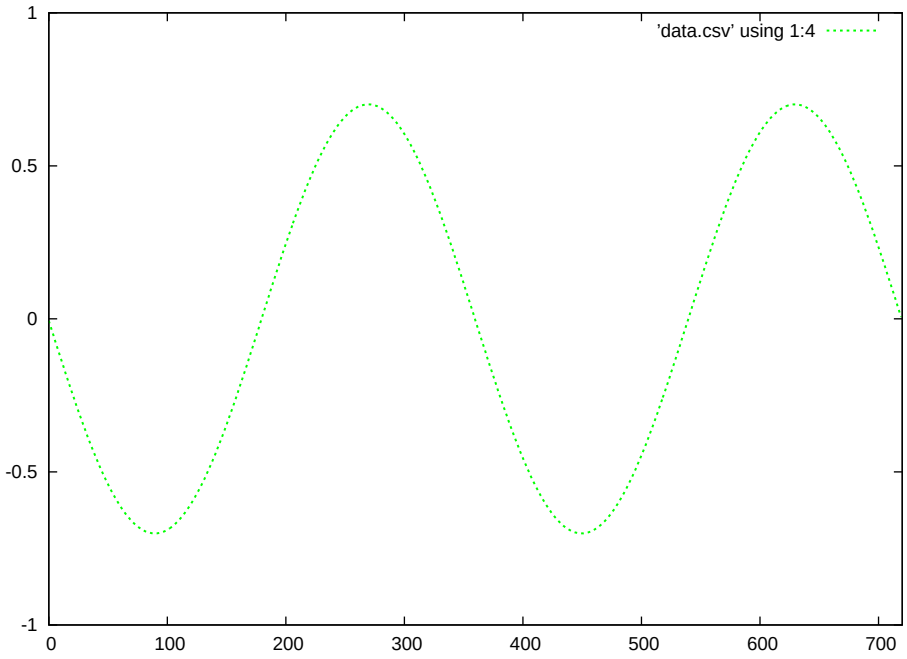
Green = standing wave



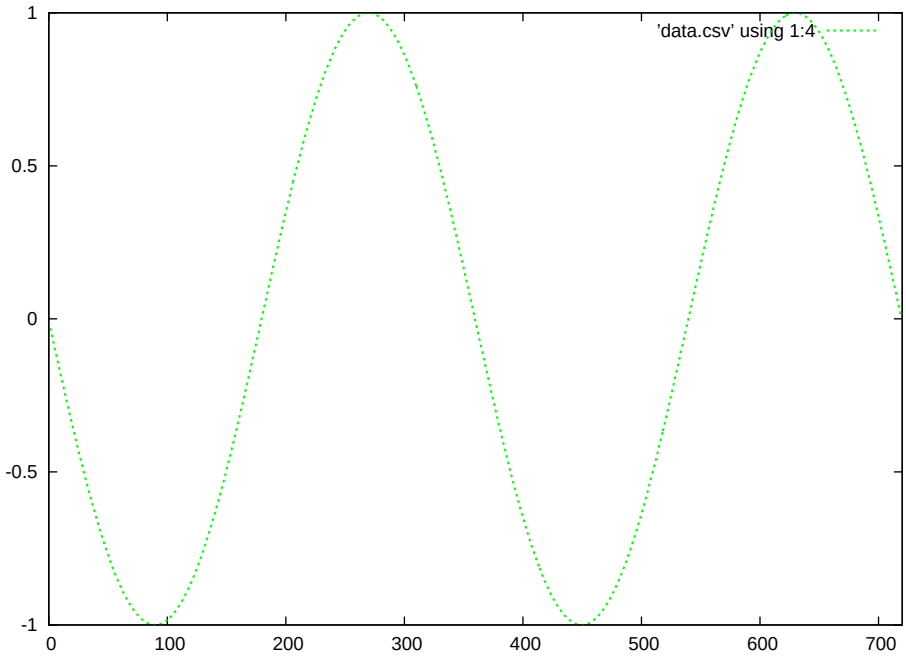
Green = standing wave



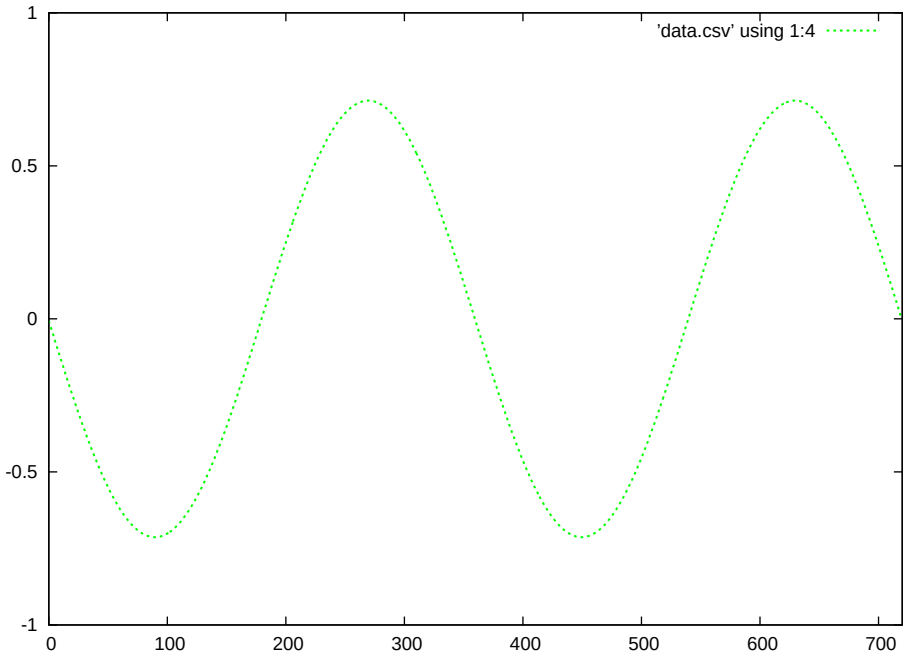
Green = standing wave



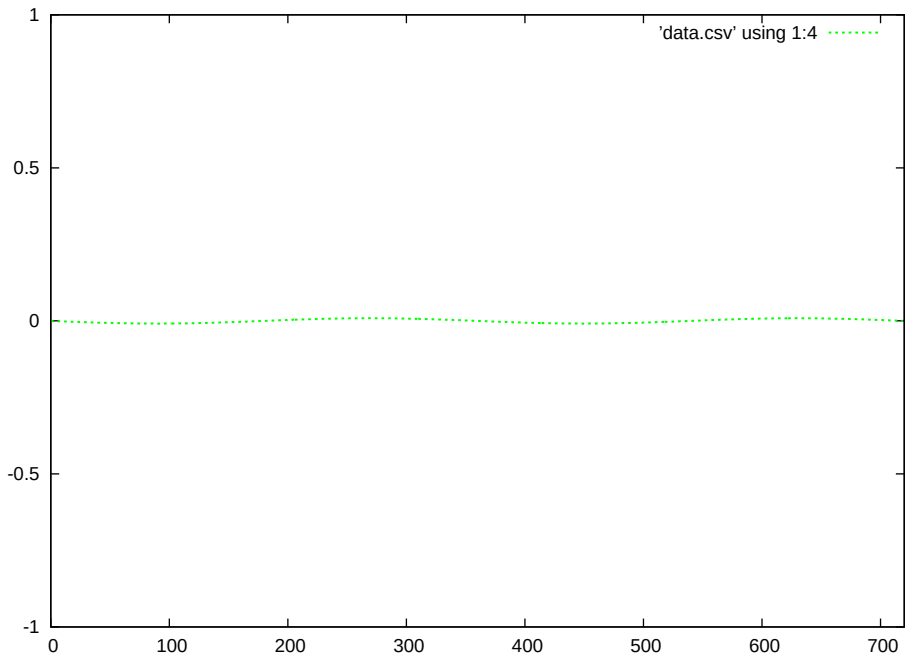
Green = standing wave



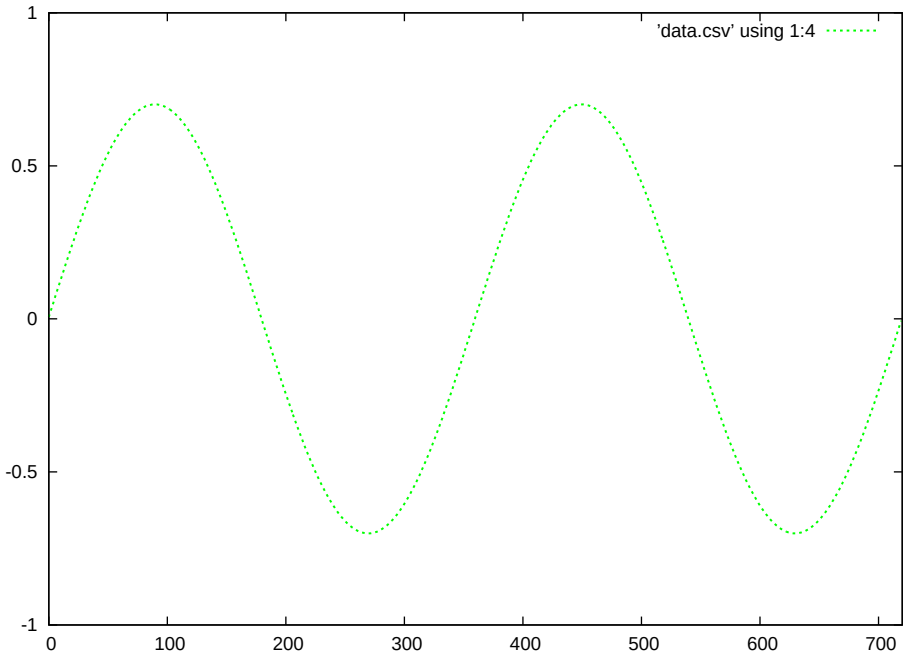
Green = standing wave



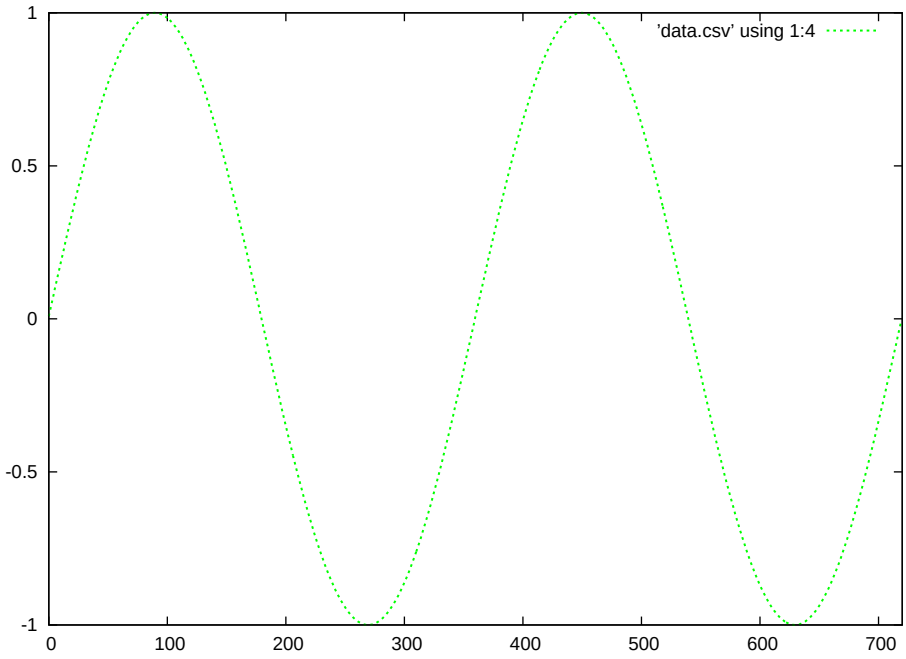
Green = standing wave



Green = standing wave



Green = standing wave



C++ source code for initial forward, reflected, and superposition waveforms

```
#include <iostream>
#include <cmath>
using namespace std;

float sinecalc (int);

int main (void)
{
  int x, offset;
  float fwd[1440];
  float ref[720];
  float sum[720];

  offset = 1440;

  for (x = 0 ; x < 1440 ; ++x)
  {
    fwd[x] = 0.5*sinecalc(x-offset);
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    if (offset <= 720)
    {
      ref[x] = 0.0;
      sum[x] = 0.0;
    }

    else
    {
      ref[719-x] = -fwd[720+x];
    }
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    sum[x] = fwd[x] + ref[x];
    cout << x << " , " << fwd[x] << " , " << ref[x] << " , " << sum[x] << endl;
  }

  return 0;
}
```

```
float sinecalc (int degrees)
{
  if (degrees < 0)
    return sin (degrees * M_PI / 180);

  else
    return 0.0;
}
```

Running this exact program produces a set of text-based data, separated by commas, representing the forward, reflected, and summation waves at an angle of 1440 degrees.

Gnuplot source code (named `gnuplot.txt`)

```
set datafile separator ","  # Interpret commas as field separators
set term postscript eps
set xrange [0:720]  # Sets bounds on the domain
set yrange [-1:1]  # Sets bounds on the domain
set style line 1 lw 3 lc rgb "red"  # Sets ls 1 thickness=3 color=red
set style line 2 lw 3 lc rgb "blue"  # Sets ls 2 thickness=3 color=blue
set style line 3 lw 3 lc rgb "green"  # Sets ls 3 thickness=3 color=green
set output 'anim_sw_fr_1440.eps'
plot 'data.csv' using 1:2 with lines ls 1, 'data.csv' using 1:3 with lines ls 2
set output 'anim_sw_sum_1440.eps'
plot 'data.csv' using 1:4 with lines ls 3
```

Note: the C++ program when compiled and run outputs comma-separated value (CVS) to the console (standard output). That C++ program must be compiled and run over and over again, each time with a different `offset` value representing the number of degrees the two waveforms have evolved over. For each run of the C++ program, I used Gnuplot to convert the comma-separated text file into graphical images, editing the gnuplot script each time to re-name the files according to the offset (angle) value. To convert this text data into graphical plots, you must redirect the output to a file (I use the filename `data.csv`) like this:

```
./a.out > data.csv
```

Then, you invoke Gnuplot to convert this text data into two Encapsulated PostScript files like so:

```
gnuplot -p gnuplot.txt
```

Gnuplot must be run once per execution of the C++ code, each time producing a pair of graphical images. The first image shows both the forward and reflected waves at that angle (`offset`), and the second image showing just the summation of the forward and reflected waves at that angle.

C++ source code for established standing waveform

```
#include <iostream>
#include <cmath>
using namespace std;

float sinecalc (int);

int main (void)
{
  int x, offset;
  float fwd[720];
  float ref[720];
  float sum[720];

  offset = 360;

  for (x = 0 ; x < 720 ; ++x)
  {
    fwd[x] = 0.5*sinecalc(x+offset);
    ref[719-x] = -0.5*sinecalc(x+offset);
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    sum[x] = fwd[x] + ref[x];
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    cout << x << " , " << fwd[x] << " , " << ref[x] << " , " << sum[x] << endl;
  }

  return 0;
}

float sinecalc (int degrees)
{
    return sin (degrees * M_PI / 180);
}
```

As before, this C++ code must be compiled and executed repeatedly, each time with a different offset value. The only difference here is that the offset values need only span from 0 to 360 degrees,

not all the way to 1440 degrees as before to simulate the initial travel of both the forward and reflected waves along the transmission line's length.

5.5 Animation of a standing wave with 50% reflection

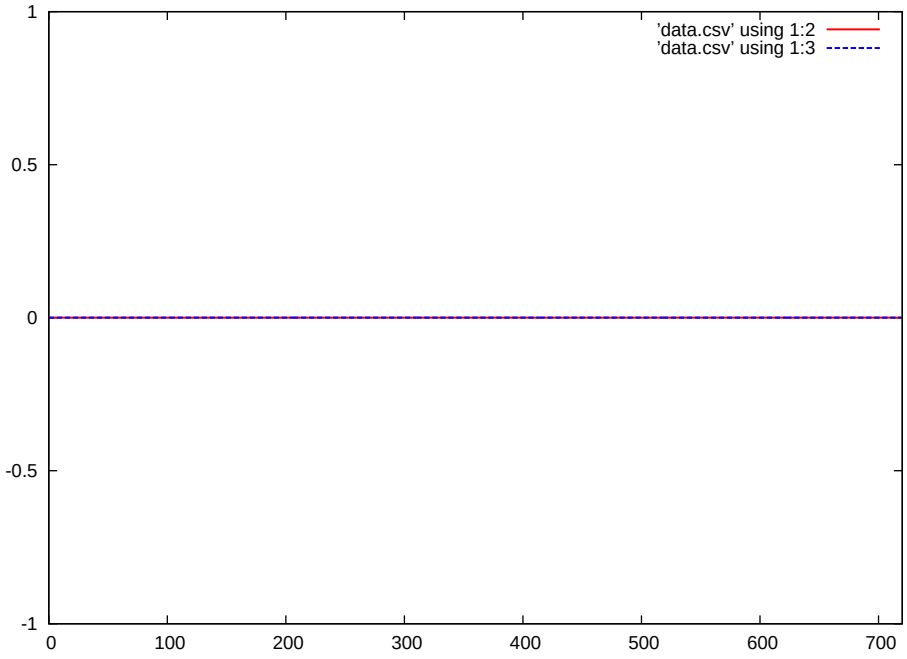
This series of animations shows waveforms traveling along a transmission line, the incident (or "forward") waveform shown in red and the reflected waveform shown in blue. It is assumed that the signal source is on the left-hand side of the graph, while the transmission line terminates on the right-hand side of the graph.

If the transmission line is terminated in such a way as to reflect 50% of the forward waveform at its end, the reflected wave will have half the amplitude as the forward wave at the line's end but negative in value. This represents the state of affairs for waves at the end of a transmission line terminated in a resistance that is half that of the line's characteristic impedance.

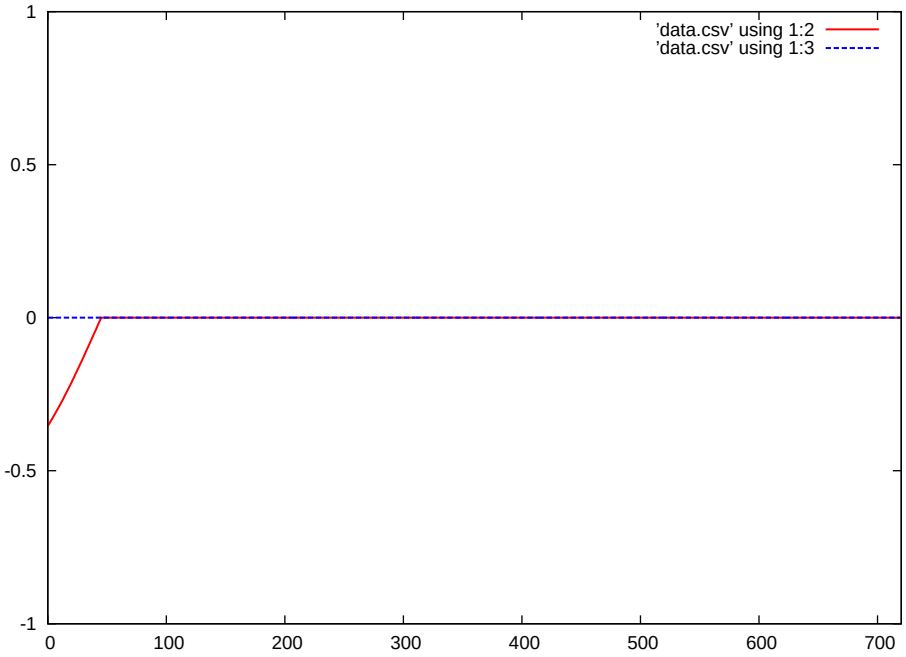
In the following flip-book animations, the first animation shows only the forward and reflected waves along the line for 1440 degrees of rotation. The second animation shows this same sequence of forward and reflected waves, but in an adjacent image shows the superposition of these two waveforms in a different color. The third animation shows that same superposition, but after the forward and reflected waves have traveled the full length of the transmission line to create an established "standing wave" along the line.

Standing wave ratio or *SWR* for any transmission line is defined as the maximum amplitude of the superposition wave divided by its minimum amplitude. In this case, where the superposition of the forward and reflected waves varies from 75% to 25% of full scale, yielding a 3:1 SWR. You will also see how with imperfect reflection the "standing wave" doesn't quite "stand" in one place as it does when the reflected signal is just as strong as the forward signal.

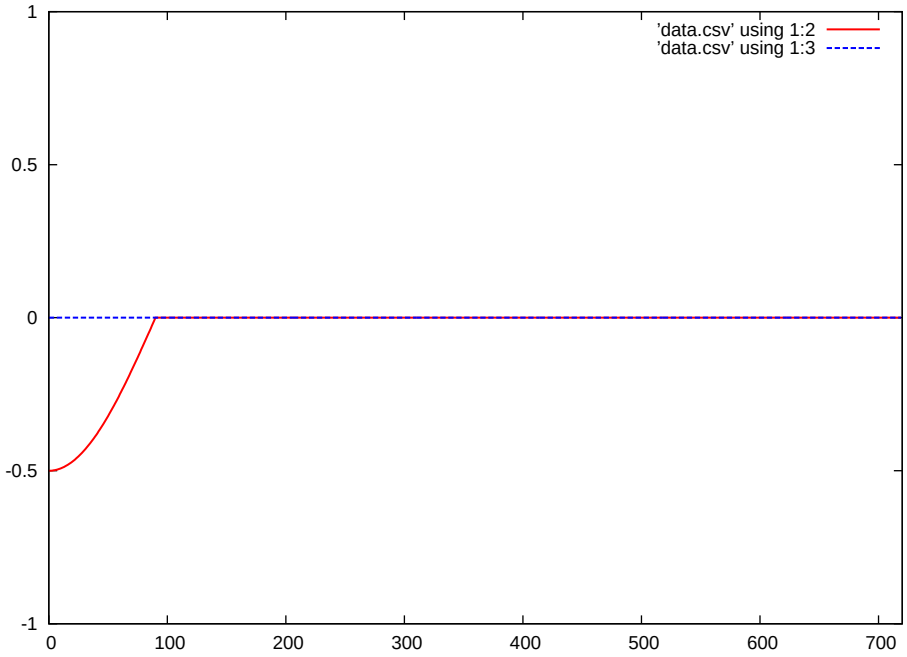
Red = forward wave Blue = reflected wave



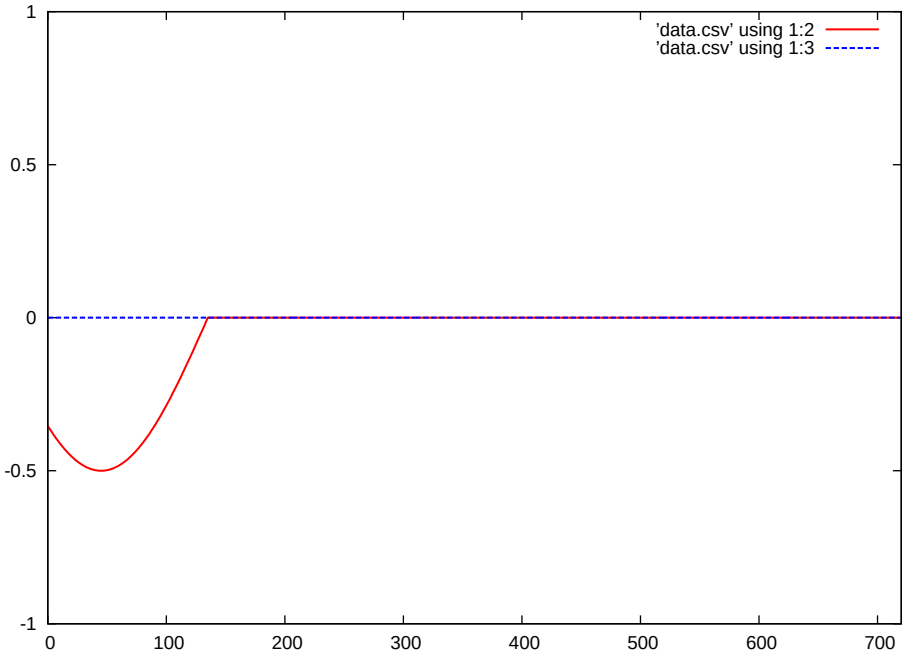
Red = forward wave Blue = reflected wave



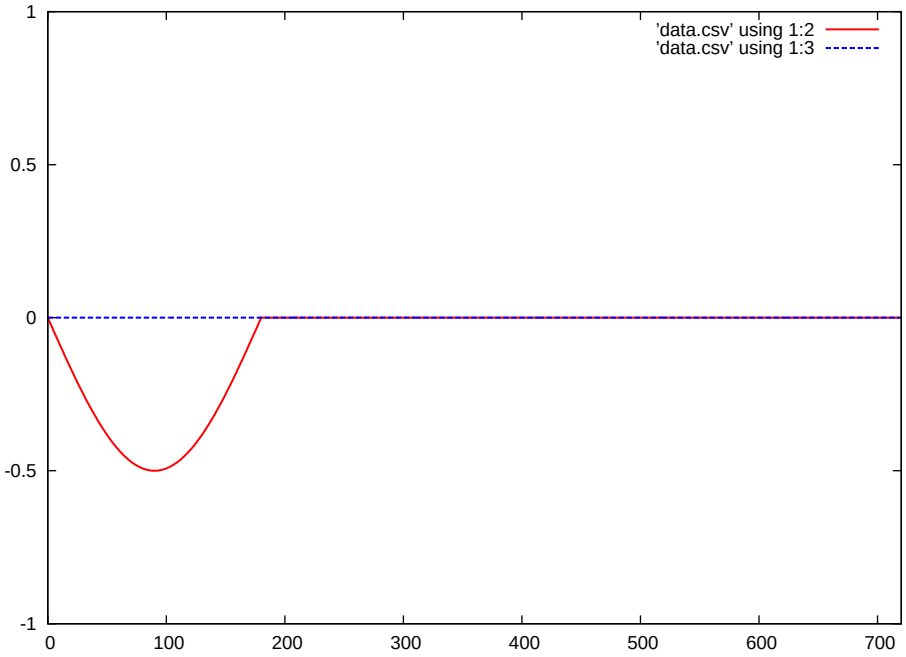
Red = forward wave Blue = reflected wave



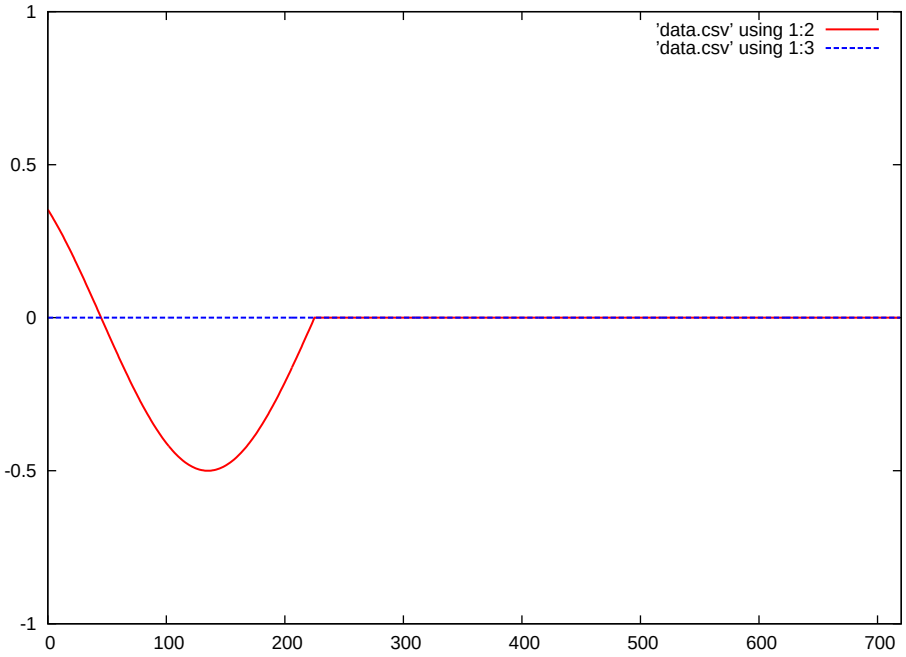
Red = forward wave Blue = reflected wave



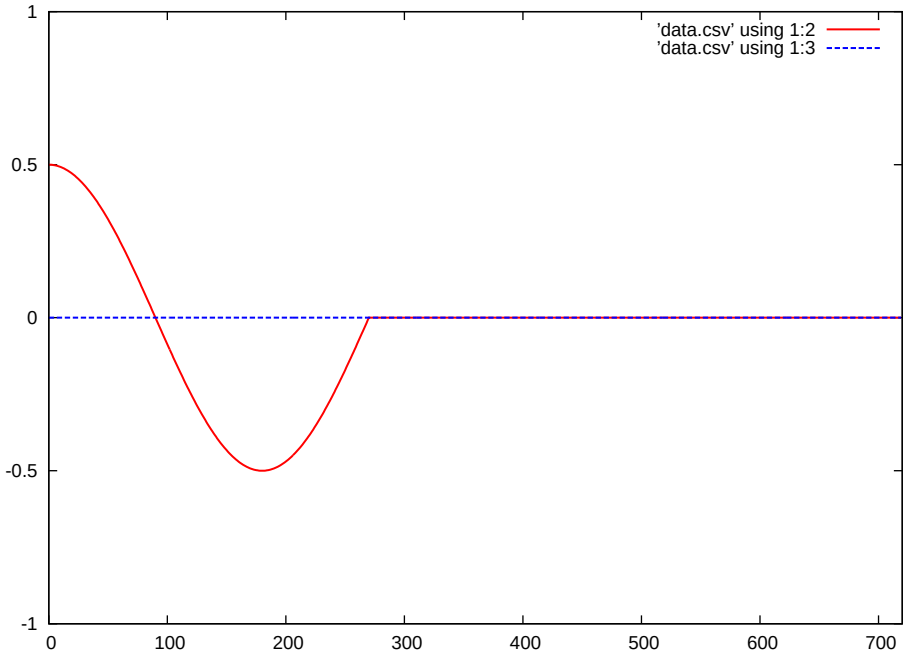
Red = forward wave Blue = reflected wave



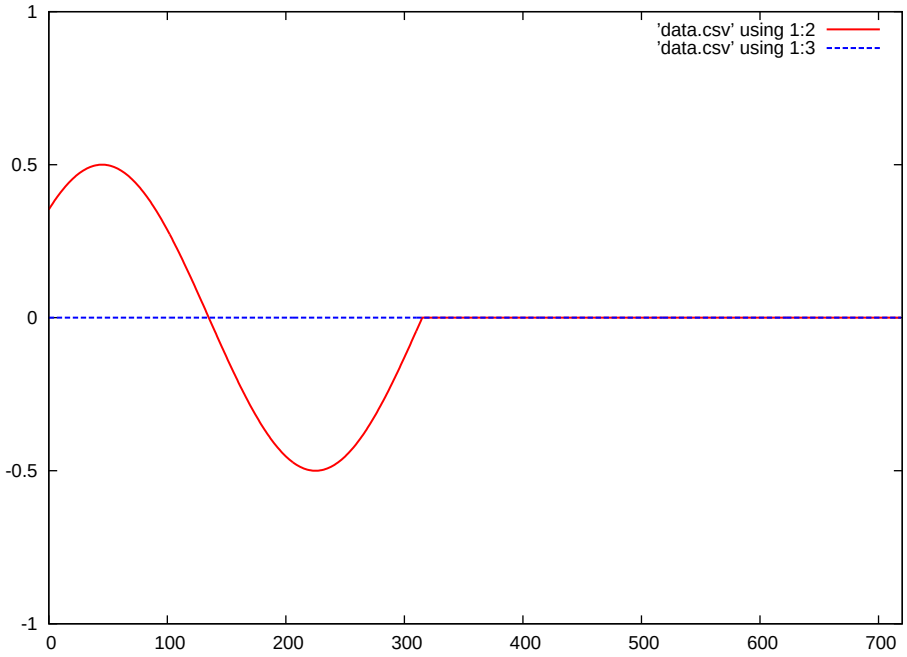
Red = forward wave Blue = reflected wave



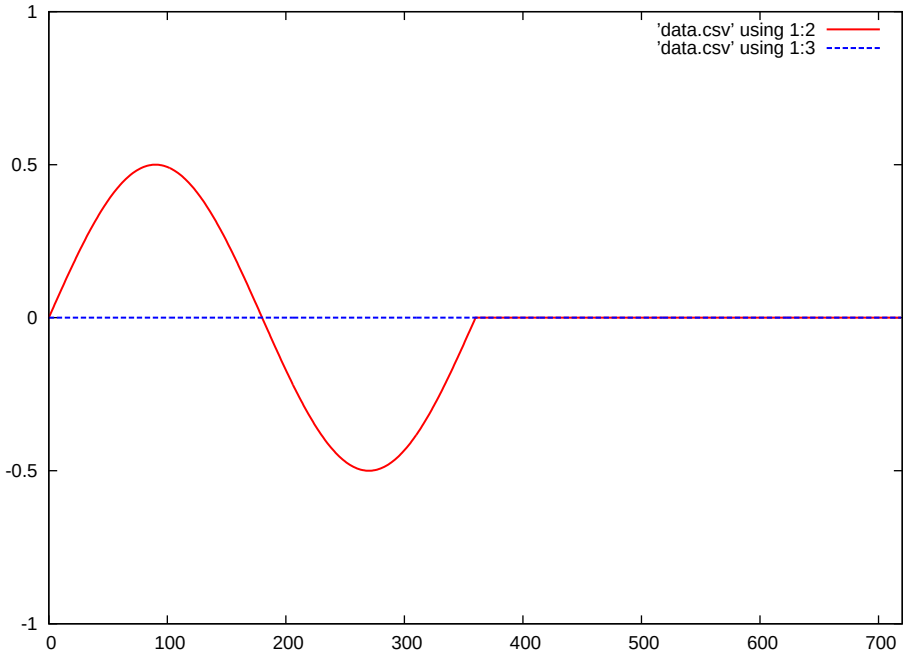
Red = forward wave Blue = reflected wave



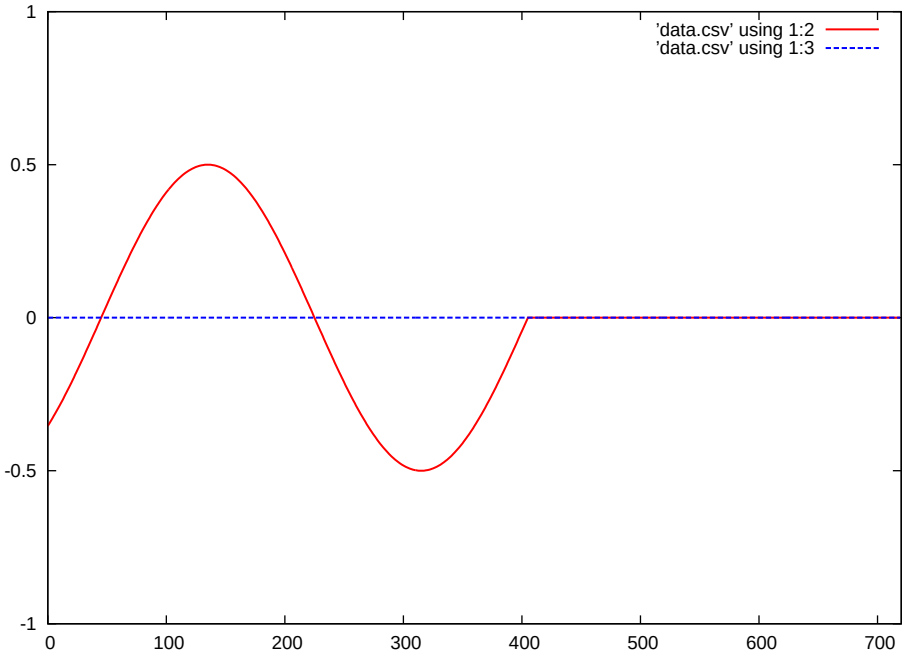
Red = forward wave Blue = reflected wave



