

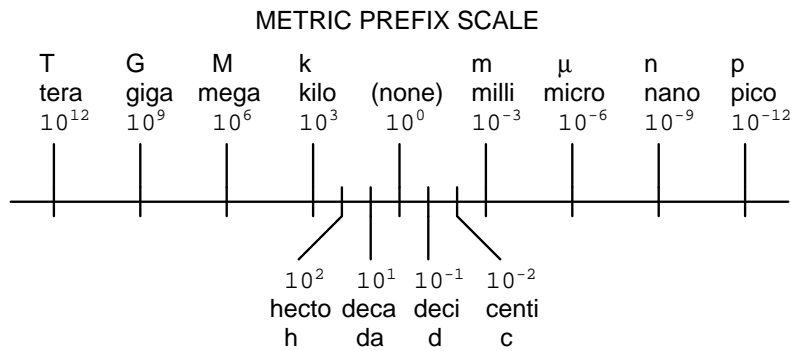
Work, energy, and power

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Metric prefixes and conversion constants

- **Metric prefixes**

- Yotta = 10^{24} Symbol: Y
- Zeta = 10^{21} Symbol: Z
- Exa = 10^{18} Symbol: E
- Peta = 10^{15} Symbol: P
- Tera = 10^{12} Symbol: T
- Giga = 10^9 Symbol: G
- Mega = 10^6 Symbol: M
- Kilo = 10^3 Symbol: k
- Hecto = 10^2 Symbol: h
- Deca = 10^1 Symbol: da
- Deci = 10^{-1} Symbol: d
- Centi = 10^{-2} Symbol: c
- Milli = 10^{-3} Symbol: m
- Micro = 10^{-6} Symbol: μ
- Nano = 10^{-9} Symbol: n
- Pico = 10^{-12} Symbol: p
- Femto = 10^{-15} Symbol: f
- Atto = 10^{-18} Symbol: a
- Zepto = 10^{-21} Symbol: z
- Yocto = 10^{-24} Symbol: y



- **Conversion formulae for temperature**

- $^{\circ}\text{F} = (^{\circ}\text{C})(9/5) + 32$
- $^{\circ}\text{C} = (^{\circ}\text{F} - 32)(5/9)$
- $^{\circ}\text{R} = ^{\circ}\text{F} + 459.67$
- $\text{K} = ^{\circ}\text{C} + 273.15$

Conversion equivalencies for distance

- 1 inch (in) = 2.540000 centimeter (cm)
- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Conversion equivalencies for volume

1 gallon (gal) = 231.0 cubic inches (in³) = 4 quarts (qt) = 8 pints (pt) = 128 fluid ounces (fl. oz.) = 3.7854 liters (l)

1 milliliter (ml) = 1 cubic centimeter (cm³)

Conversion equivalencies for velocity

1 mile per hour (mi/h) = 88 feet per minute (ft/m) = 1.46667 feet per second (ft/s) = 1.60934 kilometer per hour (km/h) = 0.44704 meter per second (m/s) = 0.868976 knot (knot – international)

Conversion equivalencies for mass

1 pound (lbm) = 0.45359 kilogram (kg) = 0.031081 slugs

Conversion equivalencies for force

1 pound-force (lbf) = 4.44822 newton (N)

Conversion equivalencies for area

1 acre = 43560 square feet (ft²) = 4840 square yards (yd²) = 4046.86 square meters (m²)

Conversion equivalencies for common pressure units (either all gauge or all absolute)

1 pound per square inch (PSI) = 2.03602 inches of mercury (in. Hg) = 27.6799 inches of water (in. W.C.) = 6.894757 kilo-pascals (kPa) = 0.06894757 bar

1 bar = 100 kilo-pascals (kPa) = 14.504 pounds per square inch (PSI)

Conversion equivalencies for absolute pressure units (only)

1 atmosphere (Atm) = 14.7 pounds per square inch absolute (PSIA) = 101.325 kilo-pascals absolute (kPaA) = 1.01325 bar (bar) = 760 millimeters of mercury absolute (mmHgA) = 760 torr (torr)

Conversion equivalencies for energy or work

1 british thermal unit (Btu – “International Table”) = 251.996 calories (cal – “International Table”) = 1055.06 joules (J) = 1055.06 watt-seconds (W-s) = 0.293071 watt-hour (W-hr) = 1.05506 x 10¹⁰ ergs (erg) = 778.169 foot-pound-force (ft-lbf)

Conversion equivalencies for power

1 horsepower (hp – 550 ft-lbf/s) = 745.7 watts (W) = 2544.43 british thermal units per hour (Btu/hr) = 0.0760181 boiler horsepower (hp – boiler)

Acceleration of gravity (free fall), Earth standard

9.806650 meters per second per second (m/s²) = 32.1740 feet per second per second (ft/s²)

Physical constants

Speed of light in a vacuum (c) = 2.9979×10^8 meters per second (m/s) = 186,281 miles per second (mi/s)

Avogadro's number (N_A) = 6.022×10^{23} per mole (mol^{-1})

Electronic charge (e) = 1.602×10^{-19} Coulomb (C)

Boltzmann's constant (k) = 1.38×10^{-23} Joules per Kelvin (J/K)

Stefan-Boltzmann constant (σ) = 5.67×10^{-8} Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2 \cdot \text{K}^4$)

Molar gas constant (R) = 8.314 Joules per mole-Kelvin (J/mol-K)

Properties of Water

Freezing point at sea level = $32^\circ\text{F} = 0^\circ\text{C}$

Boiling point at sea level = $212^\circ\text{F} = 100^\circ\text{C}$

Density of water at $4^\circ\text{C} = 1000 \text{ kg}/\text{m}^3 = 1 \text{ g}/\text{cm}^3 = 1 \text{ kg}/\text{liter} = 62.428 \text{ lb}/\text{ft}^3 = 1.94 \text{ slugs}/\text{ft}^3$