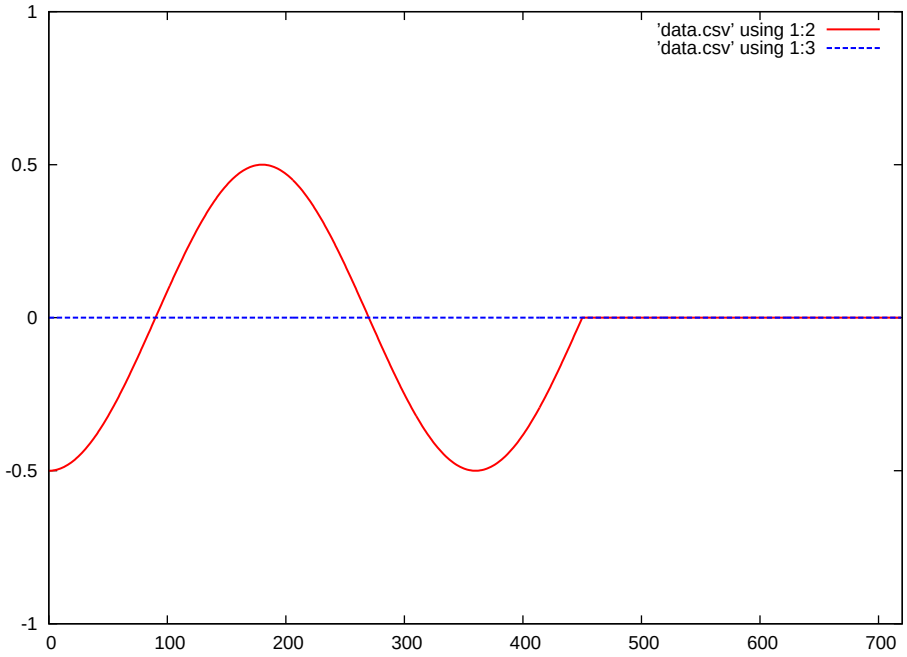
Red = forward wave Blue = reflected wave



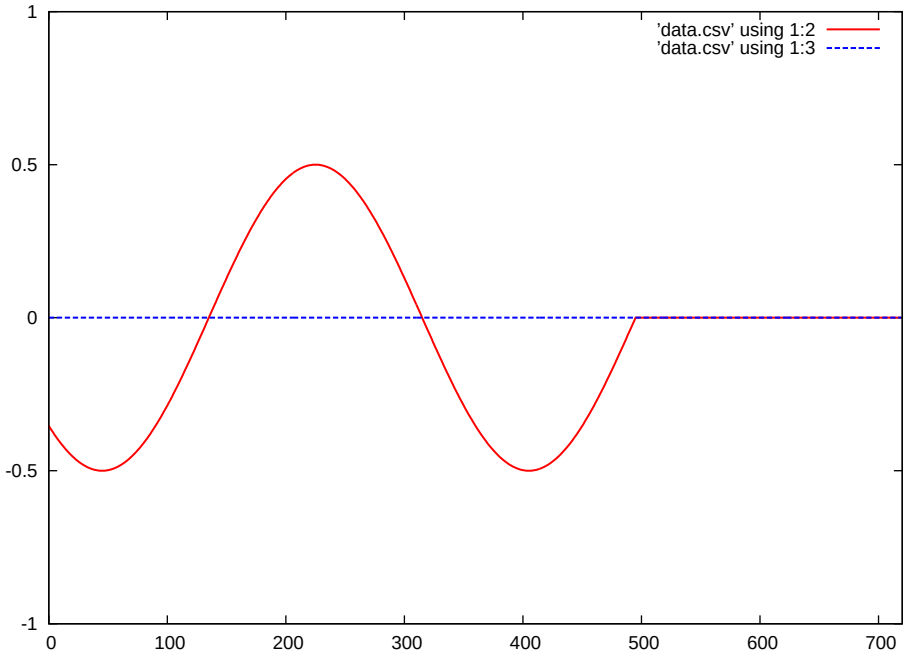
Red = forward wave Blue = reflected wave



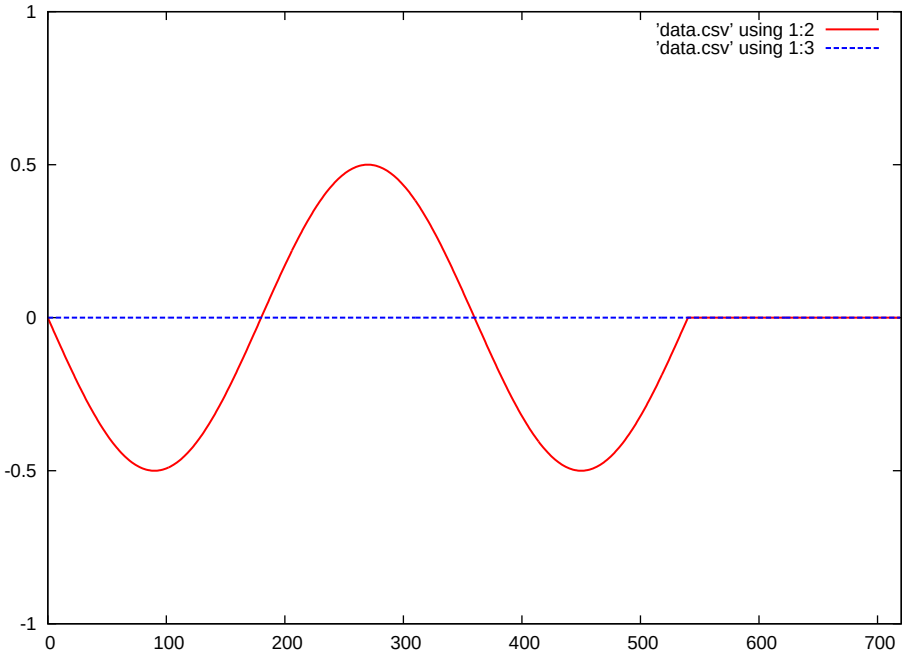
Red = forward wave Blue = reflected wave



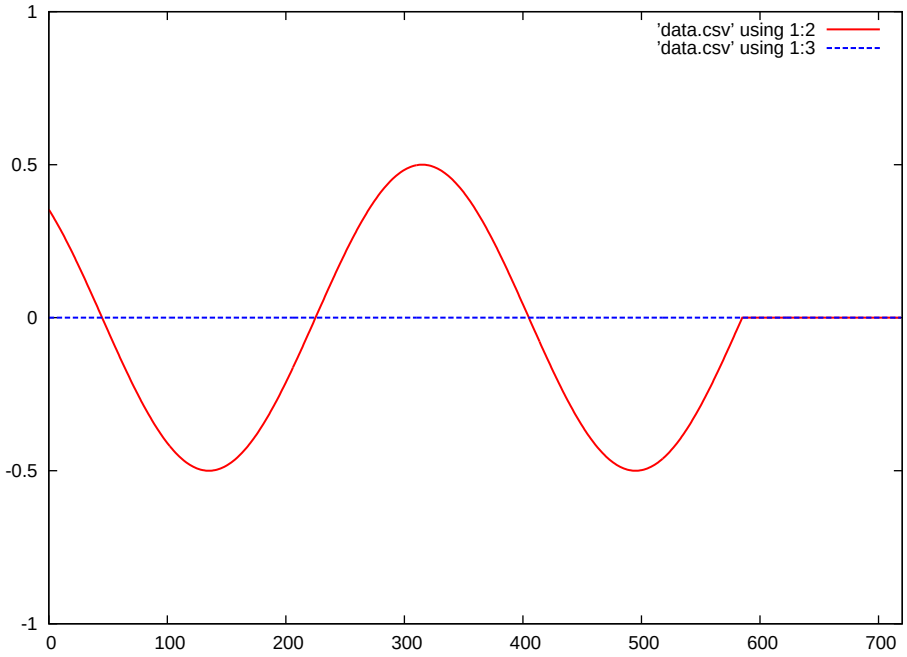
Red = forward wave Blue = reflected wave



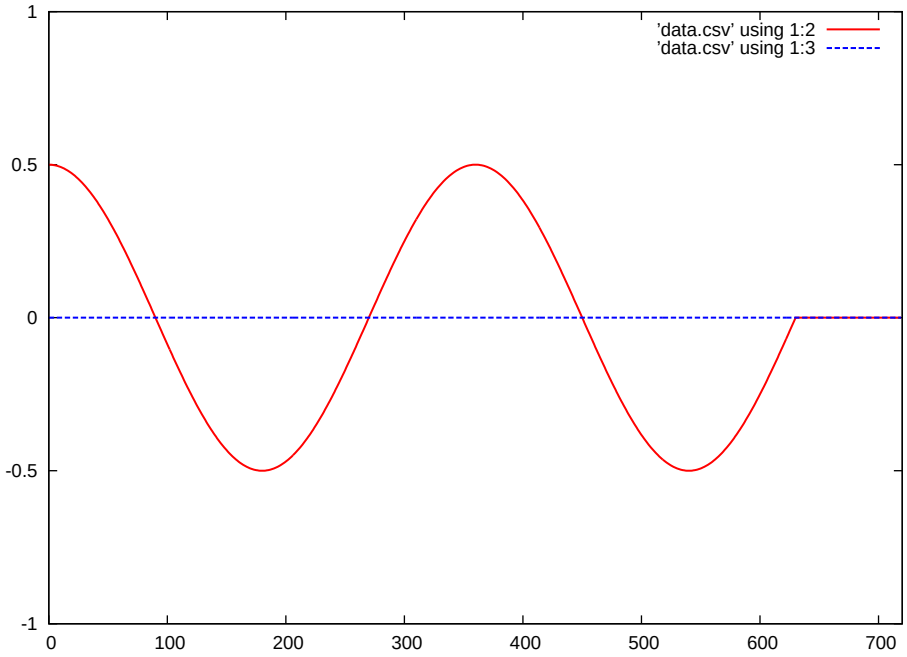
Red = forward wave Blue = reflected wave



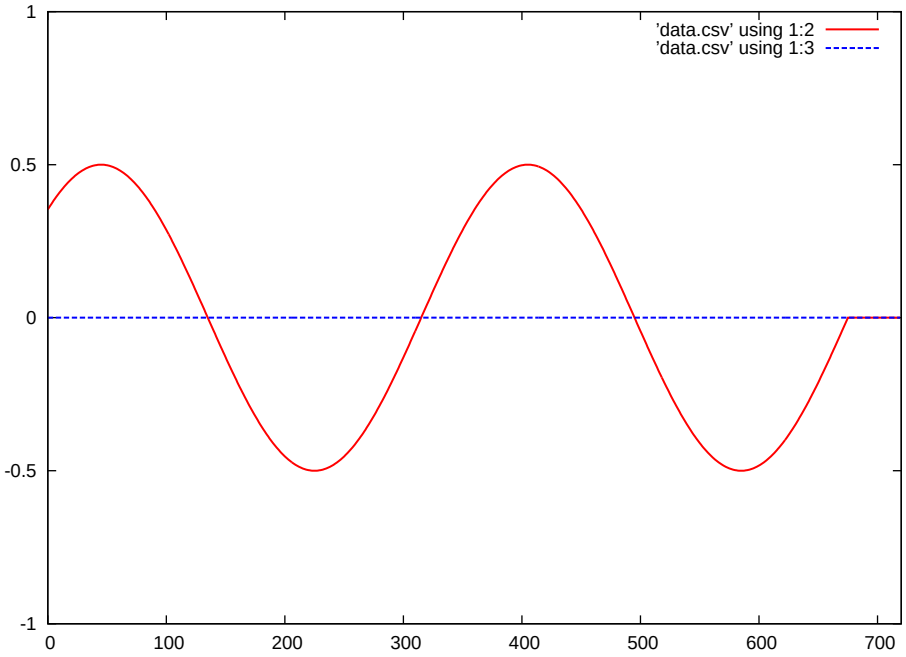
Red = forward wave Blue = reflected wave



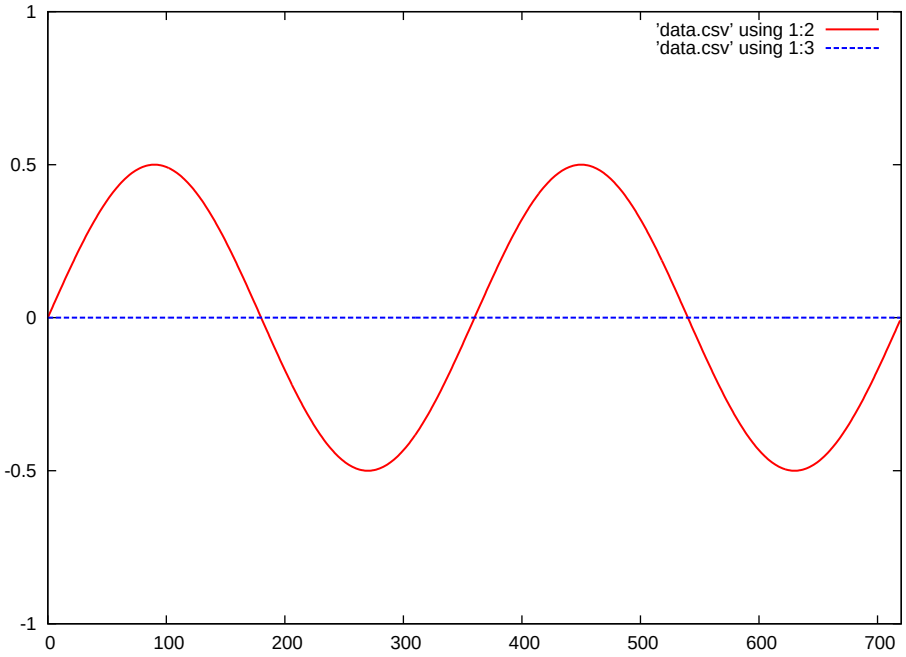
Red = forward wave Blue = reflected wave



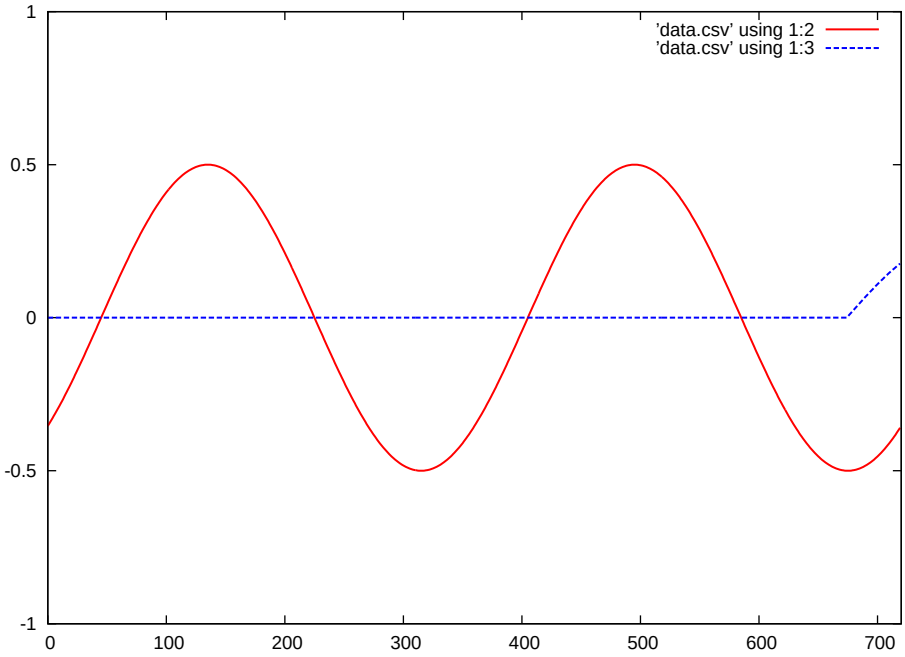
Red = forward wave Blue = reflected wave



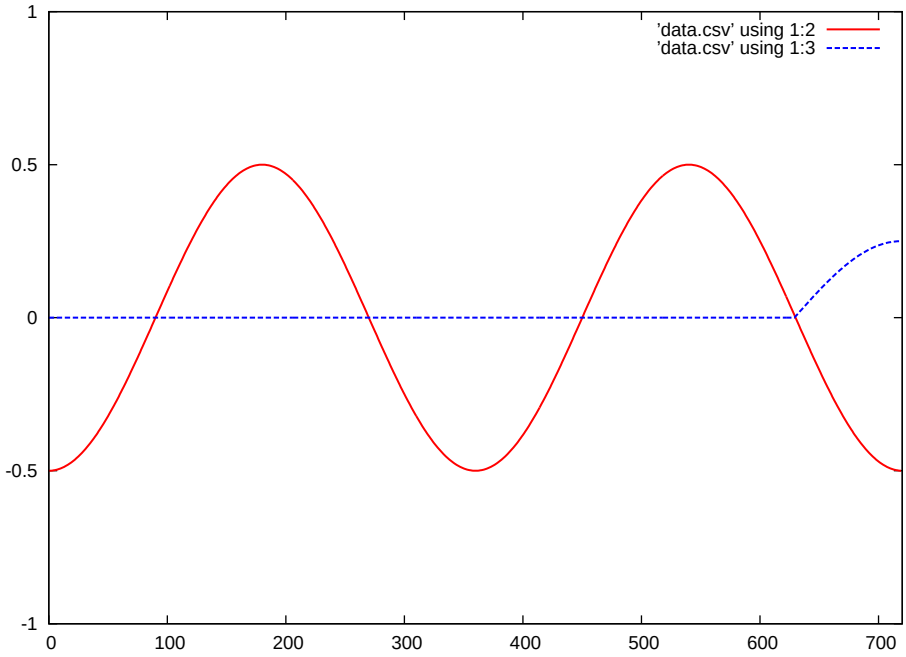
Red = forward wave Blue = reflected wave



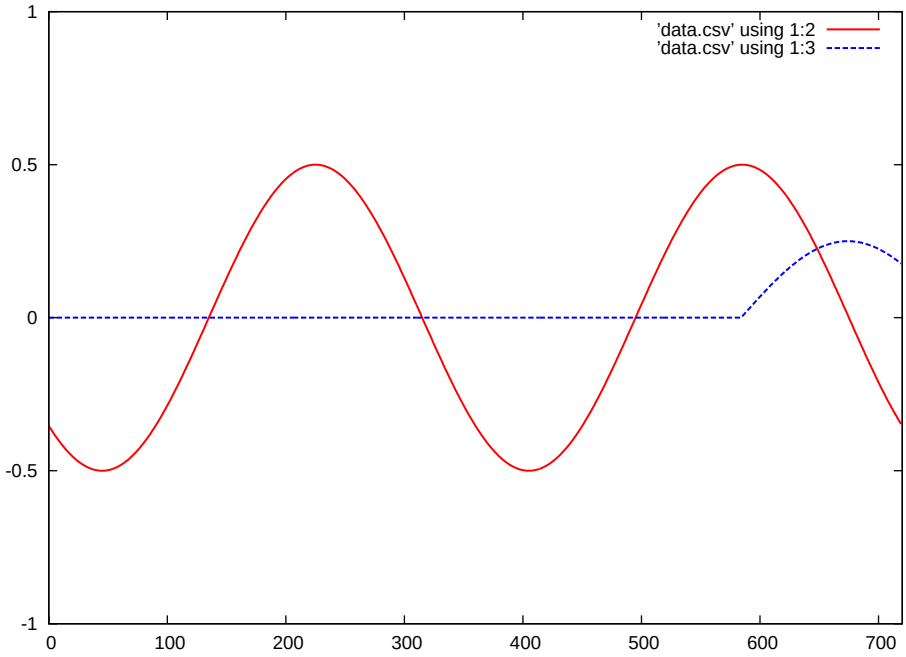
Red = forward wave Blue = reflected wave



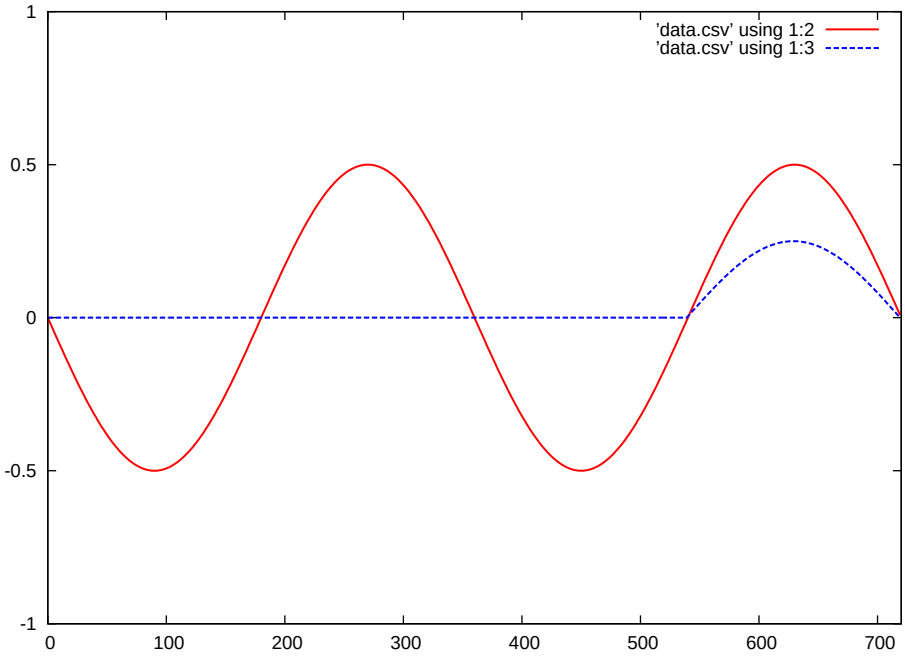
Red = forward wave Blue = reflected wave



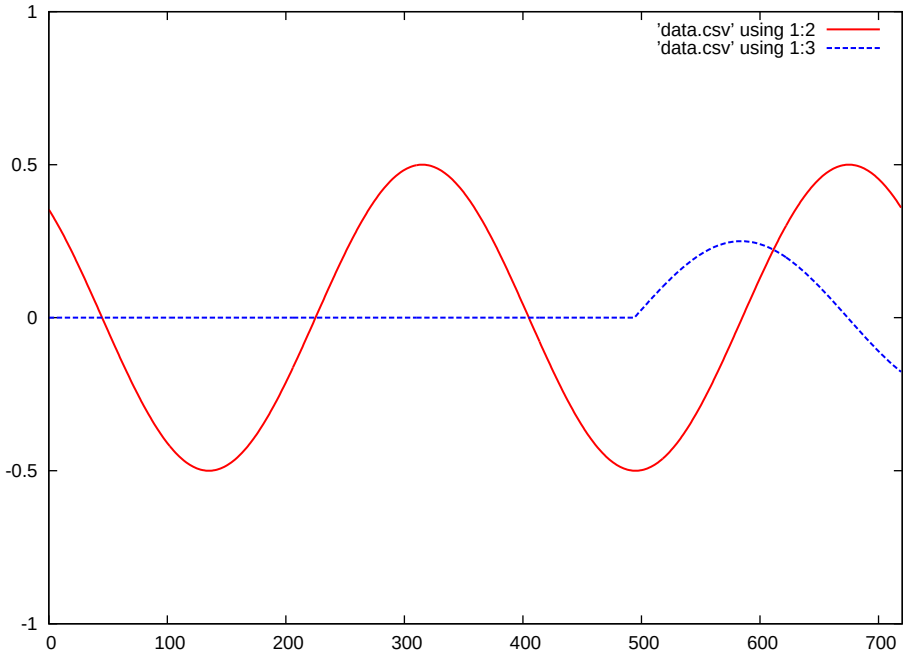
Red = forward wave Blue = reflected wave



Red = forward wave Blue = reflected wave



Red = forward wave Blue = reflected wave



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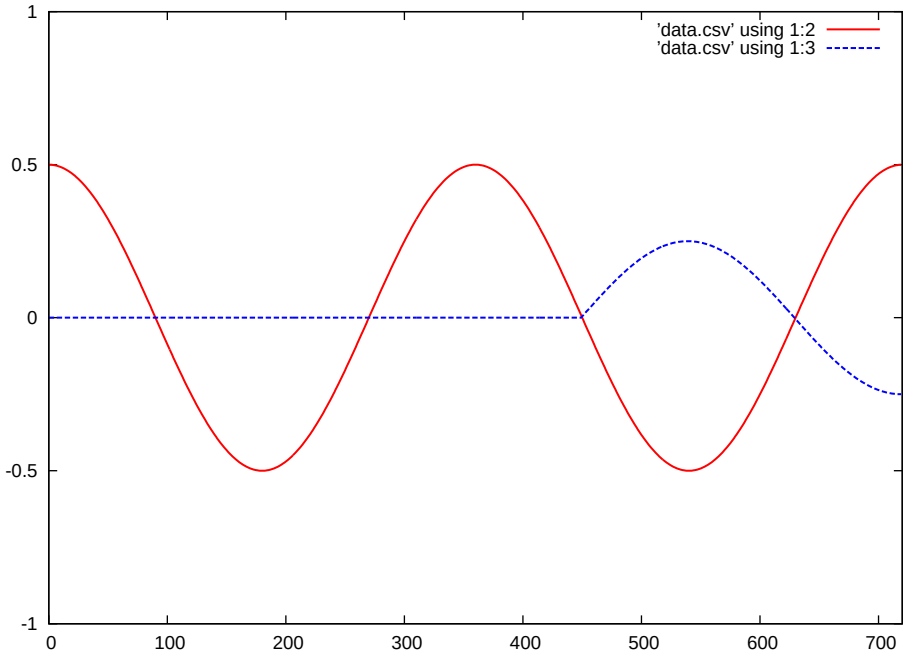

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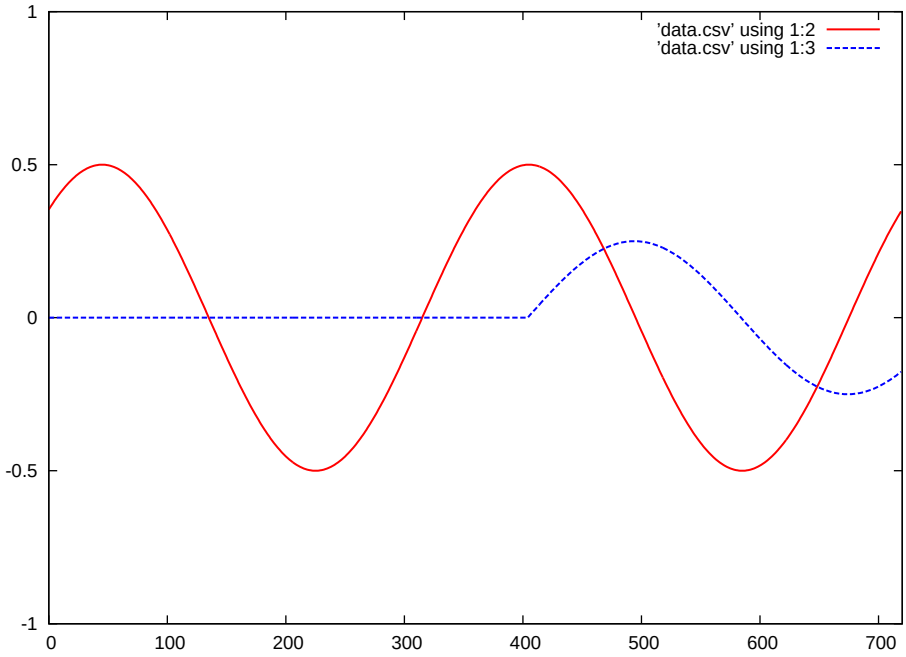

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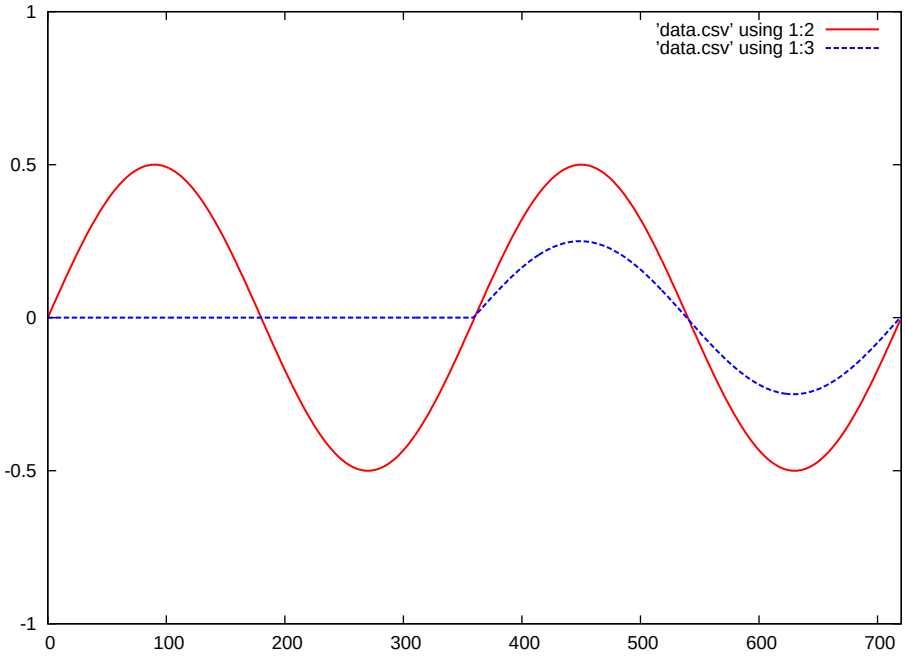

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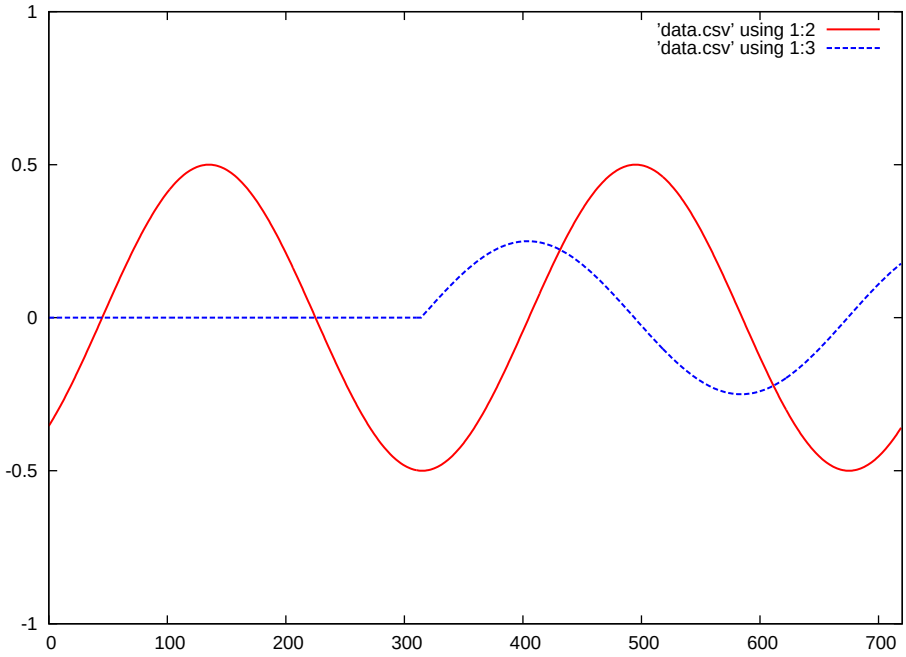

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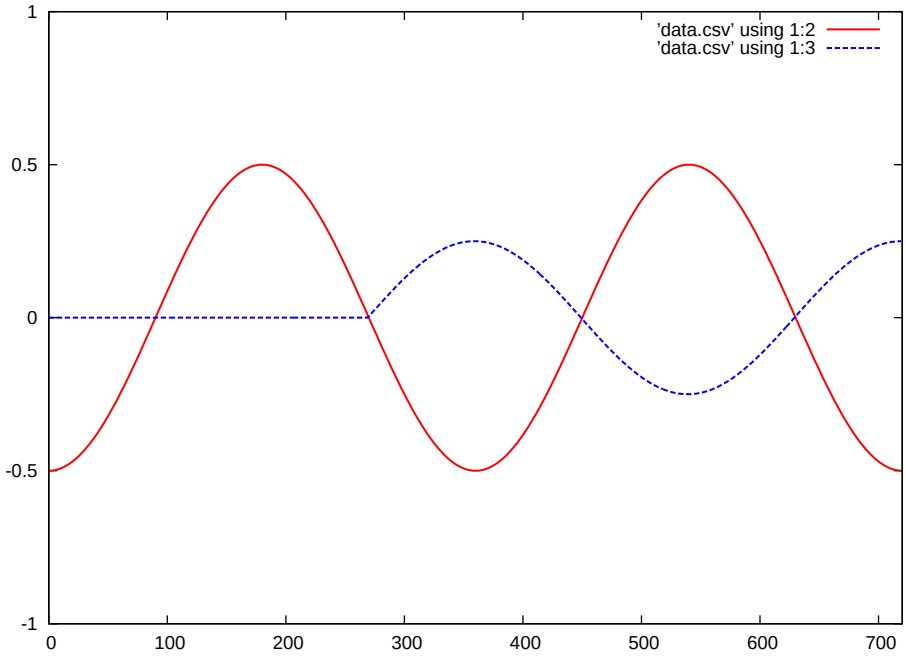

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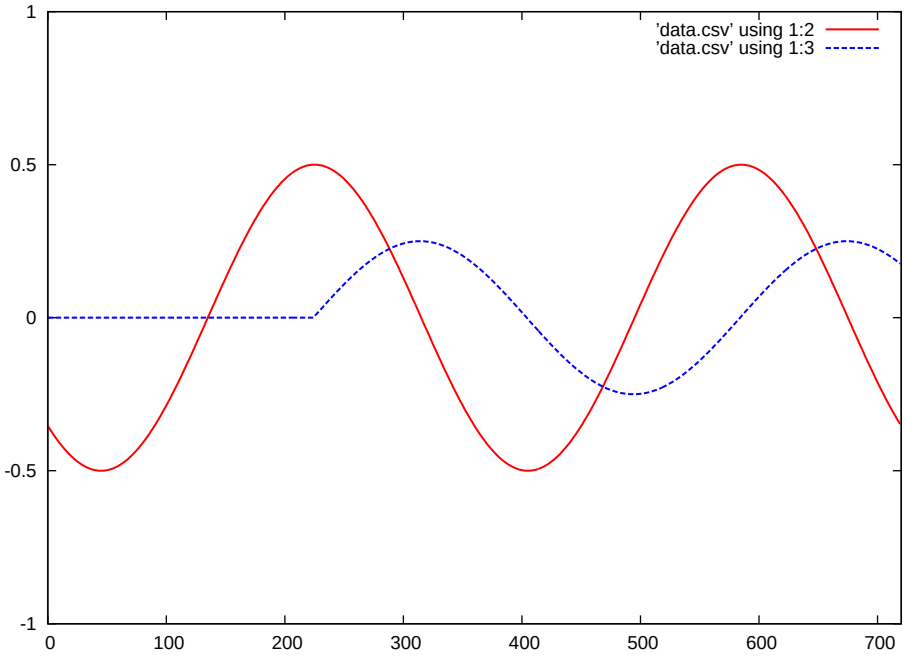

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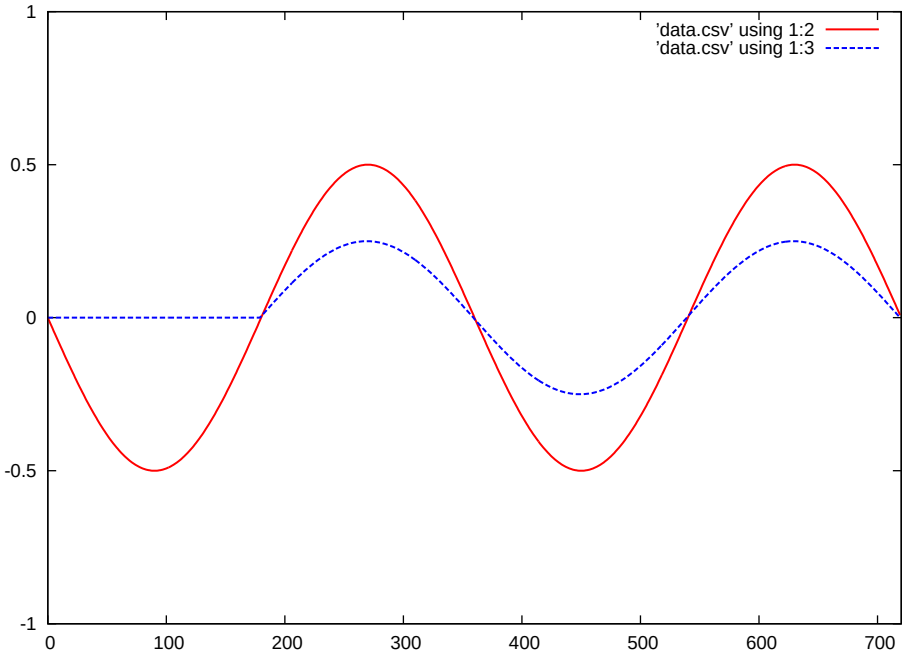

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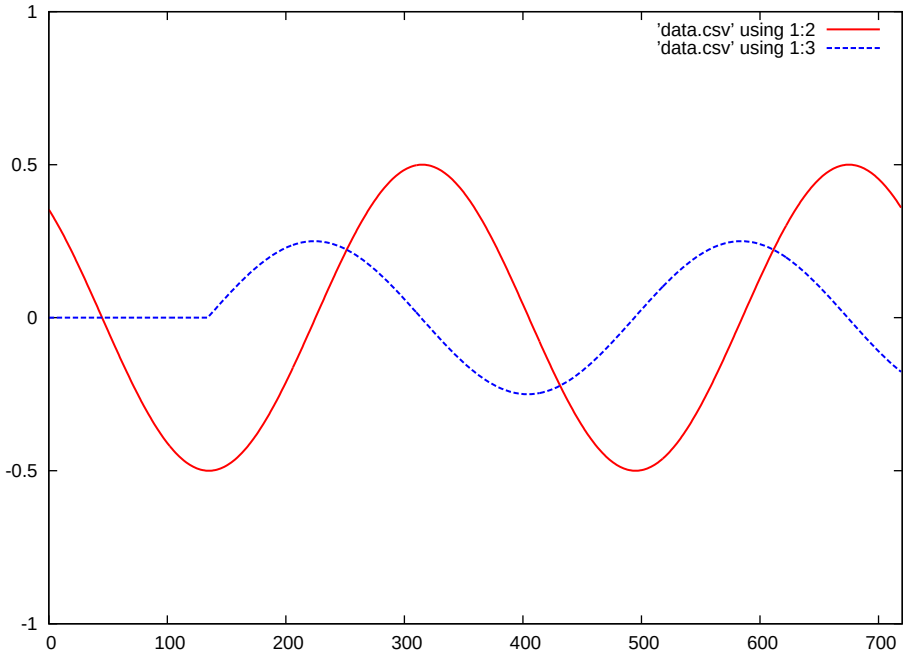

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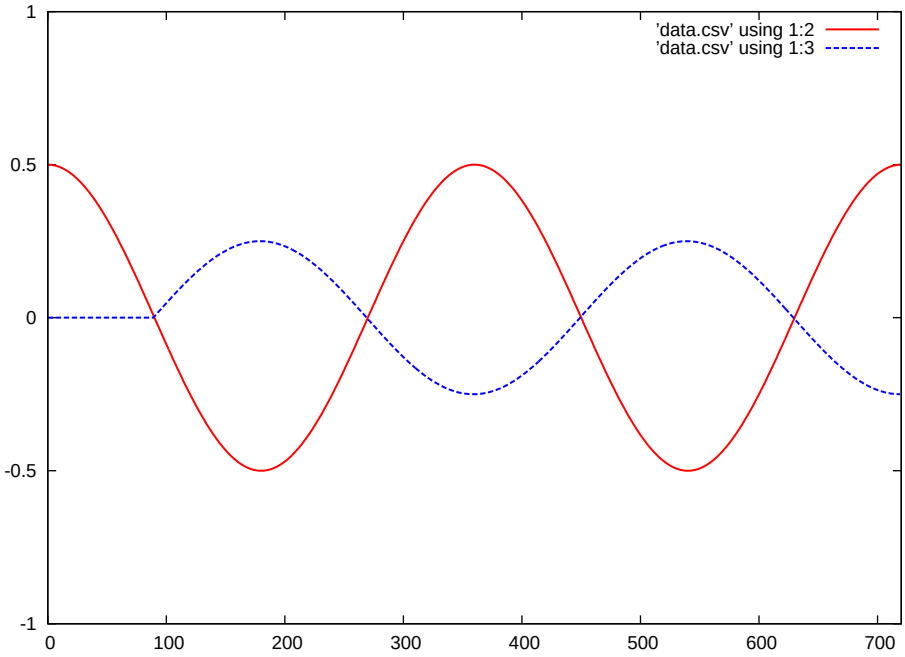

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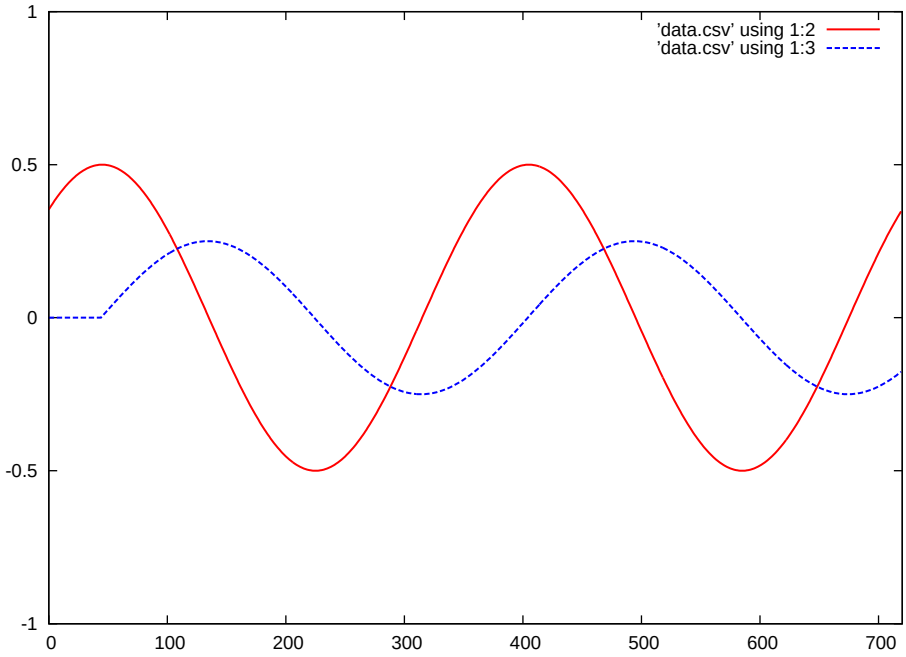

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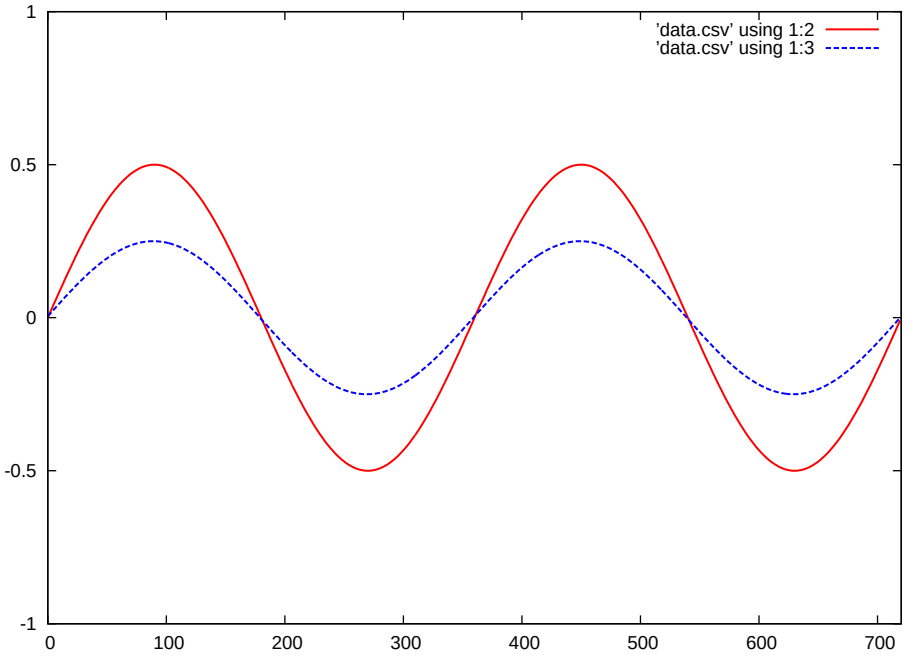
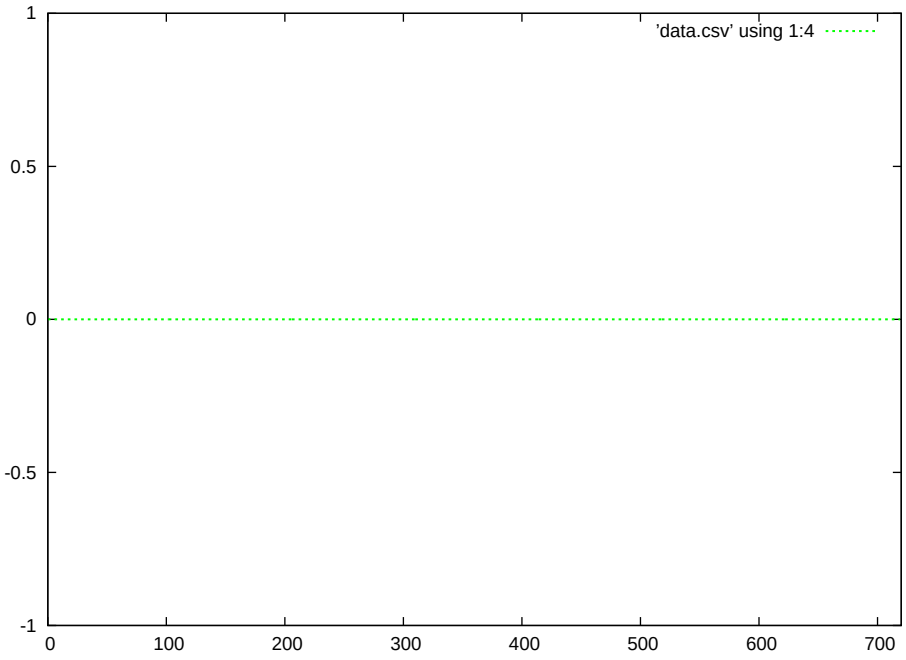


Green = superposition of the forward and reflected waves



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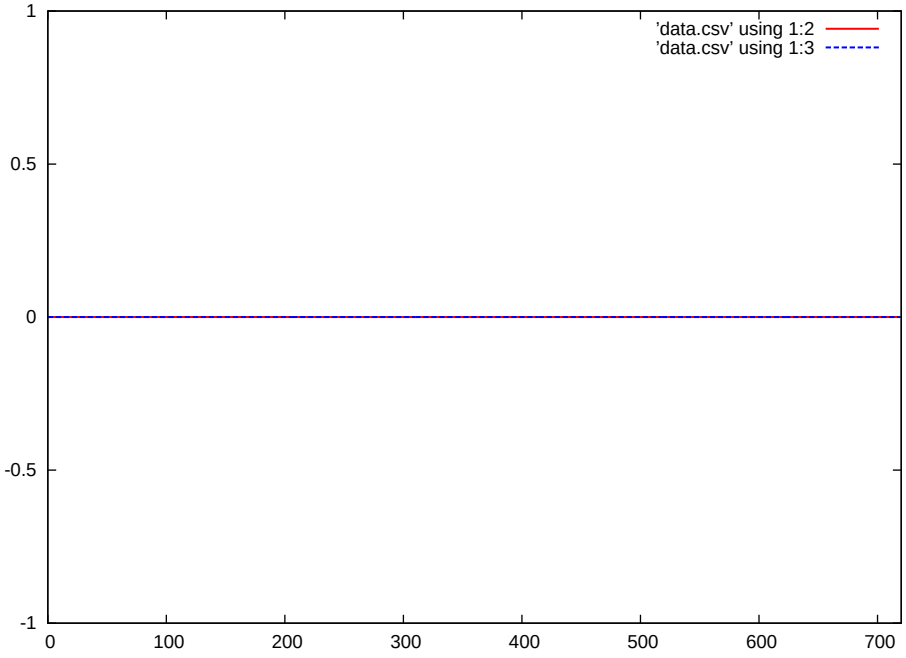

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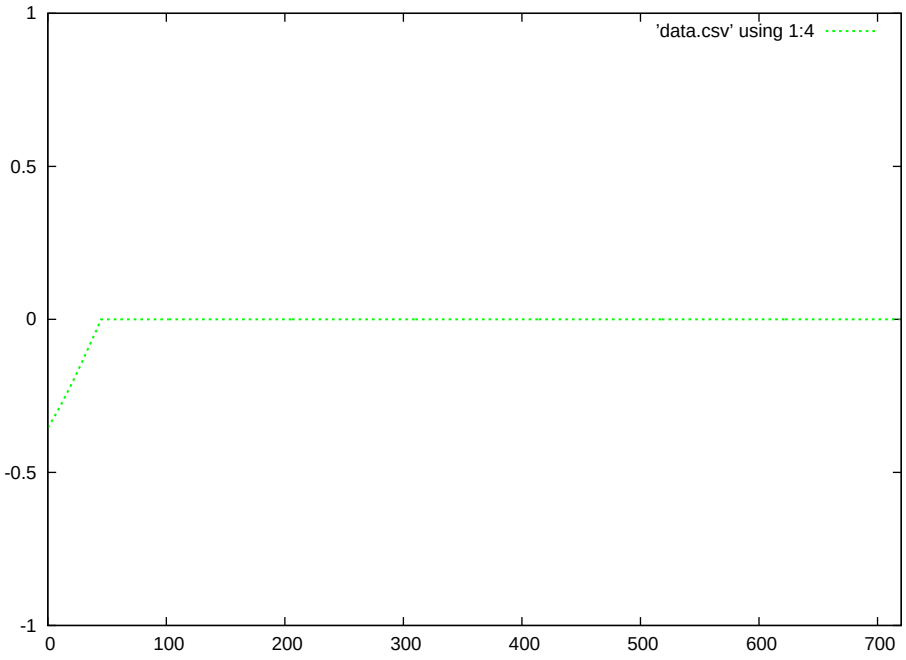

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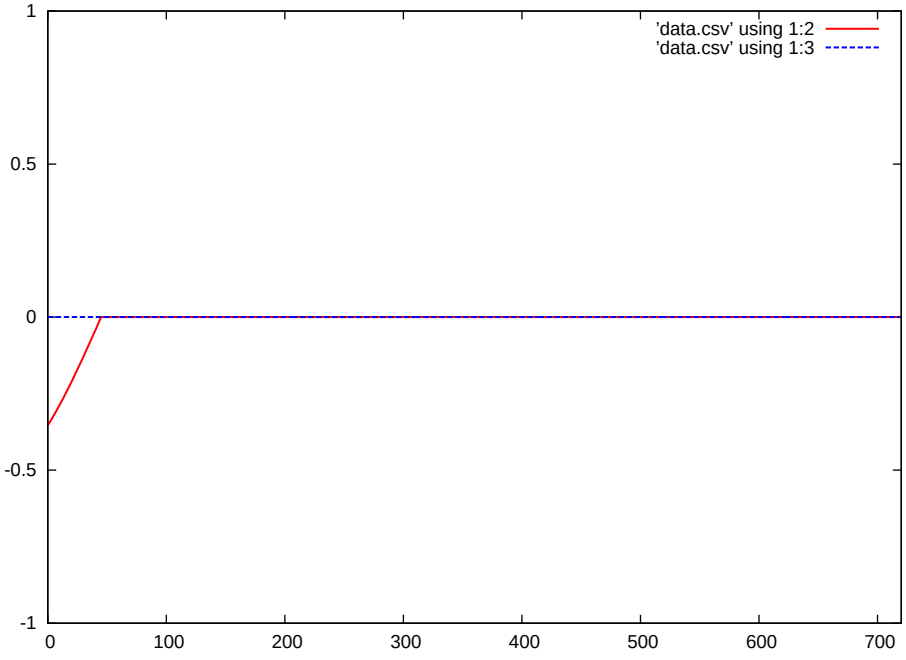

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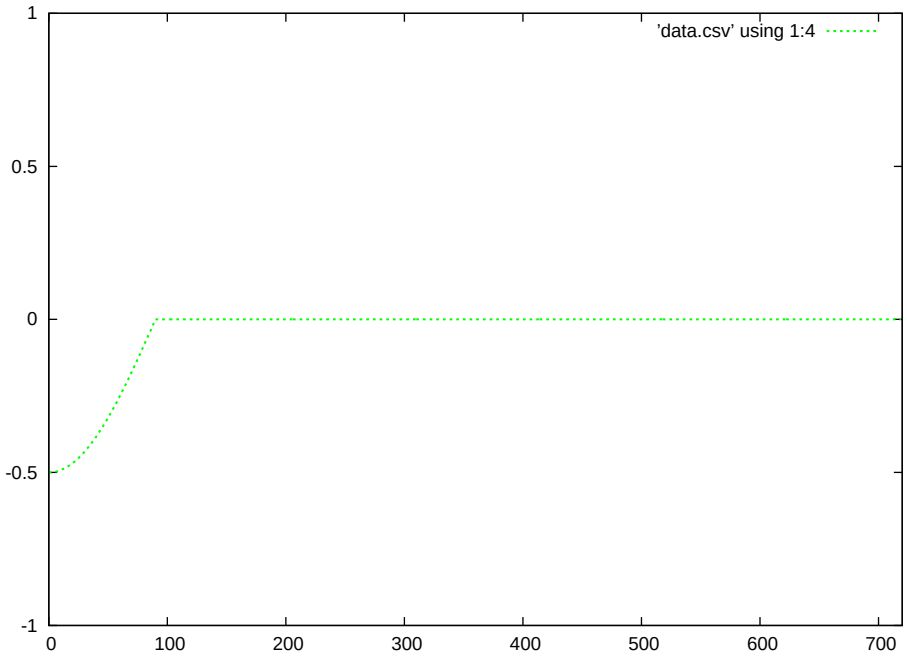

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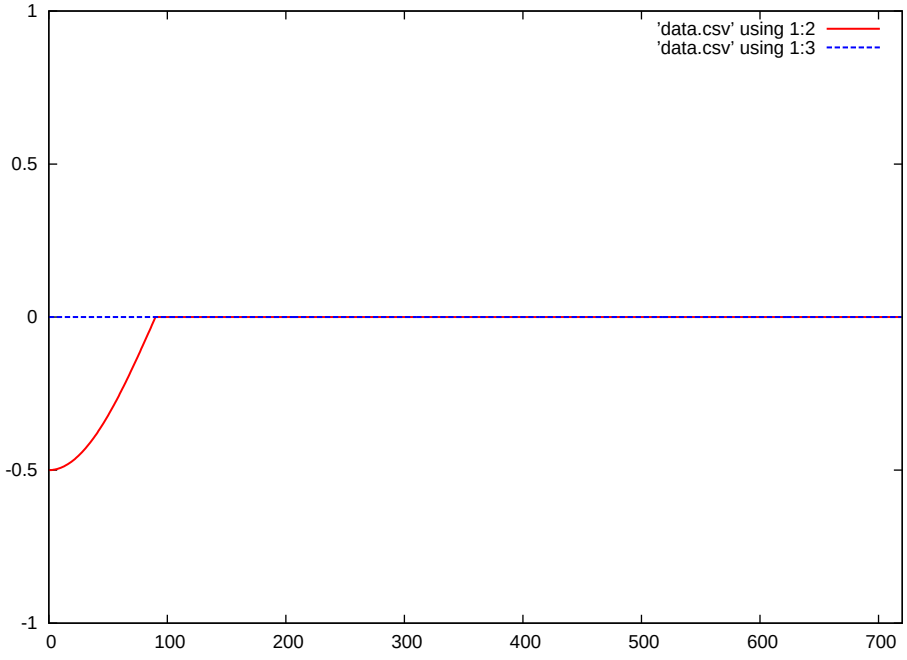

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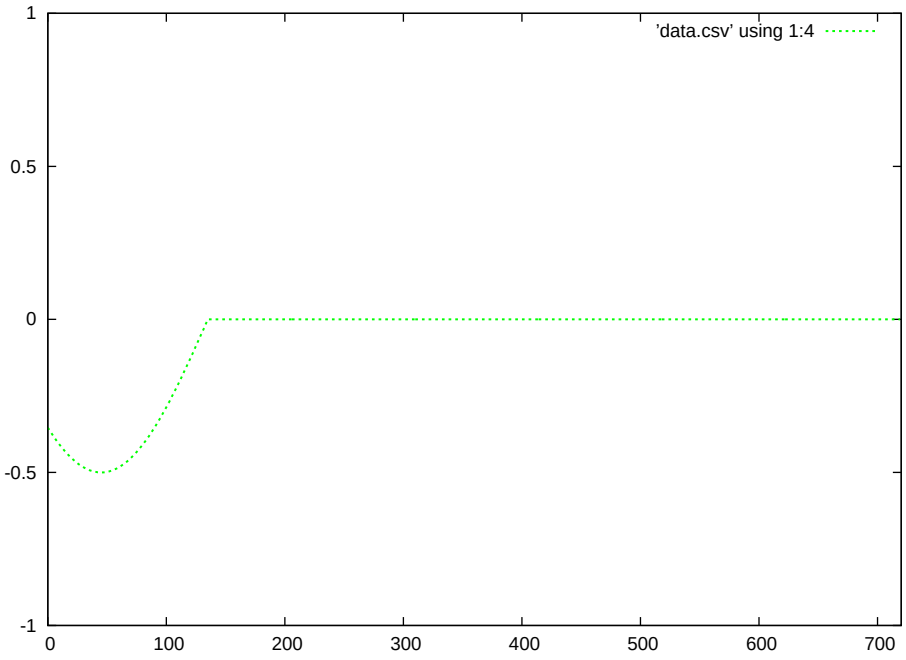

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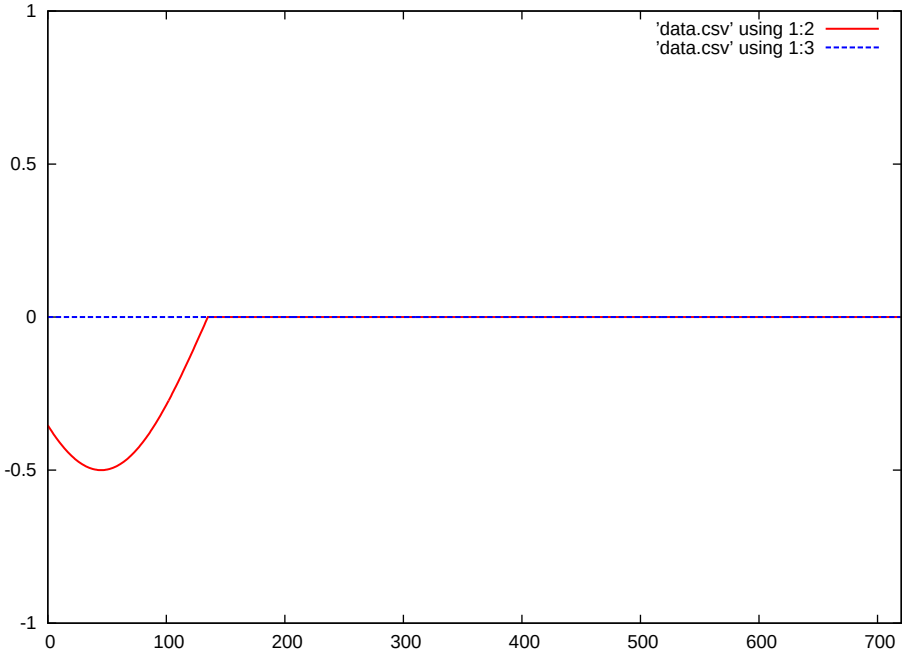

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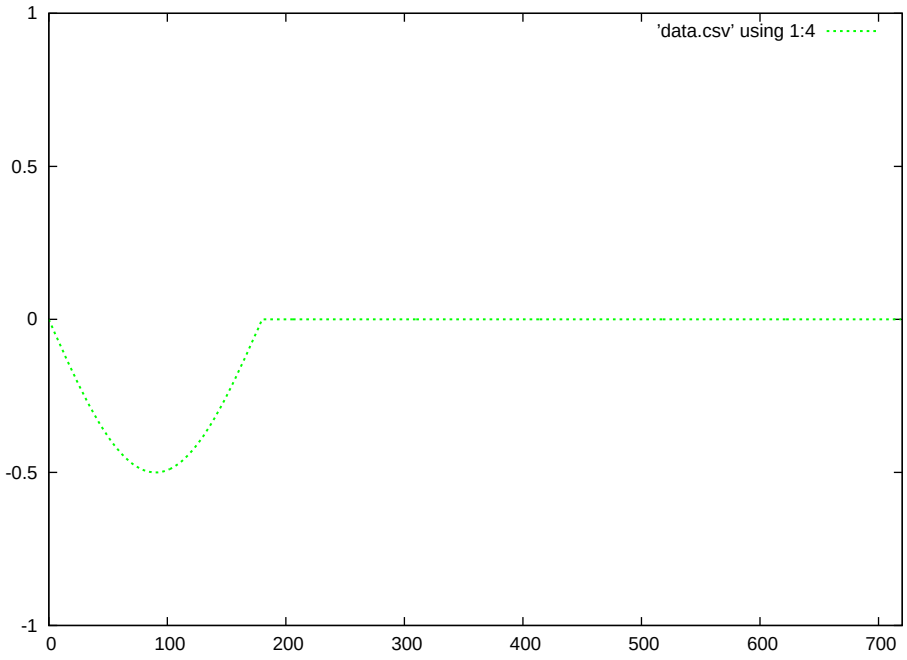

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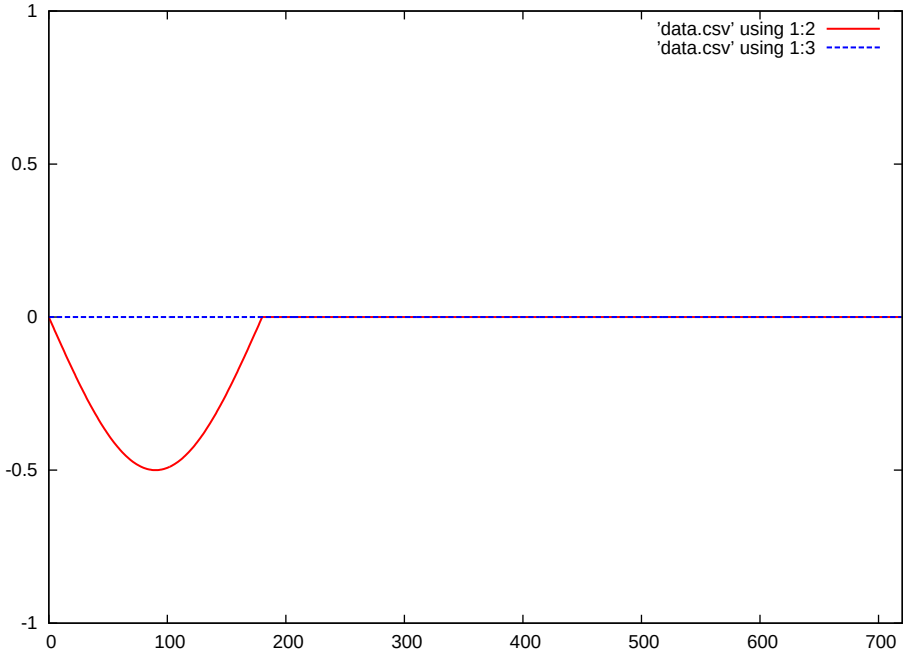

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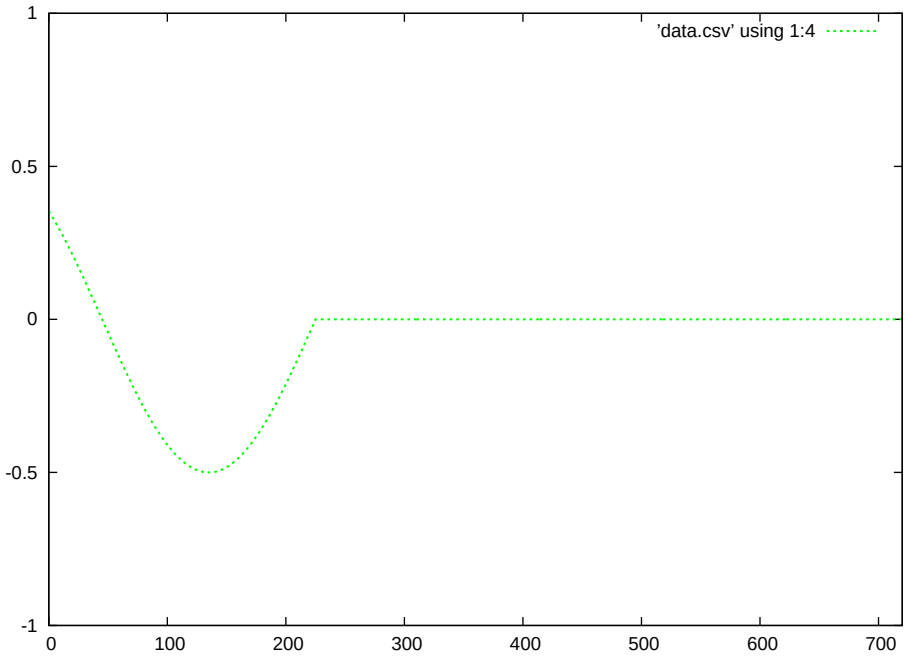

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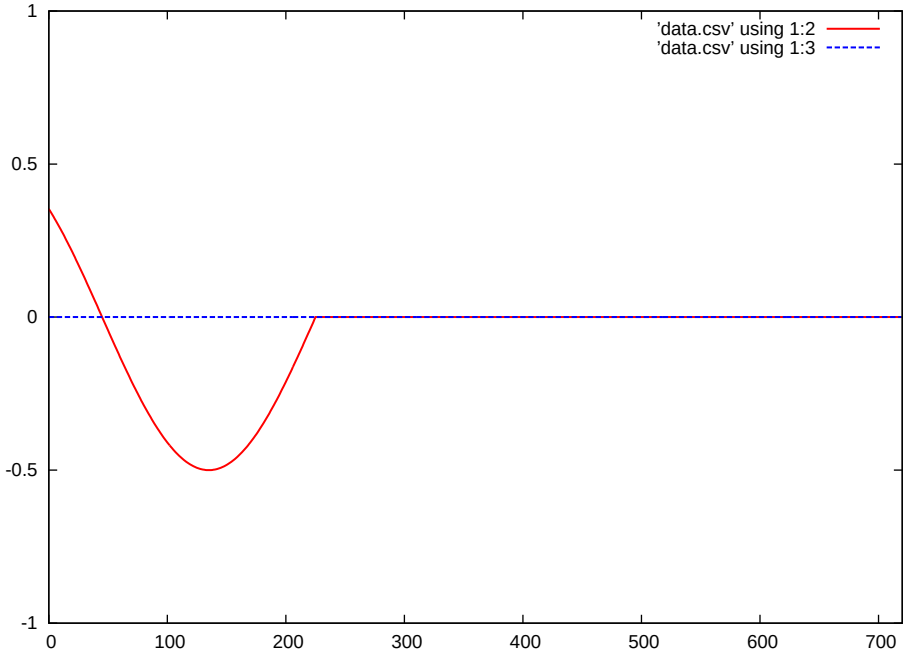

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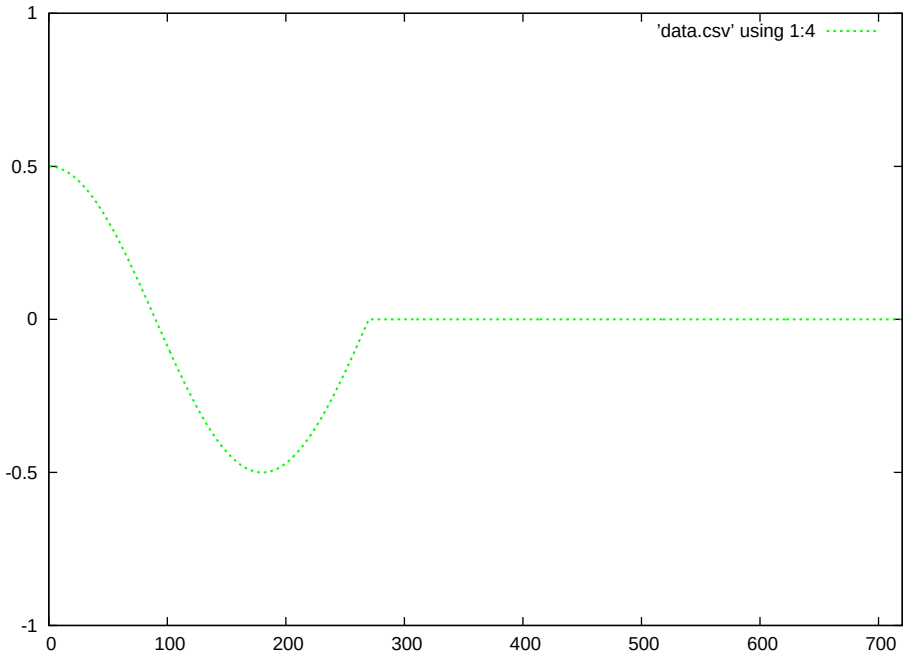

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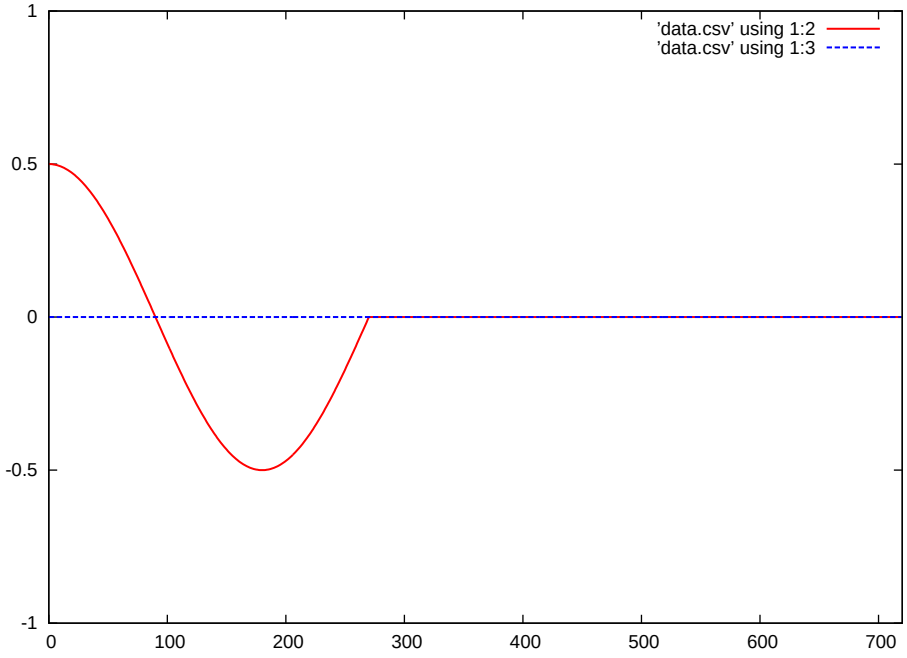
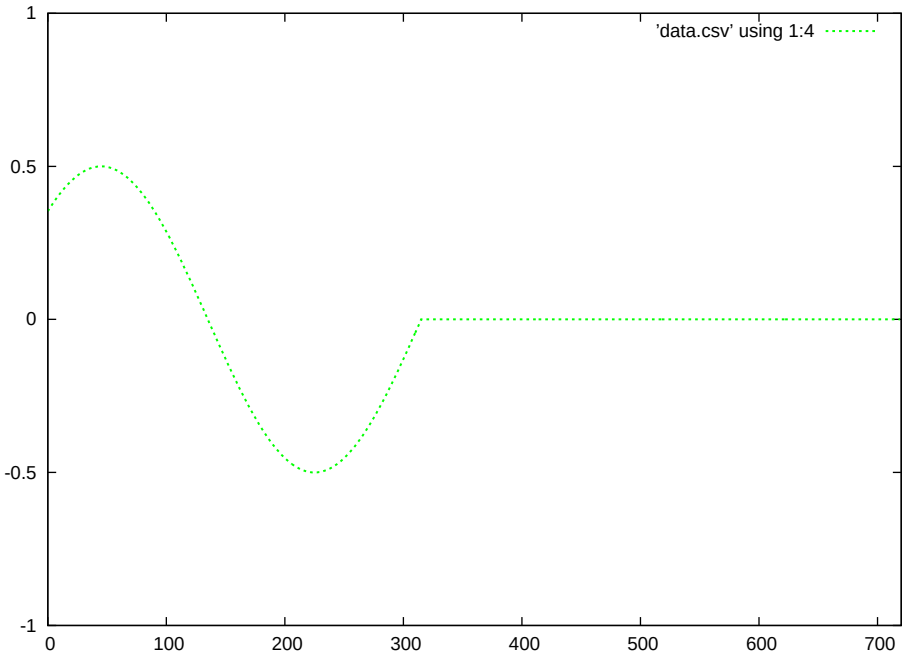