Specific heat of water at $14^\circ\text{C} = 1.00002 \text{ calories}/\text{g} \cdot ^\circ\text{C} = 1 \text{ BTU}/\text{lb} \cdot ^\circ\text{F} = 4.1869 \text{ Joules}/\text{g} \cdot ^\circ\text{C}$

Specific heat of ice $\approx 0.5 \text{ calories}/\text{g} \cdot ^\circ\text{C}$

Specific heat of steam $\approx 0.48 \text{ calories}/\text{g} \cdot ^\circ\text{C}$

Absolute viscosity of water at $20^\circ\text{C} = 1.0019 \text{ centipoise (cp)} = 0.0010019 \text{ Pascal-seconds (Pa}\cdot\text{s)}$

Surface tension of water (in contact with air) at $18^\circ\text{C} = 73.05 \text{ dynes}/\text{cm}$

pH of pure water at $25^\circ\text{C} = 7.0$ (*pH scale = 0 to 14*)

Properties of Dry Air at sea level

Density of dry air at 20°C and 760 torr = $1.204 \text{ mg}/\text{cm}^3 = 1.204 \text{ kg}/\text{m}^3 = 0.075 \text{ lb}/\text{ft}^3 = 0.00235 \text{ slugs}/\text{ft}^3$

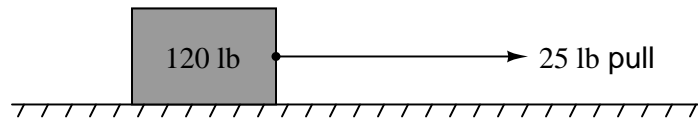
Absolute viscosity of dry air at 20°C and 760 torr = $0.018 \text{ centipoise (cp)} = 1.8 \times 10^{-5} \text{ Pascal-seconds (Pa}\cdot\text{s)}$

file conversion_constants

Questions

Question 1

A 120 pound weight is dragged along a level surface with a steady pull of 25 pounds for 10 feet. How much work is done (express your answer in both English and metric units of work)?



[file i02589](#)

Question 2

A 3500 pound automobile is hoisted 20 feet in the air by a crane. How much *work* is done in lifting the automobile to this height?

[file i02621](#)

Question 3

If 40 pounds of books are lifted from floor level to a bookshelf 5 feet above, then later those same books are taken off the shelf and returned to floor level, what is the total amount of work done by the person moving the books?

[file i02620](#)

Question 4

Suppose a forklift hoists a container weighing 2670 pounds up from ground level, setting it down on a loading dock 4.5 feet above ground level. How much work was done by the forklift in this maneuver?

$W =$ _____ ft-lbs

Now suppose the same forklift moves the container off of the loading dock and into the back of a flatbed truck 3 feet above ground level. How much work was done by the forklift in this maneuver?

$W =$ _____ ft-lbs

[file i04803](#)

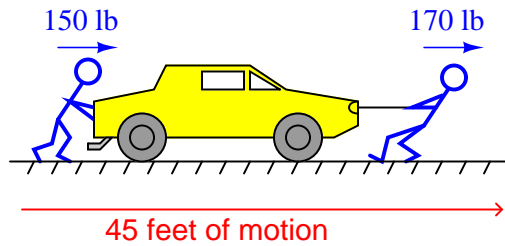
Question 5

Suppose a crane picks up a shipping container weighing 32000 pounds, lifting it 17 feet above the ground, moving it horizontally 260 feet to a warehouse, and then setting it down into a pit 5 feet below ground level. Calculate the total (net) amount of work done by the crane on the shipping container from its starting point (on the ground) to its destination (in the pit).

[file i02592](#)

Question 6

Two people are working to move a dead car, one pushing and the other pulling. The person pushing exerts 150 pounds of force, while the one pulling exerts 170 pounds of force. What is the total (resultant) force from the efforts of these two people?

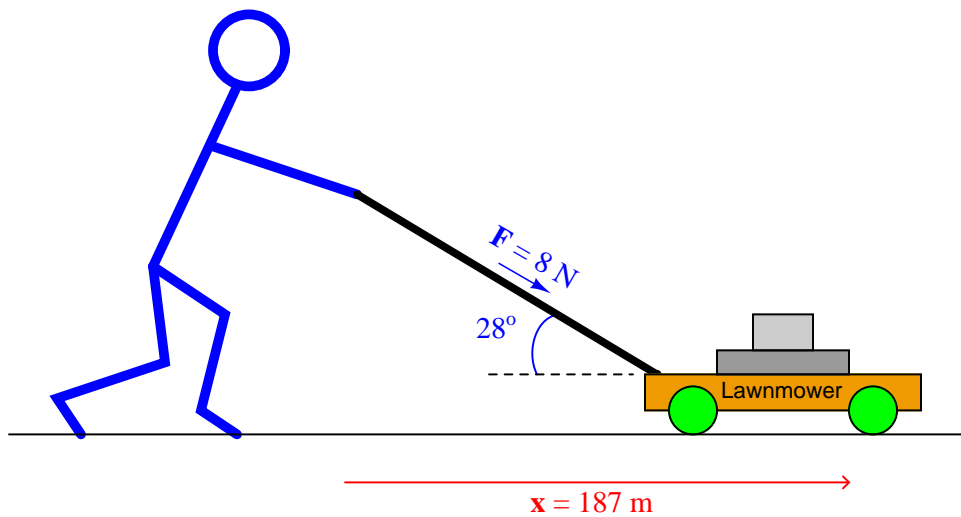


Assuming they are able to drag the car a total distance of 45 feet before collapsing in exhaustion, calculate the total work done by these two people.