Green = superposition of the forward and reflected waves



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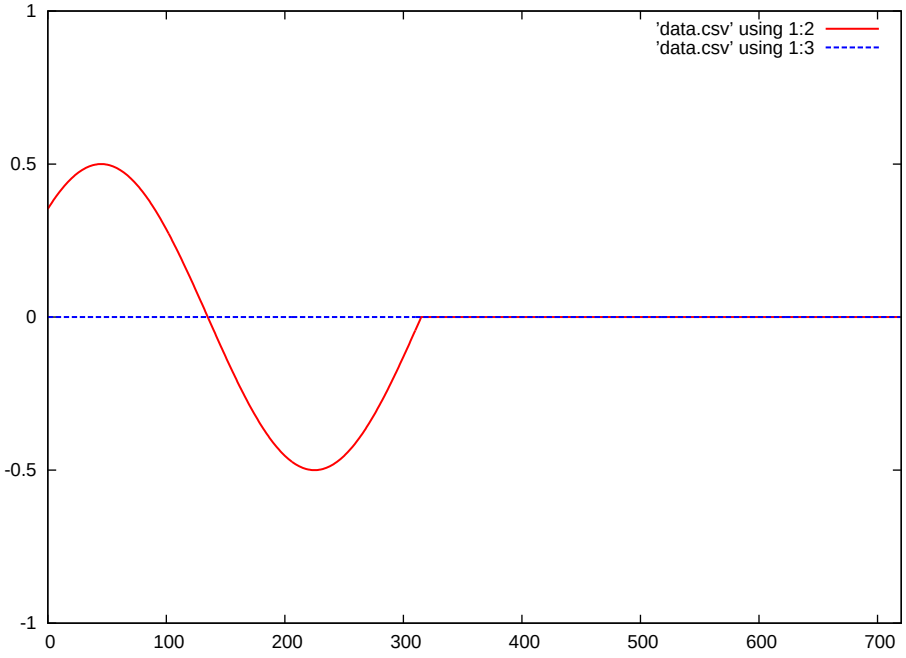

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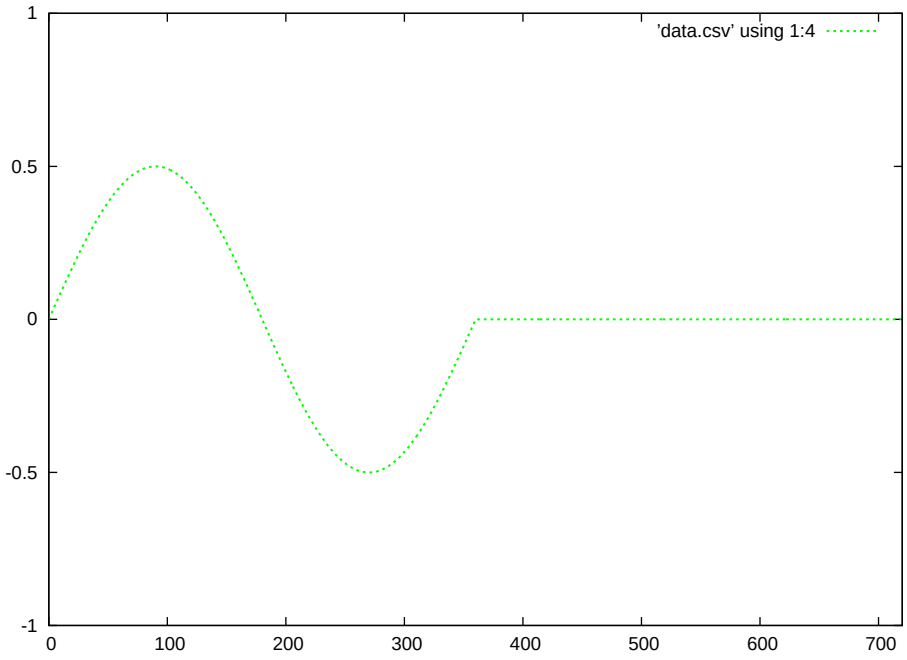

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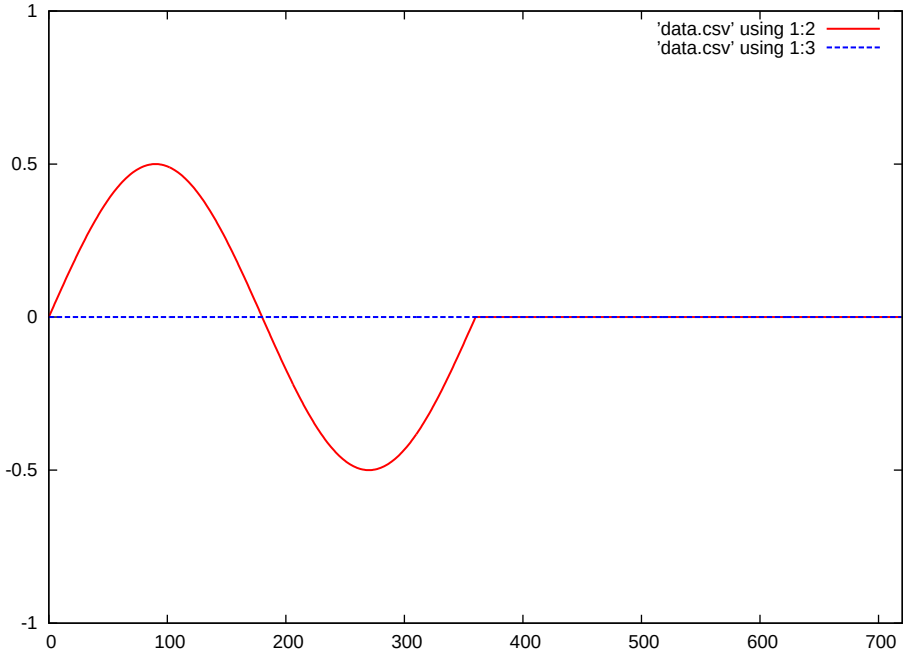

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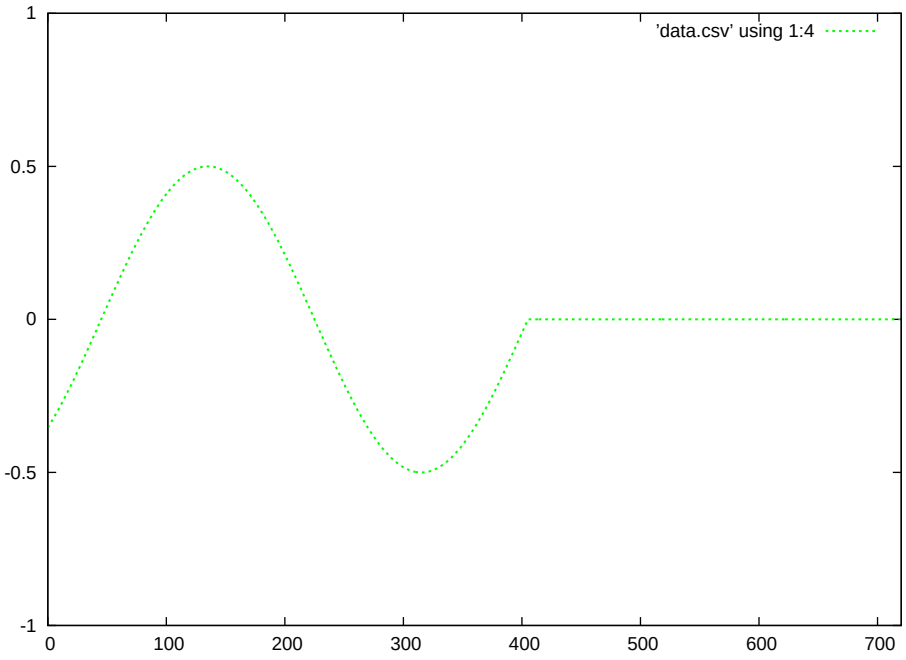

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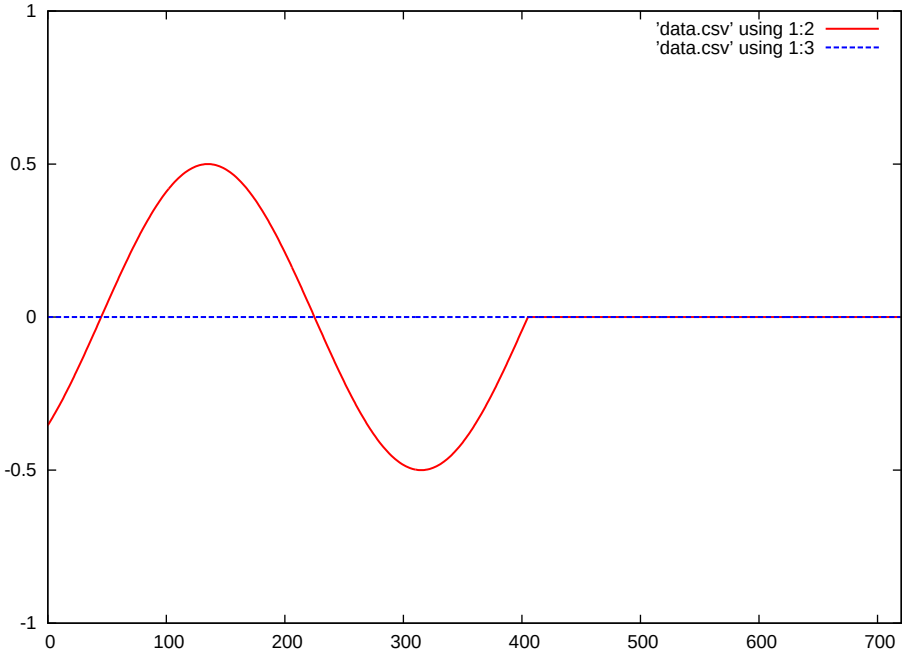

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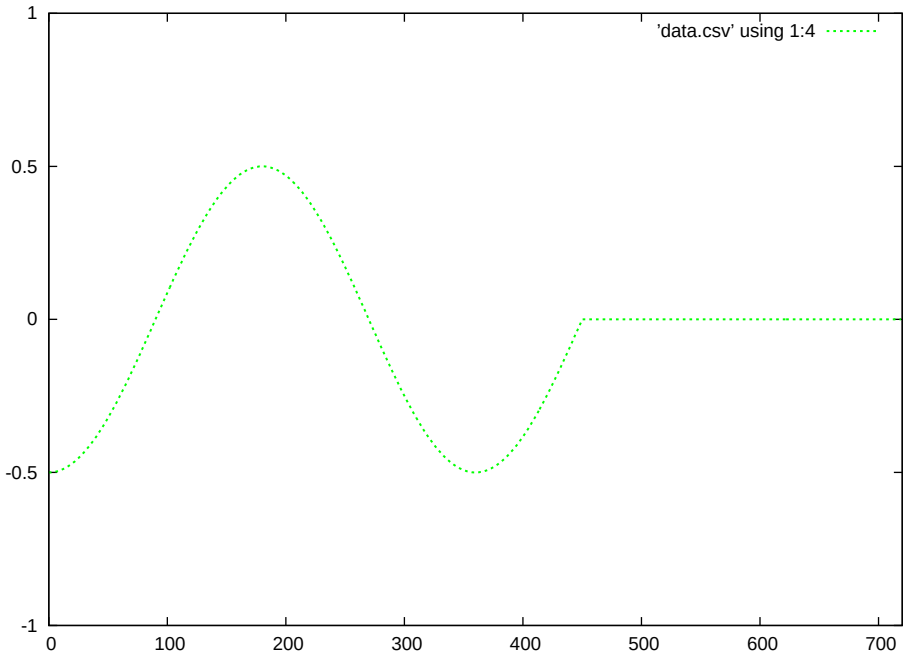

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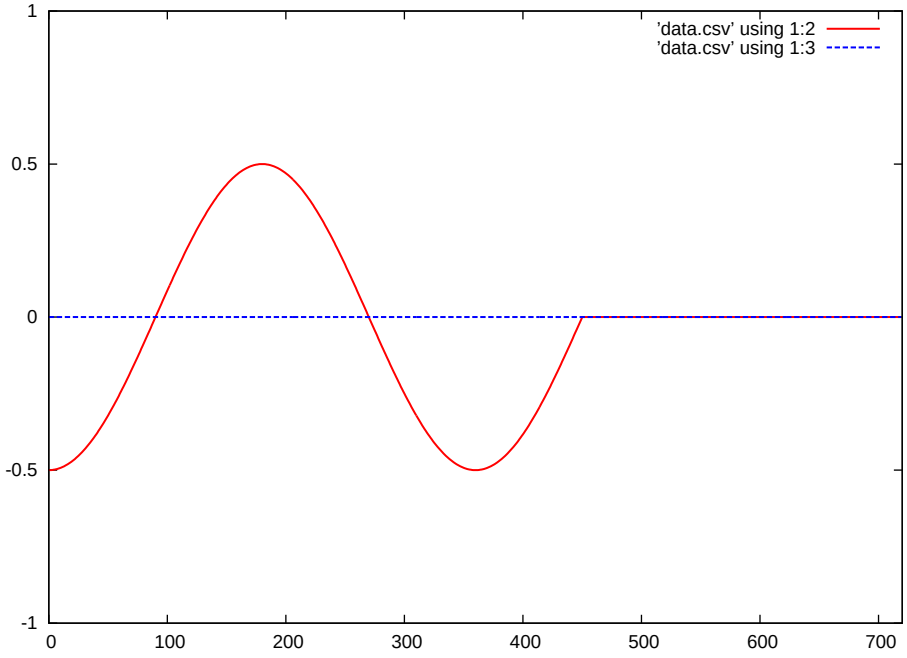

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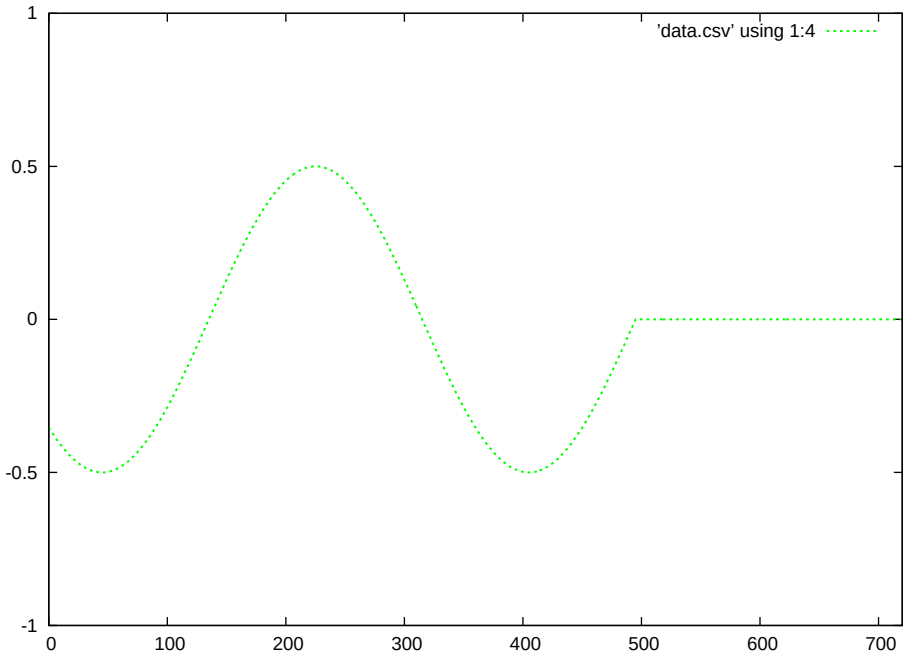

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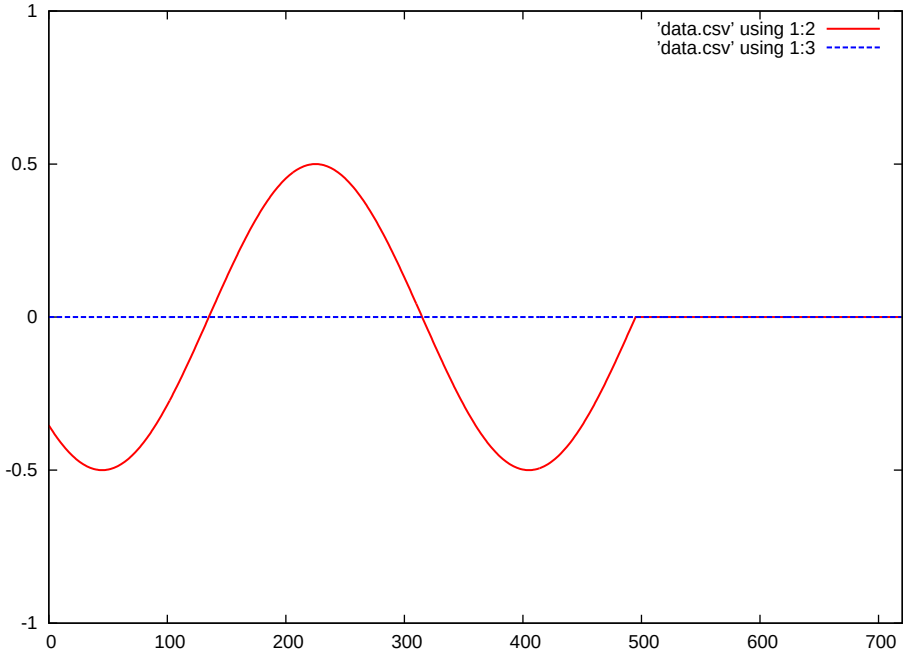

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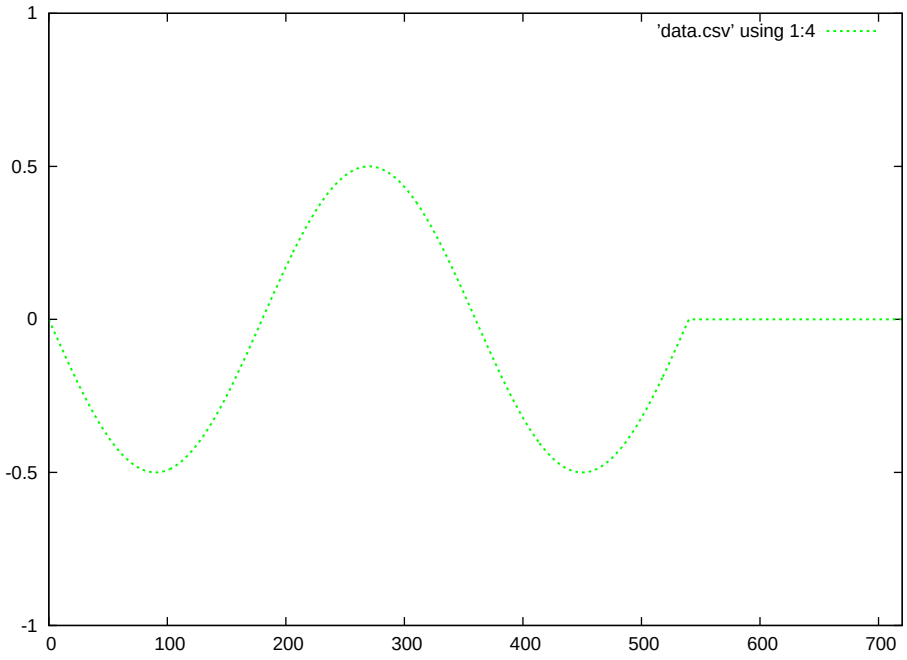

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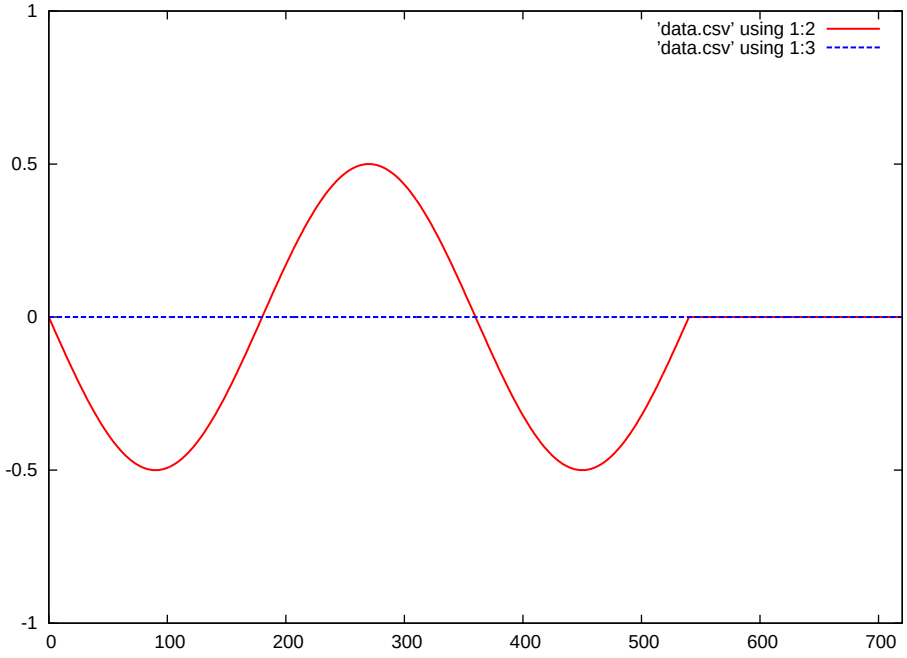

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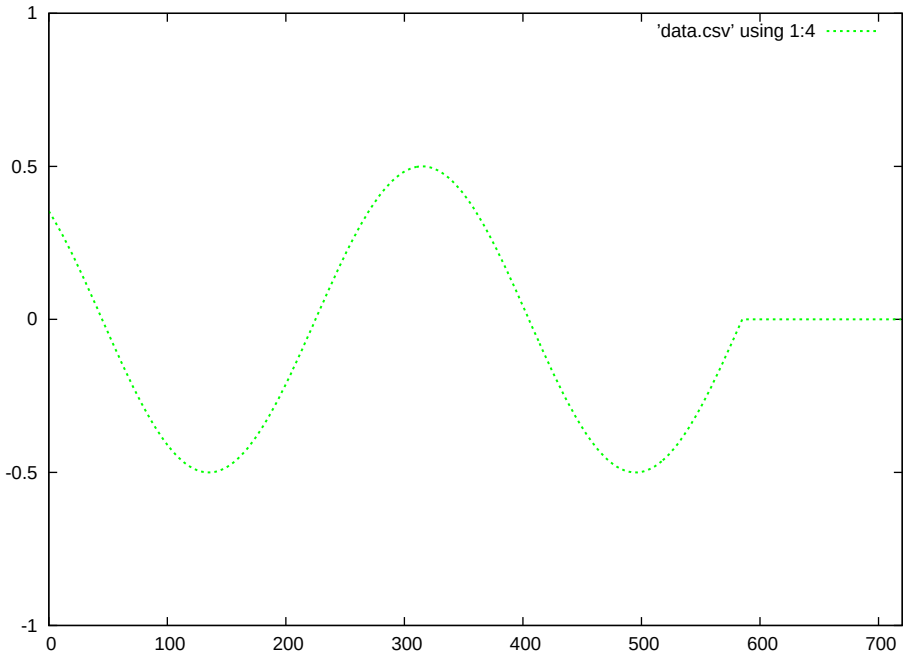

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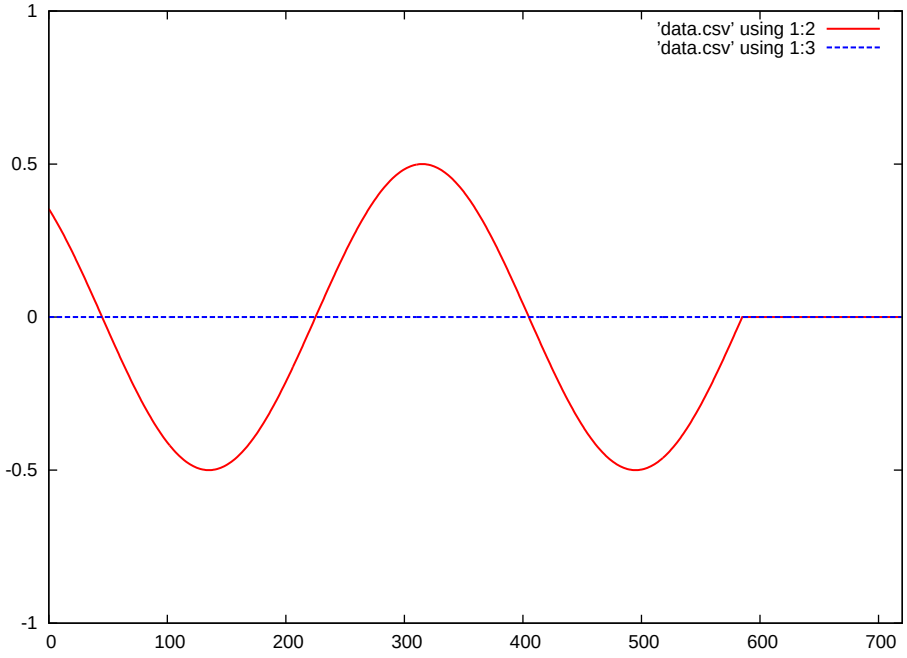

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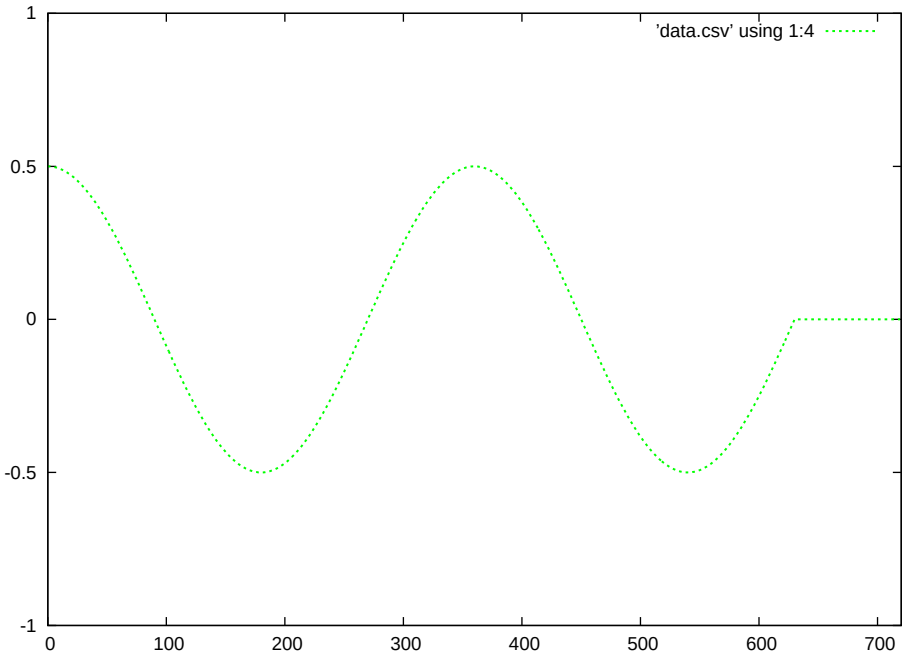

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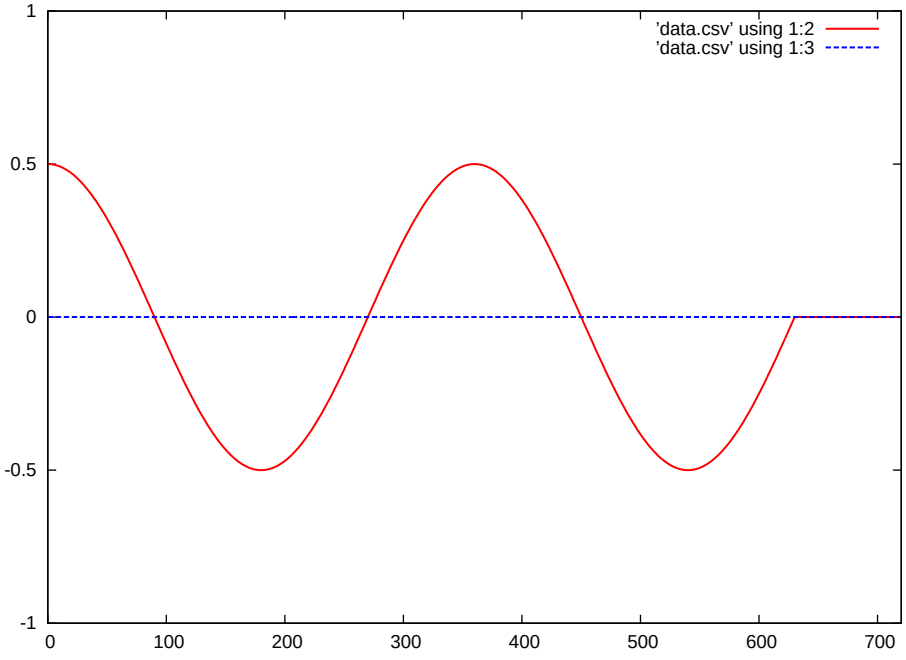

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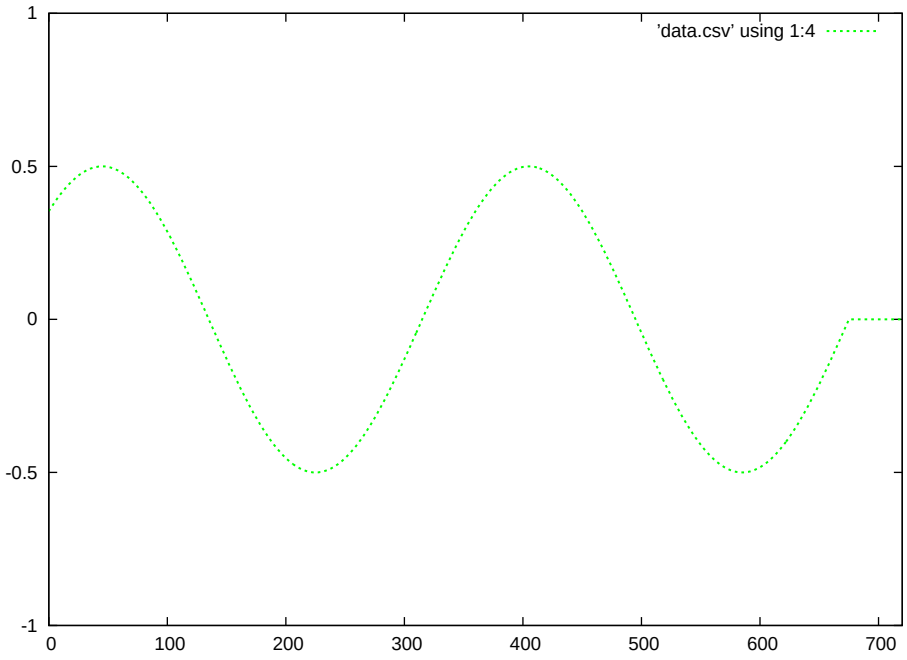

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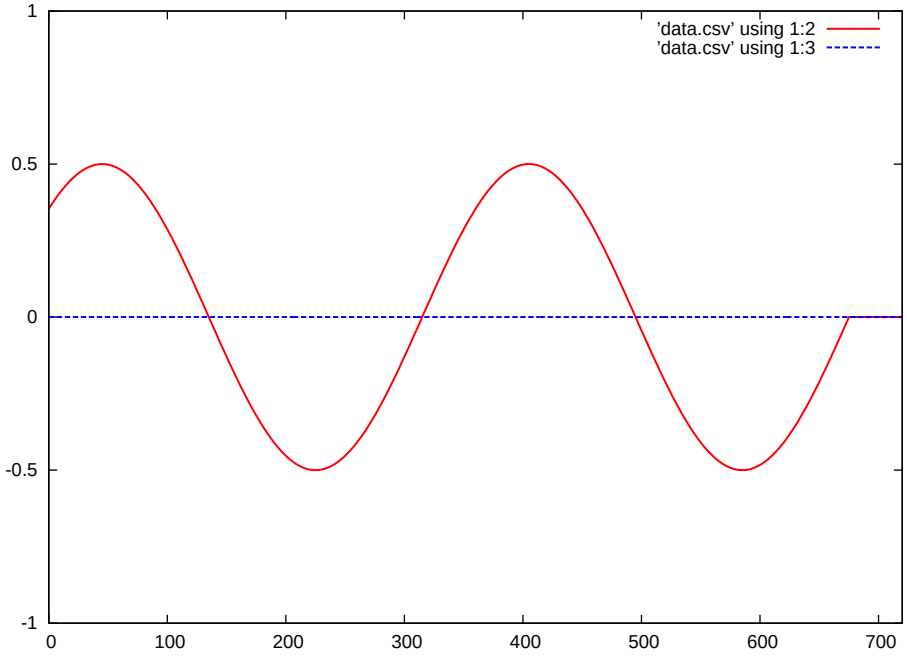

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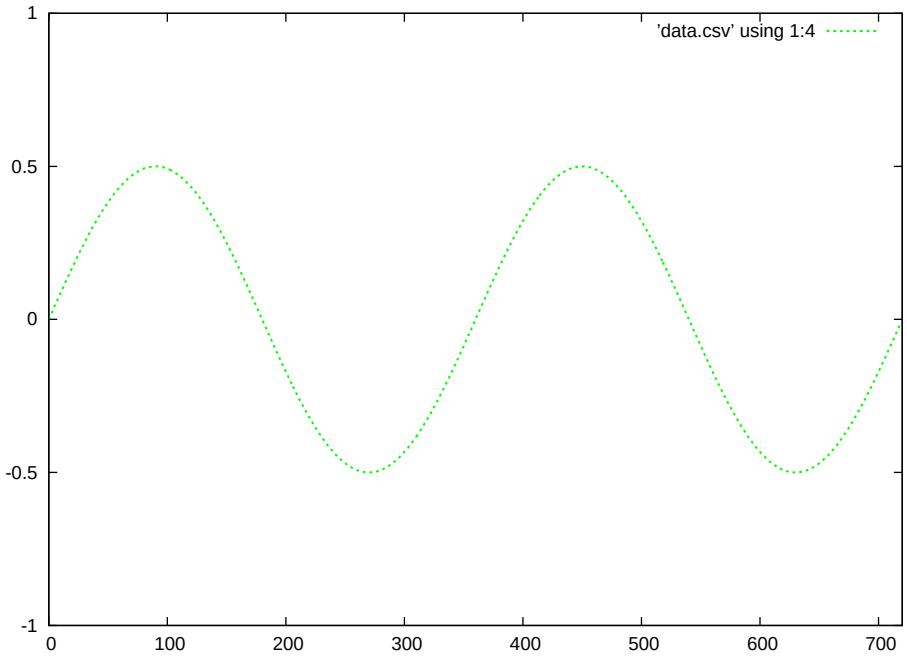