[file i02614](#)

Question 7

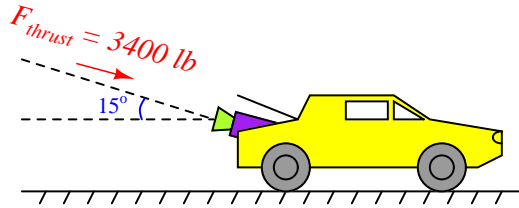
Calculate the amount of work done by this person pushing a lawnmower, given the force and displacement vectors shown:



[file i02616](#)

Question 8

Danger Dave decides to outfit his car with a solid-fuel rocket mounted in the trunk. Unfortunately, he can't get the rocket motor to fit horizontally in the trunk, and so it sits at a slight angle (15 degrees from horizontal). When the rocket is ignited, its thrust is 3400 pounds of force along the centerline of the rocket body:



One fine day, Dave is feeling particularly dangerous, so he ignites the rocket motor to see how fast his car will perform on a quarter-mile racetrack. Calculate the amount of work done by the rocket on the car during the quarter-mile run.

Assuming Dave's car weighs 3100 pounds (i.e. it has a *mass* of 96.27 slugs) with the new rocket motor installed and him in the driver's seat, calculate both his quarter-mile speed and time. Hint: you may need to use one or more of these formulae:

$$F = ma$$

$$x = vt$$

$$v = at + v_0$$

$$x = \frac{1}{2}at^2$$

Where,

F = Force applied to object

m = Mass of object

x = Position of object with reference to a starting point (x_0 , at time $t = 0$)

v = Velocity of object

a = Acceleration of object (i.e. the rate at which velocity changes)

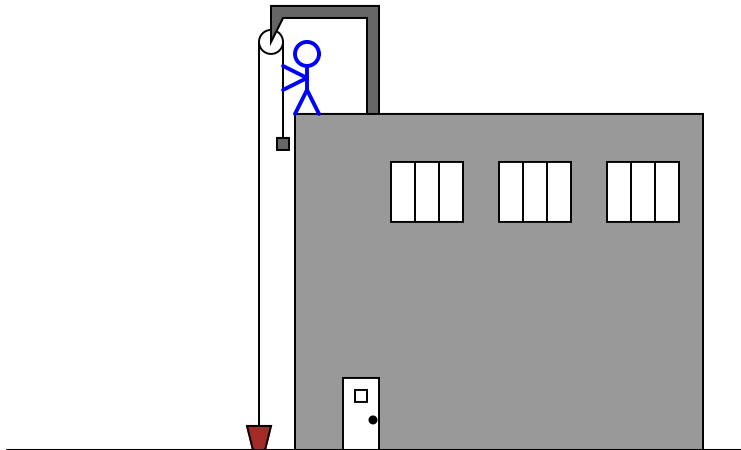
t = Elapsed time

v_0 = Initial velocity of object (at time $t = 0$)

[file i02615](#)

Question 9

A laborer working on the top of a building uses a manual hoist to lift 20 gallons of water from ground level. The height of this lift is 31 feet:



The rope is counterweighted with a mass equal to that of the bucket (empty), so that the bucket's weight does not have to be lifted, only the water inside the bucket. Assuming a vertical lift distance of 31 feet, and ignoring the weight of the rope itself, how much work does the laborer do in lifting the water up? Please express your answer in both English and metric units of work.

[file i02610](#)

Question 10

Suppose a boy working on a farm needs to draw water out of an open well. The water is 23 feet below ground level, and the boy has a 1-gallon bucket with a rope enabling him to manually lift water to the surface.

Calculate the amount of work the boy must do to lift 1 gallon of water out of the well. Assume the bucket itself weighs 1.5 pounds, but neglect the weight of the rope (for simplicity):

Work = _____ ft-lb

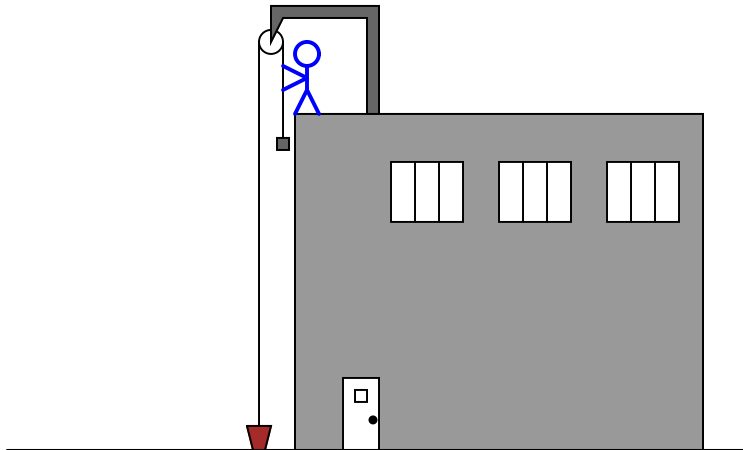
If the boy is able to lift this much water out of the well in 8 seconds, how much is his power output (in units of horsepower)?

Power = _____ HP

[file i04778](#)

Question 11

A laborer working on the top of a building uses a manual hoist to lift 20 gallons of water 31 feet up from ground level. The time required for the laborer to do this job is one minute:



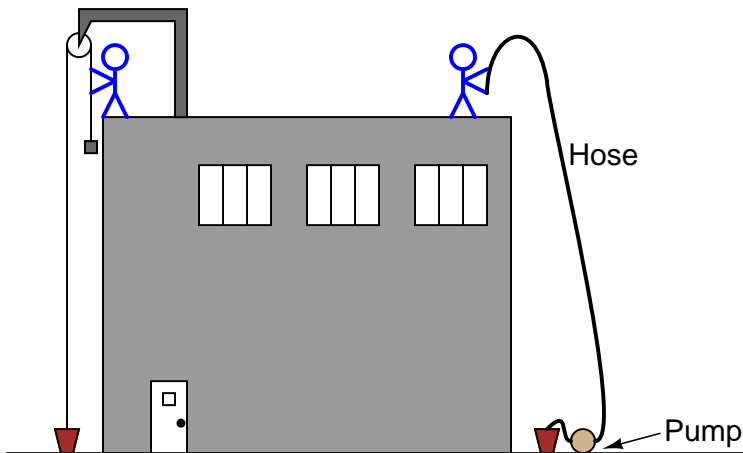
Several hours later, a different laborer does the exact same job of hoisting 20 gallons of water to the roof, but does so in only 40 seconds.