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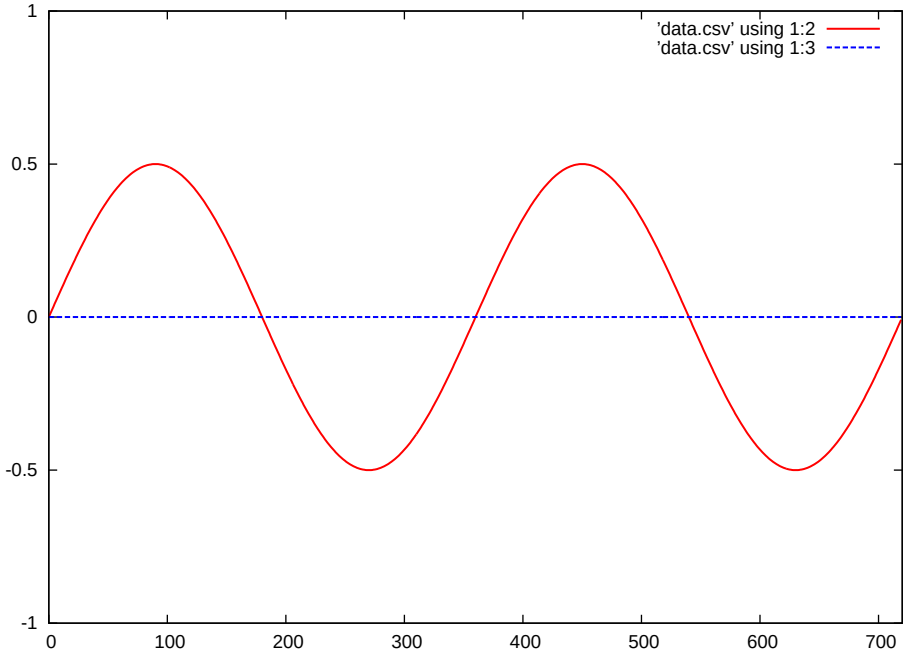

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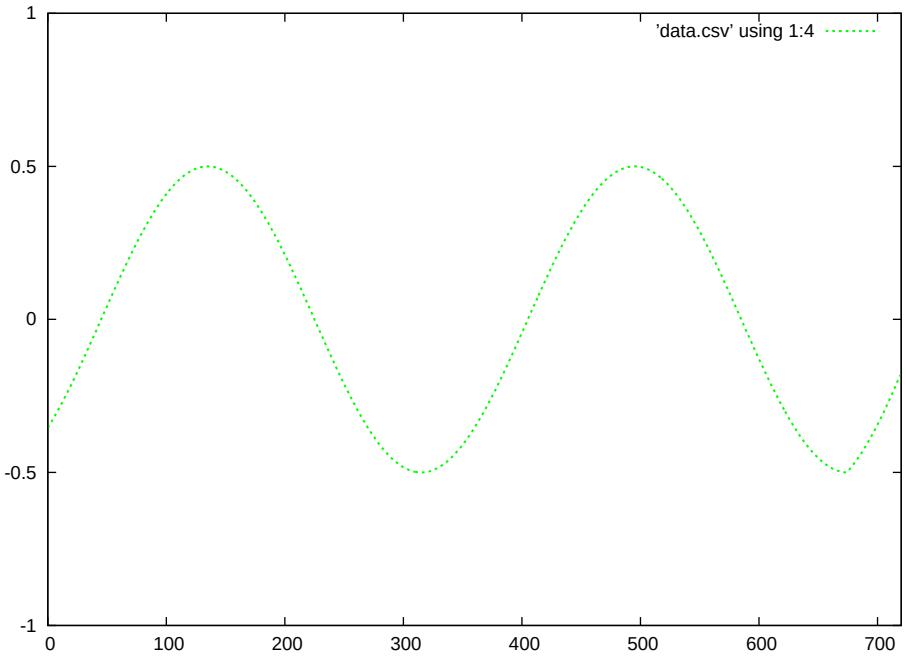

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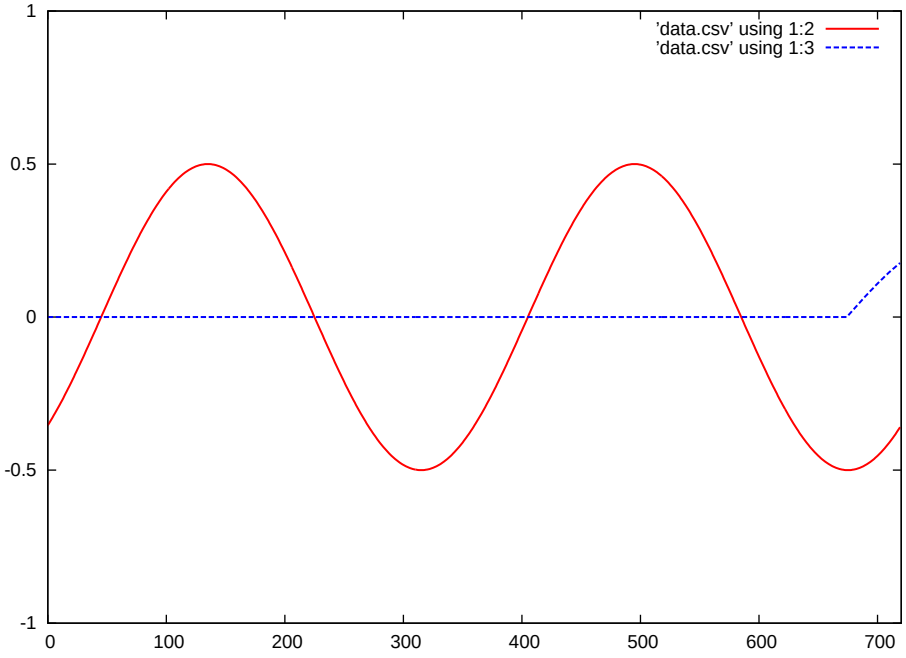

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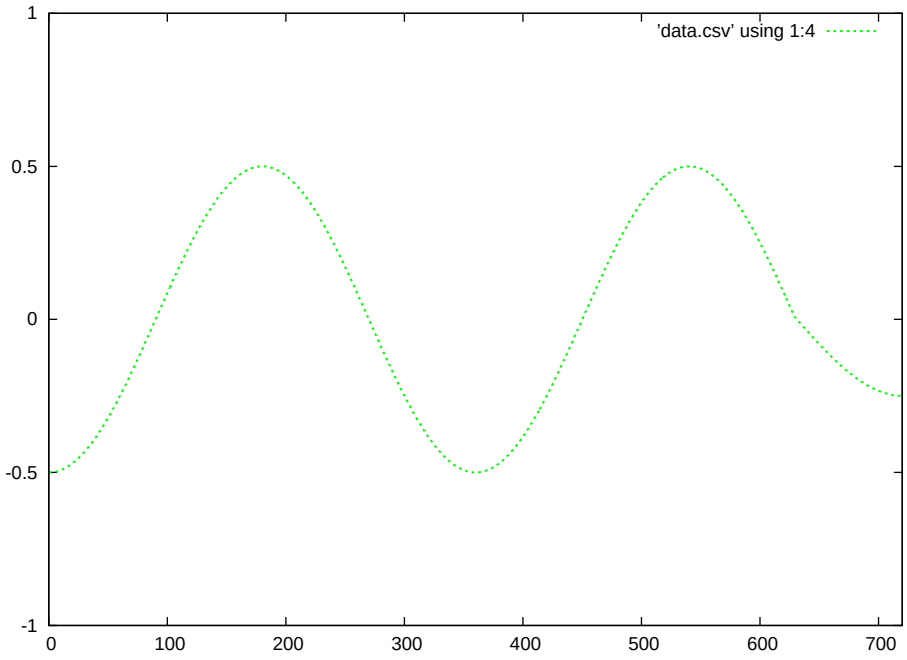

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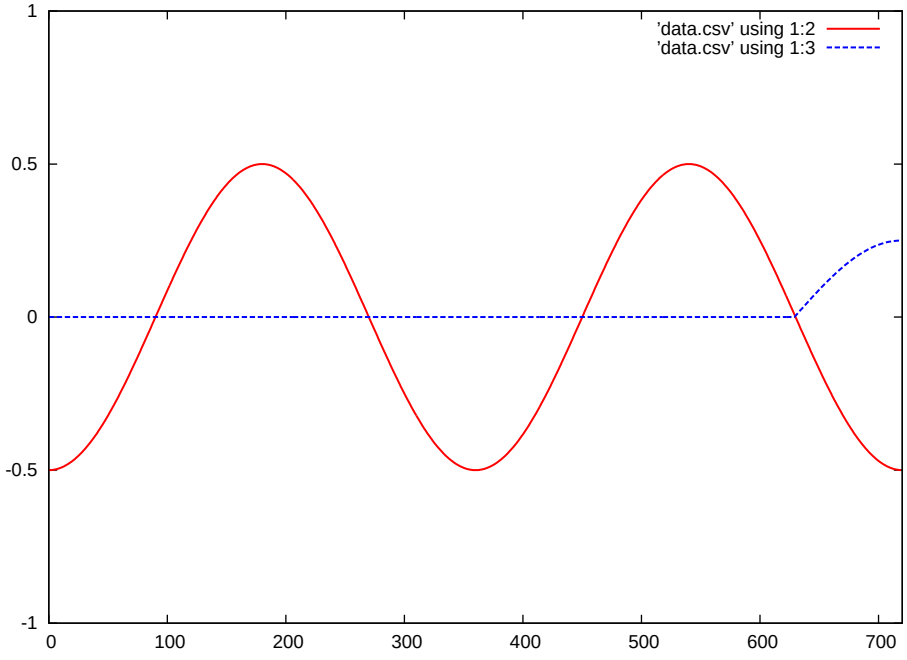

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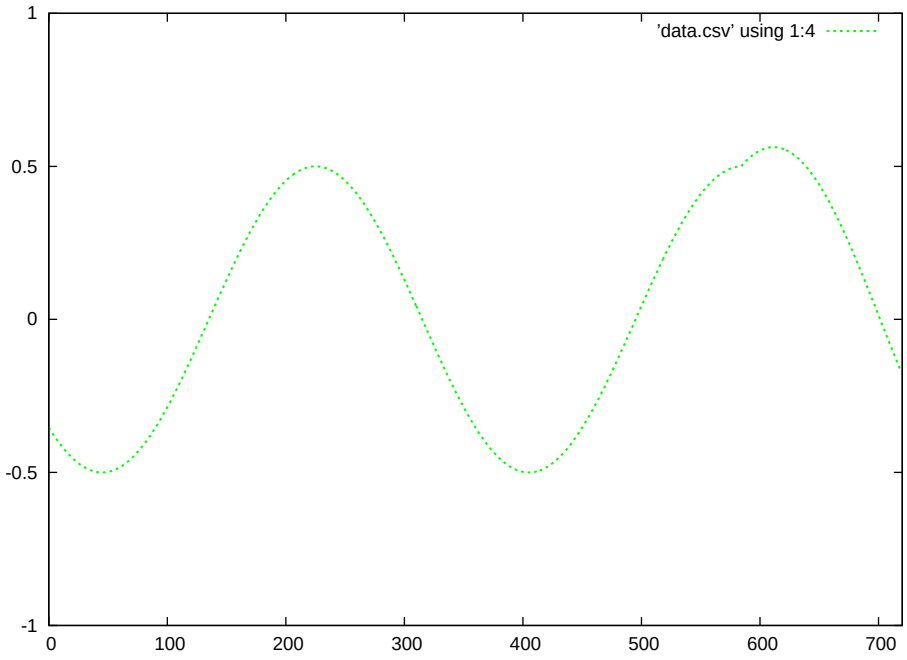

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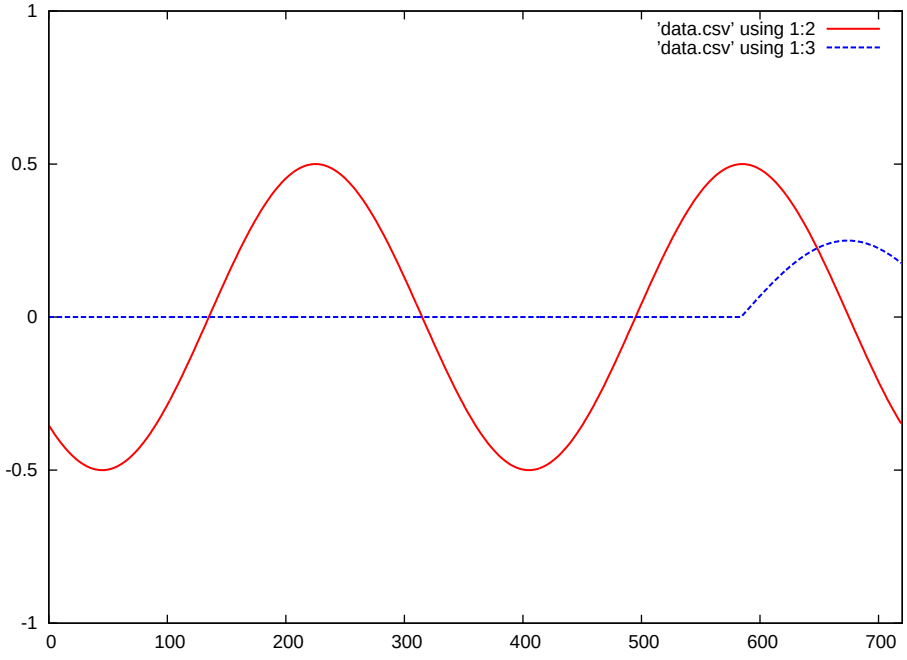

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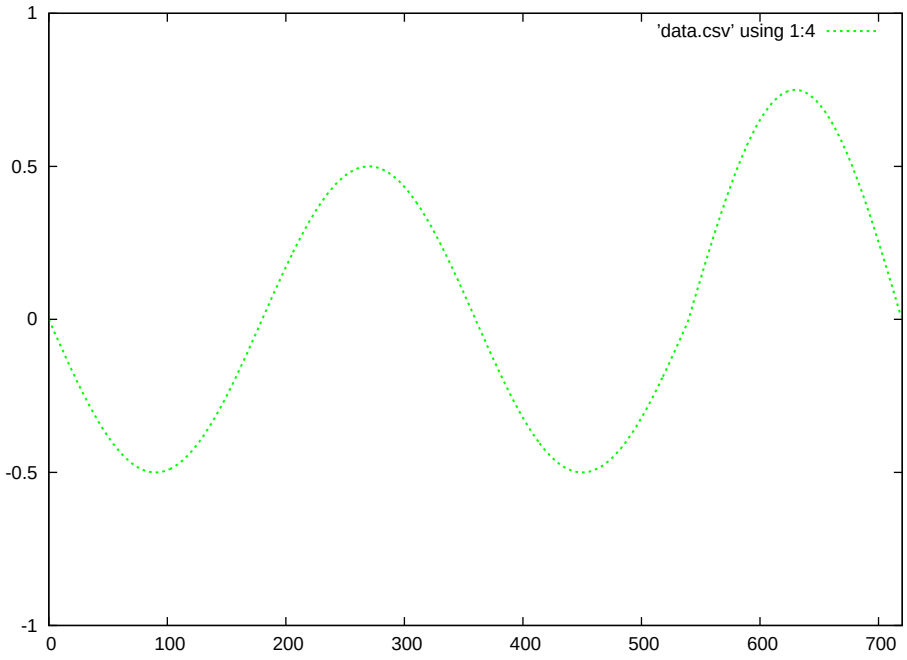
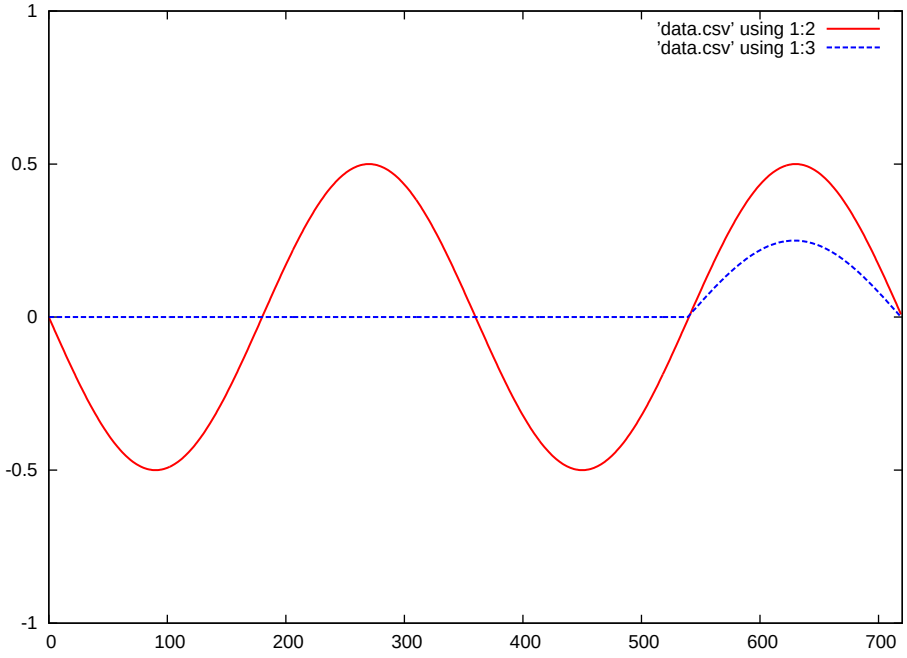
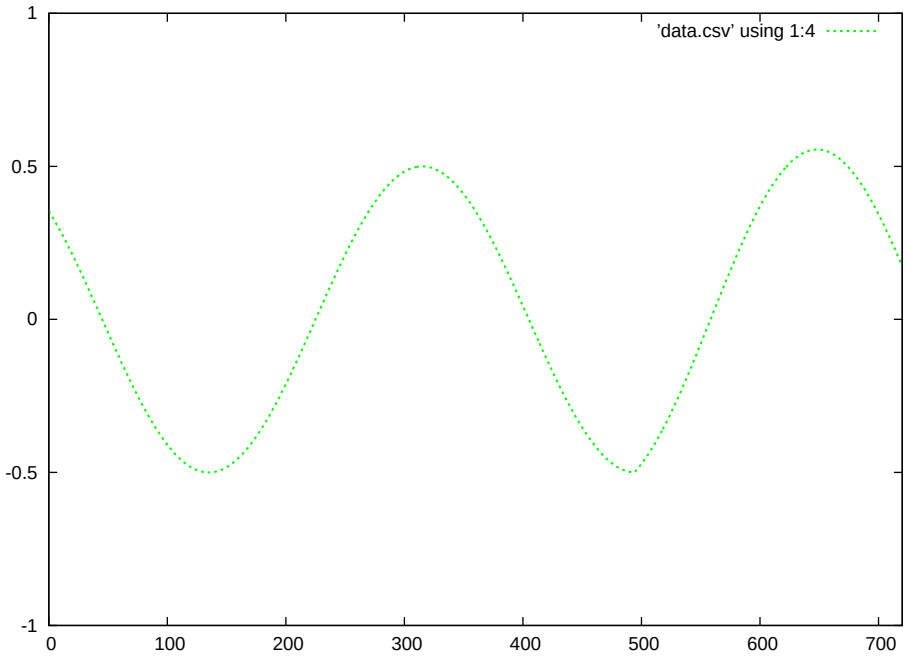


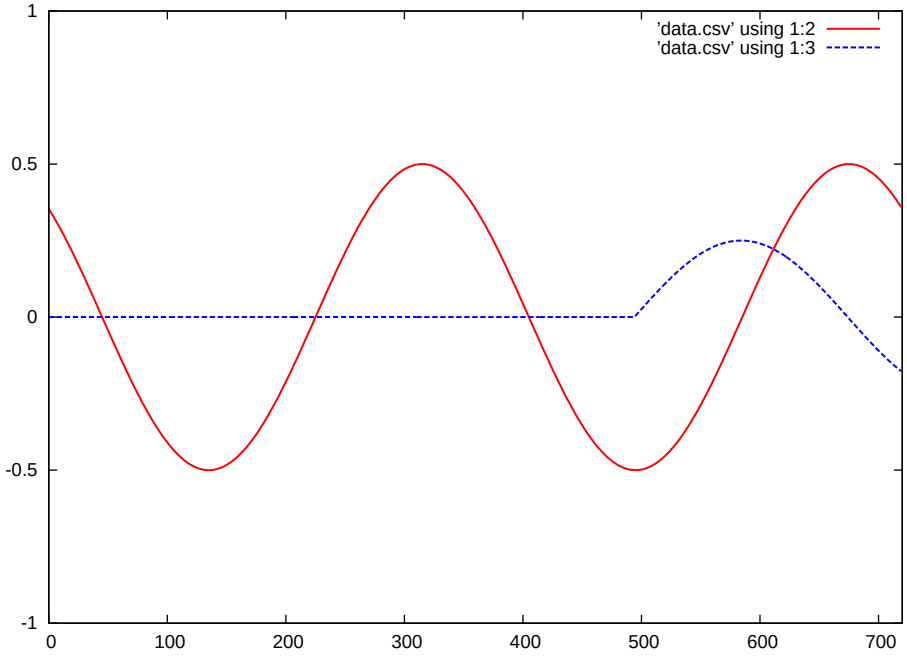
Red = forward wave Blue = reflected wave



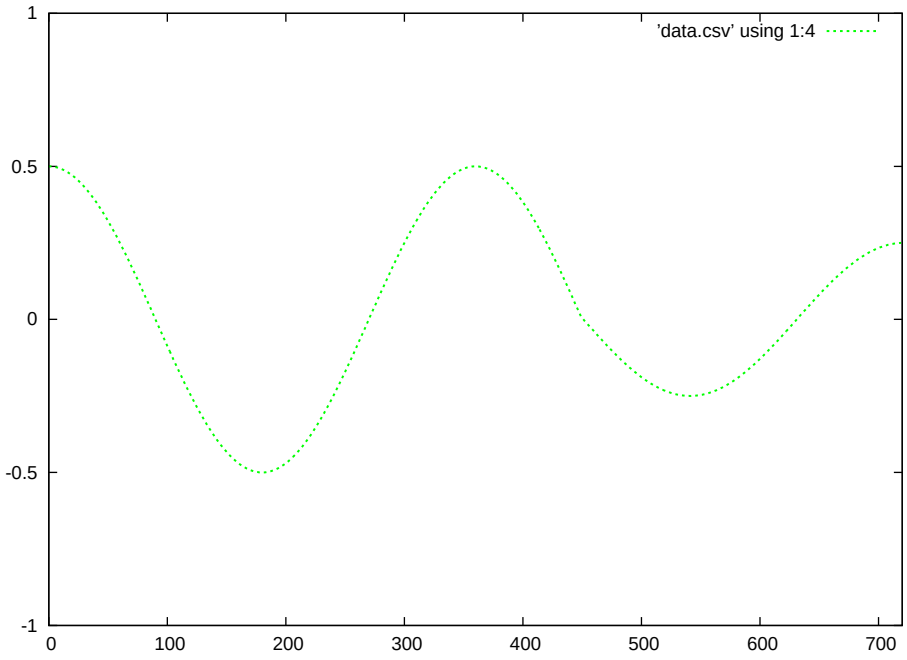
Green = superposition of the forward and reflected waves



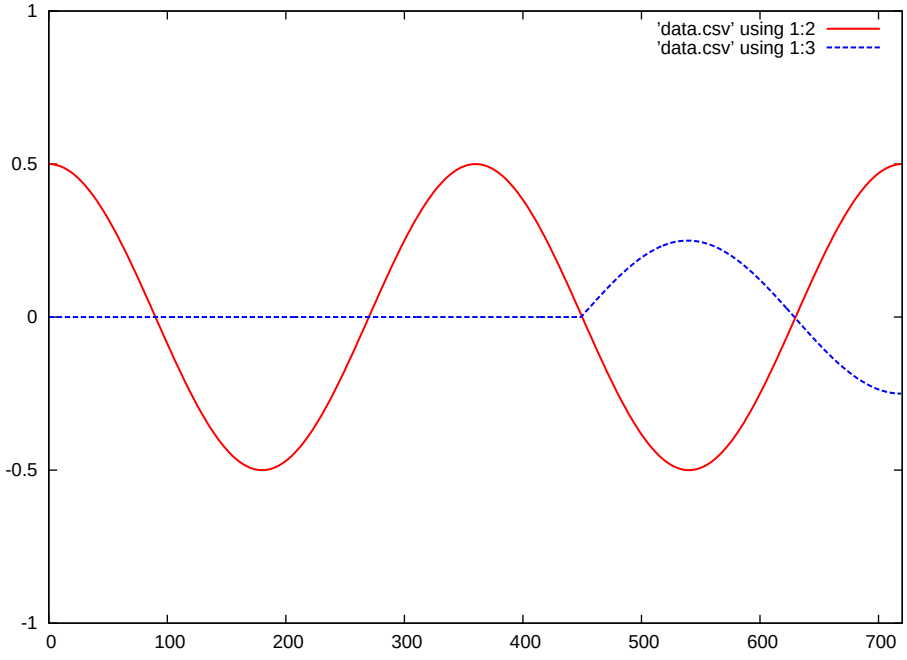
Red = forward wave Blue = reflected wave



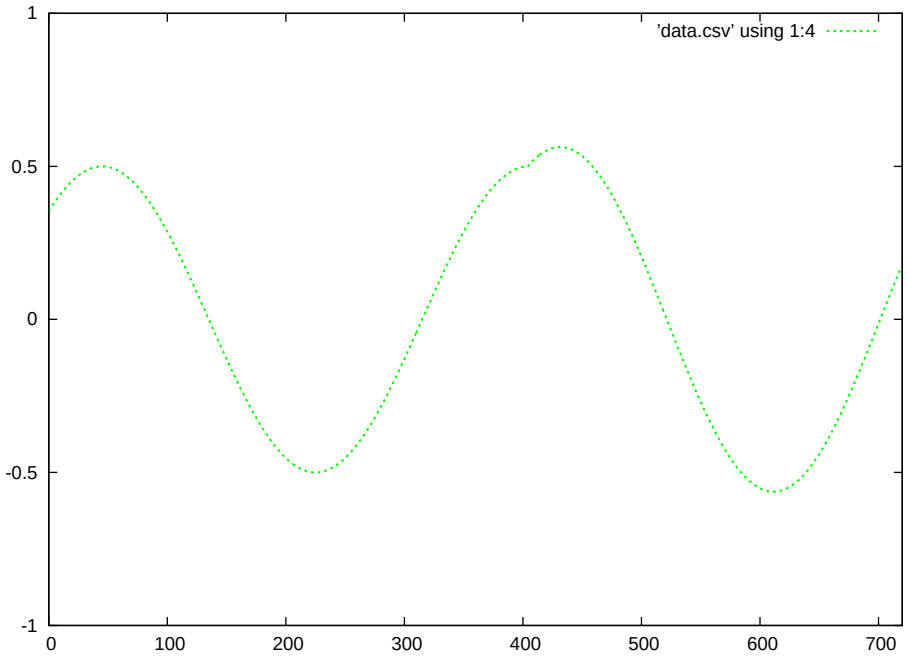
Green = superposition of the forward and reflected waves



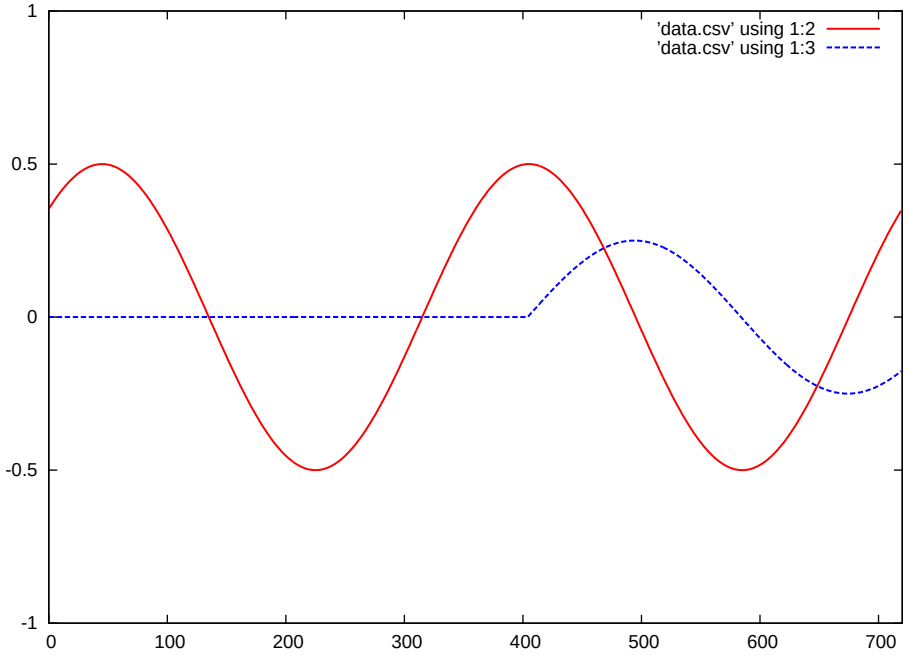
Red = forward wave Blue = reflected wave



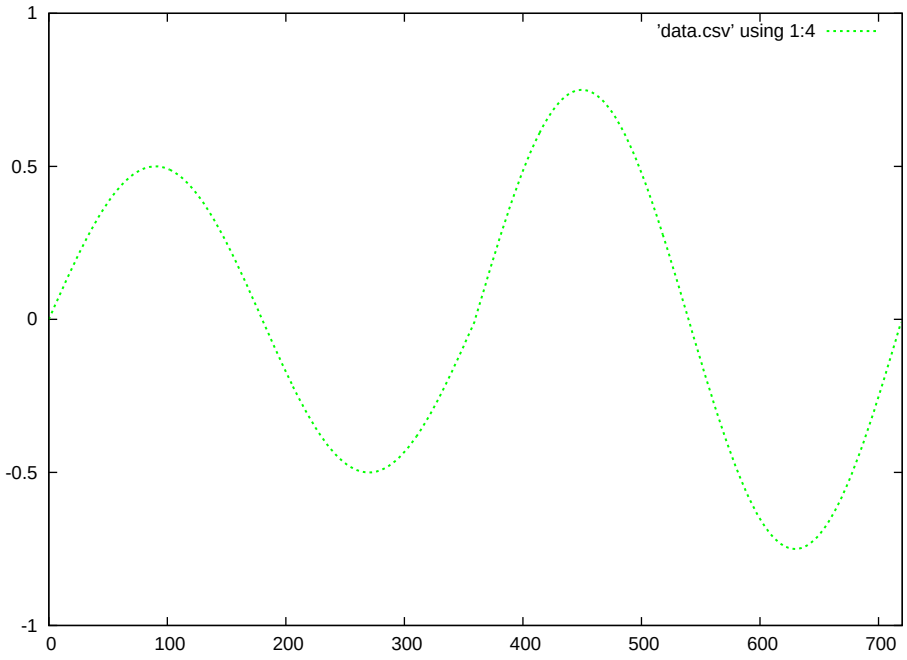
Green = superposition of the forward and reflected waves



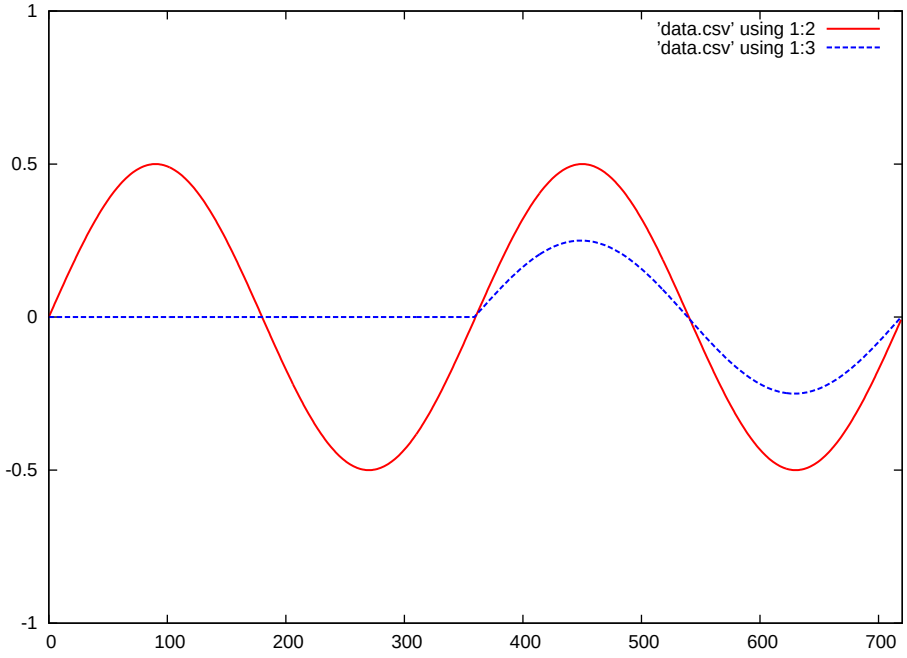
Red = forward wave Blue = reflected wave



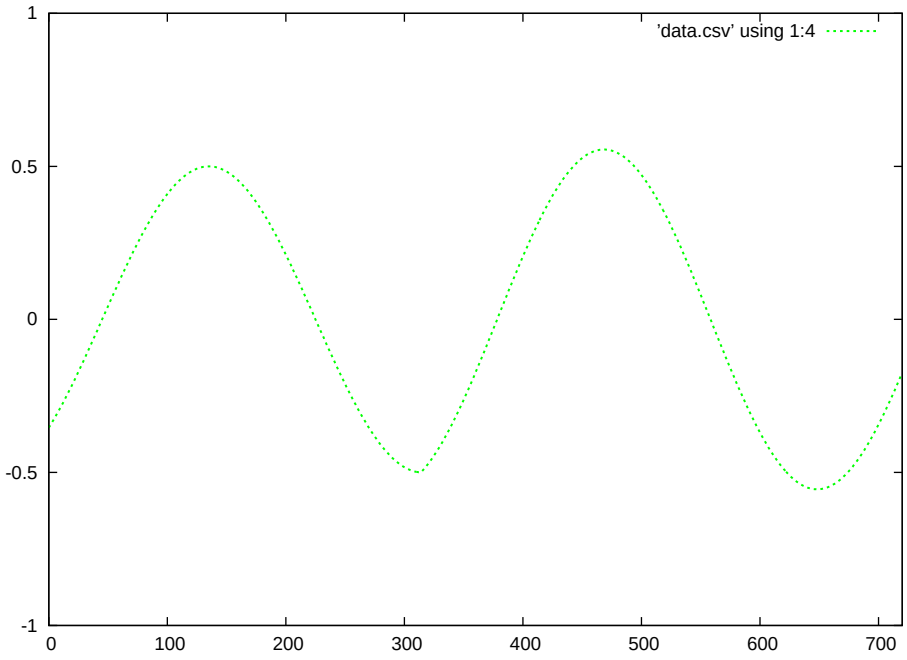
Green = superposition of the forward and reflected waves



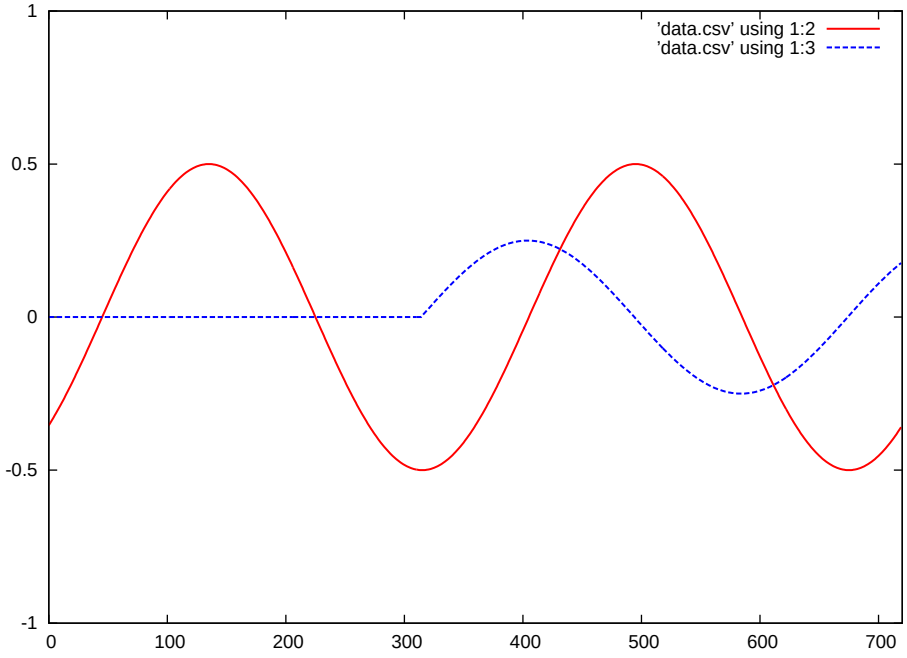
Red = forward wave Blue = reflected wave