Which laborer performs more work in lifting 20 gallons of water to the roof?

[file i02611](#)

Question 12

One laborer working on the top of a building uses a manual hoist to lift 10 gallons of water 30 feet up from ground level, while a second laborer uses an electric pump to do the same:

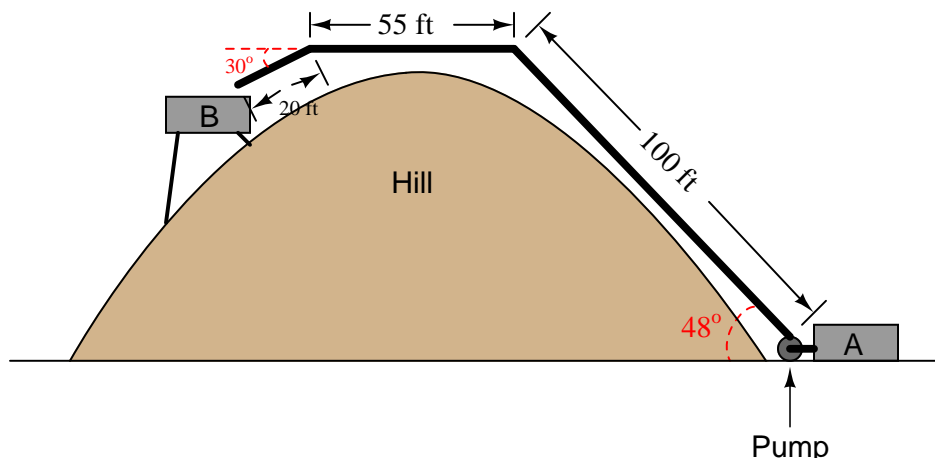


First, calculate the amount of work needed to lift 10 gallons up to the same roof. Then, calculate the time required for the pump to do this job, assuming a rating of 1.5 horsepower.

[file i02612](#)

Question 13

How much work is done pumping 3,000 gallons of water from reservoir “A” to reservoir “B” over the hill?



If the pump’s power output is 250 horsepower, how long will it take to pump all 3000 gallons to reservoir “B”?

Suggestions for Socratic discussion

- Calculate the amount of pressure at the discharge port of the pump as it lifts water up to reservoir “B”
[file i02613](#)

Question 14

Suppose a crate full of bowling balls weighing a total of 5000 newtons (i.e. the crate has a total *mass* of 509.68 kilograms) is lifted 20 meters vertically into the air and then dropped. How fast will its velocity be just before hitting the ground?

Suggestions for Socratic discussion

- What happens to all the potential energy that was “invested” in this crate after it impacts the ground?
[file i02622](#)

Question 15

At the skate park, Riley accidentally lets go of his 4 kilogram skateboard at the top of a 5-meter tall ramp. Assuming a frictionless ride down the ramp, how fast will Riley’s skateboard be rolling at the bottom of the ramp?

$$v = \text{_____ m/s}$$

How much potential energy did Riley’s skateboard have when it was 5 meters above ground level (at the top of the ramp)?

$$E_p = \text{_____ Joules}$$

[file i04801](#)

Question 16

Sam climbs a 130 foot tower with an 8 pound (0.25 slug) textbook. Tony climbs the same tower with a 5 pound (0.16 slug) textbook. Both Sam and Tony drop their textbooks from the top of the tower at exactly the same moment in time. Neglecting the effects of air friction on the books' free-fall, calculate the following:

- Work done by Sam in lifting his textbook =
- Kinetic energy of Sam's textbook just before it hits the ground =
- Velocity of Sam's textbook just before it hits the ground =

- Work done by Tony in lifting his textbook =
- Kinetic energy of Tony's textbook just before it hits the ground =
- Velocity of Tony's textbook just before it hits the ground =

[file i00430](#)

Question 17

A truck weighing 39,000 newtons (N) is traveling at 10 meters per second when its clutch blows out, disconnecting the engine from the drivetrain (transmission, axle, wheels, etc.). This failure occurs exactly at the base of a steep incline. How many meters (vertical) will the truck coast up the hill with no engine power before it stops, neglecting friction of any kind?

How high would the truck have coasted if it had been traveling twice as fast?

[file i00431](#)

Question 18

An automobile weighing 2700 pounds is traveling over level ground at a velocity of 50 miles per hour. If the driver places the vehicle's transmission in neutral (so the engine is disconnected from the wheels) and applies force to the brake pedal, the vehicle will slow down.

Assuming the braking force on the automobile is a constant 500 pounds (friction opposed to the direction of the automobile's motion), how many feet will the vehicle travel before coming to a rest? Ignore any air friction, and assume that the braking force is the only decelerating force in this mechanical system.

[file i02623](#)

Question 19

Suppose two vehicles are traveling down a highway: a pickup truck and a sports car. The truck weighs twice as much as the sports car, but is traveling only half as fast.

All other factors being equal, which vehicle can stop in the shortest distance?

[file i02624](#)

Question 20

Suppose a wrecking ball is suspended on the end of a 30 foot cable. If the wrecking ball is drawn to the side until the cable is at a 20 degree angle (from vertical) and then released to swing, how fast will the wrecking ball be traveling at its maximum velocity?

[file i04802](#)

Question 21

A stupid person fires a gun straight up into the air. The bullet leaves the gun's muzzle with a velocity of 1100 feet per second. The bullet has a mass of 150 grains. Ignoring the effects of air friction (I know, aerodynamic friction is no minor effect at supersonic velocities, but just work with me here . . .) how long will it take for the bullet to return to ground level?

[file i02655](#)

Question 22

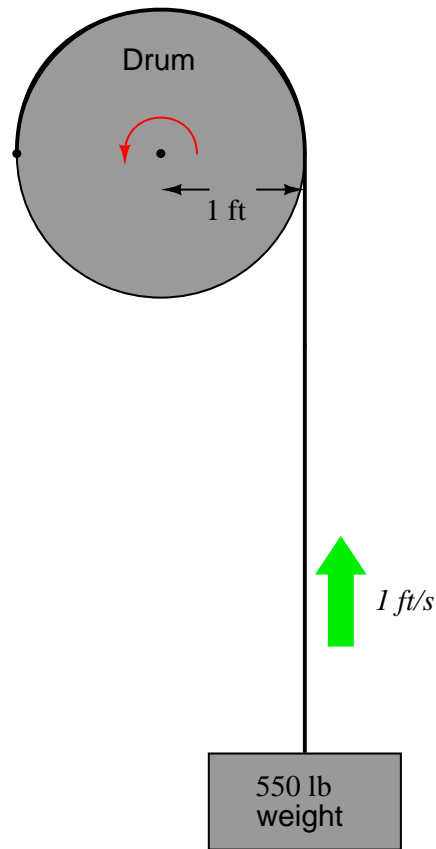
A *horsepower* is defined as 550 ft-lbs of work done in one second of time. An example of this would be a 550 pound weight lifted vertically at a speed of one foot per second, or a one pound weight lifted vertically at a speed of 550 feet per second.

There is a way to relate this to rotary motion, not just linear motion. In the case of rotary motion we must deal with torque (τ) in lb-ft and angular speed (S) in revolutions per minute (RPM) rather than force in pounds and linear speed in feet per second.

Just as linear power is proportional to the product of force and velocity ($P \propto Fv$), rotary power is proportional to torque and rotary speed ($P \propto \tau S$). What we need to turn this proportionality into an equality is a multiplying constant (k):

$$P = k\tau S$$

We may determine the value of this constant by setting up a “thought experiment” that translates between linear power and rotary power:



In this case, we have a 1 foot radius drum hoisting a 550 pound weight at a linear velocity of 1 foot per second, the definition of one horsepower. Translate the linear force and linear velocity to rotary force (torque) and rotary velocity (revolutions per minute), and then calculate the necessary k factor to make your own torque/speed/horsepower equation.

[file i01430](#)

Question 23

Calculate energy and/or power for the following scenarios:

One gallon of gasoline stores approximately 121 megajoules (MJ) of energy in chemical form. If this gallon of gasoline is burned completely and steadily over a period of 2 hours, how much power is output by the fire?

A battery charger connected to a secondary-cell battery applies a constant charging voltage of 14.3 volts at a constant charging current of 6.8 amps for 2.5 hours. How much power does this represent (in watts), and how much energy has been delivered to the battery by the charger (in Joules)?

One metric ton of bituminous/anthracite coal contains approximately 28 gigajoules of energy in chemical form. If a coal-burning power plant operating at 35% conversion efficiency (chemical to electrical energy) is to output 500 MW of power, what rate must the coal be burned in units of metric tons per minute?

[file i02463](#)

Answers

Answer 1

In this problem, the figure of 120 lb is of no consequence to our calculations. What matters is the 25 pound force *in the direction of the weight's displacement*. Since there is no displacement (motion) in the direction of the 120 pound force (straight down), there is no work performed by that force.

$$W = Fx \cos \theta$$

$$W = (25 \text{ lb})(10 \text{ ft})(\cos 0^\circ)$$

$$W = 250 \text{ ft-lb}$$

We may convert units directly from ft-lb to J in one step, forming a “unity fraction” with the equivalence :

$$\left(\frac{250 \text{ ft-lb}}{1} \right) \left(\frac{1055.06 \text{ J}}{778.169 \text{ ft-lb}} \right) = 338.96 \text{ J}$$

Answer 2

$$W = Fx \cos \theta$$

$$W = (3500 \text{ lb})(20 \text{ ft})(\cos 0^\circ)$$

$$W = 70000 \text{ ft-lb of work}$$

Answer 3

Contrary to intuition, *no work has been done*. Lifting the 40 pounds of books 5 feet up constitutes 200 ft-lb of work done on the books (i.e. potential energy invested in the books), but returning those books back to floor level constitutes 200 ft-lb of energy *released* (negative work done). Thus, in physics terms, there was no net work performed.

The person tasked with this pointless exercise, however, may beg to differ.

This same principle of storing and releasing energy is employed in electric vehicles to recover braking energy. Instead of converting electrical energy into mechanical potential and vice-versa as happens with the elevator, electric vehicles convert electrical energy into kinetic form (vehicle motion) and vice-versa. Thus, accelerating an electric car from a full stop to some speed and then regeneratively decelerating it back to full stop is another example of zero (net) work. This energy-recovering capability is what makes electric vehicles so attractive for stop-and-go travel.

Answer 4

Moving from ground level to loading dock:

$$W = \underline{12015} \text{ ft-lbs}$$

Moving from loading dock to flatbed truck:

$$W = \underline{-4005} \text{ ft-lbs}$$

Answer 5

The *total* amount of work done on the shipping container may be calculated by multiplying its weight (32000 pounds) by the total height change from beginning to end. Assuming it started at ground level, and ended up 5 feet below ground level (i.e. the displacement is *down* when the crane's force on the container is *up*), the work done on the container is:

$$W = Fx \cos \theta$$

$$W = (32000 \text{ lb})(5 \text{ ft}) \cos 180^\circ$$

$$W = -160000 \text{ ft}\cdot\text{lb of work done on the container}$$

Technically, the crane didn't do any work on the shipping container, but rather *the shipping container did work on the crane!*

Note how we are completely ignoring the crane's horizontal motion, because this displacement vector is at right angles (90°) to the container's weight vector as well as the crane's upward force vector, and therefore does not count toward or against work done on the container.