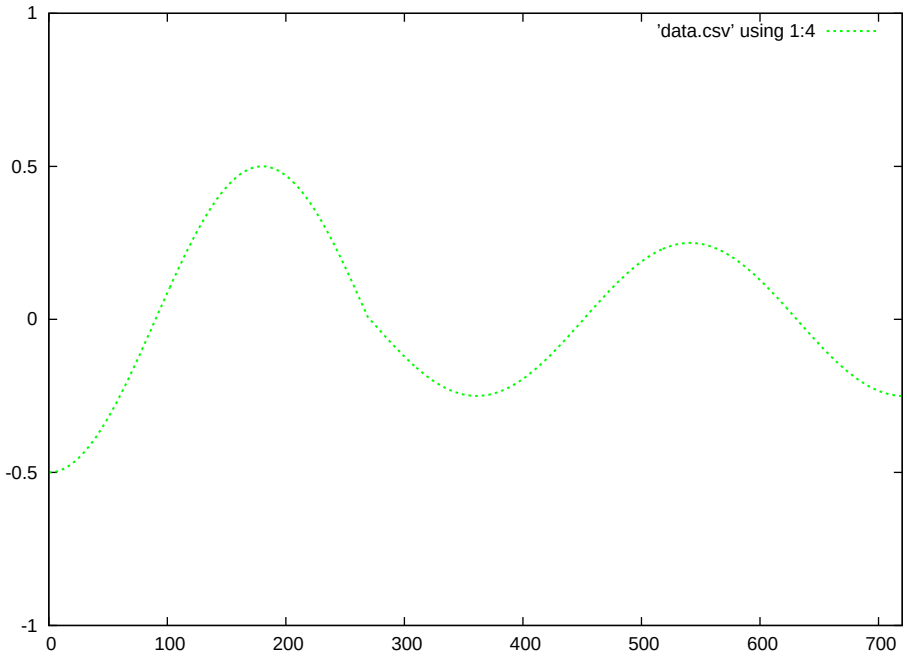
Green = superposition of the forward and reflected waves



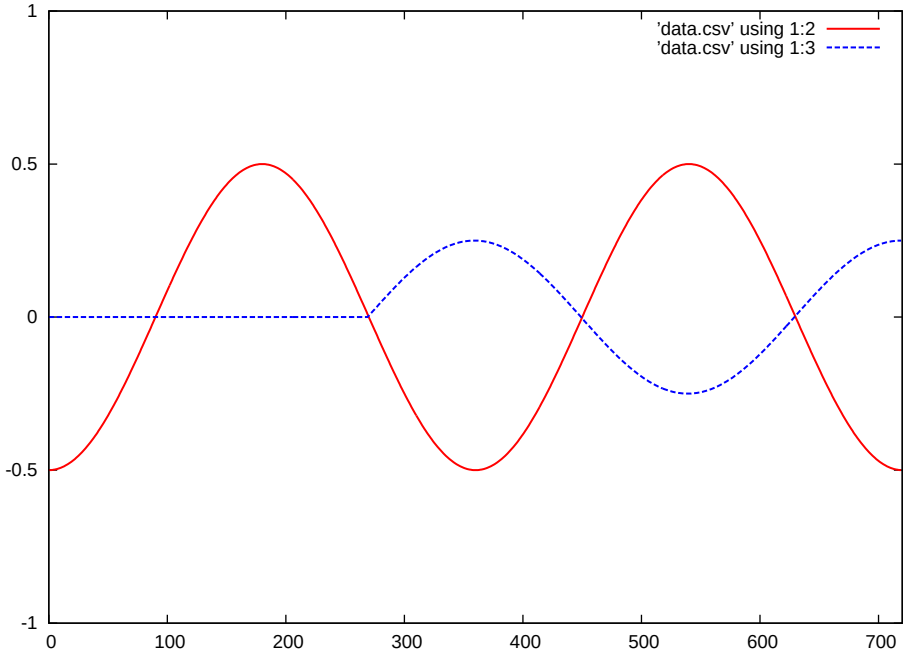
Red = forward wave Blue = reflected wave



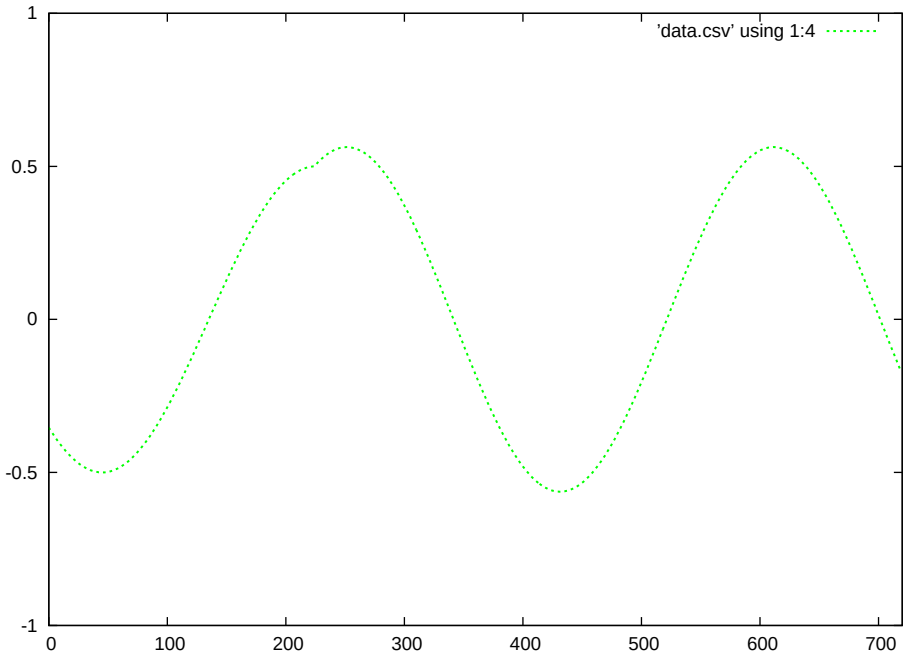
Green = superposition of the forward and reflected waves



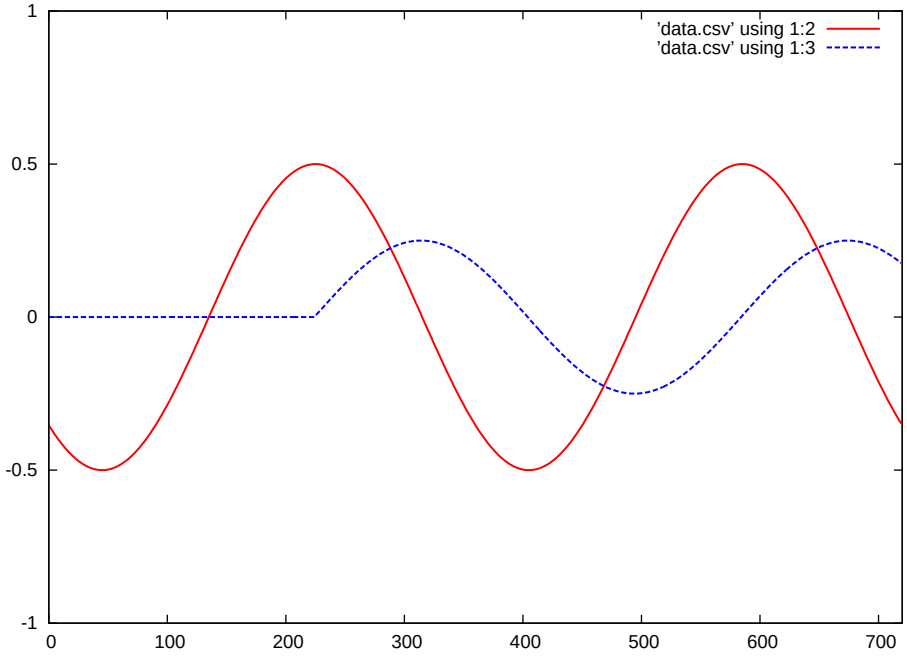
Red = forward wave Blue = reflected wave



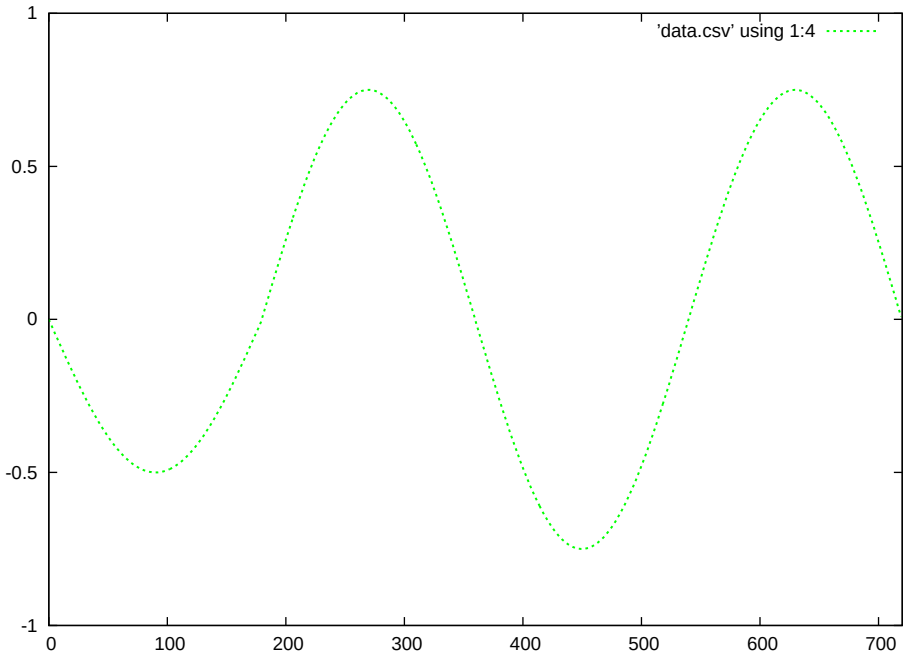
Green = superposition of the forward and reflected waves



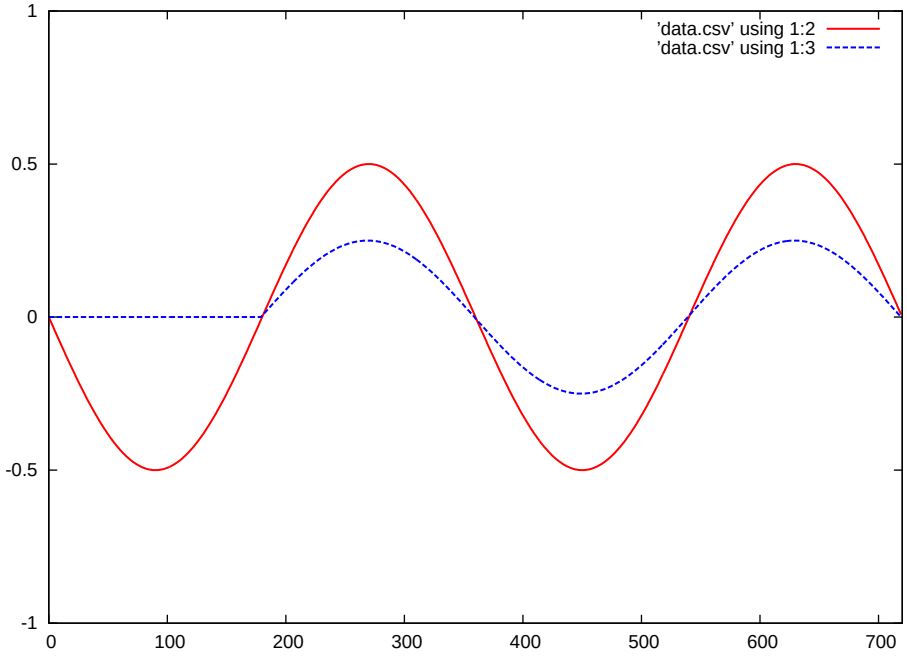
Red = forward wave Blue = reflected wave



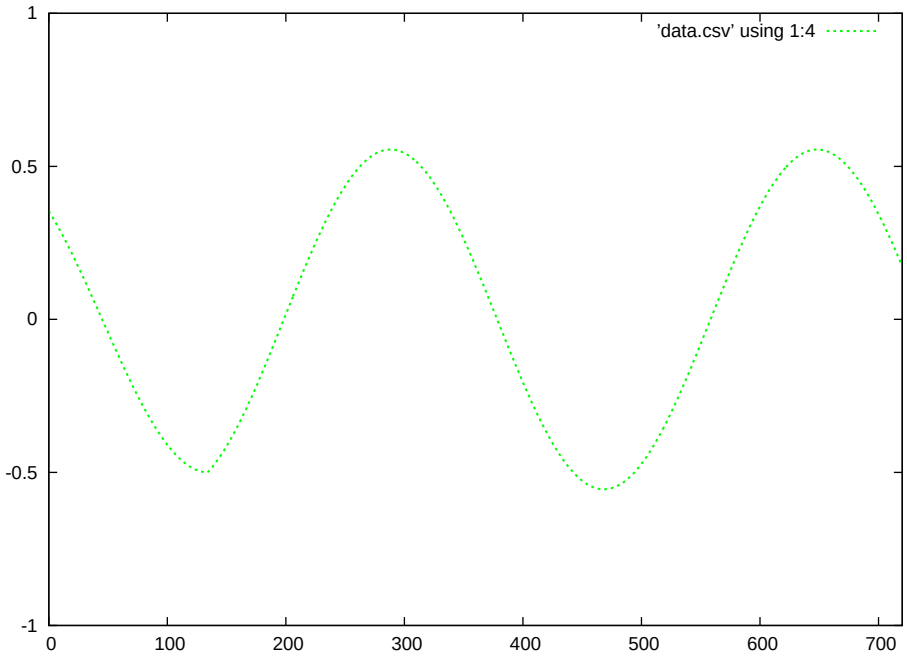
Green = superposition of the forward and reflected waves



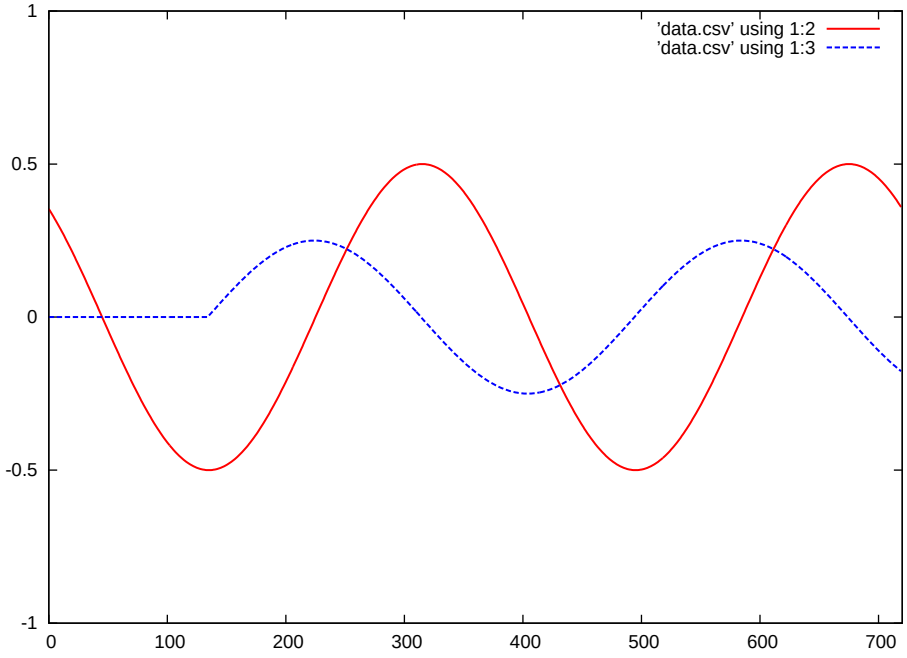
Red = forward wave Blue = reflected wave



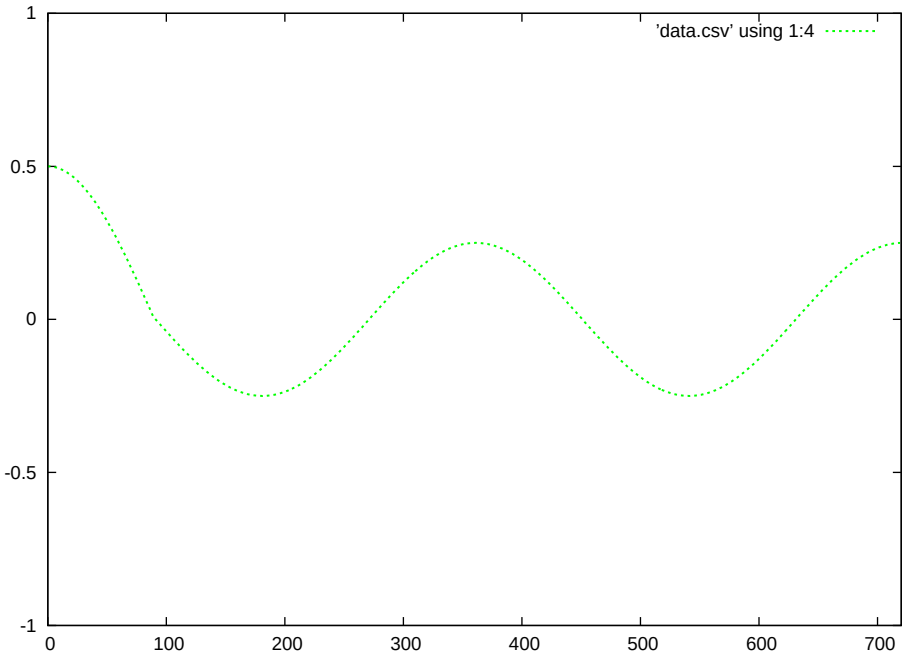
Green = superposition of the forward and reflected waves



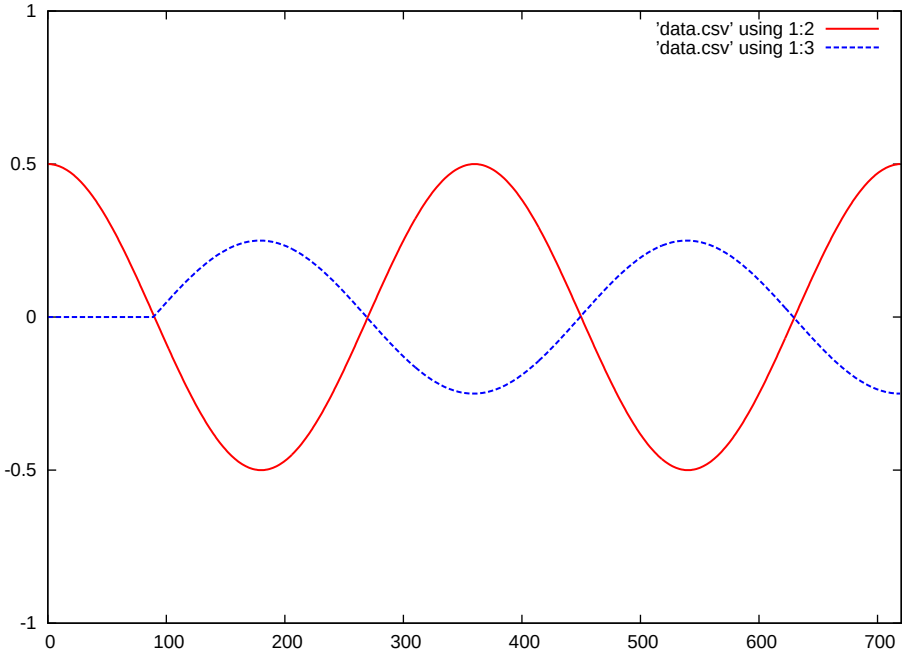
Red = forward wave Blue = reflected wave



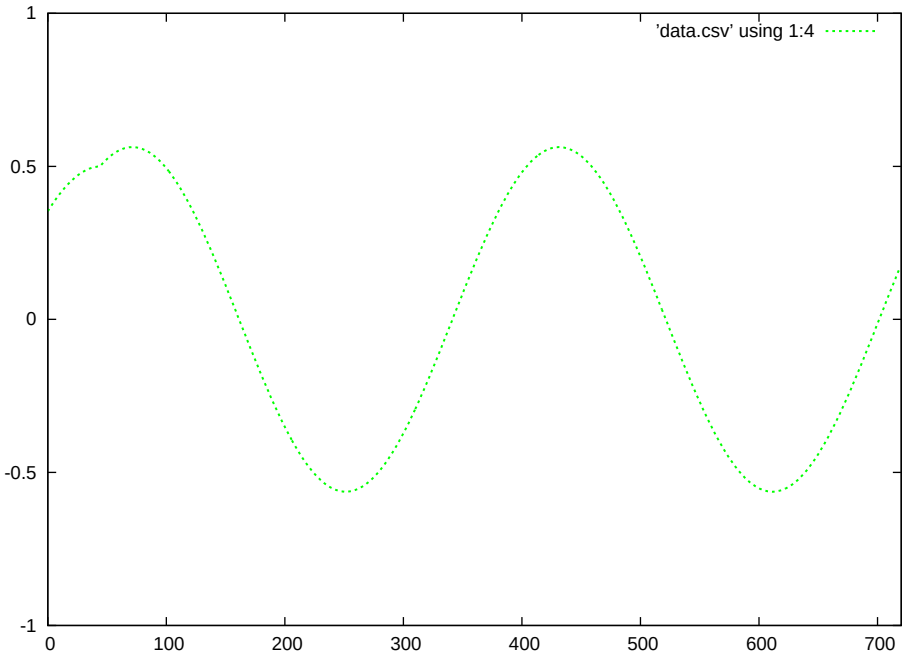
Green = superposition of the forward and reflected waves



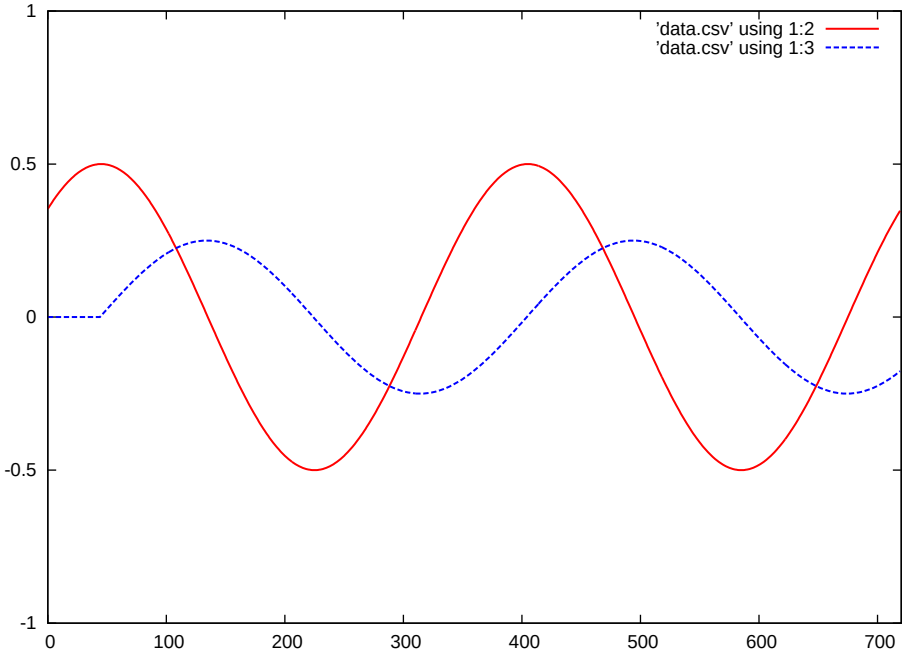
Red = forward wave Blue = reflected wave



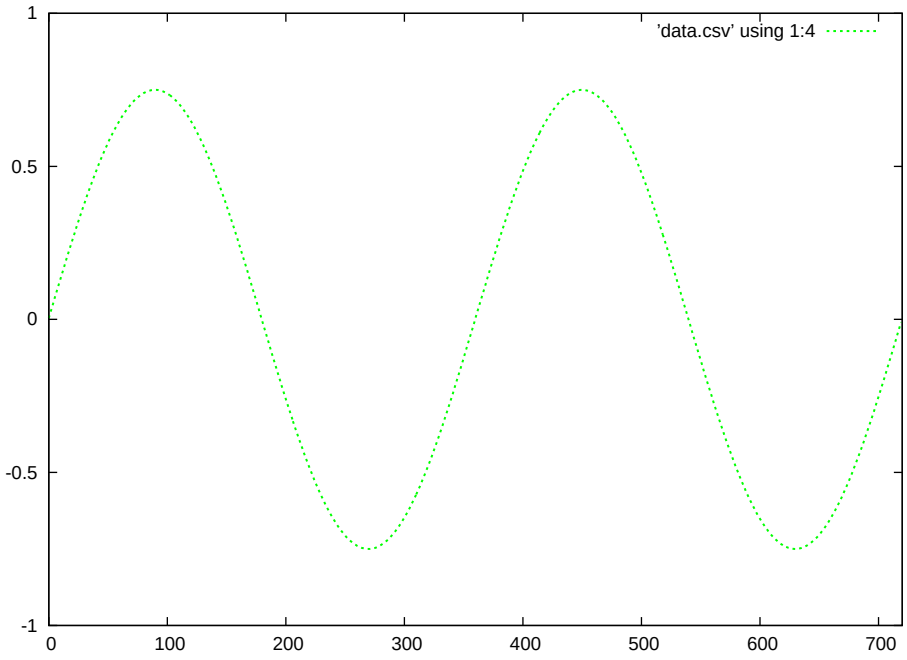
Green = superposition of the forward and reflected waves



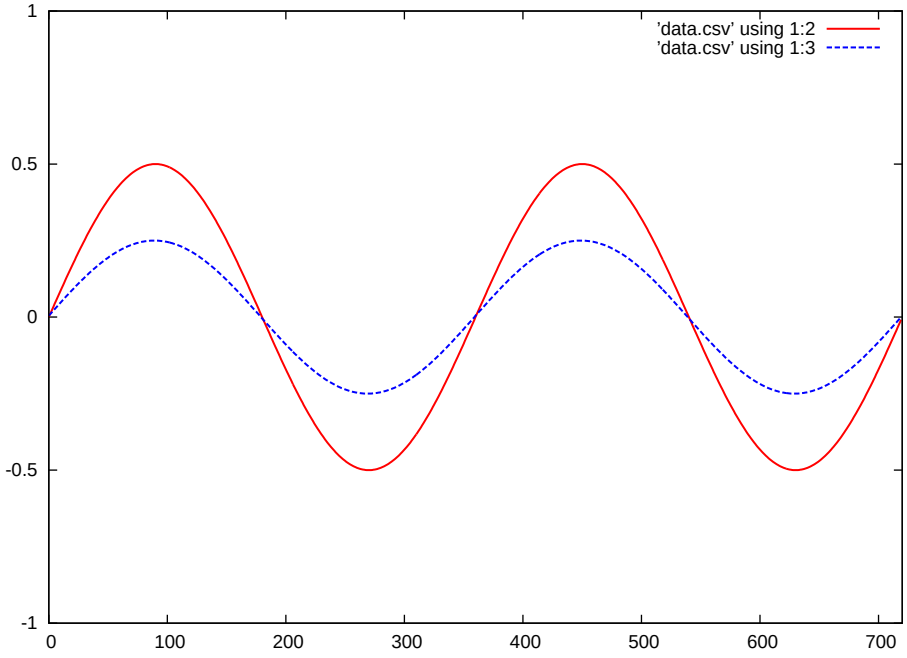
Red = forward wave Blue = reflected wave



Green = superposition of the forward and reflected waves

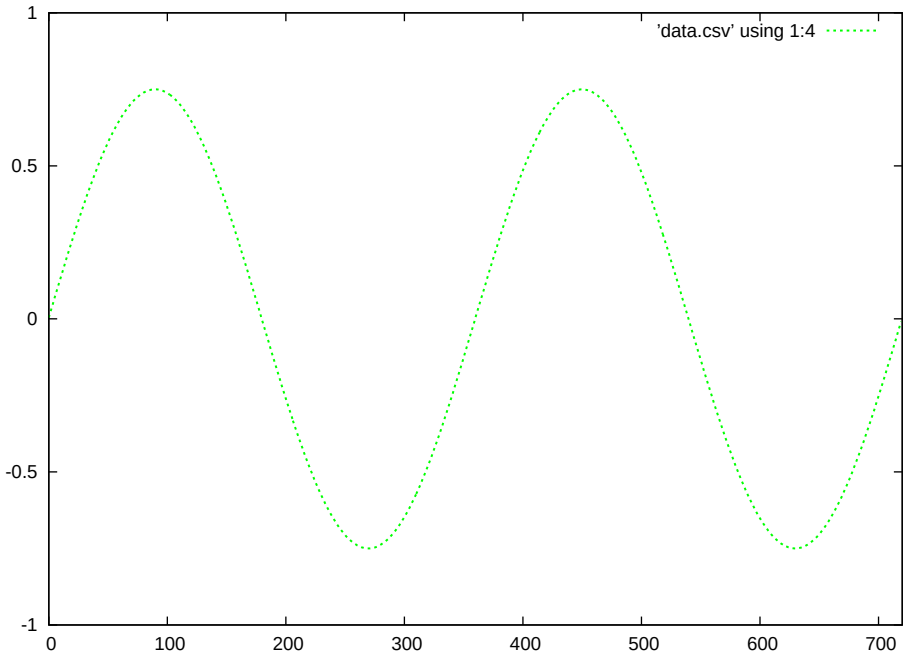


Red = forward wave Blue = reflected wave

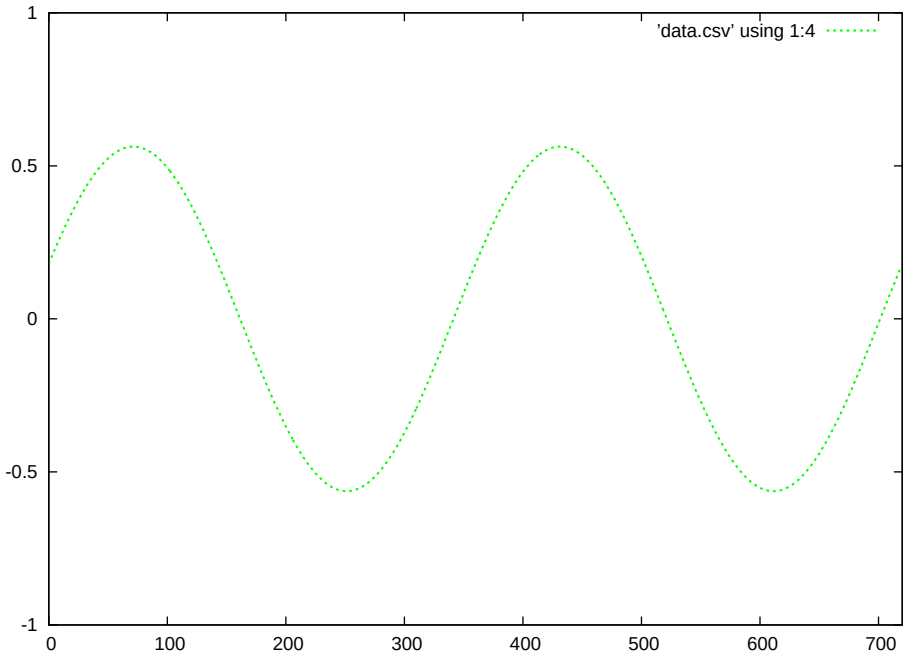


After the reflected wave has traveled all the way back to the source (left-hand end of the transmission line), the superposition of those two continuous waves results in a standing wave.