Answer 6

When multiple forces act in the same direction, the resultant force will be the simple sum of the individual forces. In this case, 150 pounds plus 170 pounds is equal to 320 pounds.

Work is calculated by taking this total (net) force of 320 pounds and multiplying by the displacement of 45 feet, since both the force and the displacement vectors are pointed in the same direction (i.e. $\theta = 0^\circ$):

$$W = Fx \cos \theta$$

$$W = (320 \text{ lb})(45 \text{ ft}) \cos 0^\circ$$

$$W = 14400 \text{ ft}\cdot\text{lb of work}$$

Answer 7

$$W = Fx \cos \theta$$

$$W = (8 \text{ N})(187 \text{ m}) \cos 28^\circ$$

$$W = 1320.9 \text{ N}\cdot\text{m}$$

Answer 8

The work done by the rocket motor on the car may be calculated with the work formula, taking the displacement (x) to be one-quarter of a mile (1320 feet):

$$W = Fx \cos \theta$$

$$W = (3400 \text{ lb})(1320 \text{ ft}) \cos 15^\circ$$

$$W = 4335075.1 \text{ ft}\cdot\text{lb of work done on Dave's car}$$

The horizontal component of the rocket's 3400 lb thrust will be the 3400 lb times the cosine of 15 degrees:

$$F_x = (3400 \text{ lb}) \cos 15^\circ = 3284.15 \text{ lb}$$

The acceleration of Dave's car with this amount of force driving it down the racetrack will be equal to force divided by mass, following the formula $F = ma$. In order to calculate a , we will need to know the mass of Dave's car in slugs ($3100 \text{ lb} / 32.3 \text{ ft/s}^2 = 96.27 \text{ slugs}$):

$$F = ma$$

$$a = \frac{F}{m} = \left(\frac{3284.15 \text{ lb}}{96.27 \text{ slugs}} \right) = 34.11 \text{ ft/s}^2$$

To put this in perspective, the car would accelerate at 32.2 ft/s^2 if pushed off a cliff, so this means Dave's car will "rocket" down the racetrack at an acceleration faster than free-fall!

The distance traveled (x) by an object under constant acceleration (a) is given by the following formula:

$$x = \frac{1}{2}at^2$$

In the case of Dave's rocket car:

$$1320 \text{ ft} = \frac{1}{2}(34.11 \text{ ft/s}^2)t^2$$

Solving for time (t):

$$t^2 = \frac{(2)(1320 \text{ ft})}{(34.11 \text{ ft/s}^2)}$$

$$t = \sqrt{\frac{(2)(1320 \text{ ft})}{(34.11 \text{ ft/s}^2)}}$$

$$t = 8.797 \text{ seconds}$$

This, my friends, is a very respectable quarter-mile time!

As for top speed (at the end of the quarter-mile run, since we assume Dave is still accelerating with the rocket motor going at full thrust), this may be calculated by the velocity-acceleration formula $v = at$:

$$v = at$$

$$v = (34.11 \text{ ft/s})(8.797 \text{ s}) = 300.1 \text{ ft/s}$$

This happens to be 204.6 miles per hour, which again is *very respectable* in a quarter-mile race!

Now, Dave's only real problem is how to stop.

Answer 9

To calculate work in this system, we need to know both the displacement (31 feet), and the force exerted in the direction of the displacement. We were not given this force, but we do know it is 20 gallons of water being lifted, and we know the density of water to be 62.4 pounds per cubic foot. So, 20 gallons of water weighs:

$$\left(\frac{20 \text{ gal}}{1}\right) \left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right) \left(\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\right) \left(\frac{62.4 \text{ lb}}{1 \text{ ft}^3}\right) = 166.83 \text{ lb of water in 20 gallons}$$

Now, knowing both the force (166.83 lb) and the displacement (31 ft), we may calculate the work done:

$$W = Fx \cos \theta$$

$$W = (166.83 \text{ lb})(31 \text{ ft})(\cos 0^\circ) = 5171.83 \text{ ft-lb of work}$$

Converting foot-pounds into joules:

$$\left(\frac{5171.83 \text{ ft-lb}}{1}\right) \left(\frac{1055.06 \text{ J}}{778.169 \text{ ft-lb}}\right) = 7012.09 \text{ J of work}$$

Answer 10

Calculating the weight of the water lifted:

$$\left(\frac{1 \text{ gal of water}}{1}\right) \left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right) \left(\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\right) \left(\frac{62.4 \text{ lb}}{1 \text{ ft}^3 \text{ of water}}\right) = 8.342 \text{ lb}$$

Added to the 1.5 pound weight of the bucket, the boy's total upward force must be 9.842 pounds.

Since the displacement of the boy's effort must be the depth of the water (23 feet), the work done is a simple product of these two figures:

$$W = Fx = (9.842 \text{ lb})(23 \text{ ft}) = 226.36 \text{ ft-lb}$$

Power is equal to work done per unit time, so:

$$P = \frac{W}{t} = \frac{226.36 \text{ ft-lb}}{8 \text{ s}} = 28.29 \text{ ft-lb/s}$$

Converting between ft-lb/s and horsepower:

$$\left(\frac{28.29 \text{ ft-lb/s}}{1}\right) \left(\frac{1 \text{ HP}}{550 \text{ ft-lb/s}}\right) = 0.0514 \text{ HP}$$

Answer 11

This is a trick question: both laborers do the exact same amount of work. The second laborer, however, does the same work with *greater power* than the first laborer, since the second laborer's work was done in a shorter amount of time.

Answer 12

Calculating the work in raising 10 gallons of water 30 feet up:

$$\left(\frac{10 \text{ gal}}{1}\right) \left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right) \left(\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\right) \left(\frac{62.4 \text{ lb}}{\text{ft}^3}\right) = 83.42 \text{ lb of water in 10 gallons}$$

Now, knowing both the force (83.42 lb) and the displacement (30 ft), we may calculate the work done:

$$W = Fx \cos \theta$$

$$W = (83.42 \text{ lb})(30 \text{ ft})(\cos 0^\circ) = 2502.5 \text{ ft-lb of work}$$

Now, since we know that 1 horsepower is 550 ft-lbs of work *per second of time*, we may take this total amount of work (2502.5 ft-lb) and divide by the pump's power in foot-pounds per second to arrive at an answer for time in seconds. Since we know the pump's power rating is 1.5 horsepower, it means it is capable of doing 825 ft-lb of work per second:

$$t = \frac{W}{P} = \frac{2502.5 \text{ ft-lb}}{825 \text{ ft-lb/s}}$$

$$t = 3.033 \text{ seconds}$$

First, let's determine the weight of 3,000 gallons of water:

$$\left(\frac{3000 \text{ gal}}{1}\right) \left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right) \left(\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\right) \left(\frac{62.4 \text{ lb}}{\text{ft}^3}\right) = 25025 \text{ lb of water in 3000 gallons}$$

This weight of water will have to be lifted to the peak of the hill through the 100 foot pipe, but we will not use the figure of 100 feet as the displacement, since it is not vertical. Instead, we will use trigonometry to calculate the vertical lift (x):

$$x = (100 \text{ ft})(\sin 48^\circ) = 74.31 \text{ ft vertical lift}$$

So, the work involved with lifting this much water to that height is:

$$W = Fx \cos \theta$$

$$W = (25025 \text{ lb})(74.31 \text{ ft}) \cos 0^\circ$$

$$W = 1,859,719.9 \text{ ft-lb of work}$$

Barring any piping friction, the horizontal section of pipe (55 ft) does not necessitate any work being done. The short, 20 foot section of downward-angled pipe, however, actually *releases* energy (performs negative work) because it lets the water drop in height. This drop is:

$$\text{drop} = (20 \text{ ft})(\sin 30^\circ) = 10 \text{ ft}$$

Since the same amount of water will drop this amount, the negative work done here is:

$$W = Fx \cos \theta$$

$$W = (25025 \text{ lb})(10 \text{ ft}) \cos 180^\circ$$

$$W = -250,250 \text{ ft-lb of work}$$

The total (net) work done, then, is the sum of these two figures:

$$W_{net} = 1,859,719.9 \text{ ft-lb} - 250,250 \text{ ft-lb} = 1,609,469.9 \text{ ft-lb of work}$$

At a pump power output of 250 HP (137,500 ft-lb per second)

$$t = \frac{W}{P}$$

$$t = \frac{1609469.9 \text{ ft-lb}}{137500 \text{ ft-lb/s}} = 11.71 \text{ seconds}$$

Answer 14

The amount of work required to lift this crate of bowling balls 20 meters into the air is 100,000 newton-meters, or 100,000 joules. This is also the amount of potential energy the crate has at its apogee.

When dropped, the crate's potential energy converts into kinetic energy, making the crate fall at a faster and faster velocity as its height approaches ground level (0). If we neglect the effects of air friction, we may say that the potential energy of the crate at its peak height will be precisely equal to its kinetic energy at the moment it contacts the ground:

$$E_p (\text{max.}) = E_k (\text{max})$$

$$mgh = \frac{1}{2}mv^2$$

Note how the term of mass (m) cancels out of both sides of the equation, letting us know that the mass of the crate will be irrelevant to its free-fall velocity:

$$gh = \frac{1}{2}v^2$$

Solving for v given its initial height of 20 meters, and Earth's acceleration of gravity being 9.81 meters per second squared:

$$2gh = v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{(2)(9.81 \text{ m/s}^2)(20 \text{ m})} = 19.81 \text{ m/s}$$

Bowling balls everywhere!!!

Answer 15

$$v = \sqrt{2gh} = \sqrt{(2)(9.81)(5)} = \underline{9.90} \text{ m/s}$$

$$E_p = mgh = (4)(9.81)(5) = \underline{196.2} \text{ Joules}$$

Answer 16

- Work done by Sam in lifting his textbook = 1040 ft-lb
- Kinetic energy of Sam's textbook just before it hits the ground = 1040 ft-lb
- Velocity of Sam's textbook just before it hits the ground = 91.2 ft/s

- Work done by Tony in lifting his textbook = 650 ft-lb
- Kinetic energy of Tony's textbook just before it hits the ground = 650 ft-lb
- Velocity of Tony's textbook just before it hits the ground = 91.2 ft/s

Knowing that kinetic energy just before the book hits the ground should be equal to potential energy when released (assuming zero energy loss due to air friction), we may solve for v quite easily:

$$E_k = \frac{1}{2}mv^2 \qquad E_p = mgh$$

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}v^2 = gh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Answer 17

5.1 meters (measured vertically) at an initial velocity of 10 m/s. At 20 m/s, the truck would have gained *four times* as much altitude (20.4 meters)!

Knowing that potential energy when the truck reaches its stopping point on the hill should be equal to kinetic energy when the clutch fails (assuming zero energy loss due to friction), we may solve for h quite easily:

$$E_p = mgh \qquad E_k = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2$$

$$h = \frac{\frac{1}{2}v^2}{g}$$

$$h = \frac{v^2}{2g}$$

Note that the slope of the hill is unspecified, because it is irrelevant to the answer of how much vertical height the truck gains by coasting. What *would* the slope of the hill affect, though?