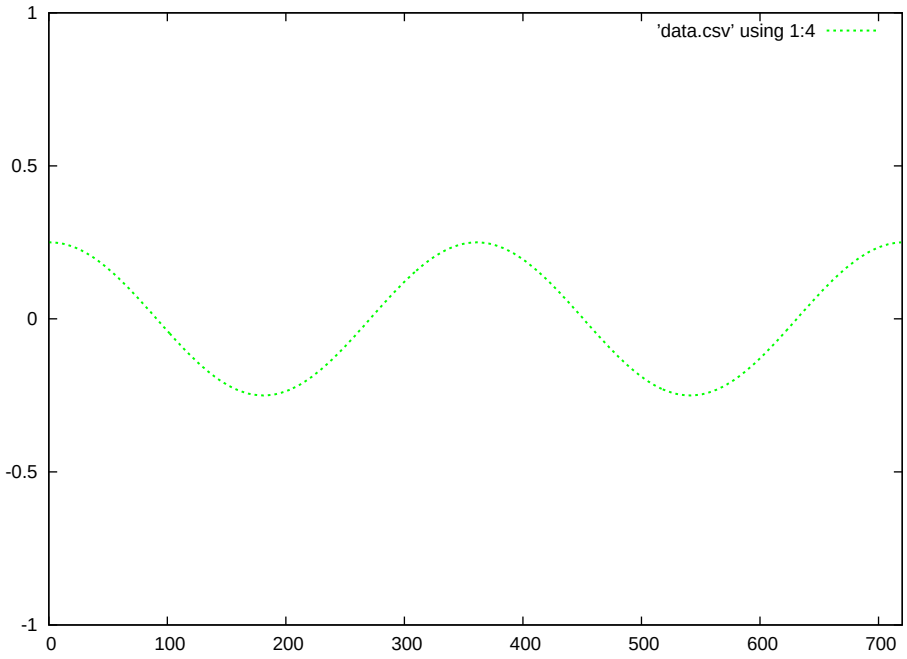
Green = standing wave



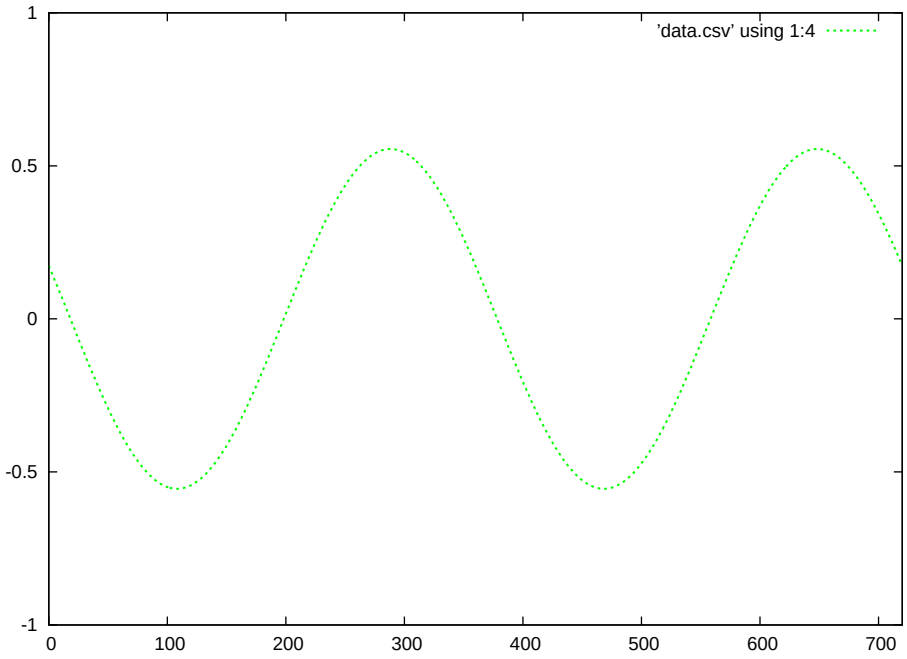
Green = standing wave



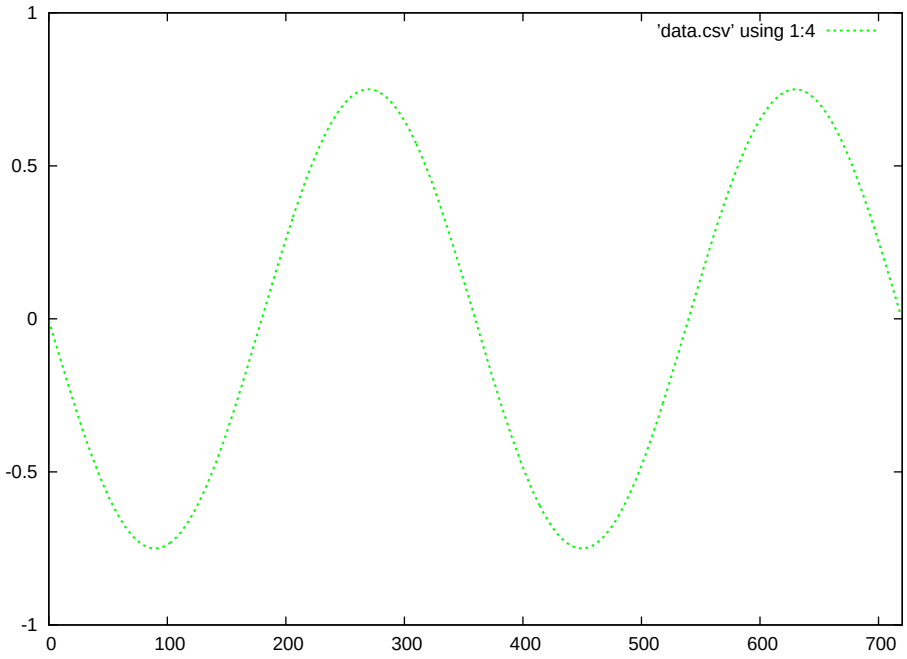
Green = standing wave



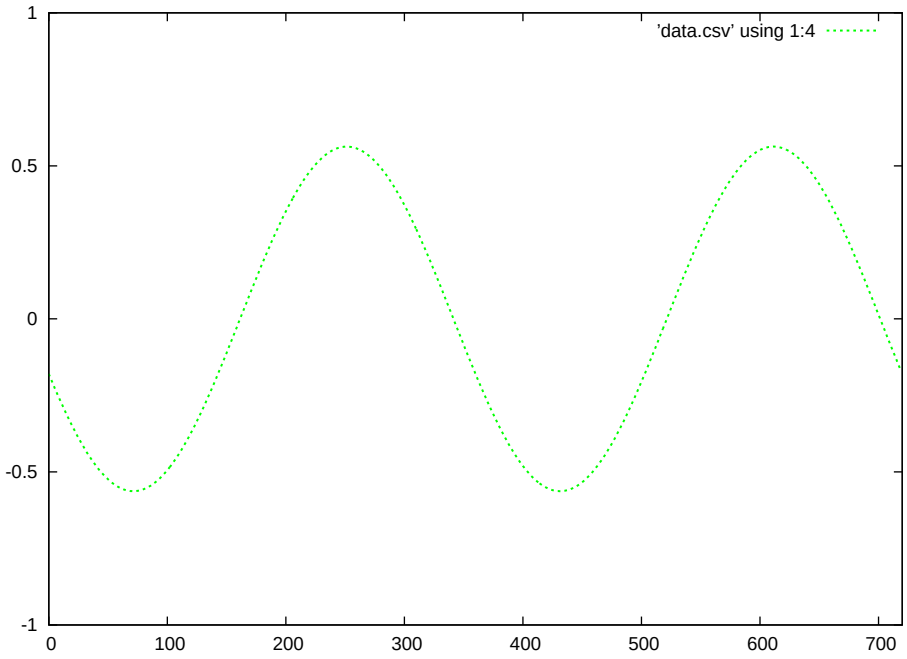
Green = standing wave



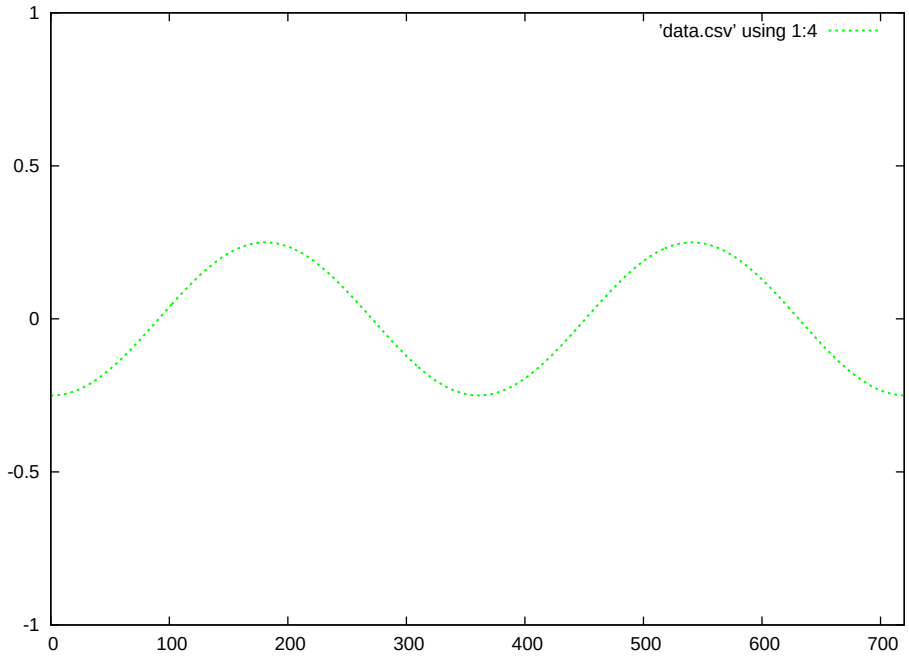
Green = standing wave



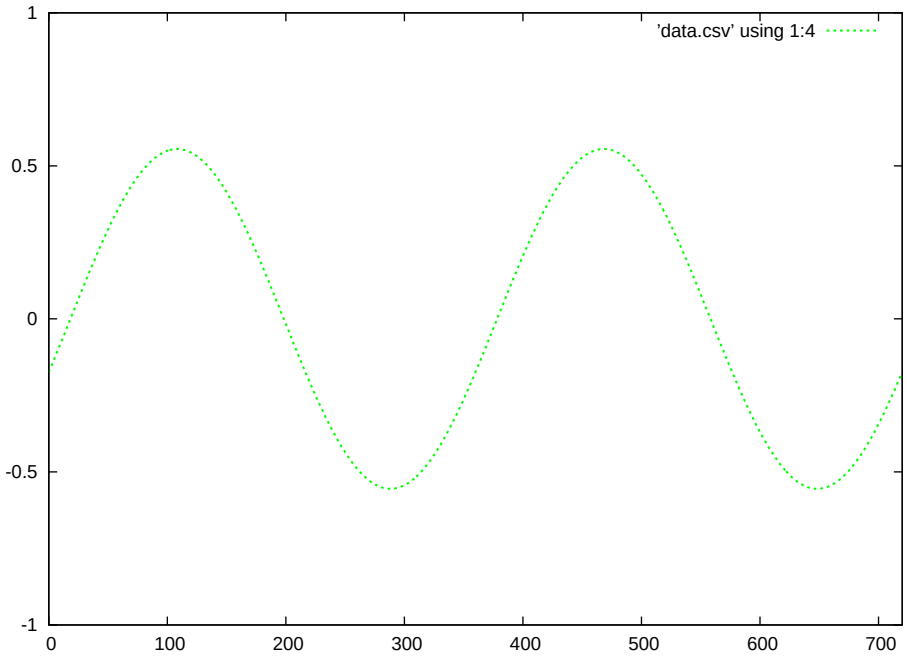
Green = standing wave



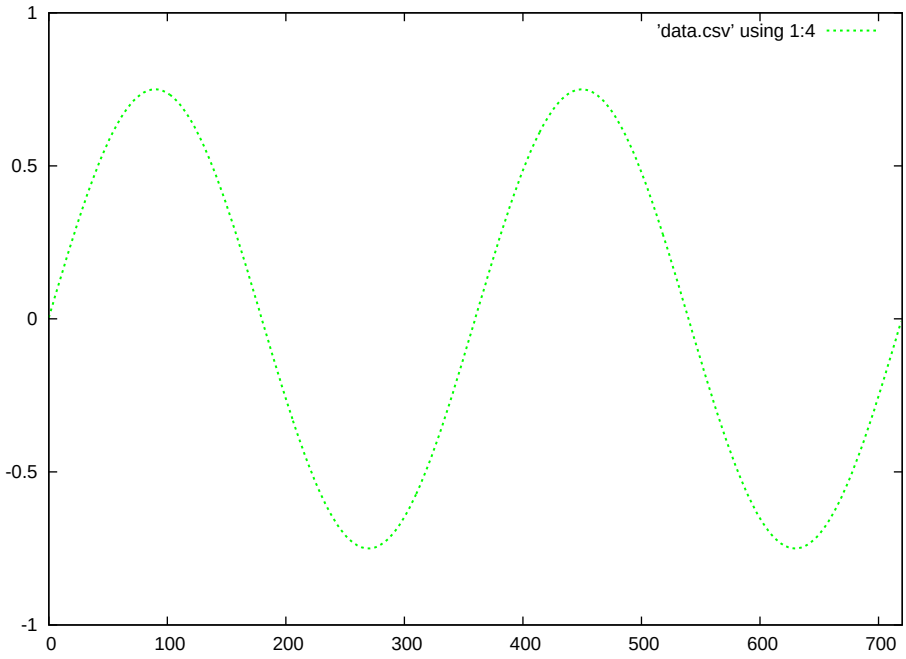
Green = standing wave



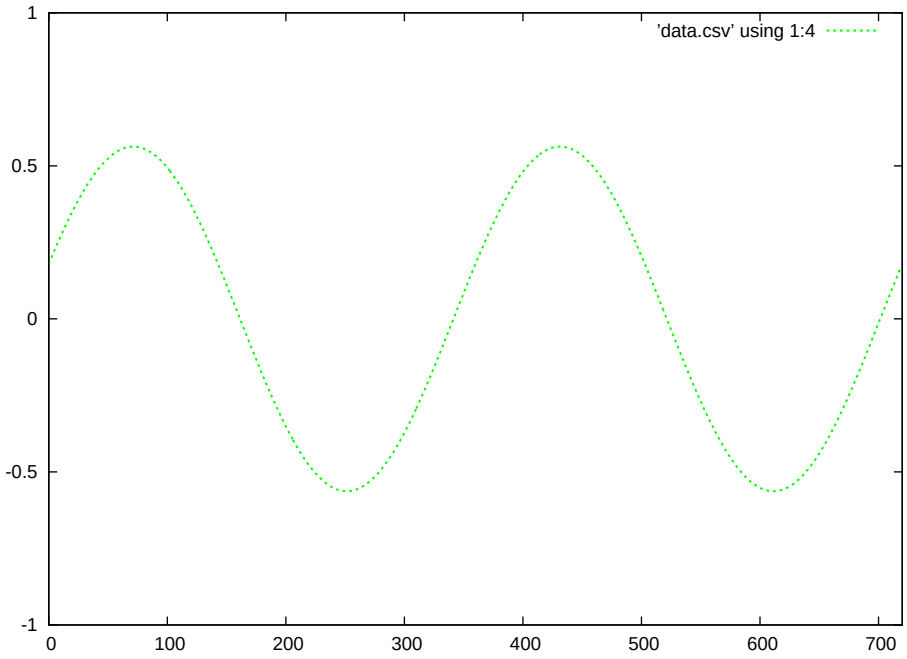
Green = standing wave



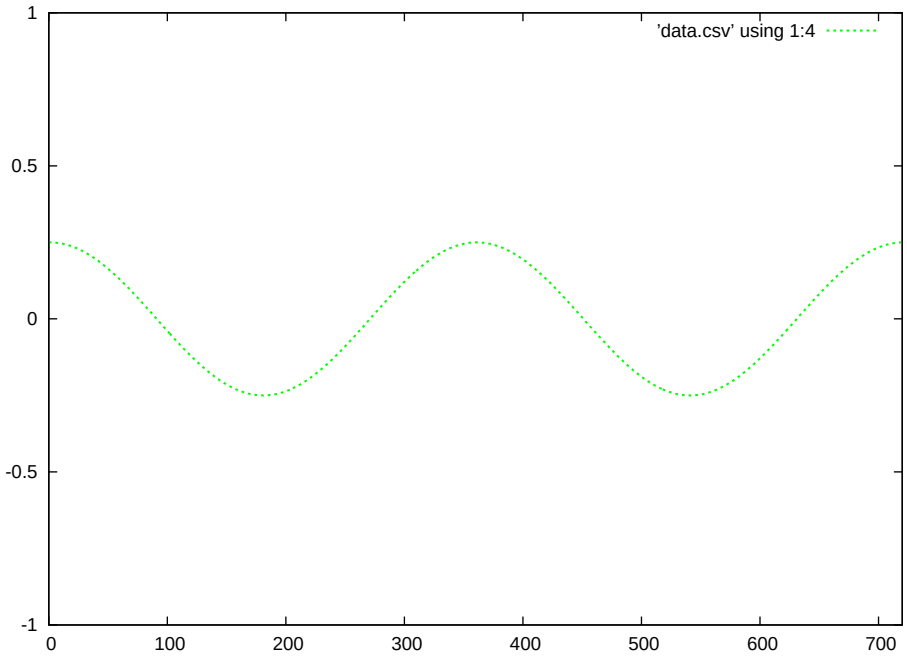
Green = standing wave



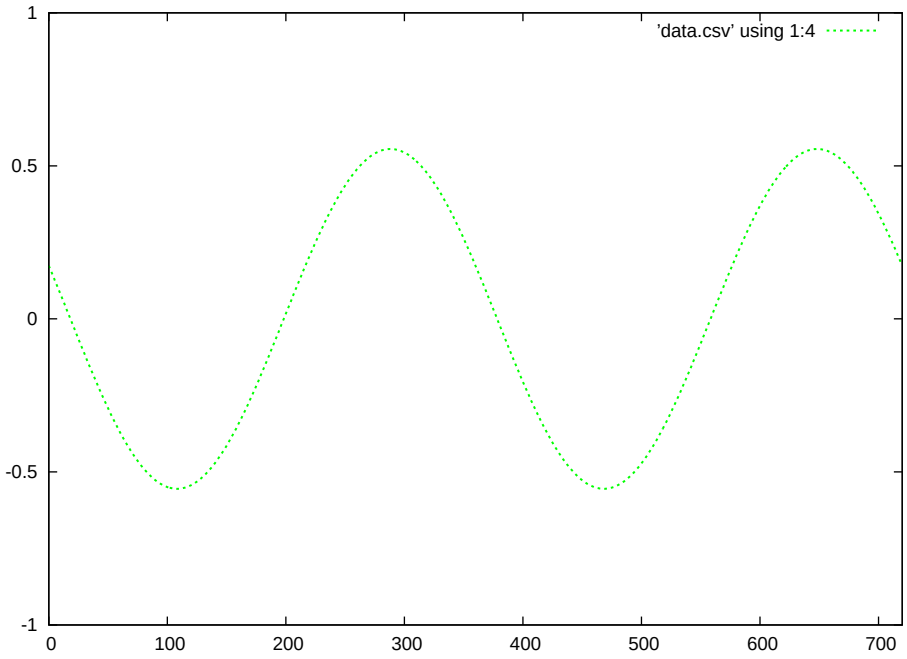
Green = standing wave



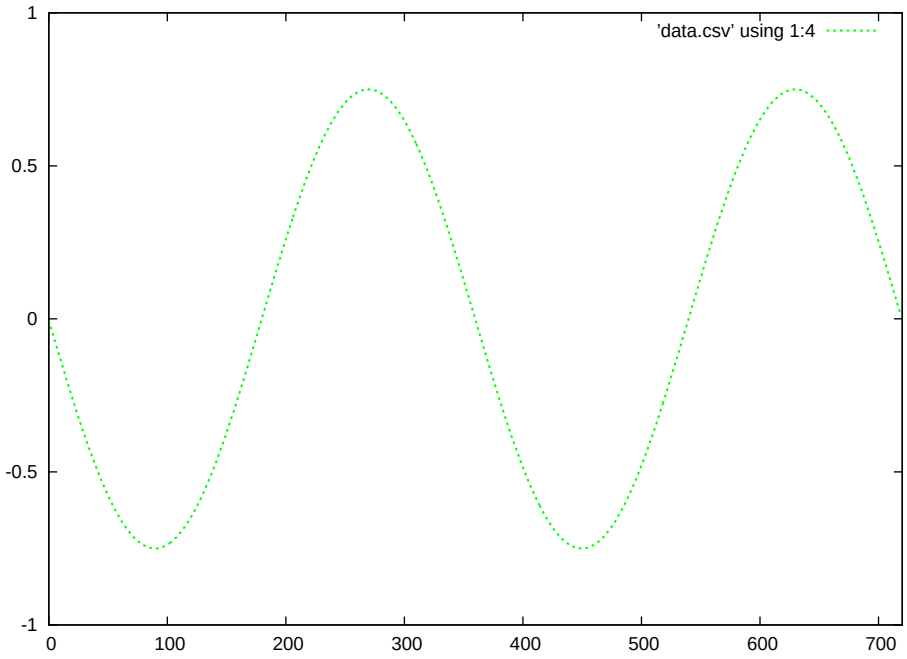
Green = standing wave



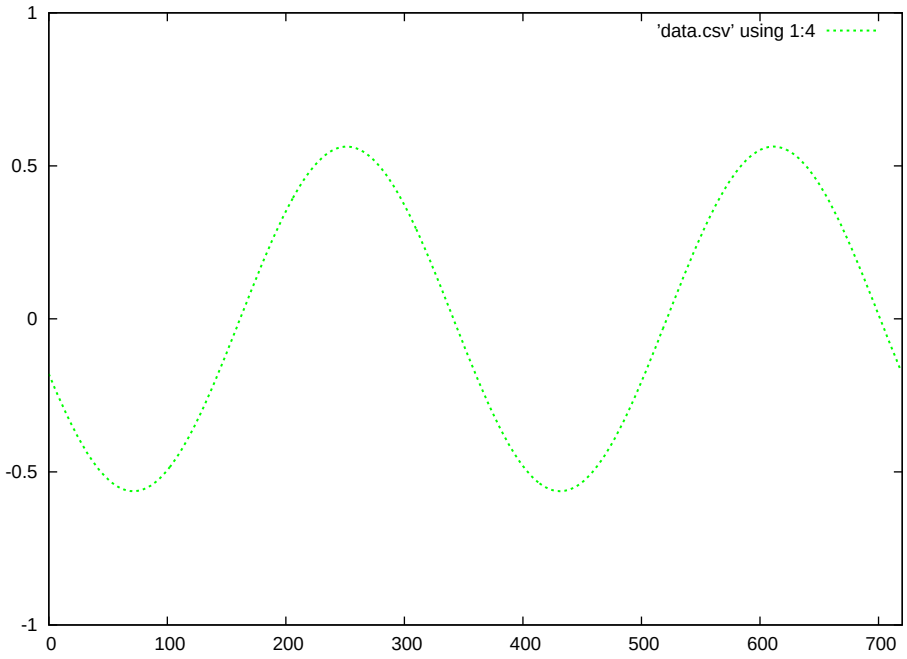
Green = standing wave



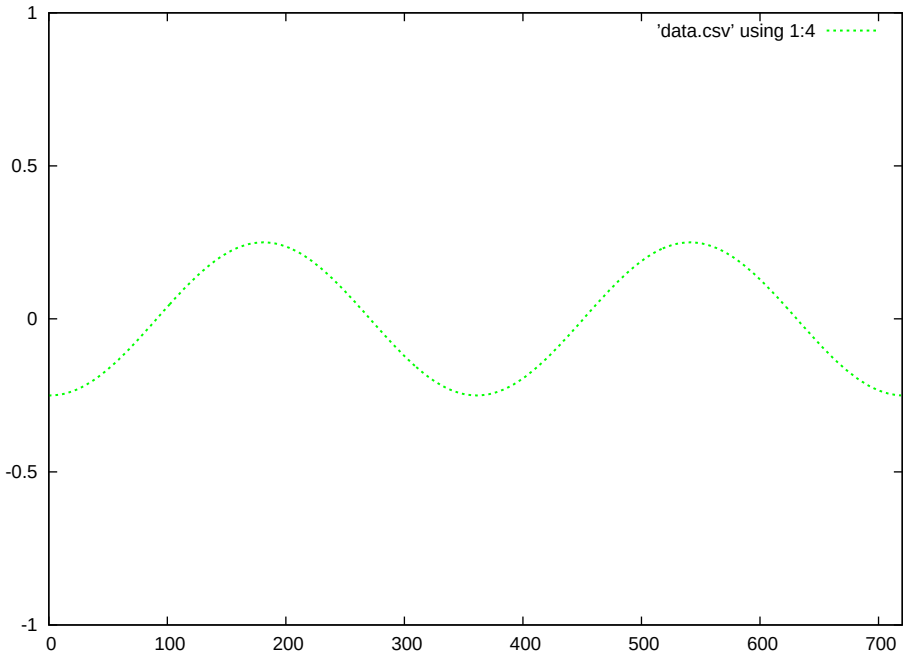
Green = standing wave



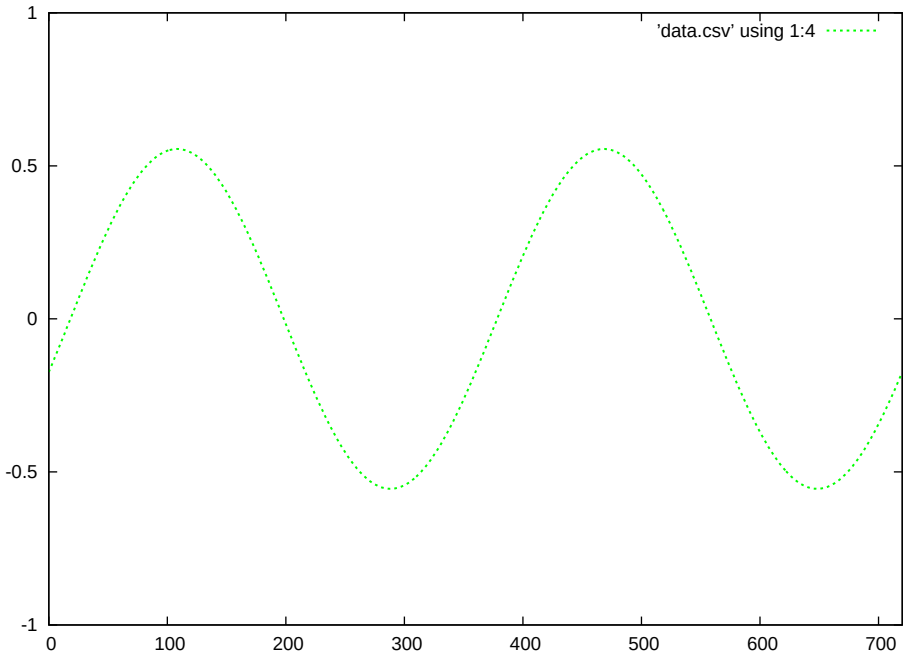
Green = standing wave



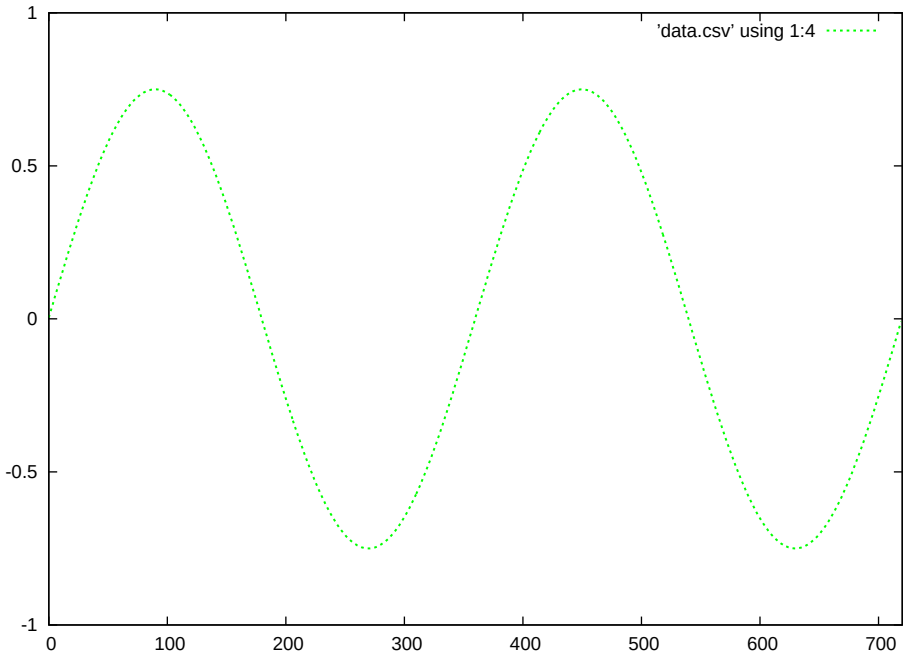
Green = standing wave



Green = standing wave



Green = standing wave



C++ source code for initial forward, reflected, and superposition waveforms with 50% reflection

```
#include <iostream>
#include <cmath>
using namespace std;

float sinecalc (int);

int main (void)
{
  int x, offset;
  float fwd[1440];
  float ref[720];
  float sum[720];

  offset = 1440;

  for (x = 0 ; x < 1440 ; ++x)
  {
    fwd[x] = 0.5*sinecalc(x-offset);
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    if (offset <= 720)
    {
      ref[x] = 0.0;
      sum[x] = 0.0;
    }

    else
    {
      ref[719-x] = -0.5 * fwd[720+x];
    }
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    sum[x] = fwd[x] + ref[x];
    cout << x << " , " << fwd[x] << " , " << ref[x] << " , " << sum[x] << endl;
  }

  return 0;
}
```

```
float sinecalc (int degrees)
{
  if (degrees < 0)
    return sin (degrees * M_PI / 180);

  else
    return 0.0;
}
```

Running this exact program produces a set of text-based data, separated by commas, representing the forward, reflected, and summation waves at an angle of 1440 degrees.

Gnuplot source code (named `gnuplot.txt`)

```
set datafile separator ","  # Interpret commas as field separators
set term postscript eps
set xrange [0:720]  # Sets bounds on the domain
set yrange [-1:1]  # Sets bounds on the domain
set style line 1 lw 3 lc rgb "red"  # Sets ls 1 thickness=3 color=red
set style line 2 lw 3 lc rgb "blue"  # Sets ls 2 thickness=3 color=blue
set style line 3 lw 3 lc rgb "green"  # Sets ls 3 thickness=3 color=green
set output 'anim_sw_fr_50_1440.eps'
plot 'data.csv' using 1:2 with lines ls 1, 'data.csv' using 1:3 with lines ls 2
set output 'anim_sw_sum_50_1440.eps'
plot 'data.csv' using 1:4 with lines ls 3
```

Note: the C++ program when compiled and run outputs comma-separated value (CVS) to the console (standard output). That C++ program must be compiled and run over and over again, each time with a different `offset` value representing the number of degrees the two waveforms have evolved over. For each run of the C++ program, I used Gnuplot to convert the comma-separated text file into graphical images, editing the gnuplot script each time to re-name the files according to the offset (angle) value. To convert this text data into graphical plots, you must redirect the output to a file (I use the filename `data.csv`) like this:

```
./a.out > data.csv
```

Then, you invoke Gnuplot to convert this text data into two Encapsulated PostScript files like so:

```
gnuplot -p gnuplot.txt
```

Gnuplot must be run once per execution of the C++ code, each time producing a pair of graphical images. The first image shows both the forward and reflected waves at that angle (`offset`), and the second image showing just the summation of the forward and reflected waves at that angle.

C++ source code for established standing waveform with 50% reflection

```
#include <iostream>
#include <cmath>
using namespace std;

float sinecalc (int);

int main (void)
{
  int x, offset;
  float fwd[720];
  float ref[720];
  float sum[720];

  offset = 360;

  for (x = 0 ; x < 720 ; ++x)
  {
    fwd[x] = 0.5*sinecalc(x+offset);
    ref[719-x] = -0.25*sinecalc(x+offset);
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    sum[x] = fwd[x] + ref[x];
  }

  for (x = 0 ; x < 720 ; ++x)
  {
    cout << x << " , " << fwd[x] << " , " << ref[x] << " , " << sum[x] << endl;
  }

  return 0;
}

float sinecalc (int degrees)
{
    return sin (degrees * M_PI / 180);
}
```

As before, this C++ code must be compiled and executed repeatedly, each time with a different offset value. The only difference here is that the offset values need only span from 0 to 360 degrees,

not all the way to 1440 degrees as before to simulate the initial travel of both the forward and reflected waves along the transmission line's length.

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Resistance

Electric field

Magnetic field

Capacitance

Inductance

Parasitic effect

Electrical load

Thévenin's theorem

Characteristic or Surge impedance

Line termination

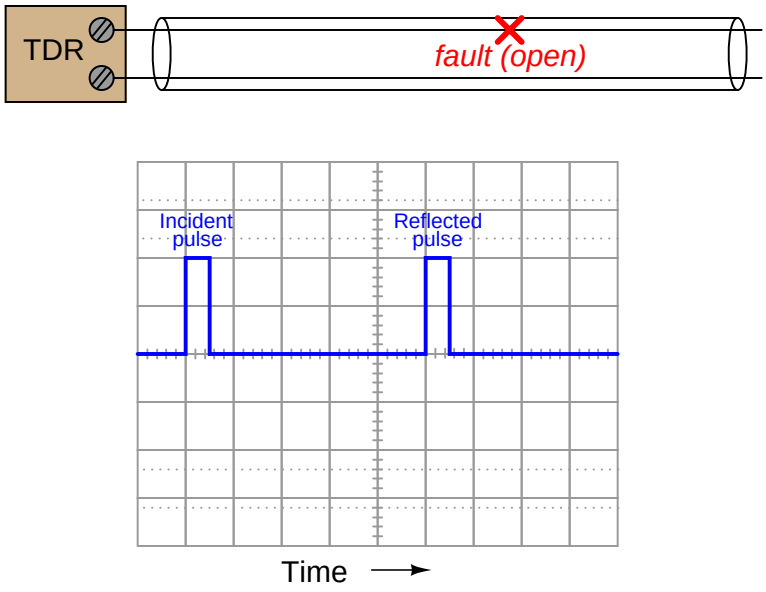
Velocity factor

Decibels

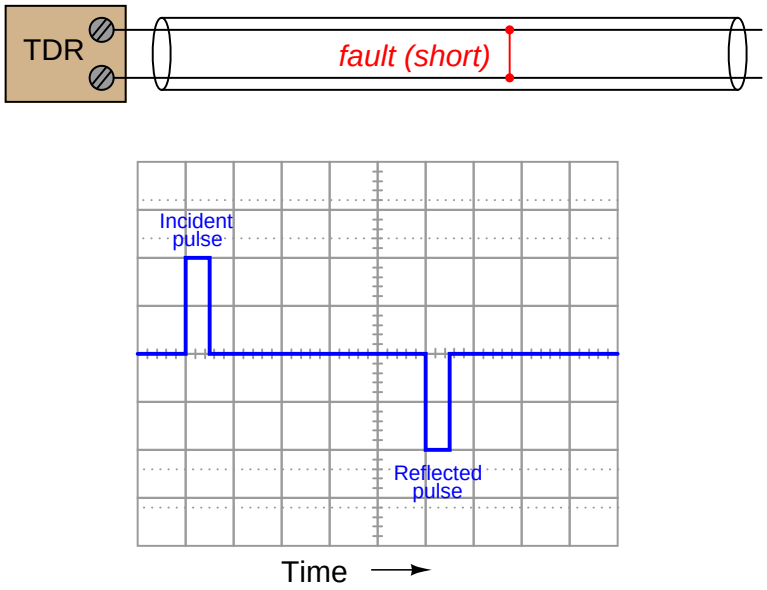
6.1.3 TDR cable fault testing

An instrument called a *Time-Domain Reflectometer*, or *TDR*, is used to check for opens or shorts along a cable length by generating a short voltage pulse and waiting to receive the reflected pulse "bounced" back from either the fault or from the unterminated cable end. This represents an interesting application of a *poorly-terminated* transmission line.

Shown below is the reflected voltage pulse for an open fault in a cable, as indicated by a TDR. It is assumed here that the pulse width is shorter in duration than the time it takes for the pulse to travel to the fault and back:



Now we see the reflected voltage pulse for an shorted fault in a cable, as indicated by a TDR. As before, it is assumed here that the pulse width is shorter in duration than the time it takes for the pulse to travel to the fault and back:



What would the technician look for in the TDR display to determine where the fault was in the cable? Also, what would the TDR display look like for a good (non-faulted) cable?

Challenges

- How would the open-fault test oscillograph differ (if at all) if there was a termination resistor at the far end of the faulted cable?
- How would the shorted-fault test oscillograph differ (if at all) if there was a termination resistor at the far end of the faulted cable?
- Suppose the cable's temperature were to rise significantly due to changes in ambient weather conditions. How would this change in cable resistance affect the results of the TDR test?

6.1.4 Traveling-wave power line fault detection

A relatively recent technological advance in power system monitoring and protection is the so-called *traveling-wave* technique, based on the fact that long-distance power lines act as signal *transmission lines* and that faults on a line (e.g. an insulator flash-over from line to ground) produce transient voltages and currents propagating along the line at nearly the speed of light.

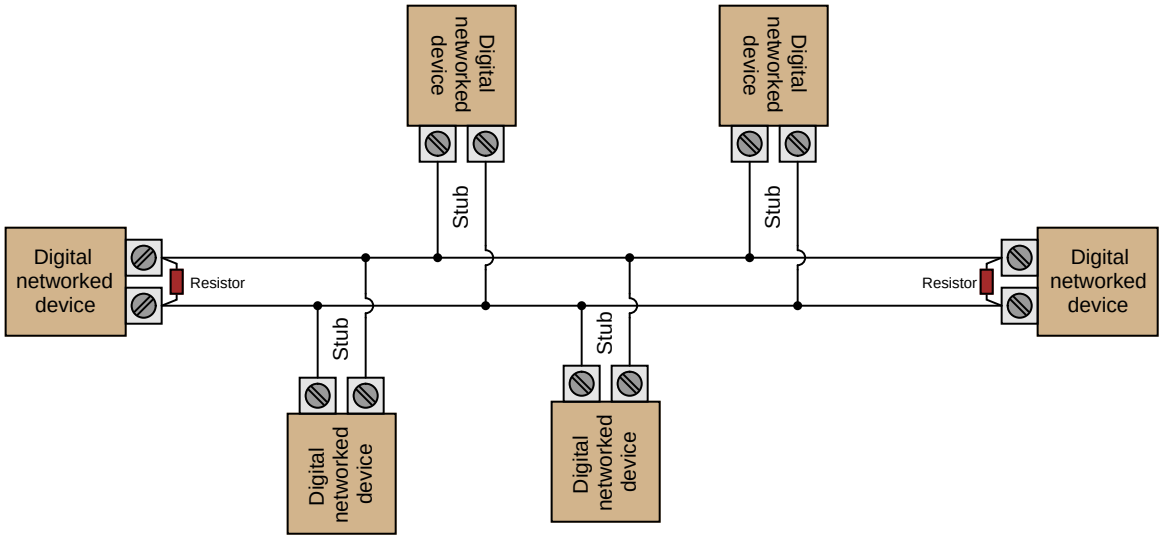
Explain how this principle of fault-signal waves traveling along a power line might be detected at the line's end(s), and what parameters would have to be known in advance in order to pinpoint the location of that fault.

Challenges

- What factor(s) dictate the surge impedance of a power line?

6.1.5 Terminator resistors in a digital network

It is common in some digital networks to connect paralleled devices to the network cable, like this:



Note that the far-end devices have terminating resistors connected in parallel to the network connection terminals, while the devices at the end of the stubs do not. Does this constitute a problem? In other words, will reflected signals result from the unterminated stubs? Why or why not?

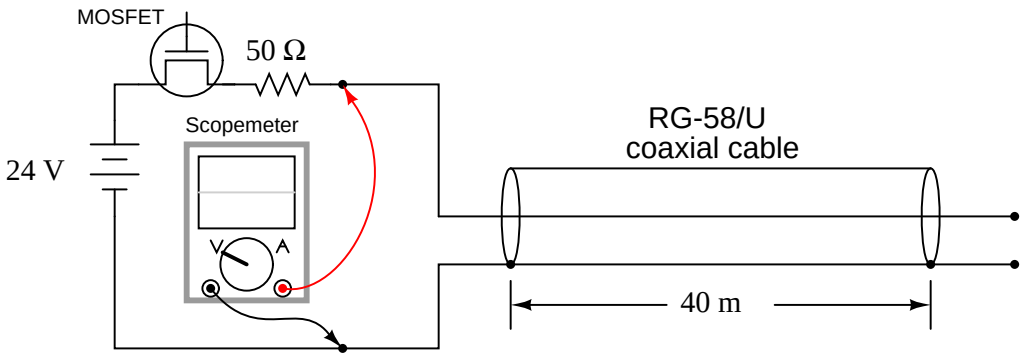
Furthermore, what would happen if each of the stub devices were terminated with the same resistance value as the end devices?