Because we are going to have to calculate kinetic energy, we know that sooner or later we will need to know the automobile's mass in units of slugs and the velocity in units of feet per second, so let's get these calculations over with now:

$$F = ma$$

$$m = \frac{F}{a}$$

$$m = \frac{2700 \text{ lb}}{32.3 \text{ ft/s}^2} = 83.85 \text{ slugs}$$

$$\left(\frac{50 \text{ mi}}{\text{h}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = 73.333 \text{ ft/s}$$

Now we are all set to calculate the automobile's initial kinetic energy:

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}(83.85 \text{ slugs})(73.33 \text{ ft/s})^2$$

$$E_k = 225,465.84 \text{ ft-lb of kinetic energy at 50 MPH}$$

The automobile's brakes convert this kinetic into heat, through friction. In order to bring the velocity to zero, the work done on the automobile by the brakes' friction must cancel out the initial kinetic energy at 50 MPH. Note that we will use 180° as the angle θ in the work calculation, because the brakes' force vector opposes the direction of the car's displacement:

$$0 = W + E_k$$

$$0 = Fx \cos \theta + E_k$$

$$0 = (500 \text{ lb})x \cos 180^\circ + 225,465.84 \text{ ft-lb}$$

$$-225,465.84 \text{ ft-lb} = (500 \text{ lb})x \cos 180^\circ$$

$$x = \frac{-225,465.84 \text{ ft-lb}}{(500 \text{ lb})(\cos 180^\circ)}$$

$$x = \frac{-225,465.84 \text{ ft-lb}}{-500 \text{ lb}}$$

$$x = 450.93 \text{ ft total braking distance}$$

Answer 19

The truck can stop in a shorter distance than the sports car (about *half* the stopping distance). The reason for this is due to how much kinetic energy each vehicle possesses as it travels down the highway. Mass has a linear effect on kinetic energy, but velocity has a square effect on kinetic energy. Thus, while doubling a vehicle's mass will double its E_k at any given velocity, doubling a vehicle's velocity will *quadruple* its E_k for any given mass.

A helpful thought experiment to try here is to imagine a vehicle traveling at the slow speed of the pickup truck, and with the light mass of the sports car, and then use this vehicle as the "standard" of comparison. The truck will have twice the kinetic energy of this third vehicle. The sports car will have four times as much kinetic energy as this third vehicle. Therefore, the sports car will have twice the kinetic energy of the truck.

Answer 20

As the wrecking ball is pulled sideways until its cable forms a 20 degree angle from vertical, it will be lifted up 1.809 feet from the height it started (a right triangle with a hypotenuse of 30 feet and an adjacent angle of 20 degrees will have a side length of 28.19 feet, or 1.809 feet less than 30 feet).

From this known amount of vertical lift, we may calculate the wrecking ball's maximum velocity by assuming all that potential energy gets converted into kinetic energy:

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}v^2 = gh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{(2)(32.2 \text{ ft/s}^2)(1.809 \text{ ft})}$$

$$v = 10.79 \text{ ft/s}$$

When the bullet is fired upward at ground level, its kinetic energy is at a maximum but its potential energy is near zero. In rising against gravity, that kinetic energy will be translated into potential energy until the bullet's apogee, where there will be zero kinetic energy and maximum potential energy. As it falls, the reverse will happen: potential energy will translate into kinetic energy again, resulting in the bullet traveling at 1100 feet per second when it returns to ground level.

The time for the bullet to rise to its apogee will be equal to the time required for it to fall back to ground level from that maximum altitude. So, if we were able to calculate that rise time (or that fall time), all we would have to do is multiply by two to obtain the total time spent in the air.

The attraction of Earth's gravity results in a downward acceleration of 32.2 feet per second squared. This means that any object dropped from altitude will accelerate at this rate, increasing in velocity by 32.2 ft/s every second. It also means that any object rising by its own inertia (no propulsive force upward) will *slow down* by 32.2 ft/s each and every second until its vertical velocity is zero and it begins to fall back down. This rate of acceleration holds true for *any* object, aerodynamic resistance notwithstanding.

For this rate of acceleration to increase a falling object's velocity from 0 ft/s to 1100 ft/s will require this much time:

$$v = at$$

$$t = \frac{v}{a}$$

$$t = \frac{1100 \text{ ft/s}}{32.2 \text{ ft/s}^2} = 34.16 \text{ seconds}$$

This is the time required to either accelerate the bullet from zero ft/s (at apogee) to 1100 ft/s downward, or to *decelerate* it from 1100 ft/s upward to 0 ft/s (at apogee). Doubling this figure gives us 68.32 seconds total time. In other words, the stupid person has a little over a minute to duck and cover before the bullet finds its way back to *terra firma*.

Answer 22

The torque (τ) in this case is obviously 550 lb-ft, since the 550 pound weight is acting on a moment arm 1 foot long (the drum's radius). All we need to do is translate the vertical velocity of 1 foot per second into drum rotation in units of RPM, and we'll have the data we need to calculate k :

$$\text{Circumference of drum} = \pi D = 2\pi r = 6.283 \text{ ft}$$

This is the amount of cable that travels in one revolution of the drum (1 rev = 6.283 ft), and this equality constitutes a conversion factor which we may use to convert the linear velocity of 1 ft/s into a rotational velocity:

$$\left(\frac{1 \text{ ft}}{\text{sec}}\right) \left(\frac{1 \text{ rev}}{6.283 \text{ ft}}\right) \left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 9.5493 \text{ RPM}$$

Therefore,

$$P = k\tau S$$

$$1 \text{ hp} = (k)(550 \text{ ft-lb})(9.5493 \text{ RPM})$$

$$k = 0.0001904$$

$$P = 0.0001904\tau S$$

... or ...

$$P = \frac{\tau S}{5252}$$

Where,

P = Shaft power in horsepower

τ = Shaft torque in lb-ft

S = Shaft speed in revolutions per minute (RPM)

By coincidence, the factor of 5252 happens to be close to the number of feet in a mile (5280 feet = 1 mile). This might come in handy as an approximation!

Answer 23

One gallon of gasoline burned over a period of 2 hours = 16.8 kW

The battery charger outputs a total of 875.16 kJ over the 2.5 hour time period, at a rate of 97.24 watts.

The power plant must burn coal at a rate of 3.061 metric tons per minute.

Data for the heat value of various fuels taken from the following web document:

http://bioenergy.ornl.gov/papers/misc/energy_conv.html