Challenges

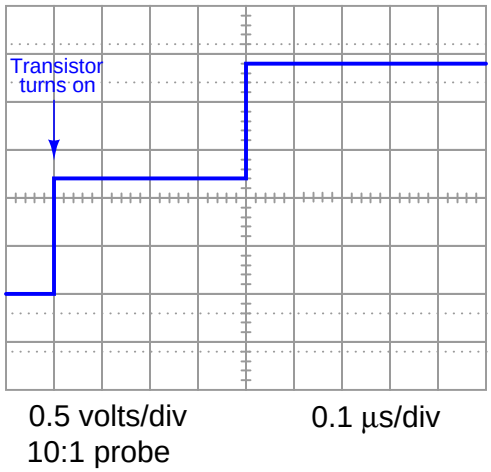
- What would happen if all terminating resistors were omitted from this circuit?

6.1.6 MOSFET-switched cable

Suppose we were to test the voltage at the source end of a long cable, driven by a 24 volt power supply and a fast-switching MOSFET transistor:

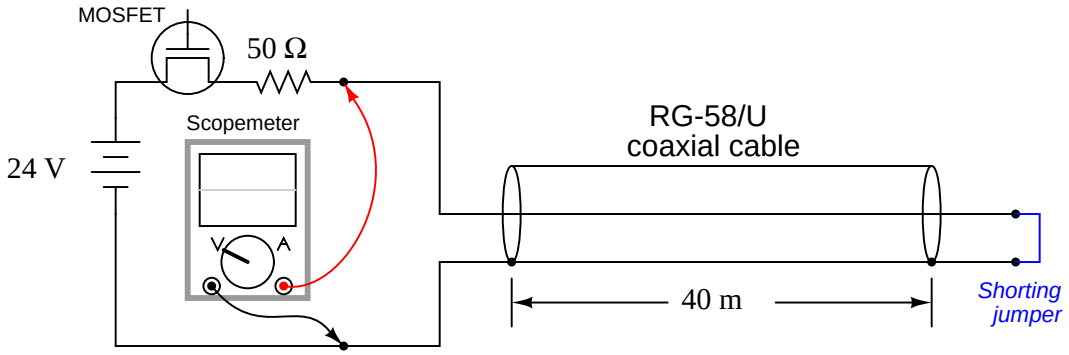


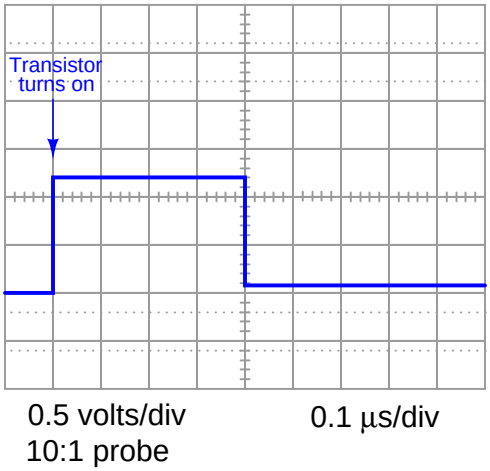
The *characteristic impedance* of an RG-58/U cable is 50 Ω (also called the "surge" impedance), and the Thévenin equivalent impedance of our power supply is 50 Ω as well. When the transistor is turned on, the scope registers an immediate step in voltage, but only to 12 Volts (one-half the supply voltage). It isn't until several tenths of a microsecond later that the voltage steps up to the full 24 Volts:



Explain why the voltage waveform steps to 12 Volts, then to 24 Volts as it does. Why doesn't it step all the way to 24 Volts immediately, given a cable that is open at the far end?

Next, suppose we were to short-circuit the end of the cable and repeat the test, once again capturing the voltage signal with the scope. This time, the voltage steps to 12 Volts and then steps back down to nearly zero Volts several tenths of a microsecond later:





Again, explain why the voltage waveform does this. Why doesn't the voltage remain nearly zero the entire time, given a dead short at the end of the cable?

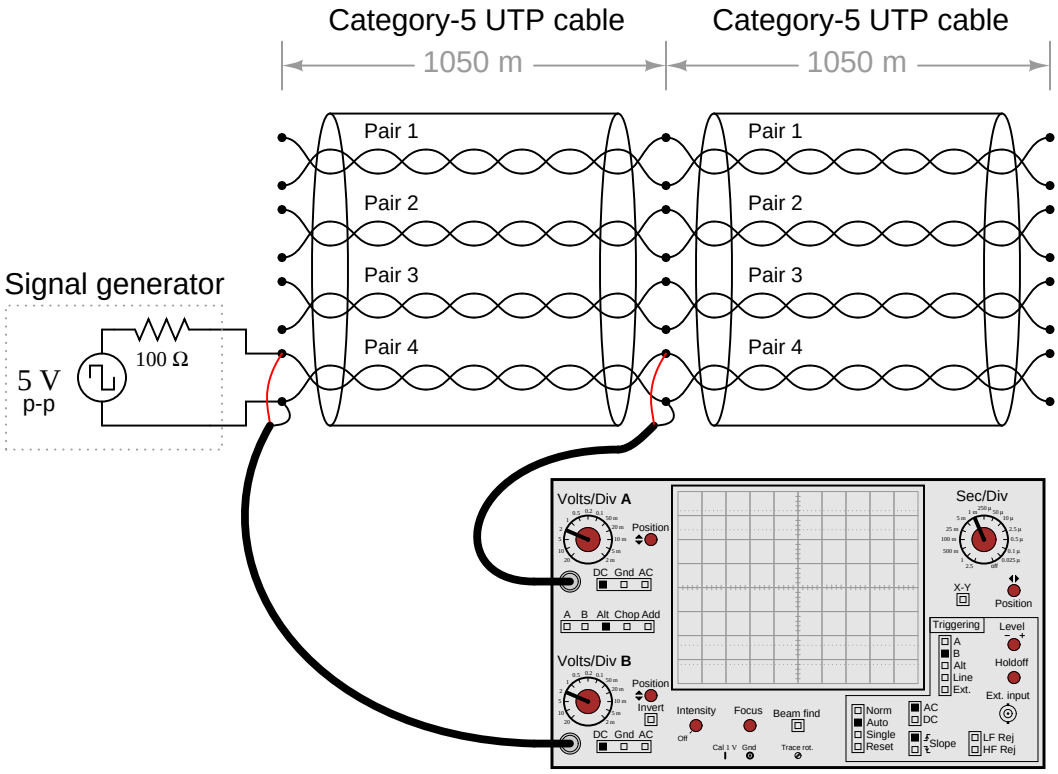
Additionally, show how the time duration of the "step" in each case is about 0.4 microseconds, given the length of the cable as 40 meters. Note: the *velocity factor* of RG-58/U cable is approximately 0.66.

Challenges

- Identify factors that would affect the duration of the "step" in this waveform.
- Would a different type of coaxial cable affect the shape of the pulse?

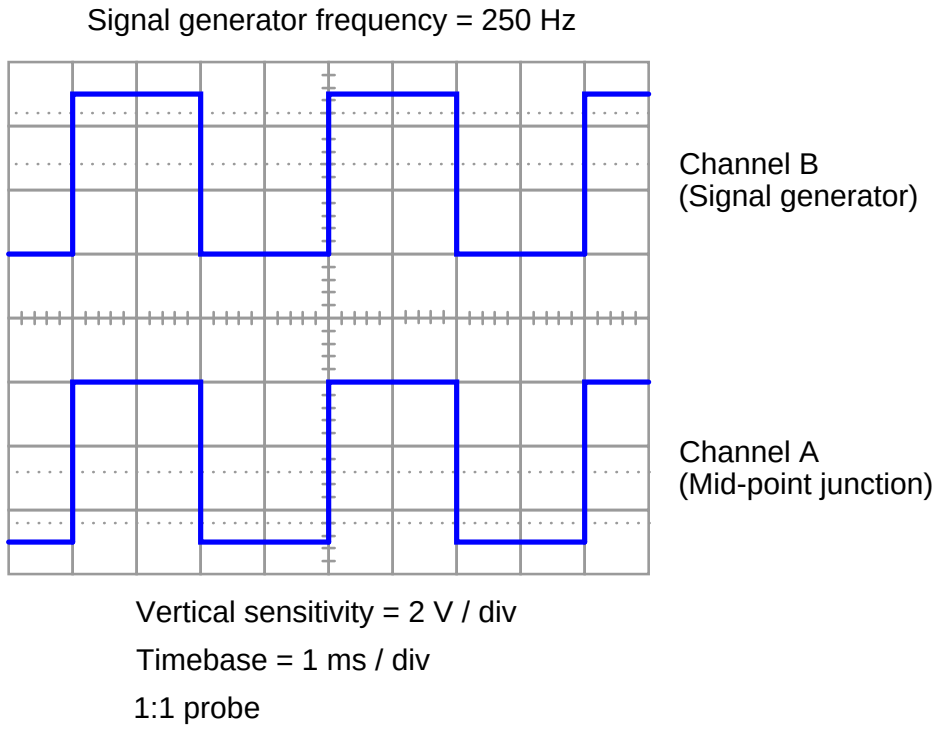
6.1.7 Cat-5 cable test

Suppose we "drive" one twisted pair of a Category-5 ("Cat-5") communications cable with a signal generator having the same Thévenin equivalent impedance as the cable (100 Ω, which is standard for the unshielded, twisted wire pairs Cat-5 cable). Actually, the cable consists of two coiled sections joined together in the middle so that we may monitor voltage mid-way as well as at the front end where the signal generator connects, with a dual-trace oscilloscope[4]:



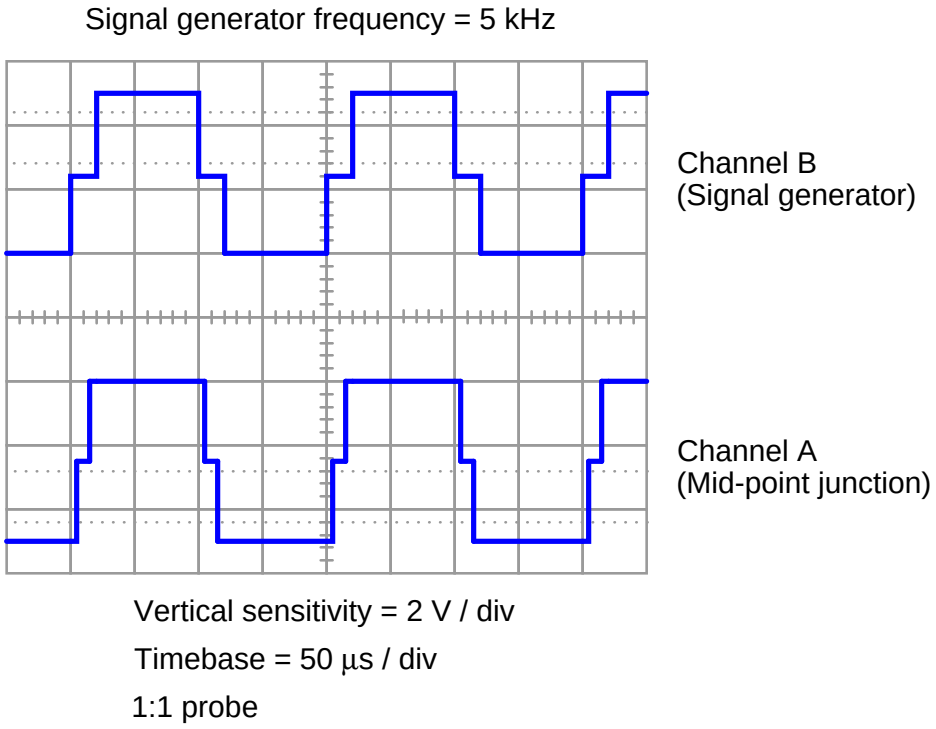
[4]One detail that might be worth mentioning is that the oscilloscope would have to have isolated inputs in order for this test to work. Otherwise, the common ground connection through both inputs (A and B) would mess things up by bypassing one wire in the twisted pair with the first 1050 meter section!

Given a typical velocity factor of 0.7, meaning that electrical signals will travel down the cable at 70% the speed of light, it should take 10 microseconds for the signal to travel from the signal generator to the far end of the cable. Given these conditions, we input a square-wave voltage signal at a frequency of 250 Hz and record the results:



So far, things look just fine. We are seeing the same voltage signal at the mid-point of the cable that we're injecting at the left-hand end. There is no noticeable distortion, and the time delay due to propagation time along the cable length is too short to be significant at this scale.

Now, we increase the signal generator frequency to 5 kHz and correspondingly increase the sweep speed of the oscilloscope. The waveforms we see are not so clean anymore!



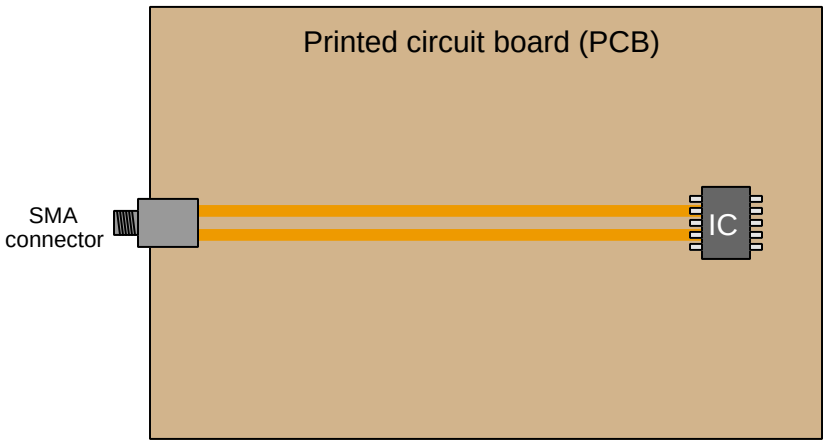
Explain what accounts for the "stepped" shape of the square wave on its leading and trailing edges.

Challenges

- What do you think the signal will look like at the far-right end of the cable?
- Why are *isolated* input channels on the oscilloscope necessary for this test.

6.1.8 Impedance of lines on a printed circuit board

Copper traces on a printed circuit board (PCB) exhibit characteristic impedance just the same as conductors within a cable. Suppose it was determined that the traces connecting a SMA-style cable connector to an integrated circuit on a PCB had too much surge impedance[5]:



First, describe the problem(s) possibly created by improper impedance between the two traces on this board.

Next, explain what would need to be done to re-design the PCB to have less characteristic impedance between these two lines.

Challenges

- Does the material the PCB is made out of matter for Z_0?

[5]This view of the PCB only shows these components and nothing else. A real PCB would likely be populated with a great many components!

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[6]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[7] on an answer key!

[6] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[7] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[8] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[8] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[9] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[10] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[11] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[9] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[10] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[11] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

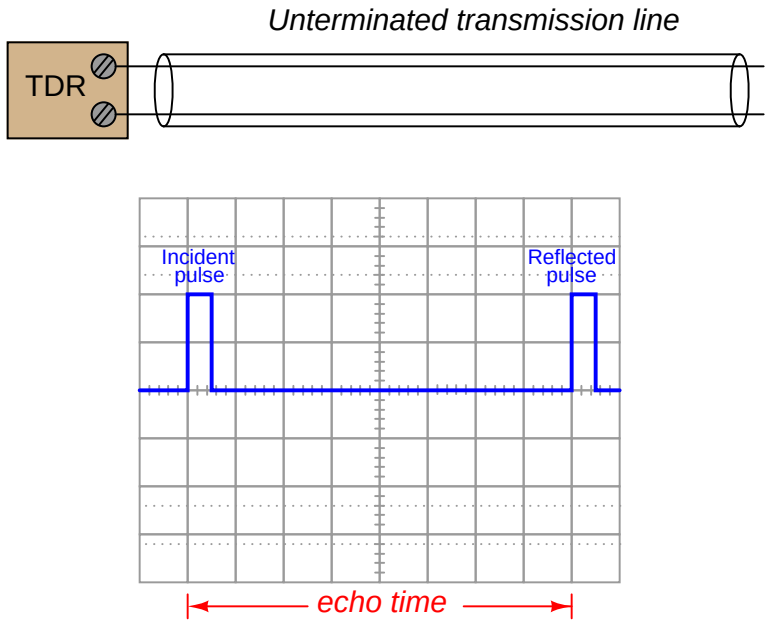
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[12] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[12] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Echo time calculations

Suppose we use a time-domain reflectometer (TDR) to test an unterminated cable, as shown in the illustration below:



Calculate the following:

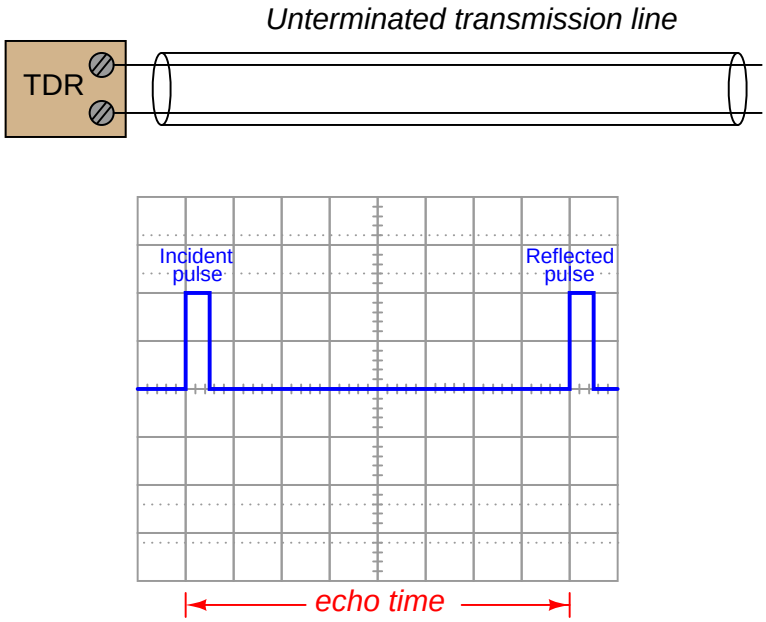
- Cable length given an echo time of 317 ns and a velocity factor of 0.8 =
- Echo time given a cable length of 125 m and a velocity factor of 0.75 =
- Velocity factor given a cable length of 81 m and an echo time of 700 ns =

Challenges

- Explain how it might be possible that a student would calculate an incorrect cable length of 76.027 meters for the first scenario (echo time 317 nanoseconds).
- Suppose a student is struggling with the arithmetic of time, velocity, and distance. Describe an every-day scenario using these same kinds of quantities which will make it clear how to mathematically solve for any of them given the other two.

6.2.4 TDR spreadsheet

An instrument called a *Time-Domain Reflectometer*, or *TDR*, may be used to measure the length of an unterminated cable by generating a very brief voltage pulse and waiting to receive the reflected pulse "bounced" back from the cable's end. The duration of this pulse is so brief that it is done and over before the signal has a chance to travel down the cable and back:



Build a computer spreadsheet to calculate cable length, given the echo time (entered in microseconds) measured by a TDR, and the velocity factor of the cable. An example spreadsheet layout is shown here (the cell fill coloring is just for looks, distinguishing the *calculated* value of cable length from the *entered* values of echo time and velocity factor):

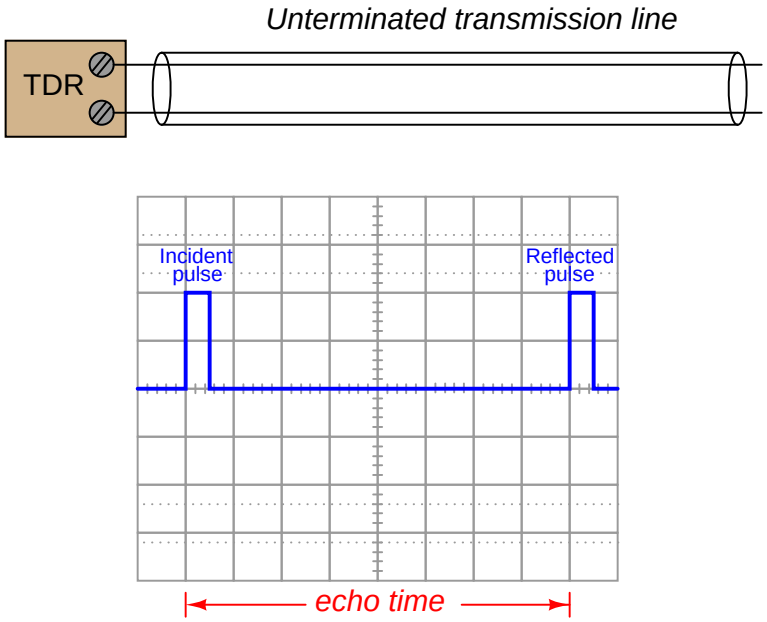
	1	2	3	4	5
1	Echo time =		(microseconds)		
2	Velocity factor =		(unitless)		
3					
4	Cable length =		(feet)		
5					

Challenges

- How could you *test* your spreadsheet cable-length calculator for accuracy (to verify you haven't made any mistakes) once you've entered all your equations?
- Note that there is no entry on this spreadsheet for the *frequency* of the TDR's test signal, or the width of that signal's pulse duration. Explain why these parameters are not a part of the cable length calculation.
- What would a *real* oscilloscope screenshot look like in this scenario, with a real cable?

6.2.5 TDR calculator program in C

An instrument called a *Time-Domain Reflectometer*, or *TDR*, may be used to test unterminated cable by generating a very brief voltage pulse and waiting to receive the reflected pulse "bounced" back from the cable's end:



Write a formula relating echo time (t), cable length (x), and velocity factor (k) together.

Complete the following program (written in the C language) to calculate either cable length or cable velocity factor given an echo time measurement obtained from a TDR:

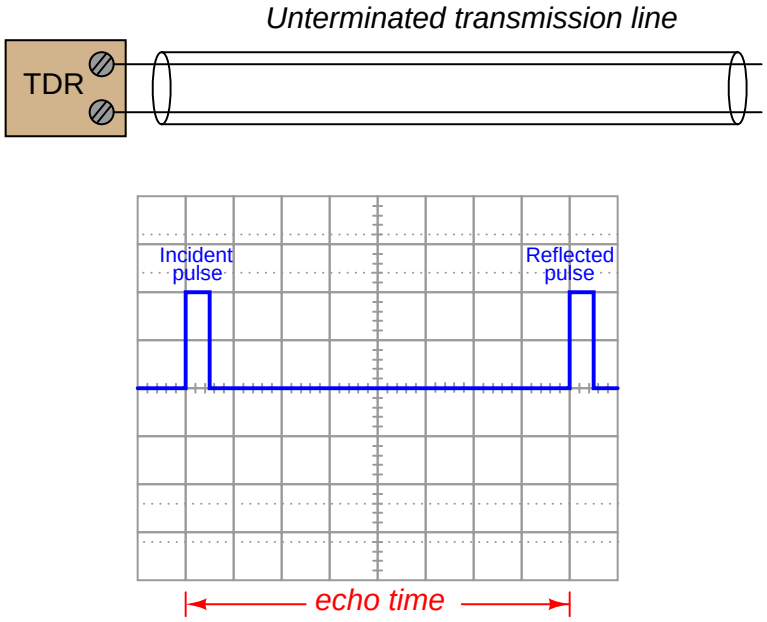
```
#include <stdio.h>

int main (void)
{
  float time, vfactor, length, c = 2.9979e8;
  int choice;

  printf("Enter echo time in microseconds: ");
  scanf("%f", &time);

  printf("Choose to calculate (1) cable length or (2) velocity factor: ");
  scanf("%i", &choice);

  if (choice == 1)
  {
    // Your code goes here
  }

  else
  {
    // Your code goes here
  }

  return 0;
}
```

Challenges

- How could you *test* your program for accuracy (to verify you haven't made any mistakes)?
- Note that there is no entry in this program for the *frequency* of the TDR's test signal, or the width of that signal's pulse duration. Explain why these parameters are not a part of the cable length calculation.
- What would a *real* oscilloscope screenshot look like in this scenario, with a real cable?

6.2.6 TDR calculator program in Python

An instrument called a *Time-Domain Reflectometer*, or *TDR*, may be used to test unterminated cable by generating a very brief voltage pulse and waiting to receive the reflected pulse "bounced" back from the cable's end:



Write a formula relating echo time (t), cable length (x), and velocity factor (k) together.

Complete the following program (written in the Python language) to calculate either cable length or cable velocity factor given an echo time measurement obtained from a TDR:

```
print("Enter echo time in microseconds:")
time = float(input())

time = time / 1e6
c = 2.9979e8

print("Choose to calculate (1) cable length of (2) velocity factor:")
choice = int(input())

if choice == 1 :
  # (your code goes here)

else :
  # (your code goes here)
```

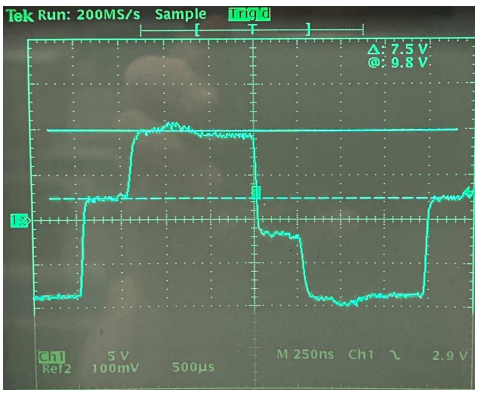
Note that the Python programming language uses indentation to identify blocks of code. That is, the line(s) of code associated with each `if` conditional need to be indented to show they belong to that conditional statement!

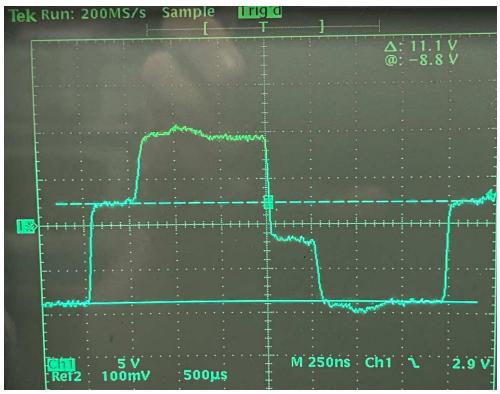
Challenges

- How could you *test* your program for accuracy (to verify you haven't made any mistakes)?
- Note that there is no entry in this program for the *frequency* of the TDR's test signal, or the width of that signal's pulse duration. Explain why these parameters are not a part of the cable length calculation.
- What would a *real* oscilloscope screenshot look like in this scenario, with a real cable?

6.2.7 Cable impedance from oscillograph

A long cable is connected to a square-wave signal generator having a 50 Ohm output impedance (and oscilloscope at the same end of the cable) with the cable's far end open-circuited. Examine the following oscillographs with cursors showing the two heights of the pulses:





From this information, determine the cable's characteristic impedance.

Challenges

- How would the waveshape differ if the cable's characteristic impedance were exactly 50 Ohms?

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 Cable testing with no oscilloscope or TDR

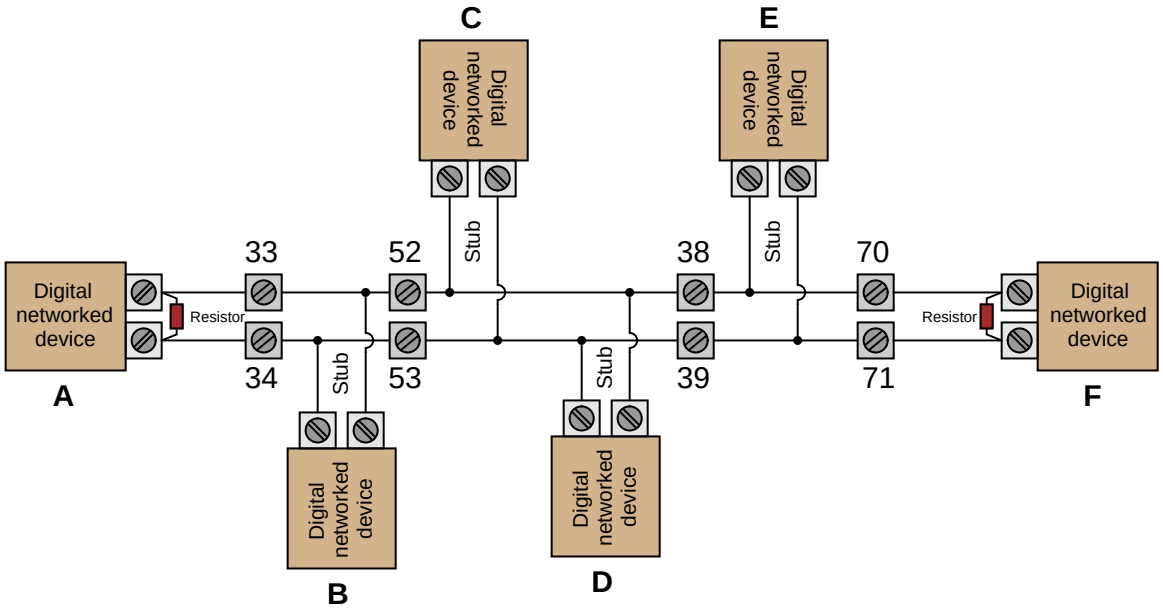
Suppose we did not have a TDR or an oscilloscope available to perform a reflected-wave test of a long cable. What other means might we employ to test its integrity?

Challenges

- Now imagine we have no electronic test equipment whatsoever. How could we test the integrity of a long cable with no multimeter, no signal generator, no measuring instruments at all?

6.3.2 Faulted digital network

Suppose a technician is troubleshooting this network of digital devices, and suspects an electrical fault somewhere in the network preventing communication. It seems *none* of the devices is able to communicate with any other device on the network:



The technician decides to loosen terminal number 38 and remove the wire entering its left-hand side. Then, she uses her ohmmeter to measure resistance between terminals 52 and 53. The ohmmeter registers 152 Ohms.

Identify the likelihood of each specified fault for this network. Consider each fault one at a time (i.e. no coincidental faults), determining whether or not each fault is compatible with *all* measurements and symptoms in this network.

- Device **A** failed shorted
- Device **B** failed open
- Right-hand terminating resistor shorted
- Device **E** failed shorted
- Short between terminals 33 and 34

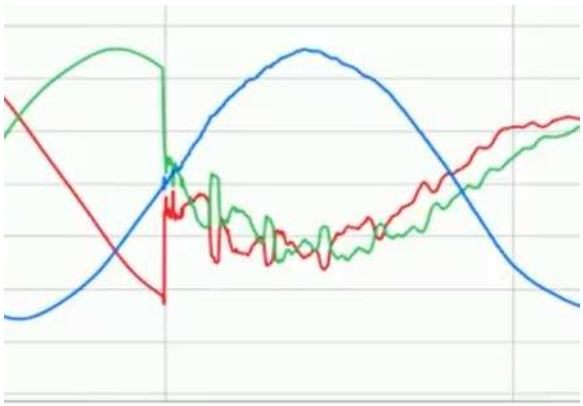
Challenges

- Suppose 152 Ohms was measured between the same terminals, but with the wire connected at terminal 38 instead of being removed. Would this alter any of your answers?

6.3.3 Traveling-wave fault location on a power line

When a fault (arc) occurs on a high-voltage power line, the low impedance of the arc presents an impedance discontinuity at that location along the power line, against which pulses may reflect. The impulses created from the onset of that fault then reflect back and forth between the fault location and the ends of the line (usually where transformers are located, themselves also presenting impedance discontinuities for pulses to reflect).

The following oscillograph was captured on a high-voltage three-phase power transmission line, showing a phase-to-phase fault (arc). Each of the sine waves show voltage between one conductor and Earth ground. The fault occurred between the green and red phases, creating an arc that made those two lines electrically common at the fault location:



A series of impulses are clearly visible immediately after the fault's onset, created as the pulses echo back and forth between the substation (where the oscillography equipment was located) and the fault location. Assuming a 60 Hz power line frequency and a velocity factor of 95% for open-air power lines, approximate the distance between the substation and the fault.

Challenges

- Why do the impulses grow weaker over time?
- Explain how we can tell that the fault occurred between the green and red phases.
- Supposing the oscillograph's timebase resolution was 1 microsecond (i.e. this digital oscilloscope samples at 1 million samples per second), what is the resolution of the fault's distance measurement?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

"Bird RF Directional Thruline Wattmeter Model 43" Operation Manual, Instruction book part number 920-43 Revision L, Bird Technologies Incorporated, (City), (Country), 2016.

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

11-12 October 2025 – minor edit to the Tutorial describing interpretation of the oscillographs for both open-ended and short-ended cables. Also made minor edits to the Tutorial discussing pulse period versus pulse travel time, as well as edits to a footnote discussing the effects of magnetic permeability on velocity factor. Also added a Challenge question and edited some instructor notes. Also added a new Quantitative Reasoning question discerning characteristic impedance from oscillographs of reflected signals.

8 October 2025 – minor edits to the Tutorial describing the experimental set-up used to measure reflections in open and shorted cables.

29 May 2025 – noted how VSWR may be calculated simply by dividing the larger impedance by the smaller impedance. Also corrected a mis-spelling of the word "dielectric".

22 October 2024 – added a Technical Reference section on decibels and a Case Tutorial on cable losses.

12 September 2024 – minor edits to the Tutorial. Also added a help programming hint for Python (regarding indentation) to the "TDR calculator program in Python" Quantitative Reasoning question.

9 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

28-30 May 2023 – moved the "Traveling waves on a power line" question from the Quantitative Reasoning section to the Diagnostic Reasoning section because at root the student is tasked to diagnose the location of a fault along a power line. Also re-named that question "Traveling-wave fault location on a power line". Finally, added a new Quantitative Reasoning question challenging

students to complete a partially-written C program for a echo-time/distance/velocity calculator.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

22 August 2022 – added a Quantitative Reasoning question centered around a high-speed oscillograph showing a phase-to-phase arc fault on a high-voltage power transmission line where impulses from traveling waves were clearly captured.

23 March 2022 – minor edits to the Tutorial for clarity on how to interpret oscilloscope screenshots showing pulse durations for open-circuited and short-circuited transmission lines. Also added an annotated oscilloscope screenshot to help students identify where the 0.2 μs figure came from in the example.

7 March 2022 – added some questions to the Introduction chapter, and made minor edits to the Tutorial.

12 October 2021 – added Case Tutorial sections showing SPICE simulations of transmission line behavior.

25 September 2021 – added an entry to the References list.

20 September 2021 – minor typo correction, plus more notes added to the Introduction chapter. Also added cross-references to the Animations chapter showing flip-book animations of open and shorted transmission lines.

14 September 2021 – added more detail to the Tutorial chapter on standing wave ratio, and added a new Tutorial section on reflection coefficient.

25 July 2021 – added more detail to the Case Tutorial sections on measuring transmission line parameters.

11 May 2021 – added Tutorial section explaining standing waves created by constructive and destructive interference of forward and reflected waves along a transmission line.

8 May 2021 – added animations showing how a standing waves are created by the constructive and destructive interference of forward and reflected waves along a transmission line.

18 March 2021 – corrected multiple instances of "volts" that should have been capitalized "Volts".

25 February 2021 – elaborated more on the deleterious effects of signal reflections in data networks, by adding some text and footnotes to the Tutorial.

11 February 2021 – added some new questions to the Introduction chapter, as well as edited the properly-terminated line illustration to read "current" and "voltage" in the bottom image. Added two more paragraphs to the Tutorial text after each example of improper cable termination, explaining in terms of energy conservation why the reflected pulse has both voltage and current

ahead of it but only one of those quantities behind. Also, corrected instances of "ohms" that should have been capitalized ("Ohms").

1 November 2020 – added some Foundational Concepts to the Conceptual Reasoning section.

27 October 2020 – added a Technical Reference section with a graphic image of a Smith chart and minimal description.

17 September 2020 – added more instructor notes to some of the questions. Also, added a Case Tutorial chapter.

1 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions. Also, added some content to the beginning of the tutorial.

29 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

28 January 2020 – added a new section at the beginning of the Tutorial, as well as a brief amount of introductory text. Also added more references to the Conservation of Energy, in explaining reflected versus non-reflected pulses.

11 January 2020 – added another Question to the Quantitative Reasoning section.

3 December 2019 – added another Question, and made minor edits to the Tutorial.

2 December 2019 – added some Questions.

1 December 2019 – document first created.

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